Indian Institute of Technology Kharagpur
Department of Electrical Engineering

Subject No.: **EE60020** **Subject:** **Machine Learning for Signal Processing**
Date of Assignment: **11 March 2024** **Semester:** **Spring 2023-24**
Assignment Number: **3 Solution** **Duration:** **1 hour 50 mins** **Full points:** **140**

Name: ______________________ **Roll No:** ______________________

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1. Here $\mathcal{I}$ is an image in RGB representation format.

$$\mathcal{I} = \begin{bmatrix} (1,3,1) & (2,2,1) & (5,6,6) & (6,5,5) \\ (3,1,2) & (1,3,3) & (4,4,4) & (4,6,4) \end{bmatrix}$$

Employ $k-$means clustering method on this dataset accordingly.

(a) (4 points) Write $\mathbf{X}$ corresponding to $\mathcal{I}$?

$$\mathbf{X} = \begin{bmatrix} 1 & 3 & 2 & 1 & 5 & 4 & 6 & 4 \\ 3 & 1 & 2 & 3 & 6 & 4 & 5 & 6 \\ 1 & 2 & 1 & 3 & 6 & 4 & 5 & 4 \end{bmatrix}$$

(b) (6 points) Assume that there are 2-clusters to be formed, and the randomly initialized seed of the first cluster denoted by the set $\mathcal{C}_0$ is $\mathbf{x}_0 = [1,3,1]^\top$ and the seed of the second cluster denoted by the set $\mathcal{C}_1$ is $\mathbf{x}_7 = [4,6,4]^\top$. Let $\boldsymbol{\mu}_0^{(i)}$ and $\boldsymbol{\mu}_1^{(i)}$ respectively denote the instanteous centroids of $\mathcal{C}_0^{(i)}$ and $\mathcal{C}_1^{(i)}$ respectively at the i^{th} iteration and the instantaneous cluster unassigned dataset is represented by $\mathbf{X}^{(i)}$. Report the following at the start of clustering with $i = 0$.

$$\mathcal{C}_0^{(i=0)} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \quad \boldsymbol{\mu}_0^{(i=0)} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

$$\mathcal{C}_1^{(i=0)} = \begin{bmatrix} 4 \\ 6 \\ 4 \end{bmatrix} \quad \boldsymbol{\mu}_1^{(i=0)} = \begin{bmatrix} 4 \\ 6 \\ 4 \end{bmatrix}$$

$$\mathbf{X}^{(i=0)} = \begin{bmatrix} 3 & 2 & 1 & 5 & 4 & 6 \\ 1 & 2 & 3 & 6 & 4 & 5 \\ 2 & 1 & 3 & 6 & 4 & 5 \end{bmatrix}$$

(c) (10 points) Now for $i = 1$ calculate the following

$$\mathbf{x} = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}^\top$$

$$d\left(\mathbf{x}, \boldsymbol{\mu}_0^{(i=0)}\right) = \left\| \begin{bmatrix} 1 & 3 & 1 \end{bmatrix}^\top - \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}^\top \right\|_2 = 3$$

$$d\left(\mathbf{x}, \boldsymbol{\mu}_1^{(i=0)}\right) = \left\| \begin{bmatrix} 4 & 6 & 4 \end{bmatrix}^\top - \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}^\top \right\|_2 = 5.47$$

$$\mathcal{C}_0^{(i=1)} = \begin{bmatrix} 1 & 3 \\ 3 & 1 \\ 1 & 2 \end{bmatrix} \quad \boldsymbol{\mu}_0^{(i=1)} = \begin{bmatrix} 2 \\ 2 \\ 1.5 \end{bmatrix}$$

$$\mathcal{C}_1^{(i=1)} = \begin{bmatrix} 4 \\ 6 \\ 4 \end{bmatrix} \quad \boldsymbol{\mu}_1^{(i=1)} = \begin{bmatrix} 4 \\ 6 \\ 4 \end{bmatrix}$$

$$\mathbf{X}^{(i=1)} = \begin{bmatrix} 2 & 1 & 5 & 4 & 6 \\ 2 & 3 & 6 & 4 & 5 \\ 1 & 3 & 6 & 4 & 5 \end{bmatrix}$$

