AE240 Spaceflight Mechanics
Course Project

Space Shuttle STS-51G/PAM-D, Arabsat-1B Satellite

Work Report - 23rd April 2021

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Overview of the Mission :

Mission Overview :

Arabsat-1B was a Saudi Arabian communications satellite which Arabsat operated. It was used to provide communication services to the Arab States. It was constructed by Aérospatiale, based on the **Spacebus 100 satellite bus**, and carried two NATO E/F-band (IEEE S-band) and twenty-five NATO G/H-Band (IEEE C band) transponders. At launch, it had a mass of 1,270 kilograms (2,800 lb) and an expected operational lifespan of seven years. Arabsat-1B was launched aboard Space Shuttle *Discovery* on mission STS-51-G. *Discovery* was launched from LC-39A at the Kennedy Space Center at 11:33:00 GMT on 17 June 1985. It was deployed from *Discovery* and boosted to a geosynchronous transfer orbit through a PAM-D upper stage. Sultan bin Salman bin Abdulaziz Al Saud flew aboard the Shuttle to supervise deployment, becoming the first Saudi citizen and the first royalty member to fly in space. Morelos 1 and Telstar 3D were also deployed on the same mission. Arabsat 1B was placed into a geosynchronous orbit at a longitude of 26° East. In October 1991, a problem developed with the spacecraft's attitude control system, causing it to drift eastward out of control. The same fault had developed aboard its sister satellite, Arabsat-1A, a month earlier. It failed in early 1992.

NASA launched Arabsat-1B from the Space Shuttle in June 1985. It was the second in a series of satellites owned by the Arabsat Satellite Communications Organization (or Arabsat). (Arabsat-1A was launched from an Ariane in February 1984.) It was a communications satellite with a coverage area that included the Arab speaking countries of North Africa and the Middle East.

Arabsat was formed in 1976. Saudi Arabia had the largest investment share.
The Arabsat Organization purchased the satellites, launch services, and major ground facilities but developed some ground equipment within the member nations. The organisation awarded a contract to Aerospatiale in May 1981 for three satellites. The satellites included equipment used for other satellites, notably the Intelsat V series and Telecom 1. It was a three-axis-stabilized design with solar arrays and antennas. The solar arrays were partially deployed in the transfer orbit;

the antennas were deployed in synchronous orbit. The satellites contained twenty-five C-band transponders and one television (C/S-band) transponder."

Figure : STS-51-G Pictures

References :

[1] https://en.wikipedia.org/wiki/Arabsat-1B
[2]https://www.google.co.in/books/edition/NASA_Historical_Data_Book/61MgAAAAIAAJ?hl=en&gbpv=1&kptab=overview
[3] https://en.wikipedia.org/wiki/STS-51-G

Mission Objectives :

The mission's objective was to establish a satellite communication link by deploying a geosynchronous satellite that would provide these services to the Arab states [Arab League] for improved communication. The system's objective was to promote economic, social, and cultural development in the Arab world by providing reliable communications links among the Arab states and rural areas, developing Arab industrial capabilities in space-related technologies, and introducing new communications services to the area.

Mission Data :

Trajectory Parameters of Space Shuttle :

Below is some data which we recovered from a different space shuttle mission STS-121. The table is given below:

Table 1: STS-121 Ascent Data

Time (s)	Altitude (m)	Velocity (m/s)	Acceleration (m/s^2)
0	-8	0	2.45
20	1244	139	18.62
40	5377	298	16.37
60	11,617	433	19.40
80	19,872	685	24.50
100	31,412	1026	24.01
120	44,726	1279	8.72
140	57,396	1373	9.70
160	67,893	1490	10.19
180	77,485	1634	10.68
200	85,662	1800	11.17
220	92,481	1986	11.86
240	98,004	2191	12.45
260	102,301	2417	13.23
280	105,321	2651	13.92
300	107,449	2915	14.90
320	108,619	3203	15.97
340	108,942	3516	17.15
360	108,543	3860	18.62
380	107,690	4216	20.29
400	106,539	4630	22.34
420	105,142	5092	24.89
440	103,775	5612	28.03
460	102,807	6184	29.01
480	102,552	6760	29.30
500	103,297	7327	29.01
520	105,069	7581	0.10

Table [1] Space Shuttle STS-121 Ascent Data

Time (s)	Space Shuttle Total Mass (kg)
0	2,049,780
10	1,932,475
20	1,795,086
30	1,676,053
40	1,562,508
50	1,468,886
60	1,374,449
70	1,264,663
80	1,163,639
90	1,061,679
100	978,131
110	902,427
120	874,457

Table [2] Space Shuttle STS-121 Ascent Data

Launch Data :

Launch Vehicle Configuration :

Mass Analysis :

1. Orbiter Weight at Liftoff - 116311 kg
2. Vehicle Weight at Liftoff - 2094060 kg
3. Total Payload Weight - 29000 kg
4. Arabsat weight - 1270 kg
5. PAM Weight - 2205 kg
6. Weight of Hydrogen in the External Tank - 102737 kg
7. Weight of Oxygen in the External Tank - 619160 kg
8. Gross Mass of the external Tank - 760000 kg
9. Total Weight of Hydrogen and Oxygen - 721897 kg
10. Structure Mass of the External Tank - 38103 kg
11. Gross Mass of Both Solid Rocket Boosters - 1180000 kg
12. Propellant Mass of Both Solid Rocket Boosters - 1000000 kg
13. Empty Structure Mass of Both Solid Rocket Boosters - 180000 kg

Time-based Event Specification :

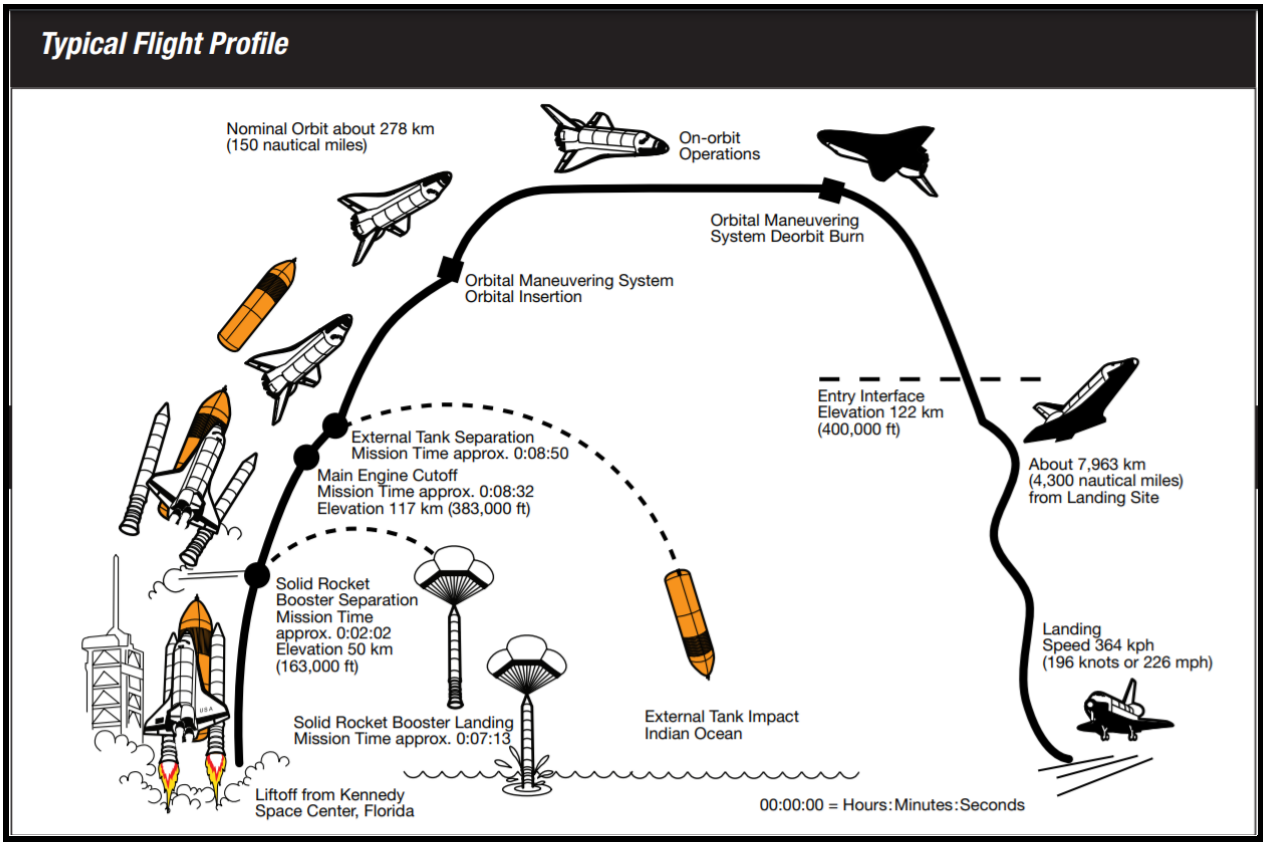


Figure [1] Space Shuttle Mission Profile

1. Time t = 00:00 -- Solid Rocket Boosters x2 and Main Engines x3 are ignited
2. Time t = 02:02 -- Solid Rocket Booster Separation, Altitude - 45~50 km
3. Time t = 08:32 -- Main Engines Cutoff, Altitude - 117 km
4. Time t = 08:50 -- External Tank Separation
5. After all this, the orbital Maneuvering system is activated and it is then inserted into the orbit, nominal orbit at about 278 km.

