Scramjet Nozzle Analysis

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Guide
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Problem Statement

- Repeat calculation demonstrated, for the off-design conditions mentioned below.
- Effect of nozzle geometry
  base expansion factor epsilon = 0, 0.1, 0.2

Overview

- Wave rider configuration
- Ram Effect
- Supersonic Combustion

General Scramjet Operation

- Freestream enters intake from left.
- Intake compresses air with oblique shock waves (acts as a compressor)
- Fuel is injected in combustor to add heat (quasi-1D flow, heat addition, friction)
- Combustion products expand through nozzle to generate thrust

Nomenclature of Parameters

$A*$	Throat area
A_{exit}	Area of cross-section of nozzle exit
M_{exit}	Mach number at nozzle exit
$\dot{m}$	mass flow rate through the duct
P	Pressure
T	Temperature
h	Enthalpy per unit of mass at the duct inlet
R	Universal gas constant
ρ	Density
x	Nozzle axis

Standard Given Values

Station 0 - Freestream
Station 1 - After 1^{st} Shock
Station 2 - After 2^{nd} Shock
Station 3 - After 3^{rd} Shock
Station 4 - Nozzle Entry
Station 5 - Nozzle Exit

θ_1	10°
θ_2	10°
θ_3	20°
R	287.1 J kg^{-1} K^{-1}
Capture area	1m $\times$ 1m
Height of intake	1m
Span(z-dimension)	1m
Height of intake exit, H_3	0.0894 m

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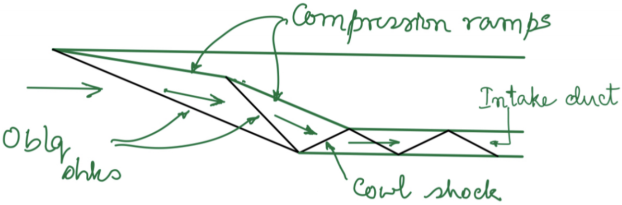
Intake of Scramjet Engine



Intake Relations

$$\tan\theta = 2\cot\beta\frac{M^2\sin^2\beta-1}{M^2(\gamma+\cos 2\beta)+2} \tag{1}$$

Component of free stream Mach numbers normal to shockwave

$$M_{1n} = M_0 sin\beta_1 \tag{2}$$

Jump in pressure and temperature

$$\frac{p_1}{p_0} = \frac{2M_{1n}^2-\gamma+1}{\gamma+1} \tag{3}$$

$$\frac{T_1}{T_0} = \frac{p_1}{p_0}\cdot\frac{(\gamma-1)^2M_{1n}^2+2}{(\gamma+1)^2M_{1n}^2} \tag{4}$$

Mach no. after 1st shockwave using oblique shock relations

$$M_1^2 sin^2\beta_1 = \frac{(\gamma-1)^2M_{1n}^2+2}{2\gamma M_{1n}^2-(\gamma-1)} \tag{5}$$

Intake Values

Ramp Angle and Wave Angle	
θ°	β°
10	17.3
10	19.9
20	33.0674912

Values calculated here

Intake Parameters						
Station	M	$T(K)$	$p(kPa)$	$\rho(kgm^{-3})$	$T_t(K)$	$P_t(MPa)$
0	6.2	230	0.805	0.0121908743	1998.24	1.56
1	4.77	360	3.06	0.0296064089	1998.2088	1.23655364
2	3.84	505.8	8.92	0.0614260851	1997.464896	1.094974229
3	2.49	893.4	44	0.1715432113	2001.233868	0.740017111

$\gamma = 1.4$ taken throughout

Intake : Conclusions

- All shockwaves converge at cowl lip
- p, T, ρ remain constant in a particular shockwave
- p, T, ρ increase across shockwaves
- Total temperature constant in intake (isentropic)
- Total pressure and Mach number decrease in intake

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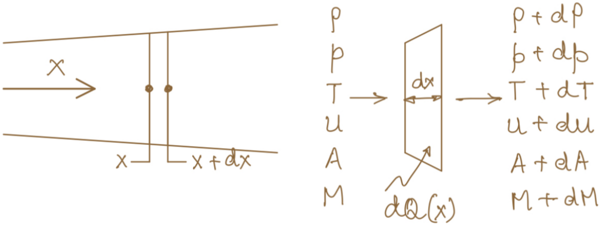
Combustor of Scramjet Engine



Assumptions

- Negligible heat losses from intake
- Combustor circular and of uniform cross section
  Area profile is constant as scramjet combustor is essentially a duct(of rectangular, circular or elliptic cross section).
- Constant mass flow rate through combustor

Combustor : Relations

$$T_{t0} = T_{t3} = T_3[1 + \frac{\gamma - 1}{2} M_3^2] \tag{6}$$

$$(\frac{1 - M^2}{B}) \frac{dM^2}{M^2} = (\gamma M^2 + 1) \frac{dT_1}{T_1} + \gamma M^2 c_f \frac{L_p}{A} dx - a \frac{dA}{A} \tag{7}$$

c_f is a skin friction coefficient at the walls, A is area of cross-section, L_p is perimeter of the duct at some x, $B = 1 + \frac{\gamma-1}{2} M^2$ is always greater than 1.

