Frequency Allocation in a SDMA Satellite Communication System with Beam Moving

Kata Kiatmanaroj, Christian Artigues
and Laurent Houssin
CNRS; LAAS; 7 avenue du Colonel Roche, F-31077, Toulouse, France
Université de Toulouse; UPS, INSA, INP, ISAE, UT1, UTM, LAAS;
F-31077 Toulouse, France
Email: kkiatman@laas.fr, artigues@laas.fr, houssin@laas.fr

Frédéric Messine
ENSEEIHT-IRIT
2 rue Charles Camichel
Université de Toulouse, UPS, INSA,
INP, ISAE, Toulouse, 31071, France
Email: frederic.messine@n7.fr

***Abstract*—Spatial Division Multiple Access (SDMA) is a principle of radio resource sharing that separates communication channels in space. It relies on adaptive and dynamic beamforming technology and well-designed algorithms for resource allocation. As satellite communication systems move towards greater capacity in both the number of users and throughput, SDMA becomes one of the most promising techniques that can achieve these two goals. This paper studies static Frequency Assignment Problem (FAP) in a satellite communication system involving a satellite and a number of users located in a service area. The objective is to maximize the number of users that the system can serve while maintaining the signal to interference plus noise ratio of each user under a predefined threshold.**

Traditionally, interference is binary and fixed. In this paper, the interference is cumulative and variable depending on how the frequency is assigned. To solve the problem, we work on both discrete and continuous optimizations. Integer linear programming formulations and greedy algorithms are proposed for solving the discrete frequency allocation problem. The solution is further improved by beam moving algorithm which involves continuous adjustment of satellite beams and deals with non-linear change of interference.

I. Introduction

Satellite communications have revolutionised the world we live in. Fixed and mobile telephone services, television broadcast, internet access, and a large number of applications have changed the way people all over the globe interact. With the continuing increase in demand, satellite communication technology continuously evolves and move towards greater capacity, higher flexibility, and better service to the end-users. Spatial Division Multiple Access (SDMA) appears to be an alternative to achieve these requirements simultaneously [1]. The technology employs antenna arrays and multi-dimensional non-linear signal processing techniques to provide significant increases in capacity and quality of many wireless communication systems [2]. The technology is not restricted to any particular modulation format or air-interface protocol, and is compatible with all currently deployed air-interfaces [3].

An SDMA satellite equips with antennas that transmit signals to numerous zones on the earth's surface. The antennas are highly directional, allowing the same frequency to be reused in other surface zones where the frequency separation is sufficiently large. To support a large number of users, frequency selection should be carefully performed. The frequency allocation strategy thus plays an important role in the system performance. This class of problem is well-known as Frequency Allocation Problem (FAP).

The satellite communication system that we study in this paper aims at establishing bi-directional communications to stationary user terminals located in a service area. We propose Integer Linear Programming (ILP) formulations and greedy algorithm for solving the problem and then use beam moving algorithm to improve the solutions.

The paper is organised as follow: Section II provides the description of the telecommunication system. In Section III, we describe ILP formulation, greedy algorithm, and beam moving algorithm. Section IV presents the experimental results while conclusions are given in Section V.

II. System Description

In general, a satellite communications system consists of a satellite, a gateway, and a number of users within a service area. The satellite provides bi-directional communication links towards users and acts as a repeater between them and the gateway, the node that connects the satellite system to the terrestrial network. In this study, we consider only the satellite, the users, and communication links between them.

Users are randomly generated and uniformly distributed within a rectangular service area defined by a set of geographic coordinates. The satellite's orthogonal projection is assumed to be at the position $(0, 0)$. The satellite utilizes SDMA technology to form beams and center them over the users. The user's perceived antenna gain, as shown in Fig. 1 (z-axis in log scale), is determined by the simplified radiation pattern of the antenna and the distance between the user and the satellite [4]. By centering the beam over the user, maximum gain is achieved.

The objective of the study is to serve as many users as possible. A user is considered "served" if it is assigned with a frequency and satisfies the link budget constraint (1) having the LHS, the user's signal to interference plus noise ratio (SINR) no less than the RHS, the required signal to noise ratio.