Thrust Data :

The entire space shuttle, fully loaded requires a combined thrust of about 35 million newtons = 35000000 N = 35 MN

Solid Rocket Boosters[1] :

1. Each Solid Rocket Boosters provide about -- 14.7 MN
2. The combined thrust of both Solid Rocket Boosters -- 29.4 MN -- they were only ignited once the three main engines reached the required 104.5% thrust level for launch. Once the SRBs were ignited, they provided about 72% of the thrust required of the entire shuttle at liftoff and through the first stage, which ended at SRB separation.
3. Burn rate -- 9.34 mm/s ~ 0.3677165 inch/s
4. Average chamber pressure -- 4.3 MPa.

External Tank :

1. Hydrogen Supply Rate -- 180,000 L/min = 3 m^3/sec = 47,000 gals/min
2. Oxygen Supply Rate ---- 67,000 L/min = 1.1167 m^3/sec = 18,000 gals/min
3. This is supplied to all three Space Shuttle Main Engines with a 6-to-1 mixture ratio of liquid hydrogen to liquid oxygen.
4. Thrust produced (by 3 main engines with fuel from ET) - 5580 kN = 5.5 MN
5. Burn time ~ 510 s = 8 minutes 30 seconds
6. Fuel - LH_2/LOx

Space Shuttle Main Engines :

1. Weight of each engine - 3200 kg
2. Total Weight of each engine - 9600 kg
3. Performance at -- 104.5% power level
4. Thrust -- 1.75 MN (394,000 pounds-force) at SL by 1 engine
5. Thrust -- 5.25 MN (1182000 pounds-force) at SL by 3 Engines
6. Thrust -- 2.20 MN (492,000 pounds-force) at vacuum by 1 engine
7. Thrust -- 6.60 MN(1476000 pounds-force) at vacuum by 3 engines
8. Throughout -- 8 minutes and 30 seconds of powered flight

[1] We now abbreviate Solid Rocket Boosters as SRBs

Orbital Parameters[2] of ArabSat 1B :

Satellite Type: Geostationary	**Inclination**: 14.3 °
NORAD ID: 15825	**Period**: 1,433.8 minutes
Int'l Code: 1985-048C	**Semi-major axis**: 42120 km
Perigee: 35,670.6 km	**RCS**: 3.4988 m^2 (large)
Apogee: 35,828.0 km	**Launch date**: June 17, 1985
Eccentricity: 0.00188	**Source**: Arab Satellite Communications Organization (AB)
RA Ascending Node : 23.263 hrs	**Launch site**: Air Force Eastern Test Range (After)
Argument Perihelion : 357.709^0	**Time Period** : 1433.8 minutes
Mean Anomaly : 286.845^0	**Longitude** : 128.8° W

Table [3] Orbital Data of ArabSat 1B

References :

[1] https://www.n2yo.com/satellite/?s=15825
[2] https://www.n2yo.com/satellites/?c=10
[3] https://in-the-sky.org/spacecraft.php?id=15825
[4] Space Shuttle Ascent - Nasa Education
[5] NSSDCA - Spacecraft - Detail

[2] Detailed information plots can be found here:
https://drive.google.com/file/d/1O0P1TGsSd3LOs6-IxxkTit99nH6-FK8p/view?usp=sharing

Launch Analysis :

Stage Analysis :

1. The First stage is from liftoff to the separation of the Solid Rocket Boosters
2. Second stage is from separation of solid rocket boosters to the main engine cut-off or the separation of the external tank
3. This now completes the ascent mission
4. Third stage is from the separation of the external tank to the orbital insertion at the required altitude and orientation and includes the orbital manoeuvring system and reaction control system.
5. Fourth stage is entirely related to the payload Arabsat and will start from deployment from Space Shuttle to the geosynchronous orbital insertion via the Payload Assist Module (PAM-D)

Nominal Burn Profile Design Launch Vehicle :

Stage 1 - From Liftoff till Separation of Solid Rocket Boosters :

Solid Rocket Boosters :

From the data gathered for Solid Rocket Boosters, each of the boosters carries a propellant mass of 5×10^5 kg of propellant and it is in active condition for about 2 minutes. The propellant mixture in each SRB motor consists of an *Ammonium Perchlorate* (oxidizer, 69.6% by weight), *Aluminum* (fuel, 16%), *Iron Oxide* (a catalyst, 0.4%), a *Polymer* (a binder that holds the mixture together, 12.04%), and an *Epoxy Curing Agent* (1.96%).

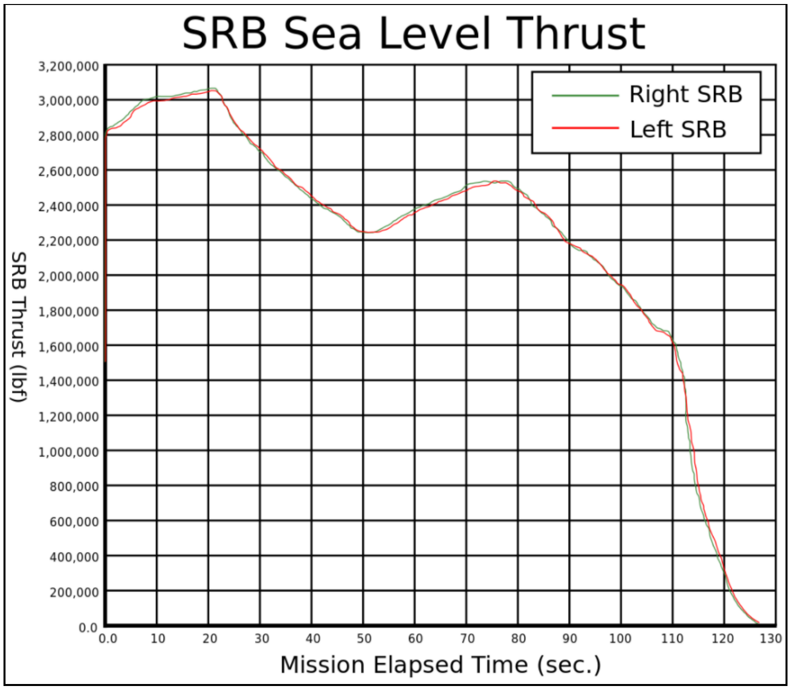


Figure [2] SRB Thrust Profile - Retrieved from Mission Database

The propellant is an 11-point star-shaped perforation in the forward motor segment and a double- truncated- cone perforation in each of the aft segments and aft closure. This configuration provides high thrust at ignition and then reduces the thrust by approximately *a third* 50 seconds after lift-off to prevent overstressing the vehicle during maximum dynamic pressure. The two SRBs provide 71.4% of the thrust at liftoff and during the first-stage ascent. The detailed thrust versus elapsed mission time behaviour of the solid rocket booster is given in the figure below which is tailored to reduce thrust during the maximum dynamic pressure region. The Specific Thrust Values (I_{sp}) of the SRB are tabulated below:

I_{sp} at Sea Level	242 s
I_{sp} at Vacuum	268 s
I_{sp} Average	255 s

Table [3] Specific Impulse Values of Solid Rocket Booster Fuel

Constant Burn Rate Assumption based Thrust and Burn Profile of SRBs:

Since the burn rate is constant [constant burn rate assumption] we get that,

$$Burn\ \ Rate\ \ \longrightarrow \beta = \frac{m_{propellant}}{t_{burn}}$$

$$m_{propellant} = 5 * 10^5 kg$$

$$t_{burn} = 127\ seconds$$

$$Burn\ \ Rate\ \ \longrightarrow \beta = \frac{5 \times 10^5}{127} = 3937.0078\frac{kg}{s}$$

From the above calculations, we present the thrust provided by the engines :

Approximated Thrust and Burn Profile :

This was completed using a tool called *Webplotdigitizer* and *Python* where **Figure [1]** was used and a simplified plot was obtained which is presented below. However, we present this as exploratory work and will go ahead with constant burn rate calculations.

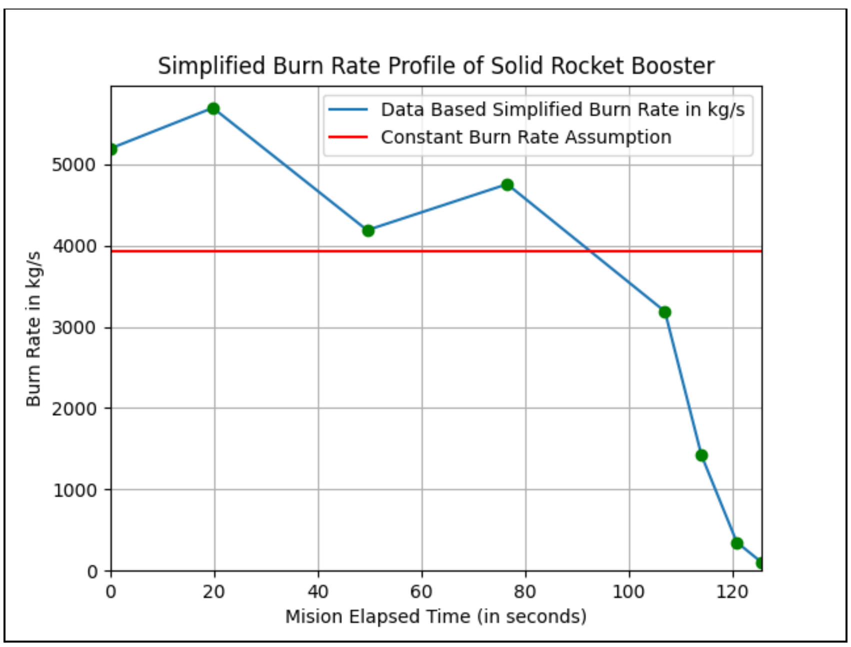


Figure [3] Simplified Thrust Profile for Solid Rocket Booster [3]

[3] The webplotdigitizer project : https://drive.google.com/file/d/1f5ly9H97Qb0YDvygl0je-ACoPej41byf/view?usp=sharing

Space Shuttle Main Engines powered by fuel in the External Tank :

From the Mass analysis section, we got the following mass details :

1. Weight of Hydrogen in the External Tank - 102737 kg
2. Weight of Oxygen in the External Tank - 619160 kg
3. Gross Mass of the external Tank - 760000 kg
4. Total Weight of Hydrogen and Oxygen - 721897 kg
5. Structure Mass of the External Tank - 38103 kg
6. Burn time of External Tank Propellant - 510 seconds
7. Constant Burn Rate Assumption -
    a. Supply Rate of Oxygen - 202.21 kg/sec
    b. Supply Rate of Hydrogen - 1213.27 kg/sec
    c. Gross Supply Rate/ Burn Rate of ET - 1415.48 kg/sec

Here, as the altitude is less than ~ 45 to 50 km we consider the space shuttle to be inside the atmosphere and use the I_{sp} for LO_2/LH_2 in atmosphere which is 363 s.

