

---

# CSS.414.1: POLYNOMIAL METHODS IN COMBINATORICS

*Instructor: Mrinal Kumar*

*TIFR 2024, Aug-Dec*

---

SCRIBE: SOHAM CHATTERJEE

SOHAMCHATTERJEE999@GMAIL.COM

WEBSITE: SOHAMCH08.GITHUB.IO

# CONTENTS

<b>SECTION 1</b>	<b>INTRODUCTION AND TARGETS</b>	<b>PAGE 3</b>
<b>SECTION 2</b>	<b>JOINTS PROBLEM</b>	<b>PAGE 4</b>
<b>SECTION 3</b>	<b>COMBINATORIAL NULLSTELLENSATZ</b>	<b>PAGE 4</b>
3.1	Chevally-Waring Theorem	4
<b>SECTION 4</b>	<b>SUM SETS</b>	<b>PAGE 4</b>
4.1	Sum Sets over Finite Fields	4
4.1.1	Cauchy-Davenport Theorem	4
4.2	Restricted Sum Sets	4
4.2.1	Erdős-Heilbronn Conjecture	4
<b>SECTION 5</b>	<b>ARITHMETIC PROGRESSION FREE SETS IN <math>\mathbb{F}_3^n</math></b>	<b>PAGE 4</b>
5.1	3AP Free sets in $\mathbb{F}_q$	4
<b>SECTION 6</b>	<b>3-TENSORS AND SLICE RANK</b>	<b>PAGE 4</b>
6.1	Rank	4
6.2	Generalization to 3-Dimension	4
6.3	Slice Rank of Diagonal 3D Tensor	4
<b>SECTION 7</b>	<b>KAKEYA AND NIKODYM PROBLEM</b>	<b>PAGE 4</b>
7.1	Lower Bound on Nikodym Sets	4
7.2	Lower Bound on Kakeya Sets	4
7.2.1	Hasse Derivative	4

# 1 Introduction and Targets

The content of this course will be the followings:

- Polynomial Methods in Combinatorics/Geometry

1. Kakeya/Nikodym Problem over finite fields
2. Joints Problem
3. Combinatorial Nullstellensatz (CN)
4. CN proof of Cauchy-Devenport, Erdős-Heilbronn Conjecture

- Polynomial Methods in Algebraic Algorithms

1. Noisy Polynomial Interpolation (Sudan, Guruswami-Sudan)
2. Multiplicative noise (Von zur Gathen-Shparlinski)
3. Coppersmith's Problem (Given an univariate  $f(x) \in \mathbb{Z}[x]$ , compute all 'small' integer roots modulo a composite)

- Polynomial Methods in Circuit Complexity

1. Razborov-Smolensky (Lower Bound for constant depth AND, OR, NOT,  $\text{mod } p$  gates)
2. Algorithmic consequences (all pairs shortest paths)
3. Upper bounds on matrix rigidity (Alman-Williams '2015, Dvir-Edelman '2017)

- Polynomial in Property Testing: Polischuk-Speilman Lemma/Variants

- Weil Bounds (Stepanov, Schmidt Bombieri)

- Rational Approximations of Algebraic Numbers (Thue[1907] - Siegel - Roth[1954])

## **2 Joints Problem**

## **3 Combinatorial Nullstellensatz**

### **3.1 Chevally-Waring Theorem**

## **4 Sum Sets**

### **4.1 Sum Sets over Finite Fields**

#### **4.1.1 Cauchy-Davenport Theorem**

### **4.2 Restricted Sum Sets**

#### **4.2.1 Erdős-Heilbronn Conjecture**

## **5 Arithmetic Progression Free Sets in $\mathbb{F}_3^n$**

### **5.1 3AP Free sets in $\mathbb{F}_q$**

## **6 3-Tensors and Slice Rank**

### **6.1 Rank**

### **6.2 Generalization to 3-Dimension**

### **6.3 Slice Rank of Diagonal 3D Tensor**

## **7 Kakeya and Nikodym Problem**

### **7.1 Lower Bound on Nikodym Sets**

### **7.2 Lower Bound on Kakeya Sets**

#### **7.2.1 Hasse Derivative**