Indian Institute of Technology Kharagpur
Department of Electrical Engineering

Subject No.: **EE60020** **Subject:** **Machine Learning for Signal Processing**
Date of Assignment: **11 March 2024** **Semester:** **Spring 2023-24**
Assignment Number: **3** **Duration:** **1 hour 50 mins** **Full points:** **140**

Name: ______________________________ **Roll No:** ____________________

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1. Here $\mathcal{I}$ is an image in RGB representation format.

$$\mathcal{I} = \begin{bmatrix} (1,3,1) & (2,2,1) & (5,6,6) & (6,5,5) \\ (3,1,2) & (1,3,3) & (4,4,4) & (4,6,4) \end{bmatrix}$$

Employ $k-$means clustering method on this dataset accordingly.

(a) (4 points) Write $\mathbf{X}$ corresponding to $\mathcal{I}$?

$$\mathbf{X} = \begin{bmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{bmatrix}$$

(b) (6 points) Assume that there are 2-clusters to be formed, and the randomly initialized seed of the first cluster denoted by the set $\mathcal{C}_0$ is $\mathbf{x}_0 = [1,3,1]^\top$ and the seed of the second cluster denoted by the set $\mathcal{C}_1$ is $\mathbf{x}_7 = [4,6,4]^\top$. Let $\boldsymbol{\mu}_0^{(i)}$ and $\boldsymbol{\mu}_1^{(i)}$ respectively denote the instanteous centroids of $\mathcal{C}_0^{(i)}$ and $\mathcal{C}_1^{(i)}$ respectively at the i^{th} iteration and the instantaneous cluster unassigned dataset is represented by $\mathbf{X}^{(i)}$. Report the following at the start of clustering with $i = 0$.

$$\mathcal{C}_0^{(i=0)} = \begin{bmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{bmatrix} \quad \boldsymbol{\mu}_0^{(i=0)} = \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix}$$

$$\mathcal{C}_1^{(i=0)} = \begin{bmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{bmatrix} \quad \boldsymbol{\mu}_1^{(i=0)} = \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix}$$

$$\mathbf{X}^{(i=0)} = \begin{bmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{bmatrix}$$

(c) (10 points) Now for $i = 1$ calculate the following

$$\mathbf{x} = \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix}^\top$$

$$d\left(\mathbf{x}, \boldsymbol{\mu}_0^{(i=0)}\right) = \left\| \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix}^\top - \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix}^\top \right\|_2 = \underline{\qquad\quad}$$

$$d\left(\mathbf{x}, \boldsymbol{\mu}_1^{(i=0)}\right) = \left\| \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix}^\top - \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix}^\top \right\|_2 = \underline{\qquad\quad}$$

$$\mathcal{C}_0^{(i=1)} = \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix} \quad \boldsymbol{\mu}_0^{(i=1)} = \begin{bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{bmatrix}$$

$$\mathcal{C}_1^{(i=1)} = \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix} \quad \boldsymbol{\mu}_1^{(i=1)} = \begin{bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{bmatrix}$$

$$\mathbf{X}^{(i=1)} = \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix}$$

(d) (10 points) Now for $i = 2$ calculate the following

$$\mathbf{x} = \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix}^\top$$

$$d\left(\mathbf{x}, \boldsymbol{\mu}_0^{(i=1)}\right) = \left\| \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix}^\top - \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix}^\top \right\|_2 = \underline{\qquad\quad}$$

$$d\left(\mathbf{x}, \boldsymbol{\mu}_1^{(i=1)}\right) = \left\| \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix}^\top - \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix}^\top \right\|_2 = \underline{\qquad\quad}$$

$$\mathcal{C}_0^{(i=2)} = \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix} \quad \boldsymbol{\mu}_0^{(i=2)} = \begin{bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{bmatrix}$$

$$\mathcal{C}_1^{(i=2)} = \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix} \quad \boldsymbol{\mu}_1^{(i=2)} = \begin{bmatrix} \underline{\qquad} \\ \underline{\qquad} \\ \underline{\qquad} \end{bmatrix}$$

$$\mathbf{X}^{(i=2)} = \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underline{\qquad} \end{bmatrix}$$

(e) (20 points) Complete the solution in the following tabular approach

	$i=3$	$i=4$	$i=5$	$i=6$
$\mathbf{x} =$	$[\_\_, \_\_, \_\_]^\top$	$[\_\_, \_\_, \_\_]^\top$	$[\_\_, \_\_, \_\_]^\top$	$[\_\_, \_\_, \_\_]^\top$
$d\left(\mathbf{x}, \boldsymbol{\mu}_0^{(i-1)}\right) =$ $d\left(\mathbf{x}, \boldsymbol{\mu}_1^{(i-1)}\right) =$	____ ____	____ ____	____ ____	____ ____
$\boldsymbol{\mu}_0^{(i)} =$ $\boldsymbol{\mu}_0^{(i)} =$	$[\_\_, \_\_, \_\_]^\top$ $[\_\_, \_\_, \_\_]^\top$	$[\_\_, \_\_, \_\_]^\top$ $[\_\_, \_\_, \_\_]^\top$	$[\_\_, \_\_, \_\_]^\top$ $[\_\_, \_\_, \_\_]^\top$	$[\_\_, \_\_, \_\_]^\top$ $[\_\_, \_\_, \_\_]^\top$

(f) (4 points) Write down the clusters at the end of the custering process

$$\mathcal{C}_0^{(i=6)} = \begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$$

$$\mathcal{C}_1^{(i=6)} = \begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$$

(g) (6 points) Find the covariance of each cluster

$$\boldsymbol{\Sigma}_0 = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

$$\boldsymbol{\Sigma}_1 = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

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