ODE Solver

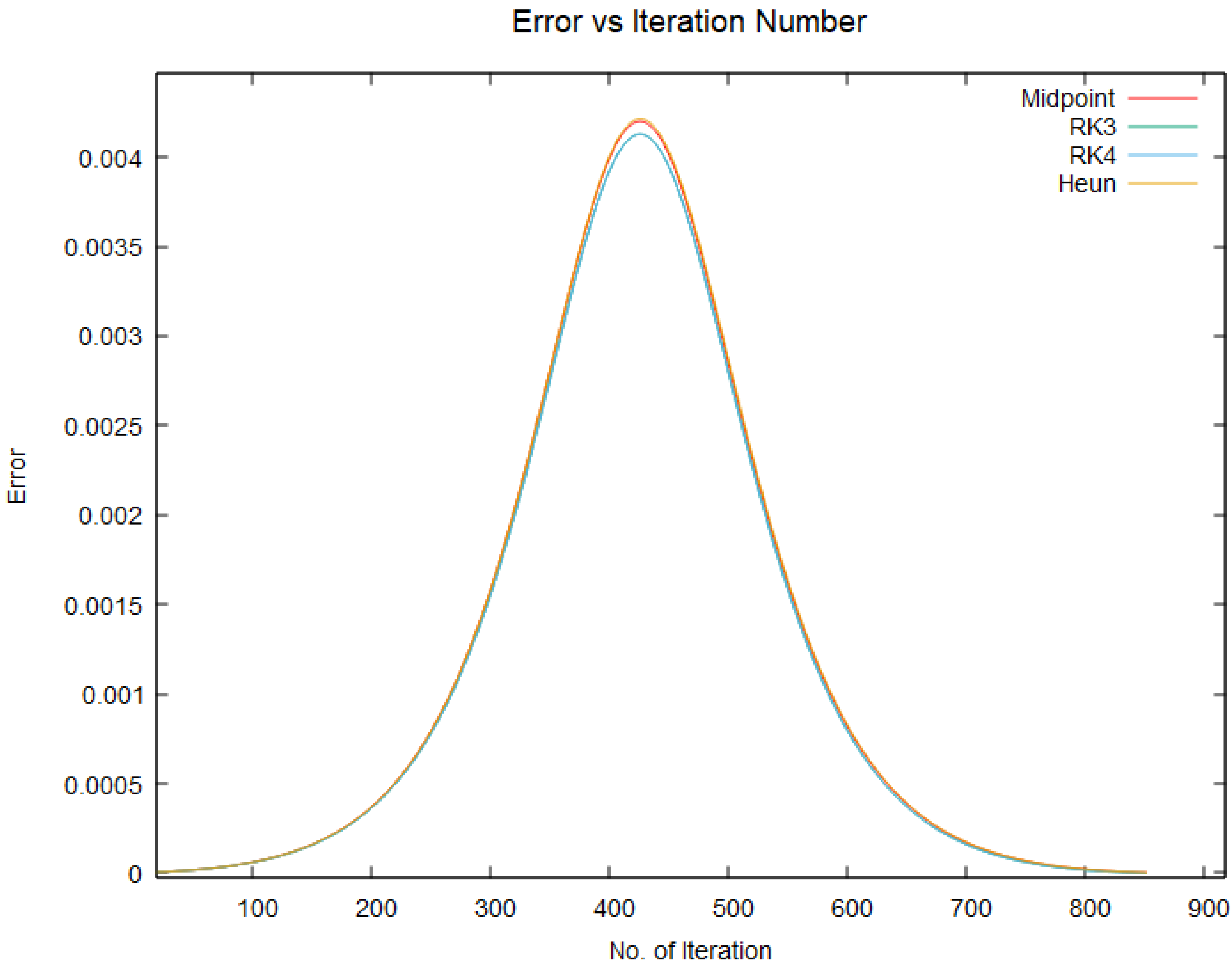
Soham Roy (EE20B130)

In this program we solve the following ODE:

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{\gamma * \alpha}{1 + \alpha^2} * H_z * \sin\theta\,, \qquad -\dot{\varphi} = \frac{\dot{\theta}}{\alpha\sin\theta} = \frac{\frac{d\theta}{dt}}{\alpha\sin\theta} = \frac{\gamma}{1+\alpha^2} * H_z$$

We can solve this as a simple ODE as $\dot{\varphi}$ is constant, making this pair of equations an ODE in θ.