978-1-4577-2053-6/12/$31.00 ©2012 IEEE

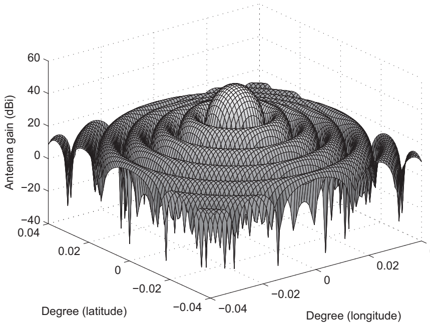

Fig. 1. An example of antenna diagram

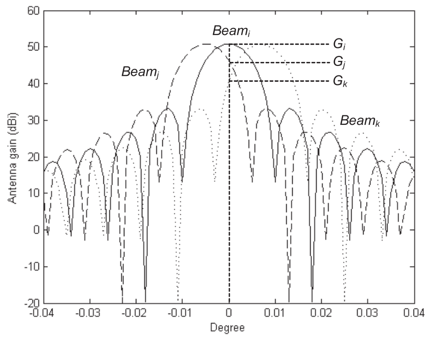

Fig. 2. Cross sections of three satellite beams

$$\frac{C}{N+I} \geq \left(\frac{C}{N}\right)_{Required} \tag{1}$$

Fig. 2 shows, cross sections ($Y = 0$) of three satellite beams associated to and centered at users i, j, k located at positions $(0,0), (-0.005, 0)$, and $(0.007, 0)$. Let's assume uniform receivers, transmitter output power and propagation loss, we can consider the received signal power from the perceived antenna gain. G_i denotes the corresponding antenna gain from $Beam_i$ at position $(0,0)$. It can be seen that, at this position, there exist also G_j and G_k from $Beam_j$ and $Beam_k$. Interference occurs if these users share the same frequency. It is cumulative in that the total interference at user i is the sum of the interferences from user j and k.

The SINR considers both interference and noise and is given by

$$\left(\frac{C}{N+I}\right)_i^{-1} = \left(\frac{C}{N}\right)_{Feeder}^{-1} + \left(\frac{C}{I}\right)_{Feeder}^{-1} + \left(\frac{C}{IM}\right)^{-1} + \left(\frac{C}{N}\right)_i^{-1} + \left(\frac{C}{I}\right)_i^{-1}. \tag{2}$$

$\left(\frac{C}{N}\right)_i$ and $\left(\frac{C}{I}\right)_i$ correspond to the user's signal to noise ratio and the user's signal to interference ratio and are defined by

$$\left(\frac{C}{N}\right)_i = K \cdot G_{Sat(Beam_i \to i)},$$
$$\left(\frac{C}{I}\right)_i = \frac{G_{Sat(Beam_i \to i)}}{\sum_{j \in Interf} \cdot G_{Sat(Beam_j \to i)}}.$$

K is a system constant while $G_{Sat(Beam_i \to i)}$ and $G_{Sat(Beam_j \to i)}$ are user i's antenna gain (regarding to its beam and position) and the interferer j 's antenna gain at user i's position.

$\left(\frac{C}{N}\right)_{Feeder}$ and $\left(\frac{C}{I}\right)_{Feeder}$ correspond to the signal to thermal noise ratio and signal to interference ratio between the gateway and the satellite. $\left(\frac{C}{IM}\right)$ concerns the satellite characteristics. These ratios are constant in our study.

Let $A = \left(\frac{C}{N}\right)_{Feeder}^{-1} + \left(\frac{C}{I}\right)_{Feeder}^{-1} + \left(\frac{C}{IM}\right)^{-1}$, $B = \left(\frac{C}{N}\right)_i^{-1}$, and $D = \left(\frac{C}{N}\right)_{Required}$. The cumulative interference constraint for user i can be written in a linear form as

$$\sum_{j \in Interf} \delta_{ij} \leq \alpha_i \tag{3}$$

where

$$\delta_{ij} = D \cdot G_{Sat(Beam_j \to i)}, \tag{4}$$
$$\alpha_i = G_{Sat(Beam_i \to i)} \cdot (1 - AD - BD). \tag{5}$$

The term α_i can be perceived as an acceptable interference threshold for the user i while δ_{ij} as an interference coefficient from users j towards the user i.

III. Modeling and solving frequency allocation problem

A. FAP literature review

Most approaches dealing with minimum interference FAP consider binary interference constraints, i.e. involving only two users. Because of the strong links between graph coloring and frequency allocation with binary interference constraints, most methods found in the literature are inspired by coloring algorithms. The graph coloring algorithms are well known to be NP-hard, thus, consequently the FAP. Among the proposed methods, the constructive (greedy) algorithms are widely used since they are simple and fast. In this category, we find the generalisation of DSATUR procedure [5]. Other more sophisticated algorithms, such as local search, metaheuristics, ILP, and constraint programming approaches, are frequently encountered [6].

