Assignment - 1

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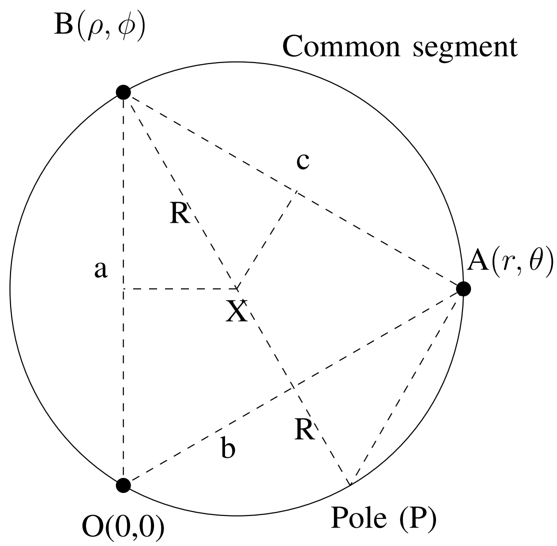
PROBLEM

1. Show that the diameter of the circum-circle formed by the points A(r,θ), B(ρ,θ) and the pole is:

$$\frac{\sqrt{r^2+\rho^2-2r\rho\cos(\theta-\phi)}}{\sin(\phi-\theta)}$$

SOLUTION



$$\mathbf{A}=\begin{pmatrix} r\ \ sin(\theta) \\ r\ \ cos(\theta)\end{pmatrix},\ \ \mathbf{B}=\begin{pmatrix} \rho\ \ sin(\phi) \\ \rho\ \ cos(\phi)\end{pmatrix},$$

$$\mathbf{O}=\begin{pmatrix} 0\ \ sin(0) \\ 0\ \ cos(0)\end{pmatrix}$$

$$\|\mathbf{A}-\mathbf{O}\|^2=(\mathbf{A}-\mathbf{O})^\top(\mathbf{A}-\mathbf{O})$$
$$\|\mathbf{A}-\mathbf{0}\|=\begin{pmatrix} r\ \ sin(\theta) \\ r\ \ cos(\theta)\end{pmatrix},$$
$$\mathbf{a}=\begin{pmatrix} r\ \ sin(\theta) \\ r\ \ cos(\theta)\end{pmatrix},$$

$$\|\mathbf{B}-\mathbf{O}\|^2=(\mathbf{B}-\mathbf{O})^\top(\mathbf{B}-\mathbf{O})$$
$$\|\mathbf{B}-\mathbf{0}\|=\begin{pmatrix} \rho\ \ sin(\phi) \\ \rho\ \ cos(\phi)\end{pmatrix}$$
$$\mathbf{b}=\begin{pmatrix} \rho\ \ sin(\phi) \\ \rho\ \ cos(\phi)\end{pmatrix}$$

From the diagram, BP is diameter with X as circum-center of the triangle ΔOAB.

Also, ΔAPB and ΔAOB share same segment. AB is same chord between them so by comparing both triangles,

$\angle APB$ = $\angle AOB$,
From that we can say,

$$\frac{c}{2R}=\frac{h}{a}$$
$$\frac{c}{D}=\frac{h}{a} \tag{1}$$
$$In\ \ \Delta AOB,\ \ Area\,[AOB]=\frac{1}{2}\,bh$$
$$h=\frac{2[AOB]}{b} \tag{2}$$

$$Area\ \ [AOB]=\frac{1}{2}\,ab\ \ sin(\theta)$$
$$=\frac{1}{2}\,\rho r\ \ sin(\theta-\phi) \tag{3}$$

From (1) and (2),

$$\frac{c}{D}=\frac{2[AOB]}{ab}$$

$$abc=2D[AOB]$$
$$2D=\frac{abc}{[AOB]} \tag{4}$$

$$c=\sqrt{(r\,sin(\theta)-\rho\,sin(\phi))^2+(r\,cos(\theta)-\rho\,cos(\phi))^2}$$
$$=\sqrt{r^2\,P+\rho^2\,P-2\,r\rho\,(sin\theta\,sin\phi+cos\theta\,cos\phi)}$$

Where P is $(sin^2(\phi) - cos^2(\phi))$ which is 1 and a = ρ and b = r

$$c = \sqrt{r^2 + \rho^2 - 2\,r\rho\,cos(\theta - \phi)} \tag{5}$$

From (4) and (5),

$$2D = \frac{\sqrt{r^2 + \rho^2 - 2\,r\rho\,cos(\theta - \phi) r\rho}}{\frac{1}{2} r\rho sin(\theta - \phi)}$$

$$\mathbf{D} = \frac{\sqrt{r^2 + \rho^2 - 2\,r\rho\,cos(\theta - \phi)}}{sin(\theta - \phi)} \tag{6}$$