References :

[1] https://en.wikipedia.org/wiki/Space_Shuttle_Solid_Rocket_Booster
[2] https://science.ksc.nasa.gov/shuttle/technology/sts-newsref/srb.html
[3] http://www.braeunig.us/space/specs.htm
[4] Webplotdigitizer - https://automeris.io/WebPlotDigitizer/
[5] SRB Profiling-Soham Phanse_19D170030.tar

Stage 2 - From Separation of Solid Rocket Boosters till Main Engine Cutoff and Separation of External Tank

Here, too we continue the assumption of constant burn rate of the Space Shuttle Main Engines. In this stage fuel is provided by the External Tank and burnt in the SSME[4]. Hence we get the following burn profile :

1. Total Fuel Left after completion of first stage in the external tank : 542131 kg
2. Hydrogen Left after completion of first stage in the external tank : 464683 kg
3. Oxygen Left after completion of first stage in the external tank : 77448 kg

Here in this stage the altitude is between ~ 50 to 117 km. Hence we can largely assume that the I_{sp} of the SSME can be taken averagely as 408 seconds. While for

[4] Space Shuttle Main Engines

more accuracy we can take I_{sp} to be 363 in Phase I Ascent while I_{sp} to be 453 in Phase II Ascent [Gravity Turn]. Same applies to Solid Rocket Boosters respectively.

Stage 3 - From Separation of External Tank to the Orbital Insertion :

Here, mainly the Space Shuttle Orbiter's Orbital Maneuvering System and Reaction Control System give the required impulses, for orbital insertion and subsequent circularization. Below given is the information[5] on the engines, fuel and specific impulses of each stage . After successful orbital insertion and

Space Shuttle (1981)	0 1 OMS RCS	Thiokol SRB (x2) Rocketdyne SSME (x3) Aerojet OMS (x2) Kaiser Marquardt R-40 & R-1E	PBAN Solid LOX/LH_2 NTO/MMH NTO/MMH	242s sl / 268s vac 363s sl / 453s vac 313s vacuum 280s vacuum

[5] Reference : http://www.braeunig.us/space/specs.htm

Nominal Trajectory Design :

Stage 1 - Parallel Staging :

We use the concept of Parallel Staging as we have parallel firing of Solid Rocket Boosters and Orbiter Main Engines. Hence we have the following equations which we obtain by modelling the parallel stages as a single stage rocket.

Parallel Staging Calculations :

$$\dot{m}_0 = \sum_{i=1}^{n} m_{0-i}$$

$$I_{sp-0} = \frac{\sum_{i=1}^{n} m_{0-i} I_{sp-i}}{\sum_{i=1}^{n} m_{0-i}}$$

$$T_0 = \sum_{i=1}^{n} T_{0-i}$$

$$T_0 = -\dot{m}_0 g_0 I_{sp,0}$$

$$\epsilon_0 = \frac{\sum_{i=1}^{n} m_{s0-i}}{\sum_{i=1}^{n} (m_{s0-i} + m_{p0-i})}$$

$$\pi_0 = \frac{m_{01}}{m_0} = \frac{m_0 - \sum_{i=1}^{n} (m_{s0-i} + m_{p0-i})}{m_0}$$

Mass Profile :

$$m(t) = m_0 - \beta_{srb} * t - \beta_{ssme} * t \quad t < t_{b1} = 127$$

$$m(t) = m_0 - \beta_{srb} * t_{b1} - \beta_{ssme} * t - m_{struct,srb} \quad 127 = t_{b1} \leq t < t_{b2}$$

$$m(t) = m_0 - \beta_{srb} * t_{b1} - \beta_{ssme} * t_{b2} - m_{struct,srb} - m_{struct,et} \quad t \geq t_{b2}$$

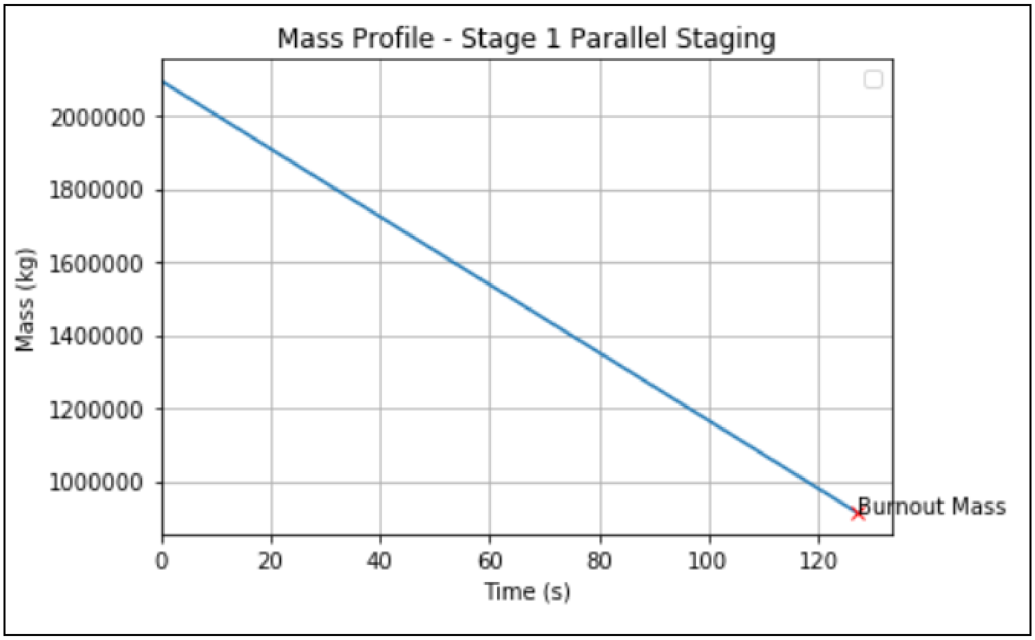


Figure [4] Mass Profile of Stage 1

Mass Derivative Profile :

$$\boxed{m'(t) = -\beta_{srb} - \beta_{ssme} = 7874.0157 + 1415.4843 = 9289.5\frac{kg}{s} \quad t < t_{b1} = 127}$$

Acceleration Profile :

$$\boxed{a(t) = \frac{dV}{dt} = \frac{-m'(t)}{m(t)} g_0 I_{sp} - g_0 \quad t < t_{b1} = 127}$$

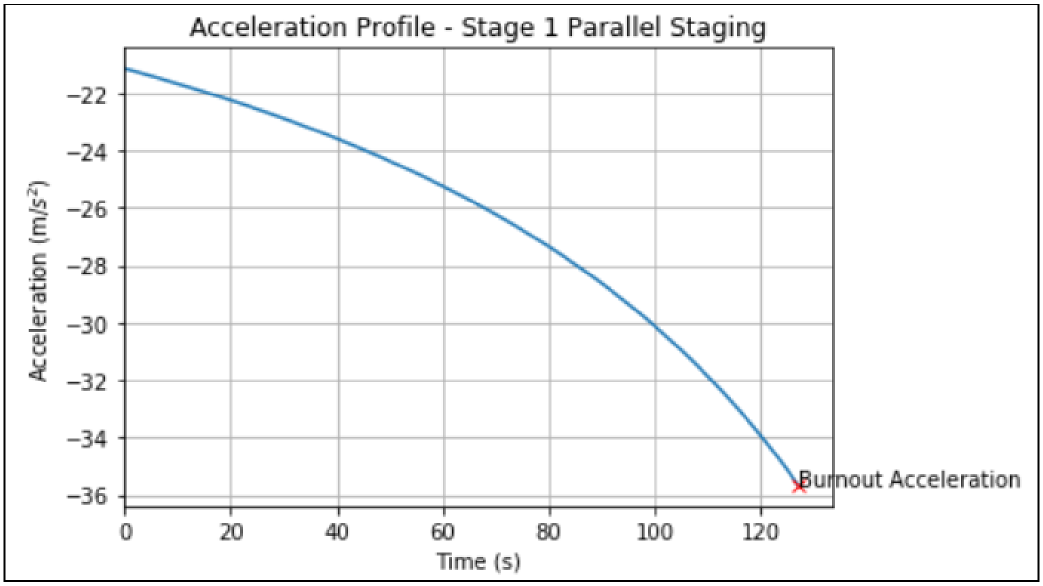


Figure [5] Acceleration Profile Stage 1

Velocity Profile :