One of the difficulties appearing in the telecommunication system considered in this study lies in the explicit consideration of non-binary interference constraints. In terms of graph coloring, deciding whether a given coloring is feasible or not cannot be made any more by checking pairwise user colors or assignments. Instead, for a given user, the cumulative interferences of the users assigned to the same color (frequency) has to be computed. The coloring is feasible if this cumulative interference remains under a threshold. In the literature, only a few approaches explicitly take into account of such interferences [7] [8] [9].

B. Integer linear programming

Taking account of hypotheses and simplifications presented in Section II, the FAP is similar to coloring problems and thus formalised as the corresponding combinatorial optimization problems. Each user has to be assigned a color, representing a frequency.

Let n denotes the number of users, $U = \{1, \ldots, n\}$ a set of users, and C the number of colors. Binary decision variables x_{ic} are defined for $i \in \{1, \ldots, n\}$ and $c \in \{1, \ldots, C\}$ in that $x_{ic} = 1$ if color c is allocated to users i and $x_{ic} = 0$ otherwise. The problem can be represented by the following ILP:

$$\max \sum_{i=1}^{n} \sum_{c=1}^{C} x_{ic}, \tag{6}$$

$$\sum_{c=1}^{C} x_{ic} \leq 1 \quad i = 1, \ldots, n, \tag{7}$$

$$\sum_{j=1}^{n} \delta_{ij} x_{jc} \leq \alpha_i + M_i(1 - x_{ic}) \quad i = 1, \ldots, n, c = 1, \ldots, C, \tag{8}$$

$$x_{ic} \in \{0, 1\} \quad i = 1, \ldots, n \quad c = 1, \ldots, C. \tag{9}$$

Objective (6) maximizes the number of accepted users while Constraints (7) restrict that at most one color has to be selected for each user. Constraints (8) are the cumulative interference constraints. The constant M_i has to be large enough to withdraw these constraints if i is not assigned a color c ($x_{ic} = 0$). More precisely, we set $M_i = \sum_{j=1}^{n} \delta_{ij} - \alpha_i$.

C. Greedy algorithm

Solving the ILP formulations provides optimal solutions only for small problems. For large-sized problems, a heuristic approach is necessary. We propose greedy algorithms to solve this problem. The principle of the greedy algorithm is, at first, to consider the users sequentially according to a given criterion named (user priority rule). Secondly, either the selected user is assigned a color or rejected according to a second criterion (frequency priority rule).

Let Q denotes a set of users that have not been assigned a color yet. Initially we have $Q = U$. At each step of the greedy algorithm, a user i is removed from Q and is either rejected or assigned a color.

For the user priority rule, we may use the frequency margin, where the margin $M(i, c)$ of a user $i \in Q$ for a color c is given by $M(i, c) = \alpha_i - \sum_{j \in U \setminus Q \cup \{i\}, F_j = c} \delta_{ij}$. This margin corresponds to the positive or negative slack of the cumulative interference constraint for user i if it is assigned a color c.

As a preliminary result, we observed that the user priority rule aimed at selecting first the most constrained users in terms of available colors while it is well known that, with this environment, the DSATUR algorithm for standard graph coloring problem gives bad results. We thus consider a kind of hybrid reverse DSATUR rule by alternately selecting the user having the largest number of available colors and the user having maximum interference with the previously assigned user. In fact, we tested two following user priority rules:

- Lexicographic: the user with the smallest number is selected,
- Hybrid: the user having the largest number of available colors is selected. A color c is available for user $i \in Q$ if $M(i, c) \geq 0$ and if for all users $j \in U \setminus Q$ that have already been assigned color c, $M(j, c) \geq 0$. In case of a tie, we select the user having the largest total margin for all its available colors. Let i denotes the selected user with this rule. For the next iteration, we select the user having maximum interference with i, i.e. the user j maximizing $\delta_{ij} + \delta_{ji}$ and we alternate the two rules.

For the frequency selection, we tested two following frequency priority rule:

- Lexicographic: the smallest available frequency is selected,
- Most used: the most used available frequency is selected. In case of a tie, we select the color c that maximizes the sum of margins $M(j, c)$ for all users $j \in Q$.

