

---

---

# CSS.414.1: POLYNOMIAL METHODS IN COMBINATORICS

*Instructor: Mrinal Kumar*

*TIFR 2024, Aug-Dec*

---

---

SCRIBE: SOHAM CHATTERJEE

SOHAMCHATTERJEE999@GMAIL.COM

WEBSITE: SOHAMCH08.GITHUB.IO

# CONTENTS

## SECTION 1

INTRODUCTION AND TARGETS \_\_\_\_\_ PAGE 3 \_\_\_\_\_

## SECTION 2

JOINTS PROBLEM \_\_\_\_\_ PAGE 4 \_\_\_\_\_

## SECTION 3

COMBINATORIAL NULLSTELLENSATZ \_\_\_\_\_ PAGE 4 \_\_\_\_\_

3.1 Chevally-Waring Theorem 4

## SECTION 4

SUM SETS \_\_\_\_\_ PAGE 4 \_\_\_\_\_

4.1 Sum Sets over Finite Fields 4

4.1.1 Cauchy-Davenport Theorem 4

4.2 Restricted Sum Sets 4

4.2.1 Erdős-Heilbronn Conjecture 4

## SECTION 5

ARITHMETIC PROGRESSION FREE SETS IN  $\mathbb{F}_3^n$  \_\_\_\_\_ PAGE 4 \_\_\_\_\_

5.1 3AP Free sets in  $\mathbb{F}_q$  4

## SECTION 6

3-TENSORS AND SLICE RANK \_\_\_\_\_ PAGE 4 \_\_\_\_\_

6.1 Rank 4

6.2 Generalization to 3-Dimension 4

6.3 Slice Rank of Diagonal 3D Tensor 4

## SECTION 7

KAKEYA AND NIKODYM PROBLEM \_\_\_\_\_ PAGE 4 \_\_\_\_\_

7.1 Lower Bound on Nikodym Sets 4

7.2 Lower Bound on Kakeya Sets 4

7.2.1 Hasse Derivative 4

# 1 Introduction and Targets

The content of this course will be the followings:

- Polynomial Methods in Combinatorics/Geometry
  1. Kakeya/Nikodym Problem over finite fields
  2. Joints Problem
  3. Combinatorial Nullstellensatz (CN)
  4. CN proof of Cauchy-Devenport, Erdős-Heilbronn Conjecture
- Polynomial Methods in Algebraic Algorithms
  1. Noisy Polynomial Interpolation (Sudan, Guruswami-Sudan)
  2. Multiplicative noise (Von zur Gathen-Shparlinski)
  3. Coppersmith's Problem (Given an univariate  $f(x) \in \mathbb{Z}[x]$ , compute all 'small' integer roots modulo a composite)
- Polynomial Methods in Circuit Complexity
  1. Razborov-Smolensky (Lower Bound for constant depth AND, OR, NOT,  $\text{mod } p$  gates)
  2. Algorithmic consequences (all pairs shortest paths)
  3. Upper bounds on matrix rigidity (Alman-Williams '2015, Dvir-Edelman '2017)
- Polynomial in Property Testing: Polischuk-Spielman Lemma/Variants
- Weil Bounds (Stepanov, Schmidt Bombieri)
- Rational Approximations of Algebraic Numbers (Thue[1907] - Siegel - Roth[1954])

## **2 Joints Problem**

## **3 Combinatorial Nullstellensatz**

### **3.1 Chevally-Waring Theorem**

## **4 Sum Sets**

### **4.1 Sum Sets over Finite Fields**

#### **4.1.1 Cauchy-Davenport Theorem**

### **4.2 Restricted Sum Sets**

#### **4.2.1 Erdős-Heilbronn Conjecture**

## **5 Arithmetic Progression Free Sets in $\mathbb{F}_3^n$**

### **5.1 3AP Free sets in $\mathbb{F}_q$**

## **6 3-Tensors and Slice Rank**

### **6.1 Rank**

### **6.2 Generalization to 3-Dimension**

### **6.3 Slice Rank of Diagonal 3D Tensor**

## **7 Kakeya and Nikodym Problem**

### **7.1 Lower Bound on Nikodym Sets**

### **7.2 Lower Bound on Kakeya Sets**

#### **7.2.1 Hasse Derivative**