Integrating $\dot{m}_3 c_p dT_t = d\dot{Q}$ from combustor entry to x,

$$T_t(x) = T_{t3} + \frac{Q(x)}{\dot{m}_3 c_p} = T_{t3} + \frac{Q(x)}{c_p} \tag{8}$$

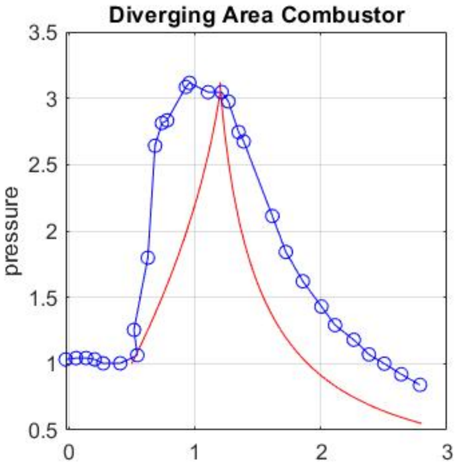
$$T(x) = T_t(x)[1 + \frac{\gamma - 1}{2} M^2]^{-1} \tag{9}$$

$$p_t(x) = p(x)[1 + \frac{\gamma - 1}{2} M^2(x)]^{\frac{\gamma}{\gamma-1}} \tag{10}$$

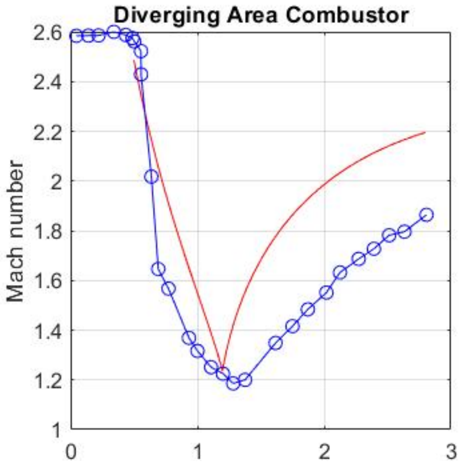
Combustor Values

Offset = 0.5 m (Portion with no heat transfer)		
Station	**3′**	**4**
Angle(°)	0	3
Length(m)	0.7	1.2
γ	1.3	1.3
Mach number	1.240174303	2.195747757
Temperature(K)	2346.885335	1882.060097
Pressure(Pa)	148588.2947	27146.78203
Density(kg m^{-3})	0.2205258653	0.05024023665
Total temperature(K)	2888.323153	3243.158867
Total Pressure(Pa)	365301.5741	286969.3713
Velocity(m s^{-1})	1160.691853	1840.295074
Area(m^2)	0.0894	0.2572448937

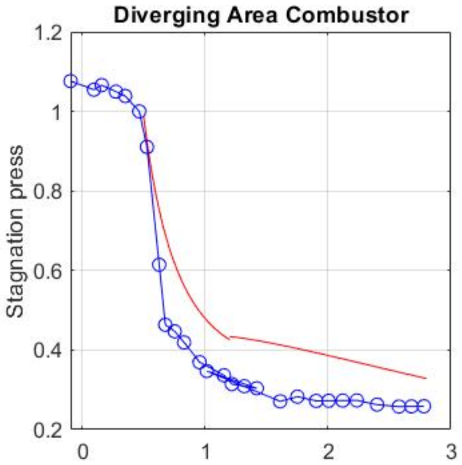
Combustor : Plots



Combustor : Plots



Combustor : Plots



Combustor : Conclusion

Constant Area Section

- Total temperature increases
- Total pressure decreases
- Mach number decreases

Diverging Area Section

- Total temperature increases
- Total pressure decreases
- Mach number increases

Choking condition : $M < 1$

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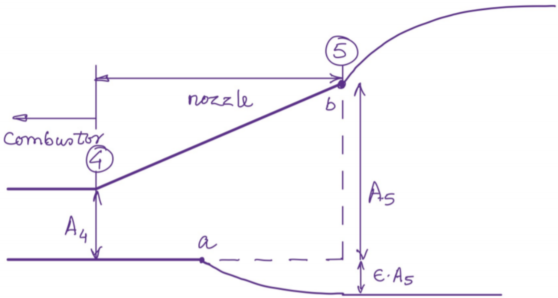
Nozzle of Scramjet Engine



Nozzle Assumptions

- Quasi 1-D flow
- No boundary layer formation
- Nozzle walls are adiabatic
- Zero friction with the walls
- Constant mass flow rate through nozzle
- Pressure along curved jet boundary on bottom side is close to ambient