(d) (10 points) Now for $i = 2$ calculate the following

$$\mathbf{x} = \begin{bmatrix} 2 & 2 & 1 \end{bmatrix}^\top$$

$$d\left(\mathbf{x}, \boldsymbol{\mu}_0^{(i=1)}\right) = \left\| \begin{bmatrix} 2 & 2 & 1.5 \end{bmatrix}^\top - \begin{bmatrix} 2 & 2 & 1 \end{bmatrix}^\top \right\|_2 = 0.5$$

$$d\left(\mathbf{x}, \boldsymbol{\mu}_1^{(i=1)}\right) = \left\| \begin{bmatrix} 4 & 6 & 4 \end{bmatrix}^\top - \begin{bmatrix} 2 & 2 & 1 \end{bmatrix}^\top \right\|_2 = 5.38$$

$$\mathcal{C}_0^{(i=2)} = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} \quad \boldsymbol{\mu}_0^{(i=2)} = \begin{bmatrix} 2 \\ 2 \\ 1.33 \end{bmatrix}$$

$$\mathcal{C}_1^{(i=2)} = \begin{bmatrix} 4 \\ 6 \\ 4 \end{bmatrix} \quad \boldsymbol{\mu}_1^{(i=2)} = \begin{bmatrix} 4 \\ 6 \\ 4 \end{bmatrix}$$

$$\mathbf{X}^{(i=2)} = \begin{bmatrix} 1 & 5 & 4 & 6 \\ 3 & 6 & 4 & 5 \\ 3 & 6 & 4 & 5 \end{bmatrix}$$

(e) (20 points) Complete the solution in the following tabular approach

	$i=3$	$i=4$	$i=5$	$i=6$
$\mathbf{x} =$	$[1,3,3]^\top$	$[5,6,6]^\top$	$[4,4,4]^\top$	$[6,5,5]^\top$
$d\left(\mathbf{x}, \boldsymbol{\mu}_0^{(i-1)}\right) =$	2.188	6.53	3.63	6.01
$d\left(\mathbf{x}, \boldsymbol{\mu}_1^{(i-1)}\right) =$	4.38	2.23	2.29	1.73
$\boldsymbol{\mu}_0^{(i)} =$	$[1.75, 2.25, 1.75]^\top$	$[1.75, 2.25, 1.75]^\top$	$[1.75, 2.25, 1.75]^\top$	$[1.75, 2.25, 1.75]^\top$
$\boldsymbol{\mu}_1^{(i)} =$	$[4,6,4]^\top$	$[4.5,6,5]^\top$	$[4.33, 5.33, 4.66]^\top$	$[4.75, 5.25, 4.75]^\top$

(f) (4 points) Write down the clusters at the end of the clustering process

$$\mathcal{C}_0^{(i=6)} = \begin{bmatrix} 1 & 3 & 2 & 1 \\ 3 & 1 & 2 & 3 \\ 1 & 2 & 1 & 3 \end{bmatrix}$$

$$\mathcal{C}_1^{(i=6)} = \begin{bmatrix} 4 & 5 & 4 & 6 \\ 6 & 6 & 4 & 5 \\ 4 & 6 & 4 & 5 \end{bmatrix}$$

(g) (6 points) Find the covariance of each cluster

$$\boldsymbol{\Sigma}_0 = \begin{bmatrix} 0.6875 & -0.6875 & -0.0625 \\ -0.6875 & 0.6875 & 0.0625 \\ -0.0625 & 0.0625 & 0.6875 \end{bmatrix}$$

$$\boldsymbol{\Sigma}_1 = \begin{bmatrix} 0.6875 & 0.0625 & 0.4375 \\ 0.0625 & 0.6875 & 0.3125 \\ 0.4375 & 0.3125 & 0.6875 \end{bmatrix}$$