$$\boxed{V(t) = g_0 I_{sp} log \frac{m_0}{m(t)} - g_0 t}$$

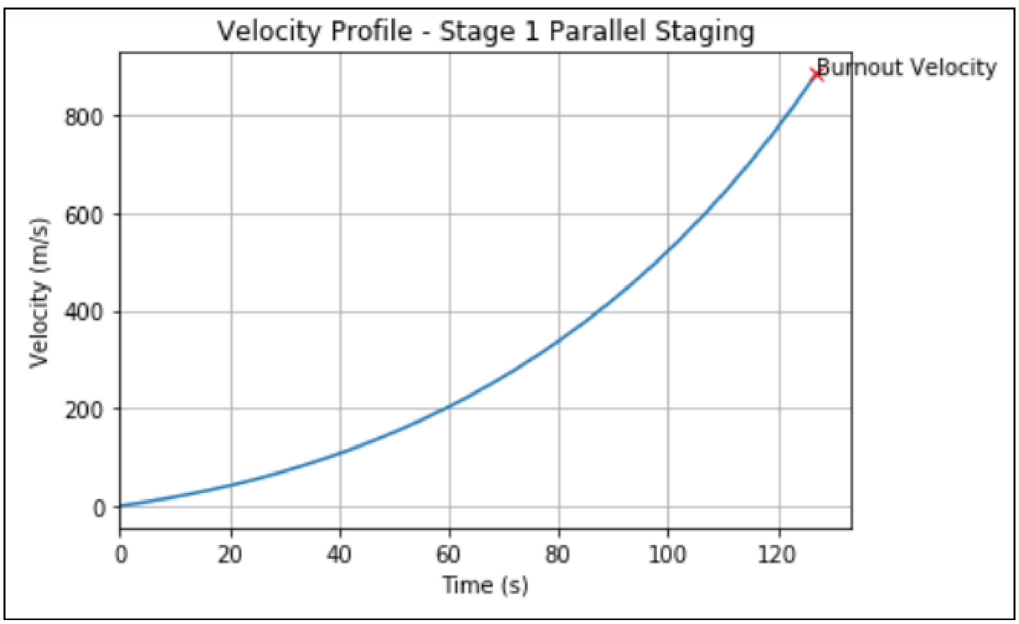


Figure [6] Velocity Profile Stage 1

Altitude Profile :

$$\boxed{h(t) = \frac{m_0 g_0 I_{sp}}{\beta_{eq}}[(1-\lambda)log(1-\lambda) - (1-\lambda)] - \frac{1}{2}g_0 t^2 + v_0 t + h_0 \quad \lambda = \frac{\beta t}{m_0}}$$

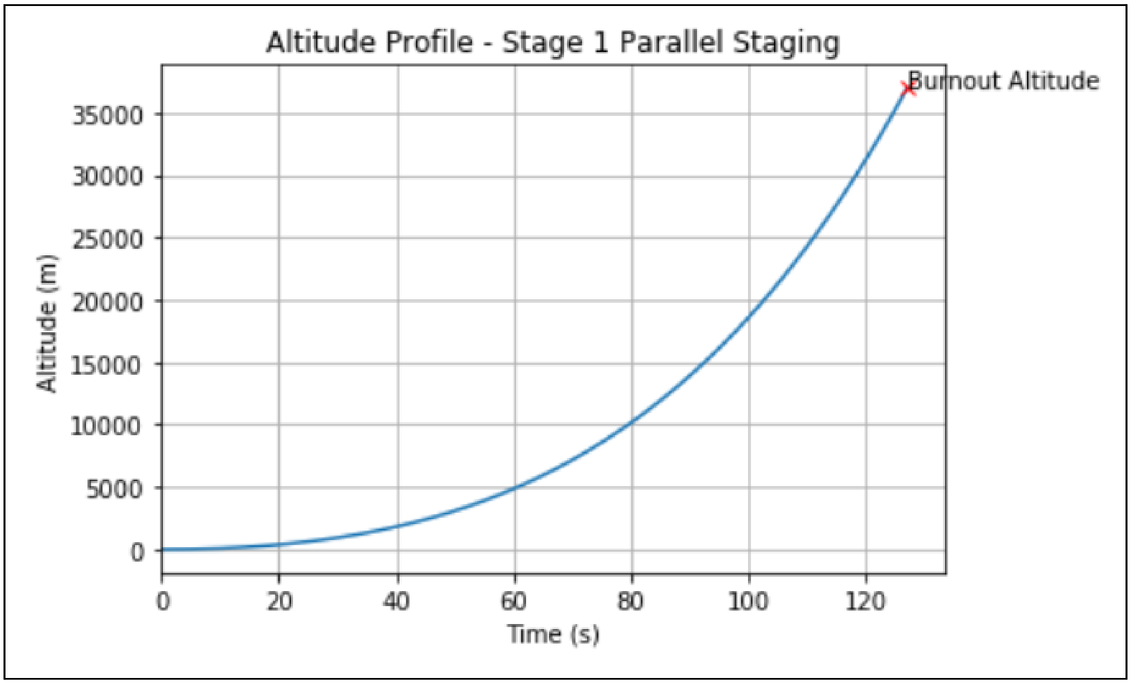


Figure [7]: Altitude Profile Stage 1

Stage 1 Burnout Features :

$$m_{burnout} = 2094060 - (127 * 9289.5) = 914293.5kg$$

$$V_{burnout} = 885.706\frac{m}{s}$$

$$|a_{burnout}| = 35.65\frac{m}{s^2}$$

$$h_{burnout} = 36972.767m = 36.972km \quad (v_0 \approx 0) \quad (h_0 \approx 0)$$

Stage 2 Analysis :

Here are the parameters at the beginning of stage 2

1. Initial Mass = Burnout Mass Stage 1 - Structure Mass of SRBs[6]
   = 914293.5 - 180000 = 734293.5 kg
2. Propellant Mass Remaining (ET[7]) = 721897 - 179766.5061 = 542130.4939 kg
3. Burn Time = 383 seconds

Constant Pitch Rate Gravity Turn Trajectory :

We model Stage 2 as a combination of constant pitch rate gravity turn trajectories. We have the following parameters obtained from the burnout parameters of parallel staging. We have $v_0 = 885.706$ m/s.

Analysis :

We have the following relation between the pitch angle of data recovered from STS-121. Please note that the pitch angle here, is not the same as the pitch angle defined in the course, however, is the angle made by the line perpendicular to the astronauts in the Shuttle and the horizontal. However as the relation between this angle and the pitch angle which we use in our case is ***linear*** we can equate the rate of change of both which is the principle which we use in Pitch Profile Estimation.

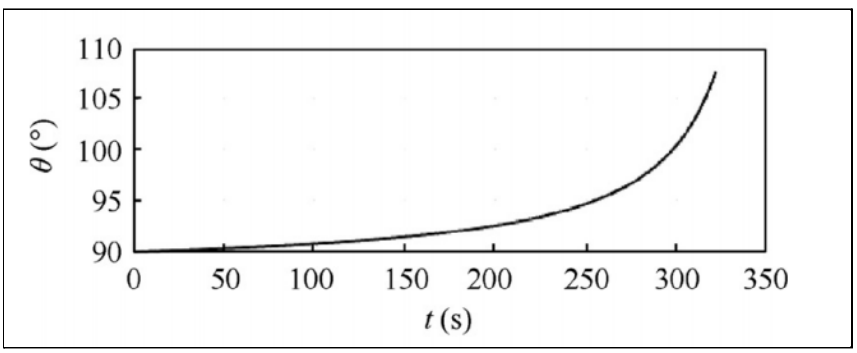


Figure [8] Pitch Angle Variation STS-121

Reference : [Mahmoodi, Ali & Kazemi, Isooda. (2020). Optimal motion cueing algorithm for accelerating phase of manned spacecraft in human centrifuge-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/). Chinese Journal of Aeronautics. 33. 1991-2001. 10.1016/j.cja.2020.02.004.]()

[6] Solid Rocket Boosters
[7] External Tank

We then used Webplotdigitizer[8] to extract data from this graph in numerical format and then approximate it using linear splines. Here's the approximated graph [Codes given in a separate document]

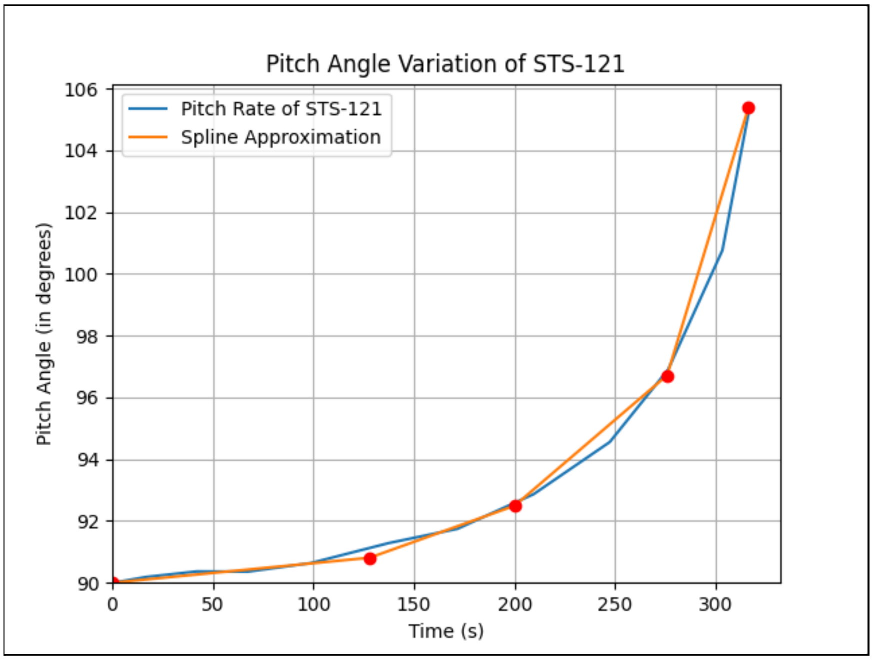


Figure [9] Spline Approximation to Pitch Angle[9] Variation of STS-121

Based on the approximate curve we have 4 gravity turn trajectory pitch rates :

$$q_{02} = \frac{92.493^0 - 90.81^0}{200 - 128} = \frac{1.683^0}{128} = 4.0797 \times 10^{-4} \approx 4.1 \times 10^{-4} \frac{rad}{s}$$

$$q_{03} = \frac{96.718^0 - 92.493^0}{275.85 - 200} = \frac{4.225^0}{75.85} = 9.72 \times 10^{-4} \frac{rad}{s}$$
$$q_{04} = \frac{105.39^0 - 96.718^0}{316.30 - 275.85} = \frac{8.672^0}{128} = 3.74 \times 10^{-3} \frac{rad}{s}$$

