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Analysing the risk of investing in options of Procter and Gamble stock

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FINANCIAL MODELLING
(BEAM046)
INDIVIDUAL ASSIGNMENT

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Executive Summary

The main task that will be undertaken within this report is to act as an analyst for an investment bank and analyze the potential risk of investing in options of Procter and Gamble stock. Risk analysis is used to estimate the uncertainty involved with carrying out a certain action. To analyze this, the value at risk (VaR) and Conditional Value at Risk (CVaR) will be calculated along with volatility. This will provide multiple measures to fully analyze the risk of the investment. The Implied volatility of the PG option is then used to conduct a sensitivity analysis for VaR. Here beyond the 70% level of significance there is no point in investing in the option as there is a chance of loosing 100% return beyond 70% Confidence Interval. This report will first outline the data used by providing important summary statistics. The method section will describe, in detail, the methods, equations and variables used to calculate value at risk and Conditional Value at Risk (Monte Carlo simulation method) and the Implied Volatility test (Black Scholes Model).

DATA

The price of the options was taken from CBOE (https://www.cboe.com/), selecting the option with a strike price close to 90% of the current market price and close to 3 months maturity. The start date taken as 25th November 2020 and the expiry date as 19th February 2021. The stock price at the start of the option period is noted as \$138.31. The Option price is \$16.45, and the Strike price is \$125. The total number of expiration days leaving out holidays are 63. The risk-free rate is taken as the current 3-month US Treasury bill yield which is 0.09%. The daily data for calculating the log-returns of Procter and Gamble stock was collected from Yahoo Finance (https://uk.finance.yahoo.com/). This provided the current stock price and a year's adjusted close price to calculate daily returns.

Value at Risk

The Value at Risk measures the possible loss in value of a risky asset or portfolio over a specified time for a given confidence interval. Thus, if the VaR on an asset is \$ 100 million at a one-week, 95% confidence level, there is an only a 5% chance that the value of the asset will drop more than \$ 100 million over any given week. In its adapted form, the measure is sometimes defined more narrowly as the possible loss in value from "normal market risk" as opposed to all risk, requiring that we draw distinctions between normal and abnormal risk as well as between market and nonmarket risk.(Damodaran, 2010). In 1995, J.P. Morgan first started to give access the approach to how to calculate risk provided public access to data on the variances of and covariances across various security and asset classes. They named the service as "RiskMetrics" and used the term Value at Risk to describe the risk measure that emerged from the data.

Conditional Value at Risk

Conditional value-at-risk acts as a measure of risk and has significant advantages over the value-at-risk. The CVaR is derived to measure the loss incurred in any financial investment that can involve discreetness. Conditional value-at-risk can analyse dangers beyond the value-at-risk, and moreover it is coherent. It provides optimization shortcuts which, through linear programming techniques, make practical many large-scale calculations that could otherwise be out of reach.(Rockafellar, Uryasev, 2002) It is also sometimes called the Expected Shortfall. In mathematical terms it can be defined as (the negative of) the expected return conditional on the return being lower than (the negative of) VaR.

$CVaR = - E(R_t \mid R_t < -VaR)$, where $E(R_t)$ is the expected return at time t and CVaR and VaR are the Conditional value at risk and the Value at Risk, respectively.

Methodology:

The Monte Carlo simulation would be the best value at risk calculation to use for this report as we need to calculate the risk of an option and the Monte Carlo simulation has the flexibility to do this. Options have a skewed distribution meaning the variance-covariance approach cannot be used as it assumes a normal distribution. It would also be difficult to use the historical simulation as this needs good historical data which is often difficult to obtain. In this report, 1,000 simulations are conducted daily, for three months of predicted data till the expiration date of 19th February 2021 taking a total of 63 working days to simulate a variety of possible outcomes. For the Monte Carlo simulation, daily returns were calculated. This was done by taking the log return of today's price divided by yesterday's price.

The modified returns equation stands as

$$lnP_t = lnP_{t-1} + \sigma_t \varepsilon_t \ldots\ldots(1)$$

where lnP_t is the log of price of PG stock at time t and lnP_{t-1} is log of price at time t-1. σ_t is the implied volatility of the option of the PG stock at time t. $\varepsilon \sim N(0,1)$.

