CSS.414.1: Polynomial Methods in Combinatorics

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TIFR 2024, Aug-Dec

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2024

Contents

1 Introduction and Targets

The content of this course will be the followings:

- Polynomial Methods in Combinatorics/Geometry

    1. Kakeya/Nikodym Problem over finite fields
    2. Joints Problem
    3. Combinatorial Nullstellensatz (CN)
    4. CN proof of Cauchy-Devenport, Erdös-Heilbronn Conjecture

- Polynomial Methods in Algebraic Algorithms

    1. Noisy Polynomial Interpolation (Sudan, Guruswami-Sudan)
    2. Multiplicative noise (Von zur Gathen-Shparlinski)
    3. Coppersmith's Problem (Given an univariate $f(x)\mathbb{Z}[x]$, compute all 'small' integer roots modulo a composite)

- Polynomial Methods in Circuit Complexity

    1. Razborov-Smolensky (Lower Bound for constant depth AND, OR, NOT, $\mod p$ gates)
    2. Algorithmic consequences (all pairs shortest paths)
    3. Upper bounds on matrix rigidity (Alman-Williams '2015, Dvir-Edelman '2017)

- Polynomial in Property Testing: Polischuk-Speilman Lemma/Variants

- Weil Bounds (Stepanov, Schmidtm Bombieri)

- Rational Approximations of Algebraic Numbers (Thue[1907] - Siegel - Roth[1954])