Now, as we have already modelled the stage 1 i.e from t = 0 seconds and t = 127 seconds as constant burn rate gravity burnout trajectory, we ignore q_{01} and

[8] Archived project webplotdigitizer https://drive.google.com/file/d/19CXnlOLW61jmOI2_uVxMU3aNwaPeRzOr/view?usp=sharing
[9] Note: The pitch angle mentioned here is not the angle we use in the calculations [as per the course], however, we equate its rate of change with the rate of change in pitch angle which we want for calculations.

consider $q_{01} \approx 0$. Now, we have 3 constant pitch rate turn trajectories which we analyze one by one.

Turn Trajectory 1 :

$$\boxed{V_0 = \frac{g_0 sin(\theta_0)}{q_0} = 855.706\frac{m}{s}}$$

$$\boxed{V_0 = \frac{\tilde{g} sin\theta_0}{q_0 1} \longrightarrow \theta_0 = sin^{-1}(\frac{V_0 \times q_{01}}{\tilde{g}}) = 0.0374 rad = 2.143^0}$$

So here we can conclude that the vehicle has been given a pitch kick of 2.143^0. Here, we can calculate the thrust required for giving the pitch kick with the Tsiolkovsky rocket equation[10].

$$\Delta v = \vec{v_2} - \vec{v_1} = |v_0|(sin\theta\hat{i} + cos\theta\hat{j}) - |v_0|\hat{j} = 33.125\frac{m}{s}$$

$$v_e = g_0 \times I_{sp} = 9.81 \times 453 = 4443.93\frac{m}{s}$$

$$m_1 = m_0 - m_{propellant} = m_0 e^{-\frac{\Delta v}{v_e}} = 728840.434 kg$$

$$m_{propellant} = 5453.0659 kg$$

Time Elapsed :

This gravity turn goes on for 73 seconds, i.e from t = 127 to t = 200 seconds.

Pitch Profile :

$$\boxed{\theta_b = \theta_0 + q_{01} \times t = 0.0374 + 4.1 \times 10^{-4} \times 73 = 0.0673 rad = 3.857^0}$$

[10] Reference : https://en.wikipedia.org/wiki/Delta-v#Orbital_maneuvers

Altitude Profile :

Here, we employ the fixed point iteration to iteratively refine the values of burnout altitude and the corresponding acceleration due to gravity. [Codes written in a separate document]

$$\boxed{h(\theta) = \frac{\tilde{g}}{4q_0^2}(cos(2\theta_0) - cos(2\theta)) + h_0}$$

$$h_b = h(\theta_b) = 124.768km$$

$$g_b = \tilde{g} = 9.437\frac{m}{s^2}$$

Mass Profile :

$$ln\frac{m_0}{m(\theta)} = \frac{2\tilde{g}}{q_0 g_0 I_{sp}}(sin\theta - sin\theta_0)$$

$$m_b = m(\theta) = m_0 e^{-0.30944} = 0.7338 \times 728840.434 = 534865.238kg$$

$$m_{propellant,used} = 728840.434 - 534865.238 = 193975.1954kg$$

$$m_{propellant,left} = 542130.4939 - 193975.1954 = 348155.2985kg$$

Velocity Profile :

$$V(t) = \frac{\tilde{g}sin\theta(t)}{q_{01}}$$

$$v_b = v(\theta_b) = \frac{9.437 \times sin(3.857)}{4.1 \times 10^{-4}} = 1548.278\frac{m}{s}$$

Turn Trajectory 2 :

Next, we have the Trajectory with Constant Pitch Rate as $q_{02} = 9.72 * 10^{-4}$ We have the following expressions.

$$V_0 = \frac{\tilde{g} sin\theta_0}{q_{02}} = 1548.278 \frac{m}{s}$$
$$\theta_0 = sin^{-1}(\frac{V_0 \times q_{02}}{\tilde{g}}) = 9.176^0 = 0.16rad$$

Hence, we have the second pitch kick of 5.319^0.

$$\Delta v = \vec{v_2} - \vec{v_1} = |v_0|(sin\theta_2\hat{i} + cos\theta_2\hat{j}) - |v_0|(sin\theta_1\hat{i} + cos\theta_1\hat{j}) = 126.446\frac{m}{s}$$
$$v_e = g_0 \times I_{sp} = 9.81 \times 453 = 4443.93\frac{m}{s}$$
$$\Delta v = v_e ln\frac{m_0}{m_1}$$
$$m_1 = m_0 - m_{propellant} = m_0 e^{-\frac{\Delta v}{v_e}} = 519860.850kg$$
$$m_{propellant,used} = 15004.387kg$$
$$m_{propellant,left} = 348155.2985 - 150004.387 = 333150.910kg$$

Time Elapsed :

This gravity turn goes on for 75.85 seconds, i.e from t = 200 to t = 275.85 seconds.

Pitch Profile :

$$\theta_b = \theta_0 + q_{02} \times t_b$$
$$\theta_b = 0.16 + (9.72 \times 10^{-4} \times 75.85) = 0.2338rad = 13.4^0$$

Altitude Profile :

$$h(\theta) = \frac{\tilde{g}}{4q_0^2}(cos(2\theta_0) - cos(2\theta)) + h_0$$

$$h(\theta_b) = 260.3km$$
$$g(\theta_b) = 9.055\frac{m}{s^2}$$

We obtain the above solutions by the fixed point iteration method which requires Lipschitz continuity of the functions. [Codes are given in a separate document]

Mass Profile :

$$m_b = m(\theta_b) = m_0 e^{-0.303} = 0.7385 \times 519860.850 = 383968.754kg$$

$$m_{propellant,used} = 519860.850 - 383968.754 = 135892.096kg$$
$$m_{propellant,left} = 197258.814kg$$

Velocity Profile :

$$V_b = \frac{\tilde{g} sin\theta_0}{q_{02}} = 2158.927\frac{m}{s}$$

Turn Trajectory 3 :

This is the final gravity turn trajectory for the ascent profile.

Time Elapsed :

This gravity turn goes on for 234.15 seconds, i.e from t = 275.85 to t = 510 seconds.

Pitch Profile :

$$\theta_b = \frac{\pi}{2} = 90^0 = \theta_0 + q_{03} \times t_b = \theta_0 + 3.74 \times 10^{-3} \times 234.15 = \theta_0 + 0.876$$

$$\theta_0 = 39.824^0$$

Altitude Profile :

$$\boxed{h(\theta) = \frac{\tilde{g}}{4q_0^2}(cos(2\theta_0) - cos(2\theta)) + h_0}$$

$$h_b = h(\theta_b) = 441.238km$$

$$\tilde{g} = g(\theta_b) = 8.58\frac{m}{s^2}$$

Mass Profile :

$$m_b = m(\theta_b) = m_0 e^{-0.3712} = 383968.754 \times 0.6899 = 264900.043kg$$

$$m_{propellant,used} = 383968.754 - 264900.043 = 119068.7106kg$$

$$m_{propellant,left} = 78190.103kg$$

Velocity Profile :

$$\boxed{V_b = \frac{\tilde{g}sin\theta_b}{q_{03}} = \frac{8.58 \times sin(90)}{3.74 \times 10^{-3}} = 2294.117\frac{m}{s}}$$

Orbital Analysis of Insertion Orbit of the Space Shuttle Orbiter :

Based on the burnout parameters, we assume the initial orbit to be an elliptic orbit. We now refer to the actual data[11] of position and velocity of the Space Shuttle Orbiter Orbit.