The proposed greedy algorithms run in $O(n^2 C)$ time.

D. Beam moving algorithm

To further improve the results from the ILP and greedy algorithm, we propose a subsequent non-linear local optimization, called beam moving algorithm. This algorithm exploits the benefit of SDMA technology by moving a number of satellite beams from their center positions.

In fact the δ_{ij} and α_i in (4) and (5) can be written as functions of user position (u, v) and beam position which are

$$\delta_{ij} = D \cdot G_{Sat}(u_i, v_i, Beam\_u_j, Beam\_v_j),$$
$$\alpha_i = G_{Sat}(u_i, v_i, Beam\_u_i, Beam\_v_i) \cdot (1 - AD - BD).$$

The terms D and $(1 - AD - BD)$ are constant. We will keep the user position fixed but alter the beam position; as a result, both δ_{ij} and α_i changes. Nonetheless, the change is non-linear according to the antenna gain shown previously in Fig. 1.

Beam moving algorithm takes the output from either ILP or greedy algorithm as its input, identifies the rejected users, and, for each rejected user, moves the most k interfering beams and tries to reassign the user a color.

Let i denotes an unassigned user, the beam moving algorithm selects a color c, i.e. sets $x_{ic} = 1$, and identifies a set of interferers S containing all users j having $x_{jc} = 1, \forall j \in S$ (unassigned user included). Let $K \subseteq S$ consists of a set of users whose beams will be moved. The parameter k defines the number of strongest interferers to the unassigned user i that are included in the set K. The parameter *UTVAR* $\in (0, 1)$, if set to 1, tells the algorithm to replace the least interferer in the set K with i thus including the user i in the move.

MAXINEG parameter provides a maximum negative margin from the required signal to noise ratio. It is based on the fact that the closer the unassigned user's signal to interference plus noise ratio is to the required signal to noise ratio, the more the possibility the algorithm has to search for a solution. Before the algorithm tries to move beams, the unassigned user is tested with this margin. If failed, the remaining colors are tried or the user is rejected.

The algorithm continuously moves the beams of users in the set K from their center positions $(u_0^{(k)}, v_0^{(k)})$ and in each move evaluates if the new positions pass the link budget constraints. This can be represented as:

$$\min \sum_{k \in K} \| (u_0^{(k)} - u_k)^2 + (v_0^{(k)} - v_k)^2 \|^2, \tag{10}$$

subject to

$$\left(\frac{C}{N+I}\right)(u_k, v_k, u_0^{(k)}, v_0^{(k)}) \geq \left(\frac{C}{N}\right)_{Required} \quad \forall k \in K. \tag{11}$$

When a beam is moved from its center, the associated user will obtain lower antenna gain and hence lower SINR. Any move that violates the link budget constraints (11) is not allowed. Nonetheless, this move could benefit the unassigned user by reducing its tentative interference level. For a selected color c, the beam moving algorithm minimises the total move distance of the interferers' beams (10), maintains their interference constraints' validity, and reduces the tentative interference of the unassigned user i to the level that the reassignment is valid.

If a decent move could not be found within a number of iterations defined by *MAXITER* each of the remaining colors is tried. If all colors are tried and there is no possible solution, the user i is rejected and the algorithm moves to next unassigned users.

IV. COMPUTATIONAL EXPERIMENTS AND RESULTS

The ILP formulation has been solved using IBM/ILOG CPLEX 12.2 [10]. The greedy algorithm has been coded in C++. We tested the proposed algorithms with C = 8; increasing stepwise the number of users by 20 from 20 to 200 users with 100 instances each. The user positions are randomly generated and uniformly distributed over the defined service area. The results were obtained on a 2.7GHz Intel Core i5 machine with 4GB RAM. The CPU times for the ILP resolutions have been limited to 60s, 120s, and 180s after which the best integer solution is obtained. The CPU times for the greedy algorithm were negligible while the beam moving was performed with the maximum of 40 iterations with no limitation on the calculation time.

The beam moving algorithm is coded in Matlab [11]. The function *fmincon* with active-set algorithm is used for computing the minimum move distance according to the given non-linear constraints.

We first present a comparison of the greedy algorithms. Table I reports the average number of accepted users over 1,000 instances. The results of the greedy algorithms are very close. It was difficult to give better results than the simple lexicographic rules. The algorithm that uses Hybrid and Most used rules gives the best result. As of this, we use it as the baseline for performance comparison with the results from ILP and beam moving.

