CSS.201.1 Algorithms

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Chapter 1

Finding Closest Pair of Points

Problem: Given a set of points find the closest pair of points in $\mathbb{R}^2$.
Input: Set $S = \{(x_i, y_i) \mid x_i, y_i \in \mathbb{R},\ \forall\ i \in [n]\}$. We denote $P_i = (x_i, y_i)$.
Output: P_i, P_j that are at minimum l_2 distance i.e. minimize $\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$.

1.1 Naive Algorithm

Now the naive algorithm for this will be checking all pairs of points and take their distance and output the minimum one. There are total $\binom{n}{2}$ possible choices of pairs of points. And calculating the distance of each pair takes $O(1)$ time. So it will take $O(n^2)$ times to find the closest pair of points.
Idea: $\forall\ P_i, P_j \in S$ find distance $d(P_i, P_j)$ and return the minimum. Time taken is $O(n^2)$.

1.2 Divide and Conquer Algorithm

Definition 1.2.1: Divide and Conquer

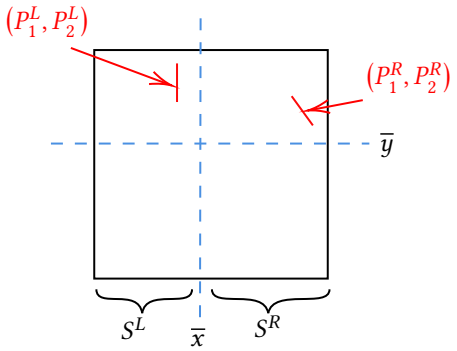
- Divide: Divide the problem into two parts (roughly equal)
- Conquer: Solve each part individually recursively. If the subproblem sizes are small enough, however, just solve the subproblems in a straightforward manner.
- Combine: Combine the solutions to the subproblems into the solution.

1.2.1 Divide

So to divide the problem into two roughly equal parts we need to divide the points into two equal sets. That we can do by sorting the points by their $x-$coordinate. Suppose S^x denote we get the new sorted array or points. And similarly we obtain S^y which denotes the array of points after sorting S by their $y-$coordinate.

Algorithm 1: Step 1 (Divide)

Function *Divide*:
  Sort S by $x-$coordinate and $y-$coordinate
  $S^x \longleftarrow S$ sorted by $x-$coordinate
  $S^y \longleftarrow S$ sorted by $y-$coordinate
  $\bar{x} \longleftarrow \lfloor \frac{n}{2} \rfloor$ highest $x-$coordinate
  $\bar{y} \longleftarrow \lfloor \frac{n}{2} \rfloor$ highest $y-$coordinate
  $S^L \longleftarrow \{P_i \mid x_i < \bar{x},\ \forall\ i \in [n]\}$
  $S^R \longleftarrow \{P_i \mid x_i \geq \bar{x},\ \forall\ i \in [n]\}$



1.2.2 Conquer

Now we will recursively get pair of closest points in S_L and S_R. Suppose the (P_1^L, P_2^L) are the closest pair of points in S^L and (P_1^R, P_2^R) are the closest pair of points in S^R.

Algorithm 2: Step 1 (Solve Subproblems)

Function *Conquer*:
    Solve for S_L, S^R.
    (P_1^L, P_2^L) are closest pair of points in S_L.
    (P_1^R, P_2^R) are closest pair of points in S_R.
    $\delta^L = d(P_1^L, P_2^L)$, $\delta^R = d(P_1^R, P_2^R)$
    $\delta_{min} \longleftarrow \min\{\delta^L, \delta^R\}$

1.2.3 Combine

Now we want to combine these two solutions.

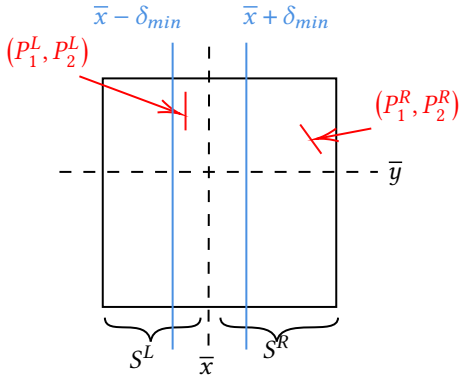
Question 1.1: We are not done

Is there a pair of points $P_i, P_j \in S$ such that $d(P_i, P_j) < \delta_{min}$

If Yes:

- One of them must be in S_L and the other is in S_R.
- $x-$coordinate$\in [\overline{x} - \delta_{min}, \overline{x} + \delta_{min}]$.
- $|y_i - y_j| \leq \delta_{min}$

So we take the strip of radius δ_{min} around $\overline{x}$. Define $T = \{P_i \in S \mid |x_i - \overline{x}| \leq \delta_{min}\}$



Chapter 2

Median Finding

Chapter 3

Bibliography