The different values of ε can now be simulated by creating multiple numbers which are distributed normally. This is done 1000 times for each day in the 63 days period of the option. The final equation of the stock price stands as

$$P_t = e^{lnP_t} \ldots\ldots\ldots\ldots\ldots(2)$$

1,000 simulations of the stock price for every day in the 63-day expiration period are carried out in Excel. (giving a simulation of 1,000 possible outcomes of investment). From these results, the expiry option price was calculated by taking the difference between the stock price and the strike price (90% of the current price of Procter and Gamble stock. When the difference is less than 0, we have taken the option price to be 0, as a rational investor would never exercise the option if the price were negative, so it would just expire. Once the 63-day simple returns of the stocks and options are found out, we calculate the VaR and CVaR at specified levels of Confidence Intervals of 50%, 60%, 70%, 80% and 90%. For each level of Confidence Interval, we can carry out a sensitivity analysis to predict the values of VaR and CVaR at different volatility levels.

The general understanding about the volatility of the stock price is that we take the standard deviation of the returns of the stock to be its volatility value. The model in this report is considered in a short horizon frame, the volatility varies with time. So, in order to get a more accurate value of the volatility parameter we underwent an estimation method in order to get the Implied Volatility of the Option prices of the PG. The most basic way of understanding the estimation of the Implied Volatility of the option pricing is to consider the volatility as a function of the stock price, the exercise price, the time to maturity the risk-free rate and the option price. Mathematically we can write it as

$$\sigma = f(S, X, T, R_f, C) \ldots\ldots\ldots(3)$$

where σ is the volatility, S is the stock price, X is the strike price, T is the time to maturity. R_f is the risk-free rate and C is the Option price. The most popular way of doing this is by using the Black Scholes Model. The mathematical formula for the Black Scholes Model has been shown below. The only problem is that the option pricing formula cannot be solved analytically for the volatility; instead, it was computed straightforward using Excel's SOLVER tool.

$$C(S_t,t) = N(d_1)S_t - N(d_2)PV(K) \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

where

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}[ln\left(\frac{S_t}{K}\right) + (r + \frac{\sigma^2}{2})(T-t) \qquad d_2 = d_1 - \sigma\sqrt{T-t} \qquad PV(K) = Ke^{-r(T-t)}$$

here

C = Option Price, N =cumulative distribution function of the standard normal distribution, σ = volatility of the returns of the underlying asset, S_t= price of the asset, (T-t) = time to maturity(in years), K = Strike price of the option, r = risk-free interest rate.

Results

From the daily returns of the PG stock, we calculate the daily Log returns and calculate the standard deviation of the log returns. The value comes as 2.04%. But as it has been said we need to estimate the volatility and for that use the solver. We use the equation no. 4 and with the help of Solver in Excel we get the value of volatility as 31%. But this value is the yearly implied volatility value of the PG option. So, in order to get the daily value of the volatility we need divide the annual value by the square root of the total number of the trading days of the year, 252. This comes as 15.87∼ 16. So, the daily implied volatility comes as 1.94%. We then simulate the values of the ε for 1000 times for each of 63 days. When these values are computed, we use those values for computing the LnP values as described in equation (1).We put the values in equation (2) so as to get the 1000 simulated values of the stock for 63 days. In the chart below, I have shown 50 simulated stock values for the 63-day period using Excel.