TABLE I
AVERAGE NUMBER OF ACCEPTED USERS OVER 1,000 INSTANCES

Lexicographic (user + frequency)	85.30
Lexicographic (user) + Most used (frequency)	85.31
Hybrid (user) + Most used (frequency)	85.63

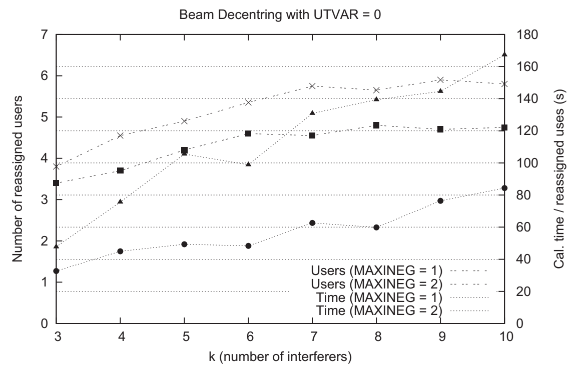

Fig. 3. Average number of reassigned users and calculation time per reassigned user for different beam moving configurations over 20 instances of 200 users with UTVAR=0

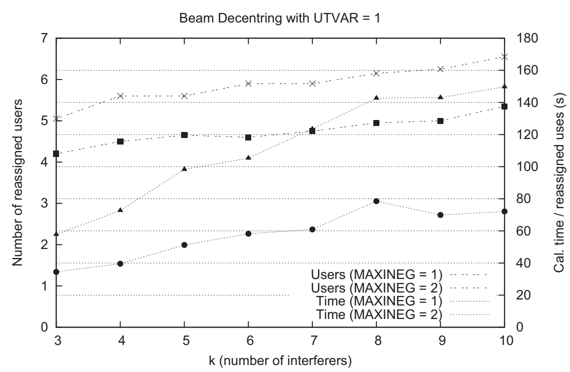

Fig. 4. Average number of reassigned users and calculation time per reassigned user for different beam moving configurations over 20 instances of 200 users with UTVAR=1

We tested 32 configurations of *k-MAXINEG-UTVAR* for the beam moving algorithm over 20 instances of 200 users. Test results are provided in Fig. 3 and 4. It can be seen that increasing any of k (from 3 to 10) or *MAXINEG* (from 1 to 2) or enabling *UTVAR* (0 or 1) yields higher number of reassigned users, at an expense of longer calculation time. Both configuration 7-2-0 and 6-2-1 provide good performances with acceptable calculation times. We chose Configuration 7-2-0 for improving the results from the ILP and greedy algorithm.

Fig. 5 and 6 display, for each algorithm and number of users, the average number of accepted users in the computed frequency allocation plans. The number of optima provided by ILPs is given in Table II. The greedy algorithm performs as good as the other two ILPs at up to 120 users (ILP can solve to optima for all or almost all of 100 instances up to this point). For 140-200 users, the performance gap becomes larger as the

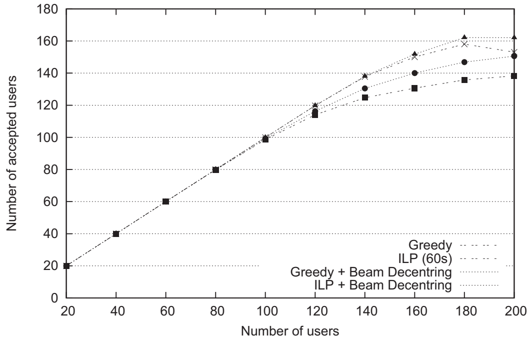

Fig. 5. Average number of accepted users before and after beam moving for Greedy algorithm and ILP 60s

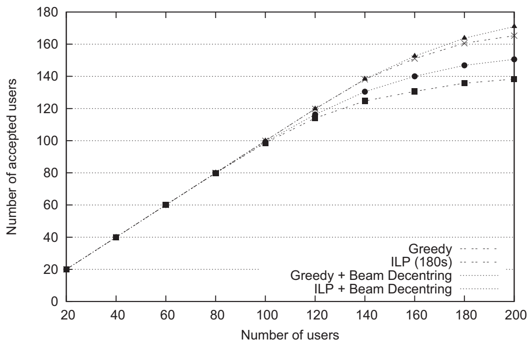

Fig. 6. Average number of accepted users before and after beam moving for Greedy algorithm and ILP 180s

number of user increases. Performance degradation is found in ILP60s at 200 user instances, contrast to that of ILP180s. This signifies that, though not reaching the optima, the ILP needs more time for a larger instance to provide a better results.