Nozzle Relations

Assuming a hypothetical throat point, we can apply the area-mach relation for compressible flow at two different stations, 4 and 5, to arrive at the following formula:

$$\frac{A_5}{A_4} = \frac{M_4}{M_5}\left[\frac{1+\frac{\gamma-1}{2}M_5^2}{1+\frac{\gamma-1}{2}M_4^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{11}$$

For thrust calculation

$$F = \dot{m}_0(\forall_5 - \forall_0) + (1+\epsilon)(p_5 - p_0)A_5 \tag{12}$$

[Momentum balance for the entire scramjet vehicle]

where $\forall_5 = M_5\sqrt{\gamma R T_5}$

$p_5 = p_{t4}(1 + \frac{\gamma-1}{2}M_5^2)^{\frac{-\gamma}{\gamma-1}}$

$T_5 = T_{t4}(1 + \frac{\gamma-1}{2}M_5^2)^{-1}$

Nozzle Calculations - without Base Expansion Factor

- $A_{exit} = A_0 = 1m^2$, where A_0 is the capture area of the engine
- $A_4 = 0.257$
- $M_4 = 2.196$
- $Mach_{exit} = 3.645$
- $T_{t4} = 3243.159$ K
- $P_{t4} = 286969.371$ Pa
- $P_{exit} = P_{t4} * (1 + \frac{\gamma-1}{2} M_{exit}^2)^{\frac{-\gamma}{\gamma-1}}$
- $T_{exit} = \frac{T_{t4}}{(1+\frac{\gamma-1}{2} M_{exit}^2)}$
- $V_{exit} = M_{exit} * \sqrt{\gamma R T_{exit}}$
- $m_\circ = \frac{P_{exit} * A_{exit} * T_{exit}}{R * T_{exit}}$

Nozzle Calculations - with Base Expansion Factor

- Base expansion factor (ϵ): Factor to account for the increase in the overall jet area at station 5
- Length of midplane (nozzle cowl) is 1m.

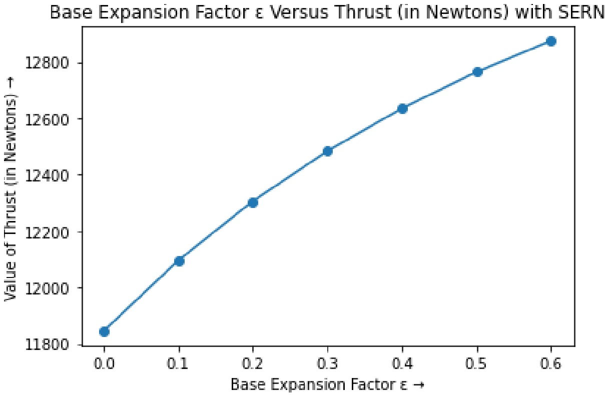
ϵ	**Mach Number**	**Thrust (in N)**	**Exit Pressure (in Pa)**
0	3.424242424	11844.9106	3532.14
0.1	3.5	12095.2653	2907.52
0.2	3.606060606	12305.5776	2639.91

Link to notebook with code

Nozzle Calculations - with Base Expansion Factor

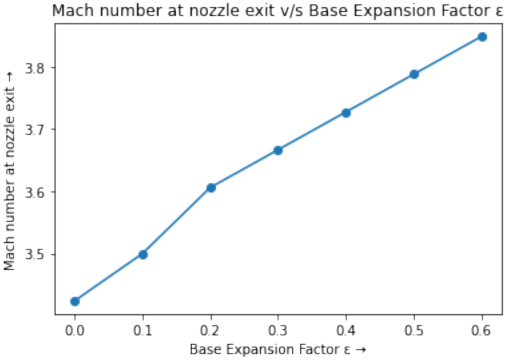
- $\epsilon = 0.1$
- $Mach_{exit} = 3.5$
- $T_{t4} = 3243.158867$ K
- $P_{t4} = 286969.3713$ Pa
- $P_{exit} = P_{t4} * (1 + \frac{\gamma-1}{2} M_{exit}^2)^{\frac{-\gamma}{\gamma-1}}$
- $T_{exit} = \frac{T_{t4}}{(1+\frac{\gamma-1}{2} M_{exit}^2)}$
- $V_{exit} = M_{exit} * \sqrt{\gamma R T_{exit}}$
- $m_{\circ} = \frac{P_{exit} * A_{exit} * T_{exit}}{R * T_{exit}}$
- Thrust = $\dot{m} * (V_{exit} - V_{fstr}) + (1+\epsilon) * (P_{exit} - P_{fstr}) * A_{exit} =$ 12095.26 N

Plots - with Base Expansion Factor



Figur: Thrust versus Base Expansion Factor

Plots : with Base Expansion Factor



Figur: Mach number versus Base Expansion Factor

Nozzle: Conclusions

- Total temperature and total pressure are constant
- Thrust increases with increase in Mach Number and base expansion factor
- Gas expansion takes place and as a result, static pressure and temperature approach ambient conditions
- Mach number has an upper limit (≈ 4.382) corresponding to the condition of static pressure becoming equal to ambient pressure

Thank you

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