Assignment - 2

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SM21MTECH14004

Problem

1. Find the in-center of the triangle formed by the following points lie,

i. 5x-12y=0, ii. 5x+12y+60=0, iii. 5x+12y-60=0

Solution

First, we need to find the intersection point of given three lines,

Let's find the intersection of line (i) and (ii),

Let's write it vector form,

$$\mathbf{AM} = \mathbf{B}$$
$$\mathbf{M} = \mathbf{A}^{-1}\mathbf{B}$$
$$\mathbf{M} = \begin{bmatrix} 5 & -12 \\ 5 & 12 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -60 \end{bmatrix}$$
$$\mathbf{M} = \frac{1}{120} \begin{bmatrix} 12 & 12 \\ -5 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ -60 \end{bmatrix}$$
$$\mathbf{M} = \begin{bmatrix} -6 \\ -2.5 \end{bmatrix}$$

In the same way, intersection of line (ii) and (iii) is,

$$\mathbf{AN} = \mathbf{B}$$
$$\mathbf{N} = \mathbf{A}^{-1}\mathbf{B}$$
$$\mathbf{N} = \begin{bmatrix} 5 & 12 \\ 12 & 5 \end{bmatrix}^{-1} \begin{bmatrix} -60 \\ 60 \end{bmatrix}$$
$$\mathbf{N} = \frac{1}{119} \begin{bmatrix} 5 & -12 \\ -12 & 5 \end{bmatrix} \begin{bmatrix} -60 \\ 60 \end{bmatrix}$$
$$\mathbf{N} = \begin{bmatrix} 8.571 \\ -8.571 \end{bmatrix}$$

In the same way, intersection of line (i) and (iii) is,

$$\mathbf{AO} = \mathbf{B}$$
$$\mathbf{O} = \mathbf{A}^{-1}\mathbf{B}$$
$$\mathbf{O} = \begin{bmatrix} 5 & -12 \\ 12 & -5 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 60 \end{bmatrix}$$
$$\mathbf{O} = \frac{1}{169} \begin{bmatrix} -5 & 12 \\ -12 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 60 \end{bmatrix}$$
$$\mathbf{O} = \begin{bmatrix} 4.260 \\ 1.775 \end{bmatrix}$$

Now we need to find difference between each vectors,
Difference between vectors N and O:

N = $\begin{pmatrix} 8.571 \\ -8.571 \end{pmatrix}$, O = $\begin{pmatrix} 4.260 \\ 1.775 \end{pmatrix}$
0-N = $\begin{pmatrix} -4.311 \\ 10.345 \end{pmatrix}$

$$\|\mathbf{O} - \mathbf{N}\|^2 = (\mathbf{O} - \mathbf{N})^\top (\mathbf{O} - \mathbf{N})$$
$$\|\mathbf{O} - \mathbf{N}\| = \begin{pmatrix} -4.311 & 10.345 \end{pmatrix} \begin{pmatrix} -4.311 \\ 10.345 \end{pmatrix}$$
$$\|\mathbf{O} - \mathbf{N}\| = \sqrt{(-4.311^2 + 10.345^2)}$$
$$\|\mathbf{O} - \mathbf{N}\| = 11.207 \qquad (\mathbf{D_1})$$

Difference between vectors M and O:

M = $\begin{pmatrix} -6 \\ -2.5 \end{pmatrix}$, O = $\begin{pmatrix} 4.260 \\ 1.775 \end{pmatrix}$
0-M = $\begin{pmatrix} 10.260 \\ 4.275 \end{pmatrix}$

$$\|\mathbf{O}-\mathbf{M}\|^2 = (\mathbf{O}-\mathbf{M})^\top(\mathbf{O}-\mathbf{M})$$
$$\|\mathbf{O}-\mathbf{M}\| = \begin{pmatrix}10.260 & 4.275\end{pmatrix}\begin{pmatrix}10.260\\4.275\end{pmatrix}$$
$$\|\mathbf{O}-\mathbf{M}\| = \sqrt{(10.260^2+4.275^2)}$$
$$\|\mathbf{O}-\mathbf{M}\| = 11.115 \qquad (\mathbf{D_2})$$

Difference between vectors M and N:

M = $\begin{pmatrix}-6\\-2.5\end{pmatrix}$, N = $\begin{pmatrix}8.571\\-8.571\end{pmatrix}$

N-M = $\begin{pmatrix}14.571\\-6.071\end{pmatrix}$

$$\|\mathbf{N}-\mathbf{M}\|^2 = (\mathbf{N}-\mathbf{M})^\top(\mathbf{N}-\mathbf{M})$$
$$\|\mathbf{N}-\mathbf{M}\| = \begin{pmatrix}14.571 & 6.071\end{pmatrix}\begin{pmatrix}14.571\\6.071\end{pmatrix}$$
$$\|\mathbf{N}-\mathbf{M}\| = \sqrt{(14.571^2+6.071^2)}$$
$$\|\mathbf{N}-\mathbf{M}\| = 15.785 \qquad (\mathbf{D_3})$$

Formula for finding in-center (I) of triangle with three given coordinates is,

$$\left(\frac{D_1M+D_2N+D_3O}{D_1+D_2+D_3}, \quad \frac{D_1M+D_2N+D_3O}{D_1+D_2+D_3}\right)$$

By putting all the values,

$$\left(\frac{(11.207)(-6)+(11.115)(8.571)+(15.785)(4.260)}{11.207+11.115+15.785},\right.$$
$$\left.\frac{(11.207)(-2.5)+(11.115)(-8.571)+(15.785)(1.775)}{11.207+11.115+15.785}\right)$$

I = $\begin{pmatrix}2.500\\-2.499\end{pmatrix}$

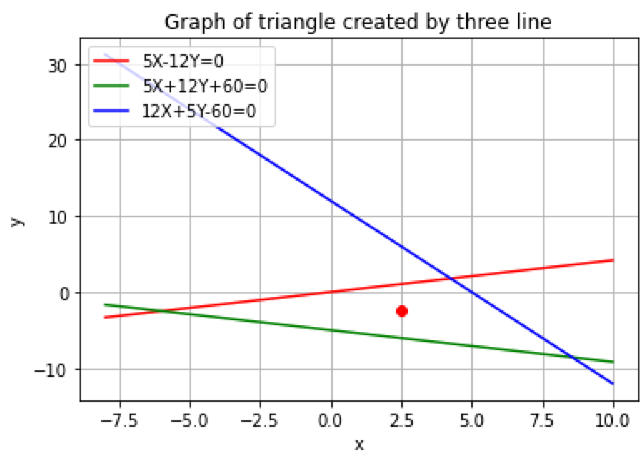

Fig. 1. Triangle created by three lines