
CSS.414.1: POLYNOMIAL METHODS IN COMBINATORICS

Instructor: Mrinal Kumar

TIFR 2024, Aug-Dec

SCRIBE: SOHAM CHATTERJEE

SOHAMCHATTERJEE999@GMAIL.COM

WEBSITE: SOHAMCH08.GITHUB.IO

CONTENTS

SECTION 1

INTRODUCTION AND TARGETS _____ PAGE 3 _____

SECTION 2

JOINTS PROBLEM _____ PAGE 4 _____

SECTION 3

COMBINATORIAL NULLSTELLENSATZ _____ PAGE 4 _____

3.1 Chevally-Waring Theorem 4

SECTION 4

SUM SETS _____ PAGE 4 _____

4.1 Sum Sets over Finite Fields 4

4.1.1 Cauchy-Davenport Theorem 4

4.2 Restricted Sum Sets 4

4.2.1 Erdős-Heilbronn Conjecture 4

SECTION 5

ARITHMETIC PROGRESSION FREE SETS IN \mathbb{F}_3^n _____ PAGE 4 _____

5.1 3AP Free sets in \mathbb{F}_q 4

SECTION 6

3-TENSORS AND SLICE RANK _____ PAGE 4 _____

6.1 Rank 4

6.2 Generalization to 3-Dimension 4

6.3 Slice Rank of Diagonal 3D Tensor 4

SECTION 7

KAKEYA AND NIKODYM PROBLEM _____ PAGE 4 _____

7.1 Lower Bound on Nikodym Sets 4

7.2 Lower Bound on Kakeya Sets 4

7.2.1 Hasse Derivative 4

1 Introduction and Targets

The content of this course will be the followings:

- Polynomial Methods in Combinatorics/Geometry

1. Kakeya/Nikodym Problem over finite fields
2. Joints Problem
3. Combinatorial Nullstellensatz (CN)
4. CN proof of Cauchy-Devenport, Erdős-Heilbronn Conjecture

- Polynomial Methods in Algebraic Algorithms

1. Noisy Polynomial Interpolation (Sudan, Guruswami-Sudan)
2. Multiplicative noise (Von zur Gathen-Shparlinski)
3. Coppersmith's Problem (Given an univariate $f(x) \in \mathbb{Z}[x]$, compute all 'small' integer roots modulo a composite)

- Polynomial Methods in Circuit Complexity

1. Razborov-Smolensky (Lower Bound for constant depth AND, OR, NOT, $\text{mod } p$ gates)
2. Algorithmic consequences (all pairs shortest paths)
3. Upper bounds on matrix rigidity (Alman-Williams '2015, Dvir-Edelman '2017)

- Polynomial in Property Testing: Polischuk-Speilman Lemma/Variants

- Weil Bounds (Stepanov, Schmidt Bombieri)

- Rational Approximations of Algebraic Numbers (Thue[1907] - Siegel - Roth[1954])

2 Joints Problem

3 Combinatorial Nullstellensatz

3.1 Chevally-Waring Theorem

4 Sum Sets

4.1 Sum Sets over Finite Fields

4.1.1 Cauchy-Davenport Theorem

4.2 Restricted Sum Sets

4.2.1 Erdős-Heilbronn Conjecture

5 Arithmetic Progression Free Sets in \mathbb{F}_3^n

5.1 3AP Free sets in \mathbb{F}_q

6 3-Tensors and Slice Rank

6.1 Rank

6.2 Generalization to 3-Dimension

6.3 Slice Rank of Diagonal 3D Tensor

7 Kakeya and Nikodym Problem

7.1 Lower Bound on Nikodym Sets

7.2 Lower Bound on Kakeya Sets

7.2.1 Hasse Derivative