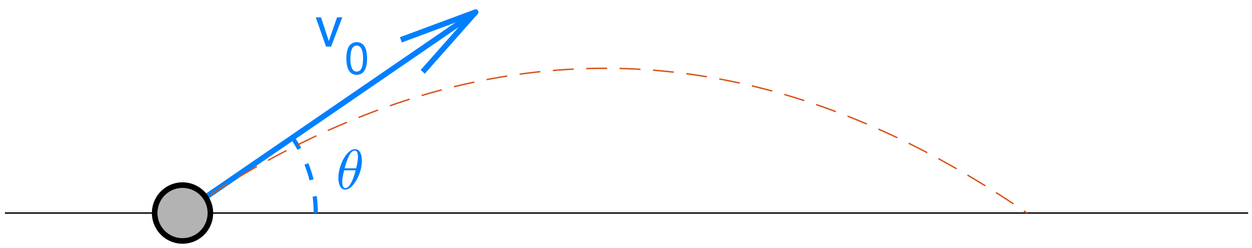
AMATH 301
Homework 6
Due: Saturday, February 16, 2019

Cannonball

A cannon can fire a cannonball at any angle θ with initial velocity $v_0 = 10$ meters per second. Depending on the angle chosen, the cannonball will follow different paths and travel some horizontal distance.



The vertical component of the cannonball's motion determines when it will impact the ground and stop, and is given by

$$y(t) = v_0 \sin(\theta)t - \frac{1}{2}gt^2.$$

where $g = 9.8$ is the acceleration due to gravity. Use the quadratic formula to work out at what time t_{impact} the cannonball impacts the ground. Note that there are two 'solutions': one will always be zero and should be thrown out.

The horizontal component of the cannonball's motion is given by

$$x(t) = v_0 \cos(\theta)t.$$

Find x_{impact} by plugging in the expression you got for t_{impact} into $x(t)$, thereby obtaining the horizontal distance at which the cannonball will impact the ground.

Problem 1 (Scorelator).

To fire the cannonball travel as far (horizontally) as possible, we need to find the optimal angle θ_{optimal} at which to launch the ball. Write a MATLAB function for x_{impact} as a function of θ and use `fminbnd` to find the optimal value of theta that maximizes x_{impact}.

Note that `fminbnd` minimizes a function, but we want to maximize this function instead. There is a trick to using minimization to maximize a function: make a new function that is the negative of the function you care about, and take the minimum of the new function,

$$\max_x f(x) = \min_x g(x) \qquad \text{where} \qquad g(x) = -f(x).$$

Use this trick and `fminbnd` to maximize your function. To do this you may need to create a very simple adapter-like function, whose only job is to change the sign of your function.

Use `fminbnd` to find the optimal value of θ. Your answer should be in radians, and be between 0 and 2π. Save the result to file as **A1.dat**. Also save the horizontal distance that the cannonball travels to file as **A2.dat**.

Problem 2 (Scorelator).

The cannon is now placed on a cliff so that its height is $y_0 = 10$ meters off of the ground. The equation for its height is now given by

$$y(t) = y_0 + v_0 \sin(\theta)t - \frac{1}{2}gt^2.$$

Solve for the new time of impact t_{impact} when the cannonball impacts the ground. Insert this time into $x(t)$ to find a function in terms of θ for the horizontal distance the cannonball travels.

Use `fminbnd` to find the optimal value of θ under these new circumstances, and save the result to file as **A3.dat**. Also save the horizontal distance that the cannonball travels to file as **A4.dat**.

Successive Over-relaxation revisited

Recall from Homework 4 where we solved the problem $Ax = b$ with $A \in \mathbb{R}^{N\times N}$ a square matrix with -2's on the main diagonal, 1's directly above and below the diagonal, and 0's everywhere else. For $N = 5$, the A looks like,

$$\begin{bmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}.$$

<u>In what follows, we will again examine this problem with $N = 30$, just as we did in Homework 4.</u>

We introduced the method of Successive Over-relaxation (SOR) for solving this system in Homework 4. In SOR, we split the matrix into parts $A = L + D + U$, where L and U were strictly lower- and upper-triangular matrices, and D was a diagonal matrix. We then performed the iteration

$$\vec{x}_{k+1} = M\vec{x}_k + g$$

where

$$M = -(D + \omega L)^{-1}(\omega U + (\omega - 1)D) \qquad\qquad g = (D + \omega L)^{-1}(\omega \vec{b}).$$

Problem 3 (Scorelator).

The eigenvalues of M depend on the choice of ω, and for some best value ω_{optim}, the largest-magnitude eigenvalue is made as small as possible. We found that $\omega = 1.82$ was a pretty good choice, but can we do better?

Write a MATLAB function that builds the matrix M for a given value of ω and returns the magnitude of the largest-magnitude eigenvalue. Use `fminbnd` to find the minimizer ω_{optim} of this function for values of $1 \leq \omega \leq 1.9$. Save the optimal value of ω to file as **A5.dat** and the magnitude of the largest-magnitude eigenvalue of M to **A6.dat**.