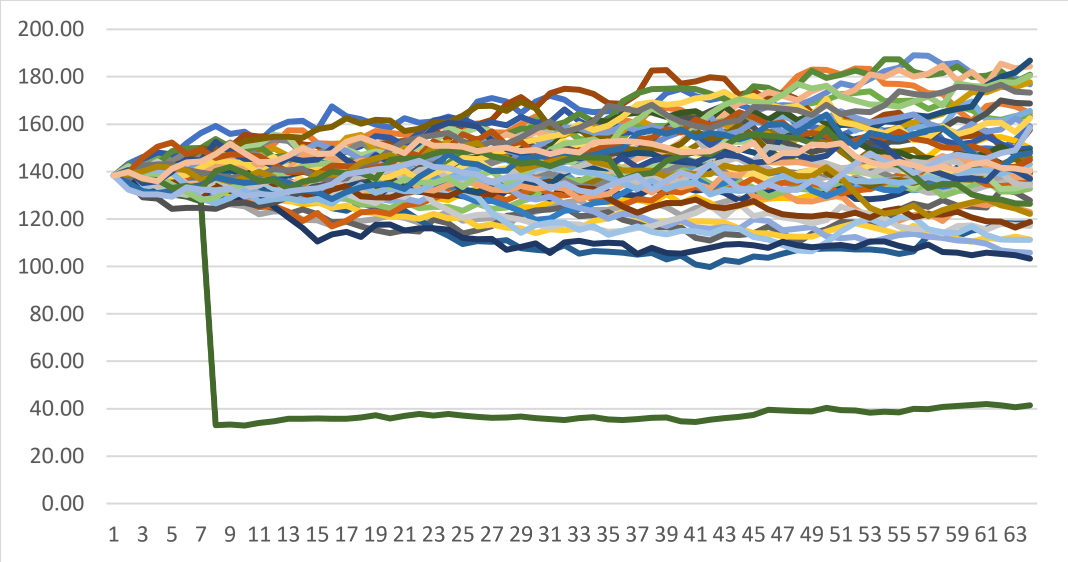

Chart1: Simulated Values of Stock Prices of PG

In the Chart 1 above, the stock prices are marked on the Y axis and days are marked on the X axis. All the 50 simulated stock prices start from just below the 150 dollars mark and end in the region between 100- and 200-dollar boundary. Only one simulated value falls below the 50-dollar mark. Using Excel, I then computed the Option prices with the help of the 1000 simulated values of the 63rd day. 63-day simple returns of the stocks and the option prices are then computed. From here we can chart two histograms for both the stocks and the options.

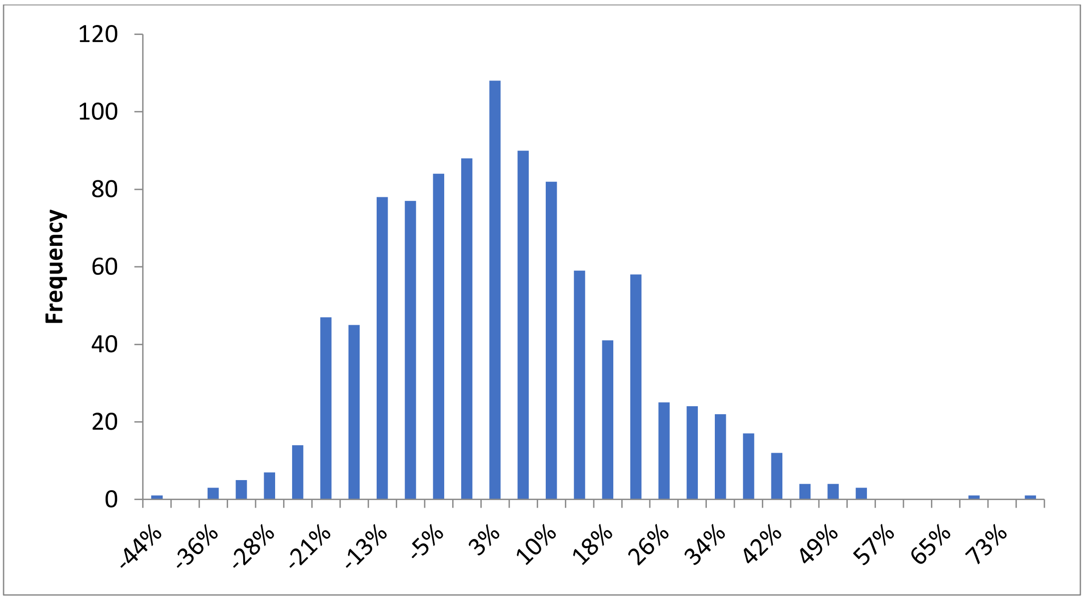

Chart 2: Histogram showing the daily simulated Stock returns for PG

Here the histogram is slightly skewed and not actually normally distributed as we have calculated the log returns instead of the simple returns. It is more of a log-normal distribution.