$$r = 6378 + 441.238 = 6819.238km = 6.819 \times 10^6 m$$

$$h = |\vec{r} \times \vec{v}| = rv = 6.819 \times 10^6 \times 2294.117 = 1.564 \times 10^{10} \frac{m^2}{s}$$

$$r = \frac{\frac{h^2}{\mu}}{1 + ecos\phi}$$

$$\epsilon = \frac{1}{2}V^2 - \frac{\mu}{r} = -5.582 \times 10^7$$

$$e = \sqrt{1 + \frac{2\epsilon h^2}{\mu^2}} = 0.91$$

$$ecos\phi = \frac{h^2}{\mu r} = 0.09 \longrightarrow \phi = cos^{-1}(\frac{0.09}{0.91}) = 84.324^0$$

$$a = \frac{h^2}{\mu(1 - e^2)} = 3569.935km$$

$$r_a = a(1 + e) = 3569.935 \times (1 + 0.91) = 6818.575km$$

$$r_p = a(1 - e) = 321.294km$$

[11] Reference : NASA Space Shuttle Education Series - Shuttle GNC

Space Shuttle Operational Orbit DataBase - retrieved from NASA Space Shuttle Education Series - Shuttle GNC Article								
Time (in minutes)	Position (X)	Position(Y)	Position(Z)	Position Magnitude	Velocity (X)	Velocity (Y)	Velocity (Z)	Velocity Magnitude
0	6521479	-734565	1458505	6722835.036	-732	5020	5798	7704.09164
5	5924596	784838	3078097	6722461.572	-3207	5010	4893	7702.363144
10	4635033	2212521	4336801	6722099.598	-5305	4415	3416	7700.928905
15	2804233	3381908	5087694	6722027.572	-6780	3305	1541	7698.448285
20	646291	4156853	5243442	6722410.5	-7464	1811	-513	7697.674064
25	-1587017	4447125	4786055	6723233.313	-7278	105	-2507	7698.399704
30	-3635191	4218696	3768833	6724292.656	-6242	-1613	-4208	7698.804907
35	-5258953	3497785	2310392	6725250.758	-4476	-3146	-5419	7700.483946
40	-6268174	2368178	581241	6725779.764	-2186	-4311	-5995	7700.866315
45	-6544580	961726	-1216040	6725711.677	361	-4973	-5868	7700.290514
50	-6055983	-557150	-2870787	6725084.7	2864	-5053	-5054	7699.235092
55	-4860024	-2010971	-4189288	6724133.52	5030	-4544	-3649	7698.313906
60	-3096963	-3230170	-5017592	6723199.204	6608	-3505	-1819	7698.015978
65	-972801	-4072773	-5259205	6722578.295	7414	-2058	224	7697.592871
70	1264767	-4440582	-4885923	6722398.949	7356	-370	2241	7698.682809
75	3354753	-4290364	-3941001	6722579.849	6440	1362	3997	7700.951435
80	5052847	-3639037	-2534456	6722895.233	4769	2938	5287	7702.439484
85	6159973	-2562141	-830686	6723085.088	2539	4171	5959	7704.118574
90	6546101	-1185374	970565	6722986.408	10	4917	5931	7704.138498
			Rmax	**6725779.764**			**Vmax**	**7704.138498**
			Rmin	**6722027.572**			**Vmin**	**7697.592871**
			Ravg	**6723423.352**			**Vavg**	**7700.307372**

Table [3] Space Shuttle Actual Orbit Data

***Orbit Circularization Maneuvers for Shuttle Orbiter*[12] :**

We can see that the apogee of our predicted orbit approximately matches with that of the apogee of the calculated orbit above. Hence we can employ a perigee raising manoeuvre for the Space Shuttle Orbiter. Here we relax two constraints in order to match the solution. One is the mass constraint and other is the External Tank Separation constraint. As our shuttle is not currently at the perigee, we will give the impulse whenever it arrives at the perigee. Also, here, as the time elapsed is around 510 seconds, the external tank separates out, however, we ignore that and assume it is attached to the orbiter.
Hence, here are the calculations for the Perigee Changing Maneuver. We assume that the resultant orbit is a circular orbit with Radius r = (6818.575 + 6723.9km)*0.5 which is r = 6771.237 km.

$$r_0 \longrightarrow perigee \quad r_1 \longrightarrow apogee$$

$$r_0 = 321.294km = 321294m$$

$$r_1 = \frac{6723.9 + 6818.575}{2} = 6771.237km = 6771237m$$

$$a_{ellipse} = \frac{r_0 + r_1}{2} = 6723.9km$$

$$v_{circle} = \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{3.986 \times 10^{14}}{6771.2375 \times 10^3}} = 7672.459\frac{m}{s}$$

$$v_{ellipse,apogee} = \sqrt{\frac{\mu r_0}{r_1 a}} = 1677.163\frac{m}{s}$$

$$\Delta v = v_{circle} - v_{ellipse,apogee} = 5995.2958 \approx 5995.3\frac{m}{s}$$

[12] Please Note that these manoeuvres are for the Orbiter, manoeuvres for SpaceCraft [ArabSat 1B] are documented in a later section.

Here, we complete the ascent phase and orbit circularization of the Space Shuttle Orbiter. Now, in the next section, we will start with the orbital manoeuvres of the SpaceCraft which is ArabSat-1B.

Orbit Analysis :

Orbital Parameters :

The SpaceCraft which is ArabSat-1B is deployed by the Space Shuttle Orbiter Discovery in a Circular Orbit at 345.9 km of altitude and deployment average velocity of 7700 m/s. It will be boosted to a Geosynchronous Orbit with a PAM-D Orbital Booster. We discuss the orbital manoeuvres in the subsequent sections.

Initial Orbit Parameters :

Here, we directly use the available data from the Space Shuttle Mission Portfolio and do not include the earlier results since it will cause accumulation of errors.

- Altitude : 355 km = 355000 m
- Inclination : 28.47^0
- Radius : 6723.9 km = 6723900 m
- Velocity : 7700.3 m/s
- Nature of Orbit : Low Earth Circular Orbit (LEO)

Final Orbit Parameters[13] :

- Altitude : 35749.3 km = 35749300 m
- Inclination : 14.3^0
- Radius : 42127.3 km = 42127300 m
- Velocity = 3076 m/s
- Nature of Orbit: Geosynchronous Circular Orbit (GSO)

[13] Refer Section : Orbital Parameters of ArabSat 1B

Orbital Manoeuvres Performed :

It was deployed from *Discovery*, and boosted to a geosynchronous transfer orbit by means of a PAM-D upper stage. Arabsat 1B was placed into a geosynchronous orbit at a longitude of 26° East. A GTO is highly elliptic. Its perigee (closest point to Earth) is typically as high as low Earth orbit (LEO), while its apogee (furthest point from Earth) is as high as geostationary (or equally, a geosynchronous) orbit. That makes it a Hohmann transfer orbit between LEO and GSO.

Hohmann Transfer Maneuver :

Here are the calculations required for a Hohmann Transfer between the Initial and Final Orbits. The SpaceCraft ArabSat 1B (1270 kg) along with the PAM-D (2205 kg)and fuel weighed about 3475 kg. It uses a Star-48B[14] rocket motor. We have the following specifications [15]for Star-48B rocket motor -

$$\textit{Specific Impulse} = 286 \textit{ seconds} = I_{sp}$$

Apogee Kick :

Generally, for geosynchronous satellites, the second velocity impulse which transfers the satellite from the transfer elliptical orbit to the final circular orbit is done via a '***Apogee Kick Motor***' which is a special motor which is only fired at the apogee. When the satellite reaches its orbit's apogee position, the AKM is ignited, transforming the elliptical orbit into a circular orbit, while at the same time bringing the inclination to around zero degrees, thereby accomplishing the insertion into a geostationary orbit. This process is called an apogee kick[16].
Here, however, we assume that the specific impulse is the same as the Star-48B solid rocket motor as we have limited amounts of data available at our disposal.

[14] Reference : https://en.wikipedia.org/wiki/Payload_Assist_Module
[15] Reference : https://www.northropgrumman.com/wp-content/uploads/NG-Propulsion-Products-Catalog.pdf
[16] Reference : https://en.wikipedia.org/wiki/Apogee_kick_motor

Below we present the Hohmann Transfer Calculations :

$$a_{eq} = \frac{a_1 + a_2}{2} = \frac{r_1 + r_2}{2} = \frac{6723.9 + 42127.3}{2} = 24425.6\,km$$

$$v_{TO,p} = \sqrt{\mu\{\frac{2}{r_{o1}} - \frac{1}{a}\}} = 10111.537\frac{m}{s}$$

$$v_{o1} = \sqrt{\frac{\mu}{r_{o1}}} = 7700.3\frac{m}{s}$$

$$\Delta v_1 = v_{TO,p} - v_{o1} = 2412.116\frac{m}{s}$$

$$m_1 = m_0 e^{-\frac{\Delta v}{v_e}} = m_0 e^{-\frac{\Delta v}{g_0 I_{sp}}} = 3475 \times e^{-\frac{2412.116}{9.81 \times 286}} = 1470.882\,kg$$

