Assignment - 2

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Problem

1. Find the in-center of the triangle formed by the following points lie,

i. 5x-12y=0, ii. 5x+12y+60=0, iii. 5x+12y-60=0

Solution

First, we need to find the intersection point of given three lines,

Let's find the intersection of line (i) and (ii),

Let's write it in AX = B form,

$$\begin{pmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

$$\begin{pmatrix} 5 & -12 \\ 5 & 12 \end{pmatrix} \begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ -60 \end{pmatrix}$$

By solving above equation, we are getting the intersection point of line (i) and (ii),
Let's call it as M:

$$\text{M} = \begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} = \begin{pmatrix} -6 \\ -2.5 \end{pmatrix} \qquad (1)$$

In the same way, intersection of line (ii) and (iii) is,

$$\begin{pmatrix} 5 & 12 \\ 12 & 5 \end{pmatrix} \begin{pmatrix} X_2 \\ Y_2 \end{pmatrix} = \begin{pmatrix} -60 \\ 60 \end{pmatrix}$$

By solving above equation, we are getting the intersection point of line (ii) and (iii),
Let's call it as N:

$$\text{N} = \begin{pmatrix} X_2 \\ Y_2 \end{pmatrix} = \begin{pmatrix} 8.571 \\ -8.571 \end{pmatrix} \qquad (2)$$

In the same way, intersection of line (i) and (iii) is,

$$\begin{pmatrix} 5 & -12 \\ 12 & -5 \end{pmatrix} \begin{pmatrix} X_3 \\ Y_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 60 \end{pmatrix}$$

By solving above equation, we are getting the intersection point of line (i) and (iii),
Let's call it as O:

$$\text{O} = \begin{pmatrix} X_3 \\ Y_3 \end{pmatrix} = \begin{pmatrix} 4.260 \\ 1.775 \end{pmatrix} \qquad (3)$$

Now we need to find difference between each vectors,

Difference between vectors N and O:

$$\text{N} = \begin{pmatrix} 8.571 \\ -8.571 \end{pmatrix}, \text{O} = \begin{pmatrix} 4.260 \\ 1.775 \end{pmatrix}$$

$$\text{0-N} = \begin{pmatrix} -4.311 \\ 10.345 \end{pmatrix}$$

$$\|\mathbf{O} - \mathbf{N}\| = \sqrt{(-4.311^2 + 10.345^2)}$$

$$\|\mathbf{O} - \mathbf{N}\| = 11.207 \qquad (\mathbf{D_1})$$

Difference between vectors M and O:

$$\text{M} = \begin{pmatrix} -6 \\ -2.5 \end{pmatrix}, \text{O} = \begin{pmatrix} 4.260 \\ 1.775 \end{pmatrix}$$

$$\text{0-M} = \begin{pmatrix} 10.26 \\ 4.275 \end{pmatrix}$$

$$\|\mathbf{O} - \mathbf{M}\| = \sqrt{(10.260^2 + 4.275^2)}$$

$$\|\mathbf{O} - \mathbf{M}\| = 11.115 \qquad (\mathbf{D_2})$$

Difference between vectors M and N:

$$\text{M} = \begin{pmatrix} -6 \\ -2.5 \end{pmatrix}, \text{N} = \begin{pmatrix} 8.571 \\ -8.571 \end{pmatrix}$$

$$\text{N-M} = \begin{pmatrix} 14.571 \\ -6.071 \end{pmatrix}$$

$$\|\mathbf{N} - \mathbf{M}\| = \sqrt{(14.571^2 + 6.071^2)}$$

$$\|\mathbf{N} - \mathbf{M}\| = 15.785 \qquad (\mathbf{D_3})$$

Formula for finding in-center (I) of triangle with three given coordinates is,

$$\left(\frac{D_1X_1 + D_2X_2 + D_3X_3}{D_1 + D_2 + D_3}, \quad \frac{D_1Y_1 + D_2Y_2 + D_3Y_3}{D_1 + D_2 + D_3}\right)$$

By putting all the values,

$$\left(\frac{(11.207)(-6) + (11.115)(8.571) + (15.785)(4.260)}{11.207 + 11.115 + 15.785}, \right.$$
$$\left. \frac{(11.207)(-2.5) + (11.115)(-8.571) + (15.785)(1.775)}{11.207 + 11.115 + 15.785}\right)$$

I = $\begin{pmatrix} 2.500 \\ -2.499 \end{pmatrix}$

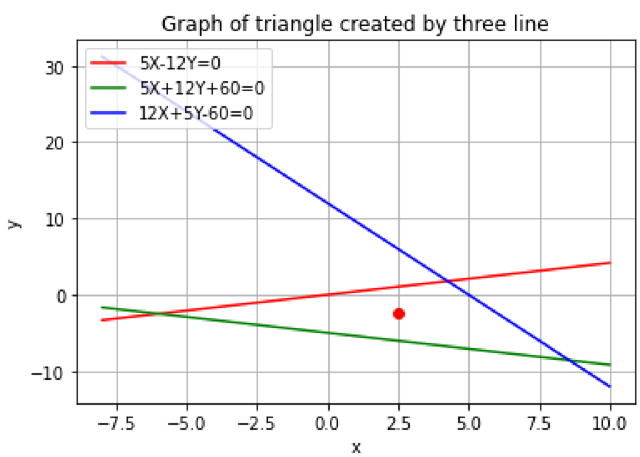

Fig. 1. Triangle created by three lines