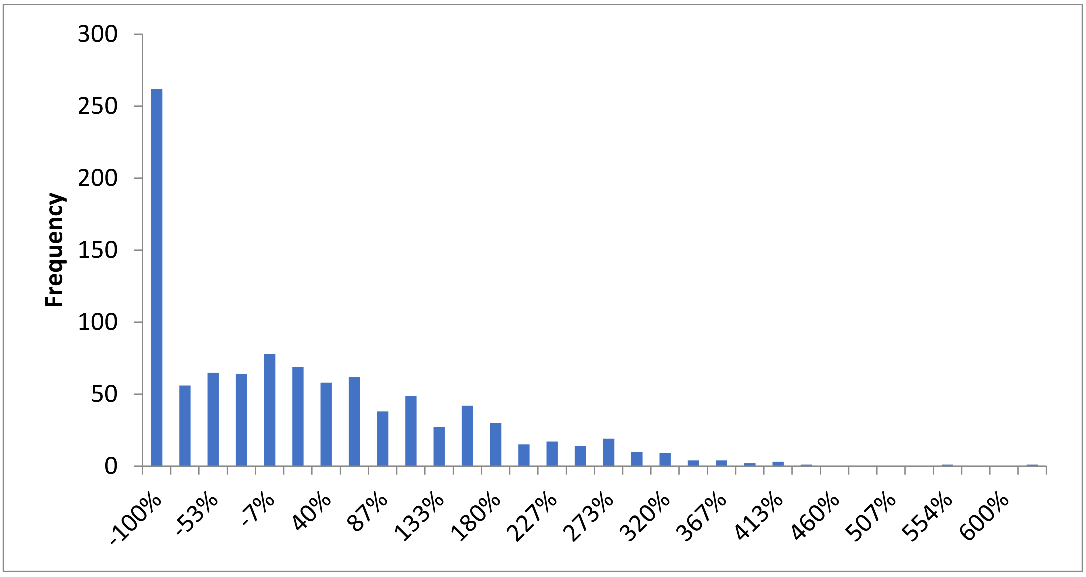

Chart 3: Histogram showing the daily simulated option returns for PG

Chart 3 shows the non-normal distribution of the option returns with a very high spike at the beginning and very high return percentages at the tail with very low frequency. After this we now estimate the VaR and CVaR for the Option prices at different significance levels of 50%, 60%, 70%, 80% and 90% respectively using Excel. In the table below I have summarized the values for VaR and CVaR.

Confidence Interval	**VaR**	**CVaR**
	Options	Options
50%	14.19%	142.11%
60%	45.86%	172.03%
70%	82.46%	206.43%
80%	100%	249.33%
90%	100%	312.59%

Table 1: Table representing Value at Risk and Conditional Value at Risk of Option prices at different CI.

Now I tried to estimate the VaR and CVaR values at each Confidence Intervals over a range of Volatility levels as the option prices depend on the volatility levels and the volatility levels vary over time. In the table below I have shown the sensitivity analysis of the values at 50% and 70% Confidence Intervals for understanding purpose.

Volatility Levels	VaR at 50% CI	Volatility Levels	CVaR at 50% CI
	14.19%		142.11%
1.90%	14.29%	1.90%	139.41%
1.91%	14.27%	1.91%	140.08%
1.92%	14.24%	1.92%	140.76%
1.93%	14.22%	1.93%	141.44%
1.94%	14.19%	1.94%	142.11%
1.95%	14.17%	1.95%	142.79%
1.96%	14.14%	1.96%	143.47%
1.97%	14.11%	1.97%	144.15%
1.98%	14.09%	1.98%	144.83%

Table 2: Sensitivity Analysis at 50% CI

Table 2 Shows that as we vary the volatility levels from 1.90% to 1.98% the VaR values will vary from 14.09% to 14.29%. That for CVaR, the values will range between 139.41% to 144.83% at 50% level of significance.

Volatility Levels	VaR at 70% CI	Volatility Levels	CVaR at 70% CI
	82.46%		206.43%
1.90%	81.21%	1.90%	202.50%
1.91%	81.52%	1.91%	203.48%
1.92%	81.83%	1.92%	204.46%
1.93%	82.15%	1.93%	205.45%
1.94%	82.46%	1.94%	206.43%
1.95%	82.78%	1.95%	207.42%
1.96%	83.09%	1.96%	208.40%
1.97%	83.40%	1.97%	209.39%
1.98%	83.72%	1.98%	210.38%

Table 3: Sensitivity Analysis at 70% CI

In this table the at 70% CI we see that the VaR ranges between 81.21% to 83.72% and CVaR ranges between 202.50% to 210.38%.

Conclusion

So, it can be concluded that with 50% certainty, over the next 63 days the largest loss that should be expected from the Option Pricing of PG is 14.29% and with 70% certainty this option investment will lose 82.46%. At 80% and 90% level of significance VaR predicts that such an investment will lose up to 100% of the investment. As for the CVaR, it measures the maximum possible loss or the weighted average of the extreme losses. From Table 1 it is clear that the values are extremely high, above 100%. But theoretically any loss above 100% is not possible. So as an analyst I would make a call till 70% CI as after at any level of CI there is a high percentage of losing all the money. One of the issues with using the Monte Carlo simulation for risk analysis is the computational costs of running multiple simulations, this would cause bigger problems when trying to predict larger horizons of maturity, however, for this report this limitation would not be problematic. Also, in the estimation of the volatility of the option pricing, Exponentially Weighted Moving Average (EWMA) gives a better estimation of the volatility by assigning weights to the stocks.

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