$$m_{propellant,used} = 3475 - 1470.882 = 2004.118\,kg$$

$$v_{TO,a} = \sqrt{\mu\{\frac{2}{r_{02}} - \frac{1}{a}\}} = 1613.89\frac{m}{s}$$

$$v_{o2} = \sqrt{\frac{\mu}{r_{o2}}} = 3076\frac{m}{s}$$

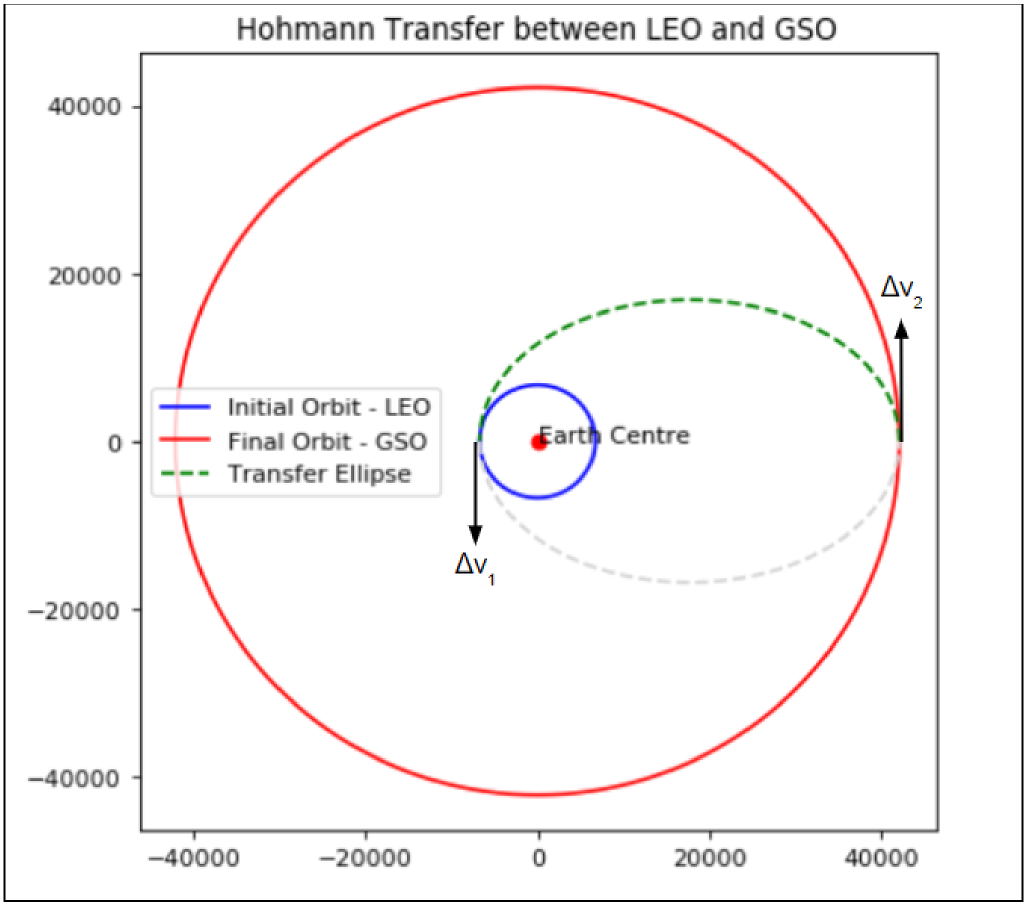
$$\Delta v_2 = v_{TO,a} - v_{o2} = 1462.113\frac{m}{s}$$

$$m_1 = m_0 e^{-\frac{\Delta v}{v_e}} = m_0 e^{-\frac{\Delta v}{g_0 I_{sp}}} = 1470.882 \times e^{-\frac{1462.113}{9.81 \times 286}} = 873.475\,kg$$

$$m_{propellant,used} = 597.4\,kg$$

$$\Delta v = \Delta v_1 + \Delta v_2 = 3874.229\frac{m}{s}$$

$$\Delta t = \pi \times \sqrt{\frac{a^3}{\mu}} = 18995.428\,seconds = 316.59\,minutes = 5.27\,hours$$



Figure[10] Hohmann Transfer Maneuver between LEO and GSO

Plane Change Maneuver :

We also have to change the inclination from 28.47^0 to 14.3^0 for which we use a Plane Change Maneuver. We provide the solution via both the methods:

Direct Method Velocity Impulse :

$$\Delta v = 2vsin\frac{\Delta i}{2} = 2 \times 3076 \times sin\frac{(14.3 - 28.47}{2} = -758.798\frac{m}{s}$$

$$m_1 = m_0 e^{-\frac{\Delta v}{v_e}} = m_0 e^{-\frac{\Delta v}{g_0 I_{sp}}} = 873.475 \times e^{-\frac{758.798}{9.81 \times 286}} = 666.47\ kg$$

$$m_{propellant,used} = 207\ kg$$

Plane Change Maneuver via Parabola :

$$\Delta v = 2(\sqrt{2}-1)v_{circle} = 2548.244\frac{m}{s}$$

$$m_1 = m_0 e^{-\frac{\Delta v}{v_e}} = m_0 e^{-\frac{\Delta 2528.24}{g_0 I_{sp}}} = 873.475 \times e^{-\frac{2528.24}{9.81\times 286}} = 352.210\, kg$$

$$m_{propellant,used} = 521.264\, kg$$

Nature of Orbit relation with Overall Mission Objective :

Since, ArabSat 1B was meant to be a communication satellite which was to be utilized by the Arab League, it was imperative for the SpaceCraft to be placed in the geosynchronous/geostationary orbit for efficient satellite communication. To an observer on Earth, a satellite in a geostationary orbit appears motionless, in a fixed position in the sky. This is because it revolves around the Earth with Earth's angular velocity (one revolution per sidereal day, in an equatorial orbit). A geostationary orbit is useful for communications because ground antennas can be aimed at the satellite without tracking the satellite's motion. This is relatively inexpensive. In applications that require many ground antennas, such as DirecTV distribution, the savings in ground equipment can more than outweigh the cost and complexity of placing a satellite into orbit.

Reference : https://en.wikipedia.org/wiki/Communications_satellite

Alternate Approach :

We have the altitude, velocity, acceleration profiles. While using this approach we must remember this analysis will be valid only until the shuttle motion is in the vertical plane. Hence the plots after a certain point will not depict the actual trajectory. Nevertheless we try to use this approach for trajectory estimation. We present the plots over here:

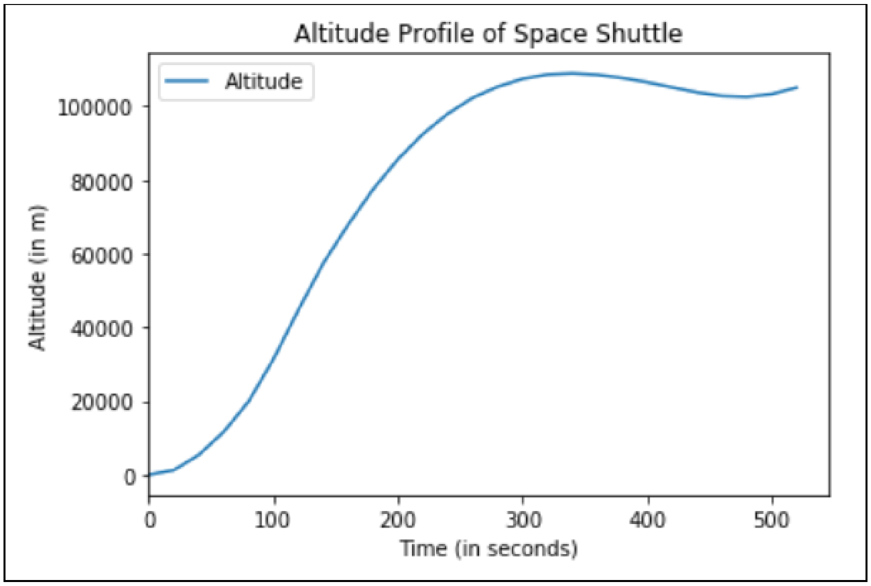


Figure [11] Altitude Profile of Space Shuttle

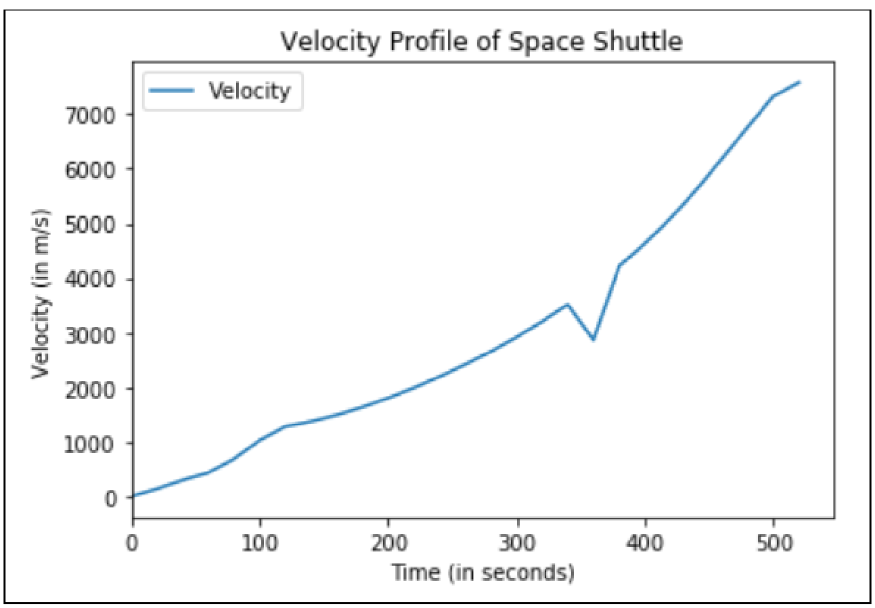


Figure [12] Velocity Profile of Space Shuttle

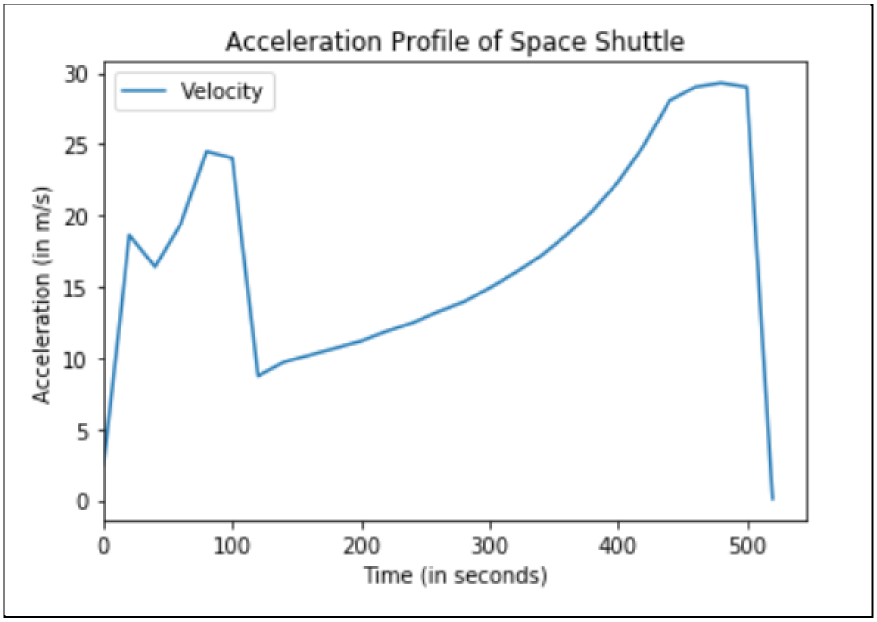


Figure [13] Acceleration Profile of Space Shuttle

Now we have the following relations :