Beam moving gives performance improvement for both greedy algorithm and ILP. Significant improvements can be seen in the greedy algorithm case. It could provide comparable results at 200 users compared to ILP60s. Nonetheless, the algorithm's calculation time is high, see Table III.

TABLE II
NUMBER OF OPTIMA PROVIDED BY ILPS

n	20	40	60	80	100	120	140	160	180
ILP60s	100	100	100	100	100	97	54	0	0
ILP120s	100	100	100	100	100	98	61	0	0
ILP180s	100	100	100	100	100	100	67	0	0

TABLE III
AVERAGE CALCULATION TIME (S) PERFORMED BY BEAM MOVING ALGORITHM

n	60	80	100	120	140	160	180	200
Greedy	-	9.2	22.9	67.6	241.7	570.7	1017.3	1542.5
ILP60s	-	-	-	13.6	29.7	125.3	365.2	1032.0
ILP180s	-	-	-	-	28.4	114.9	272.9	622.0

V. CONCLUSION

In this paper we have developed integer linear programming formulation and greedy algorithms for Frequency Assignment Problems involving cumulative interference. The greedy algorithm, though simple, is very fast and efficient enough to provide comparable results to ILP up to a certain number of users. By utilising SDMA, the beam moving algorithm, with an expense of processing time, offers performance improvement for both ILP and greedy algorithm; the latter gains significant improvement.

We have considered frequency assignment problems based on single frequency over a total period of time. We can further generalise the problem in both domains in that a user could occupy more than one frequency over a fraction of time. The problem with frequency demand of cardinality n but fixed in time could be treated as 1-dimensional bin packing problem with additional constraints on cumulative interference between different bins. Further generalisation on time gives rise to 2-dimensional bin packing problem with cumulative interference constraints between different bins based on overlapping of $frequency \times time$.

REFERENCES

[1] J. C. Liberti and T. S. Rappaport, *Smart Antennas for Wireless Communications: IS-95 and Third Generation CDMA Applications*. Upper Saddle River, NJ, USA: Prentice Hall PTR, 1999.
[2] R. Roy, "An overview of smart antenna technology: the next wave in wireless communications," in *Aerospace Conference, 1998. Proceedings., IEEE*, vol. 3, mar 1998, pp. 339 –345 vol.3.
[3] ——, "Spatial division multiple access technology and its application to wireless communication systems," in *Vehicular Technology Conference, 1997 IEEE 47th*, vol. 2, may 1997, pp. 730 –734 vol.2.
[4] L. Houssin, C. Artigues, and E. Corbel, "Frequency allocation problem in a sdma satellite communication system," *Computers and Industrial Engineering*, vol. In Press, Corrected Proof, pp. –, 2010.
[5] D. Brélaz, "New methods to color the vertices of a graph," *Communications of the ACM*, vol. 22, pp. 251–256, 1979.
[6] K. I. Aardal, C. P. M. van Hoesel, A. M. C. A. Koster, C. Mannino, and A. Sassano, "Models and solution techniques for the frequency assignment problem," *4OR: Quarterly Journal of the Belgian, French and Italian Operations Research Societies*, vol. 1, no. 4, pp. 261–317, 2003.
[7] N. Dunkin, J. Bater, P. Jeavons, and D. Cohen, "Towards high order constraint representations for the frequency assignment problem," Royal Holloway University of London, Tech. Rep. CSD-TR-98-05, 1998.
[8] C. Mannino and A. Sassano, "An enumerative algorithm for the frequency assignment problem," *Discrete Applied Mathematics*, vol. 129, no. 1, pp. 155–169, 2003, previously published as technical report 1096, Università di Roma La Sapienza, 1998.
[9] M. Palpant, C. Oliva, C. Artigues, P. Michelon, and M. D. Biha, "Models and methods for frequency allocation with cumulative interference constraints," *International Transactions in Operational Research*, vol. 15, no. 3, pp. 307–324, 2008.
[10] *ILOG CPLEX C++ API 12.2 Reference Manual*, IBM Corp., Armonk, New York, 2010.
[11] MATLAB, *version 7.7.0 (R2008b)*. Natick, Massachusetts: The MathWorks Inc., 2008.