Deterministic List Decoding of Reed Solomon Codes

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Introduction

Introduction to Coding Theory

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- Distance of Code: $\Delta(\mathcal{C}) = \min_{c_1 \neq c_2 \in \mathcal{C}} \Delta(c_1, c_2)$
- Relative Distance: $\delta(\mathcal{C}) = \frac{\Delta(\mathcal{C})}{n}$

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$$k \leq n - d + 1$$

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Codes achieving this bound are called **Maximum Distance Separable (MDS)** codes.

- Reed Solomon Codes are MDS codes.

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- If we go more than $d/2$ distance, multiple codewords may lie in the radius of hamming ball.

List Decoding

Definition ((ρ, L)-List Decodable)

C is called (ρ, L)-list decodable if for every $v \in \Sigma^n$,

$$|\{c \in C \mid \Delta(c, v) \leq \rho n\}| \leq L$$

We denote the list for v by $L(v)$.

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We will talk in terms of agreement.

$$1 - t \text{ fraction of errors} \implies t \text{ fraction of agreement}$$

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Their Deterministic variant has polynomial dependence on field characteristic

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Sudan	Guruswami–Sudan
$m = 1, \quad D, t \approx \sqrt{2n(k-1)}$	$m = \sqrt{n(k-1)}, \quad D \approx m\sqrt{n(k-1)},$ $t = \sqrt{n(k-1)}$

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Want: Do the factorization step deterministically in $\mathrm{poly}(n, \log q)$ time.

Derandomization of Sudan

Newton Iteration

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Observe: $\partial_Y(Q(X, f(X)))$ has more than $\sqrt{2n(k-1)}$ roots but degree at most $\sqrt{2n(k-1)}$. Implying $\partial_Y(Q(X, f(X))) \equiv 0$

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So recurse on $\partial_Y(Q(X, f(X)))$.

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Theorem

There is a deterministic algorithm that, for every finite field $\mathbb{F}$ *and parameters* $n, k \in \mathbb{N}$ *runs in time* $\mathrm{poly}(n, \log|\mathbb{F}|)$ *list decodes Reed Solomon code* $RS[n, k]$ *from agreement more than* $\sqrt{2n(k-1)}$.

Derandomization of Guruswami-Sudan

Local Splitting

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Suppose we are given the factorization $P = \prod_{i=1}^{s} P_i$ into irreducibles (with multiplicity)

Eg: $P(X, Y) = (Y^2 + X)(Y^2 + X + 1)^2$ then

$$P_1 = Y^2 + X, \quad P_2 = Y^2 + X + 1, \quad P_3 = Y^2 + X + 1$$

We partition $[s]$ into four sets which defines four types of factors at (α, β):

$$A(\alpha, \beta), \quad B(\alpha, \beta), \quad C(\alpha, \beta), \quad D(\alpha, \beta)$$

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  So $Y^2 + X^2 + Y \in C(0, 0)$

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  So $Y^2+X^2+Y \in C(0,0)$
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  So $Y^2 + X^2 + Y \in C(0, 0)$
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- $C(\alpha, \beta) = \{i \in [s] \mid P_i(\alpha, Y) = (Y - \beta)^m \hat{P}_i(Y), m \geq 1, \hat{P}_i(\beta) \neq 0, \deg \hat{P}_i \geq 1\}$
  So $Y^2 + X^2 + Y \in C(0, 0)$
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  So $XY + 1 \in D(0, 0)$

I will use A, B, C, D to denote these. Define $P_A = \prod_{i \in A} P_i$ and similarly P_B, P_C, P_D.

Local Splitting

Let $P(X,Y) = (Y^2+X+1)(Y^2+X)(Y^2+X^2+Y)(XY+1)$ and $(\alpha,\beta) = (0,0)$

- $A(\alpha,\beta) = \{i \in [s] \mid P_i(\alpha,\beta) \neq 0, \deg(P_i(\alpha,Y)) \geq 1\}$
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- $B(\alpha,\beta) = \{i \in [s] \mid P_i(\alpha,Y) = \gamma(Y-\beta)^m, m \geq 1, \gamma \neq 0\}$
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Observe: If P is monic in Y then D is empty.

A Nice Observation

$w = \{(\alpha_j, \beta_j)\}_{j\in[n]}$ denote the received word.

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5. Do some interpolations to recover list

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Observe: If f is in the list there is one factor $g \in S$, $Y - f(X) \mid g$.

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You will notice $\Phi(S)$ decreases by at least 1 in each update of S.

Finally

Final Algorithm:

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Theorem

There is a deterministic algorithm that, for every finite field $\mathbb{F}$ *and parameters* $n, k \in \mathbb{N}$ *runs in time* $\mathrm{poly}(n, \log |\mathbb{F}|)$ *list decodes Reed Solomon code* $RS[n, k]$ *from agreement more than* $\sqrt{n(k-1)}$.

Splitting Algorithm

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We gave degree bounds for multiple iteration of general Hensel Lifting

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Otherwise:

Use Hensel Lifting $t = 2\log(\deg_Y P)$ times to get g_t, h_t such that

$$P \equiv g_t h_t \bmod (X - \alpha)^{2^t}, \quad g_t \equiv g_0 \bmod (X - \alpha), \quad h_t \equiv h_0 \bmod (X - \alpha)$$

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Recurse on $P_1, P/P_1$ (or $P_2, P/P_2$). Then combine them to get P_A, P_B.

Final Theorem

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If the algorithm passes initial checks and has no solution of linear systems. Then $P = P_C$.

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Here we mention the full version of the theorem we proved:

Theorem

For every bivariate polynomial $P(X, Y) \in \mathbb{F}_q[X, Y]$ and point $(\alpha, \beta) \in \mathbb{F}_q^2$ the above algorithm outputs (P_1, P_2) such that $P_1 = P_A \cdot R_1$ and $P_2 = P_B \cdot R_2$ where $R_1 R_2 \mid P_D$.

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- For beyond Johnson Bound even finding one element of the list is open.
- List decode Folded Reed Solomon Codes and Univariate Multiplicity Codes deterministically upto list decoding capacity in $\mathrm{poly}(1/\epsilon)\tilde{O}(n)$ time.

Thank You