$$\boxed{\frac{dh}{dt} = V(t)cos(\theta(t))}$$

Now, with methods derived from numerical methods to solve ordinary differential equations, we set up an Euler's scheme as follows :

$$\boxed{\begin{gathered} h_{k+1} = h_k + (t_{k+1} - t_k) \times (V_k cos\theta_k) \\ \theta_k = cos^{-1}\left(\frac{h_{k+1} - h_k}{(t_{k+1} - t_k) \times V_k}\right) \\ x_{k+1} = x_k + (t_{k+1} - t_k) \times (V_k sin\theta_k) \\ \frac{dx}{dt} = V(t)sin\theta(t) \end{gathered}}$$

Tabulated Results :

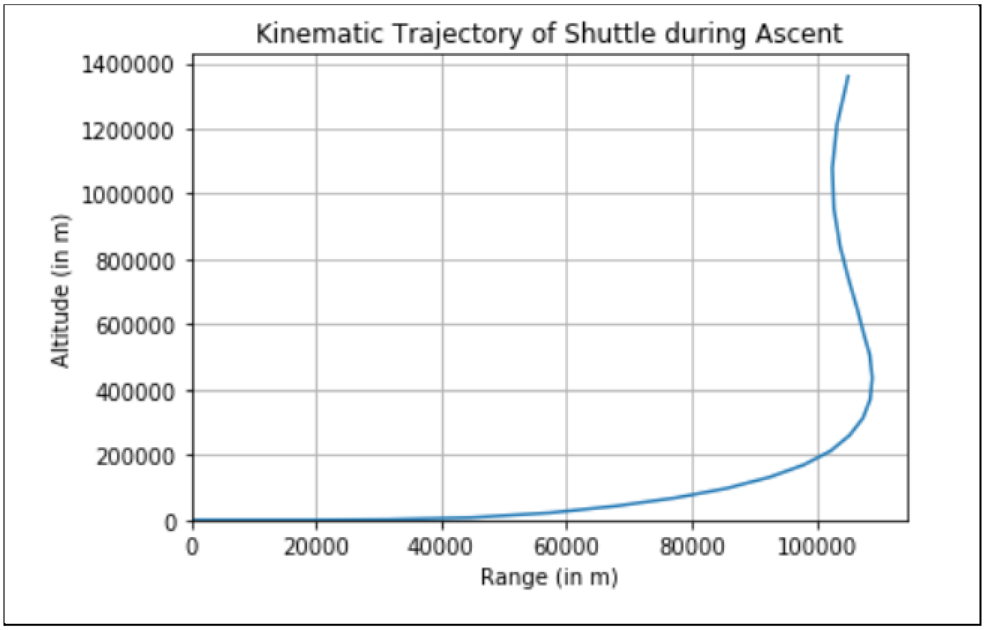
We used a spreadsheet based framework to implement the above relations:

Range Estimation Via Euler's Integration Method - Methodology explained in report								
Time (in seconds)	Altitude (in metres)	Velocity Magnitude (in m/s)	COS(theta)	Theta (in radians)	Theta (in degrees)	SIN (theta)	Horizontal Velocity (in m/s)	Range
0	-8	2	1	0	0	0	0	0
20	1244	139	31.3	0	0	0	0	0
40	5377	298	1.486690647	0	0	0	0	0
60	11617	433	1.046979866	0	0	0	0	0
80	19872	685	0.9532332564	0.3070370679	17.59192814	0.3022356018	207.0313872	0
100	31412	1026	0.8423357664	0.5691937749	32.61240102	0.5389531117	552.9658926	1251.445118
120	44726	1279	0.6488304094	0.8647499473	49.54652231	0.7609330456	973.2333653	7211.898888
140	57396	1373	0.495308835	1.052606024	60.30988267	0.8687169608	1192.748387	22023.20746
160	67893	1490	0.3822651129	1.178549988	67.52594028	0.9240526952	1376.838516	42746.42254
180	77485	1634	0.3218791946	1.243082682	71.22339126	0.9467807476	1547.039742	68191.84937
200	85662	1800	0.2502141983	1.317894842	75.50981231	0.9681905055	1742.74291	97485.99823
220	92481	1986	0.1894166667	1.380228303	79.0812565	0.9818968003	1950.047045	131232.141
240	98004	2191	0.1390483384	1.431295974	82.00721856	0.990285595	2169.715739	169527.0401
260	102301	2417	0.09806024646	1.472578242	84.37251824	0.9951804801	2405.351221	212499.8049
280	105321	2651	0.0624741415	1.508281474	86.41816279	0.9980465829	2645.821491	260374.9766
300	107449	2915	0.04013579781	1.530649745	87.69977033	0.9991942342	2912.651193	313188.0385
320	108619	3203	0.02006861063	1.550726369	88.85007611	0.9997986052	3202.354932	371394.1241

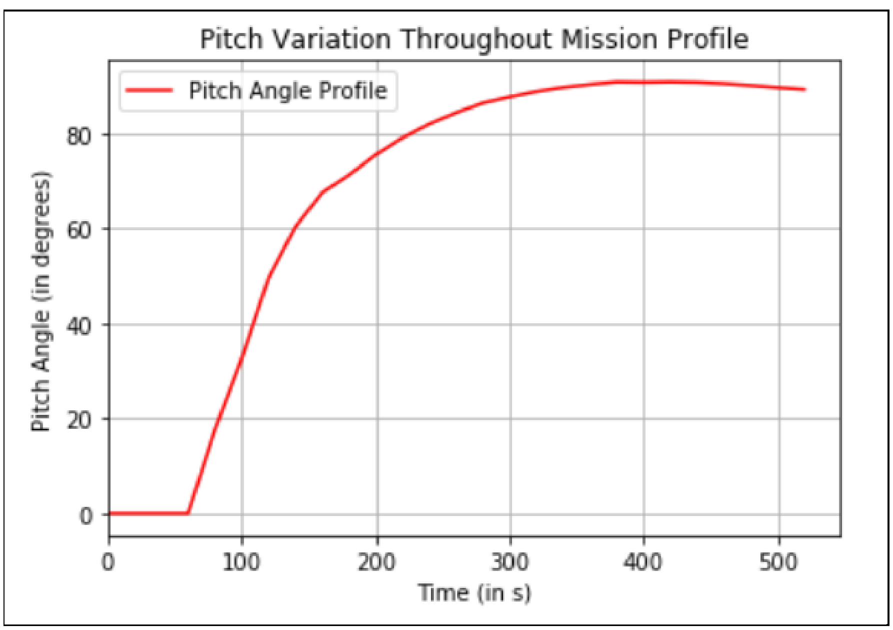
340	108942	3516	0.005042147986	1.565754157	89.71110498	0.9999872883	3515.955306	435428.324
360	108543	2860	-0.005674061433	1.576470419	90.32510152	0.9999839024	2859.953961	505746.5362
380	107690	4216	-0.01491258741	1.585709467	90.85445999	0.9998888012	4215.531186	562944.6947
400	106539	4630	-0.01365037951	1.58444713	90.78213343	0.9999068292	4629.568619	647245.9431
420	105142	5092	-0.01508639309	1.585883292	90.86441944	0.9998861939	5091.420499	739828.6887
440	103775	5612	-0.0134230165	1.584219746	90.76910529	0.9999099073	5611.4944	841645.51
460	102807	6184	-0.008624376336	1.57942081	90.49414649	0.9999628094	6183.770013	953865.2869
480	102552	6760	-0.002061772316	1.572858101	90.11813094	0.9999978745	6759.985632	1077536.088
500	103297	7327	0.00551035503	1.565285944	89.68427832	0.9999848179	7326.888761	1212735.513
520	105069	7581	0.0120922615	1.558703771	89.30714757	0.9999268859	7580.445722	1359271.063

Table [4] Kinematic Trajectory Estimation using Euler's Discrete Ordinary Differential Equation Quadrature

Now, we present a plot of the kinematic trajectory the Shuttle Follows while **Ascent.** Please note that to obtain reasonable results, we have to fix certain initial conditions like initial pitch angle, velocity, altitude to be zero and so on.



Figure[14] Kinematic Shuttle Ascent Trajectory based on Data Retrieved from Nasa Space Shuttle Education Series Articles



Figure[15] Pitch Angle Variation of Shuttle based on Euler's Numerical Scheme and Data Retrieved from Nasa Space Shuttle Education Series Articles

Here, the important thing to note is that this will only be valid until the shuttle traverses in the vertical plane. Nevertheless, assuming the shuttle travels only in the vertical plane during the Ascent Phase, we present this graph.