Assignment - 1

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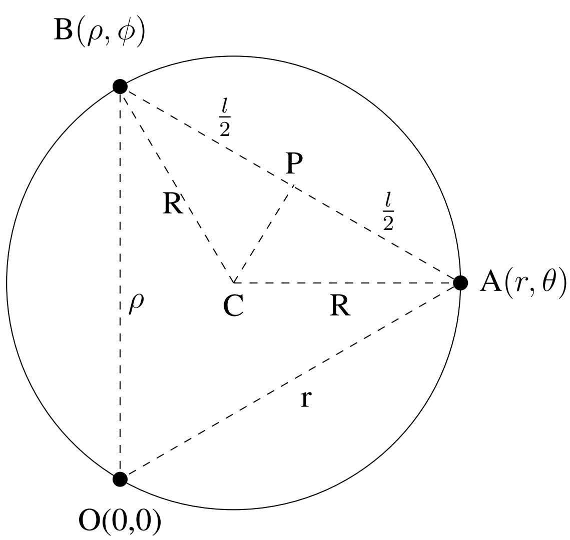
SM21MTECH14004

Problem

1. Show that the diameter of the circum-circle formed by the points A(r,θ), B(ρ,θ) and the pole is:

$$\frac{\sqrt{r^2+\rho^2-2r\rho\cos{(\theta-\phi)}}}{\sin{(\phi-\theta)}}$$

Solution



We know that area of ΔOAB with vertices $(0,0)$, (θ,ϕ), (r,ϕ) is given by

$$A_t=\frac{1}{2}(OA)(OB)(\sin(\phi-\theta))$$

$$A_t=\frac{1}{2}r\rho\sin(\phi-\theta) \qquad (1)$$

Now, by right angle triangle ΔPCB,

$$\sin(\phi-\theta)=\frac{\frac{l}{2}}{R}=\frac{l}{2R} \qquad (2)$$

$$(where R is radius of the circle)$$

Substituting values of (2) in (1), we get

$$A_t=\frac{1}{2}r\rho(\frac{l}{2R})$$

$$\frac{1}{2}r\rho\sin(\phi-\theta)=\frac{1}{2}r\rho(\frac{l}{2R})$$

$$\sin(\phi-\theta)=\frac{l}{2R}$$

$$R=\frac{l}{2\sin(\phi-\theta)}=Radius \quad of \quad circum \quad circle$$

$$D=2R=\frac{l}{\sin(\phi-\theta)} \qquad (3)$$

$$(Where D is Diameter of circum circle)$$

But we know that, the distance between two points $A(r,\theta)$ and $B(\rho,\phi)$ is,

$$AB=l=\sqrt{r^2+\rho^2-2r\rho\cos(\theta-\phi)}$$

From (3),

$$D=\frac{\sqrt{r^2+\rho^2-2r\rho\cos(\theta-\phi)}}{\sin(\phi-\theta)}$$