In the program, the solution has been found by RK45, euler's, heun's, midpoint, RK3 and RK4 methods. Euler's hasn't been considered for analysis here as it is significantly less accurate.



$\alpha = 0.08 \quad$ step-size $= 10^{-17}$

This is the graph of absolute error at each point vs the number of iteration the point is at, for $\alpha = 0.08$ and step-size $= 10^{-17}$.

We can see that the error is maximum around the middle of the graph, where the vector **m** varies the most.

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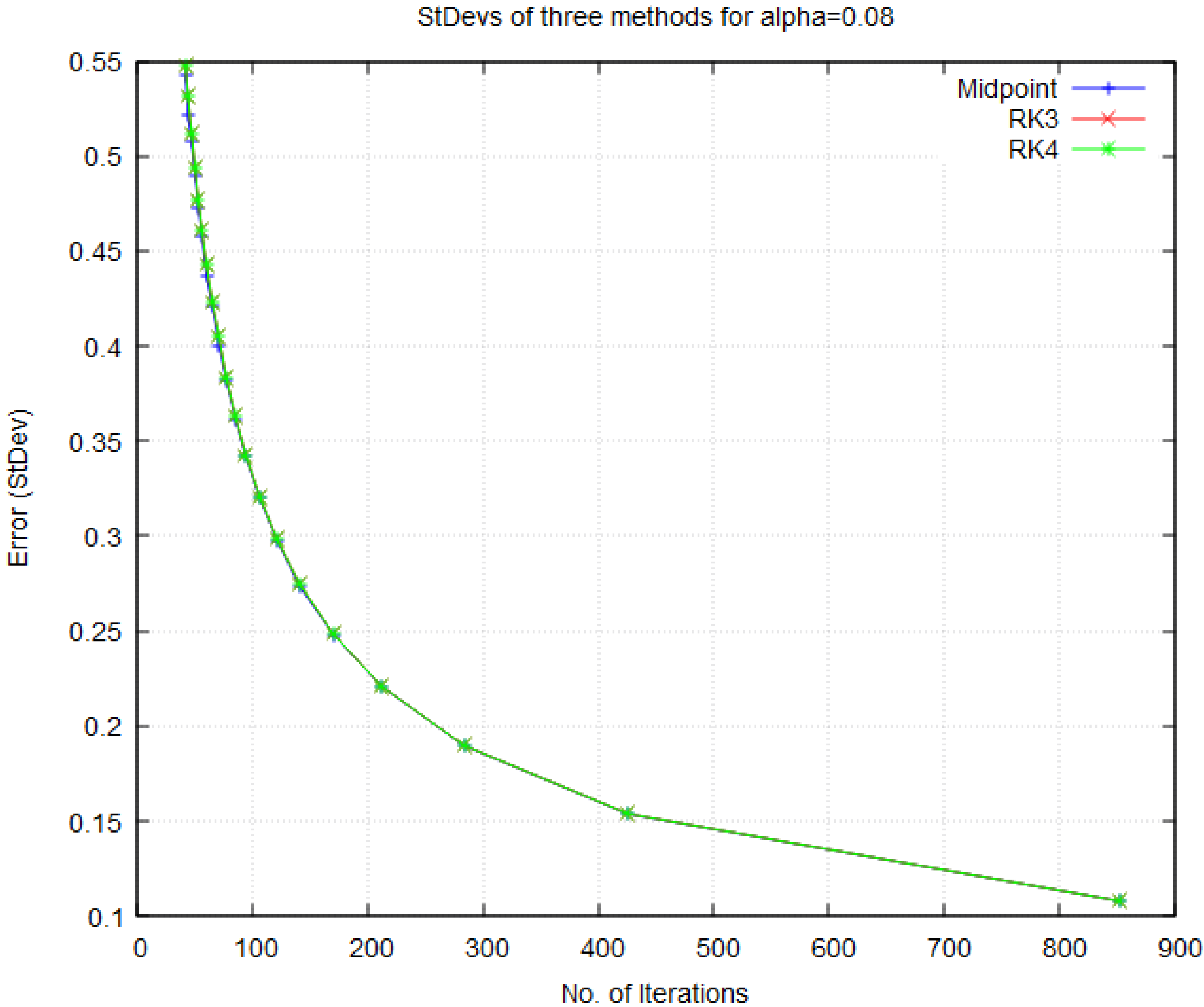
Plot of Error vs No. of Total Iterations Needed curve for each specific Step-Size

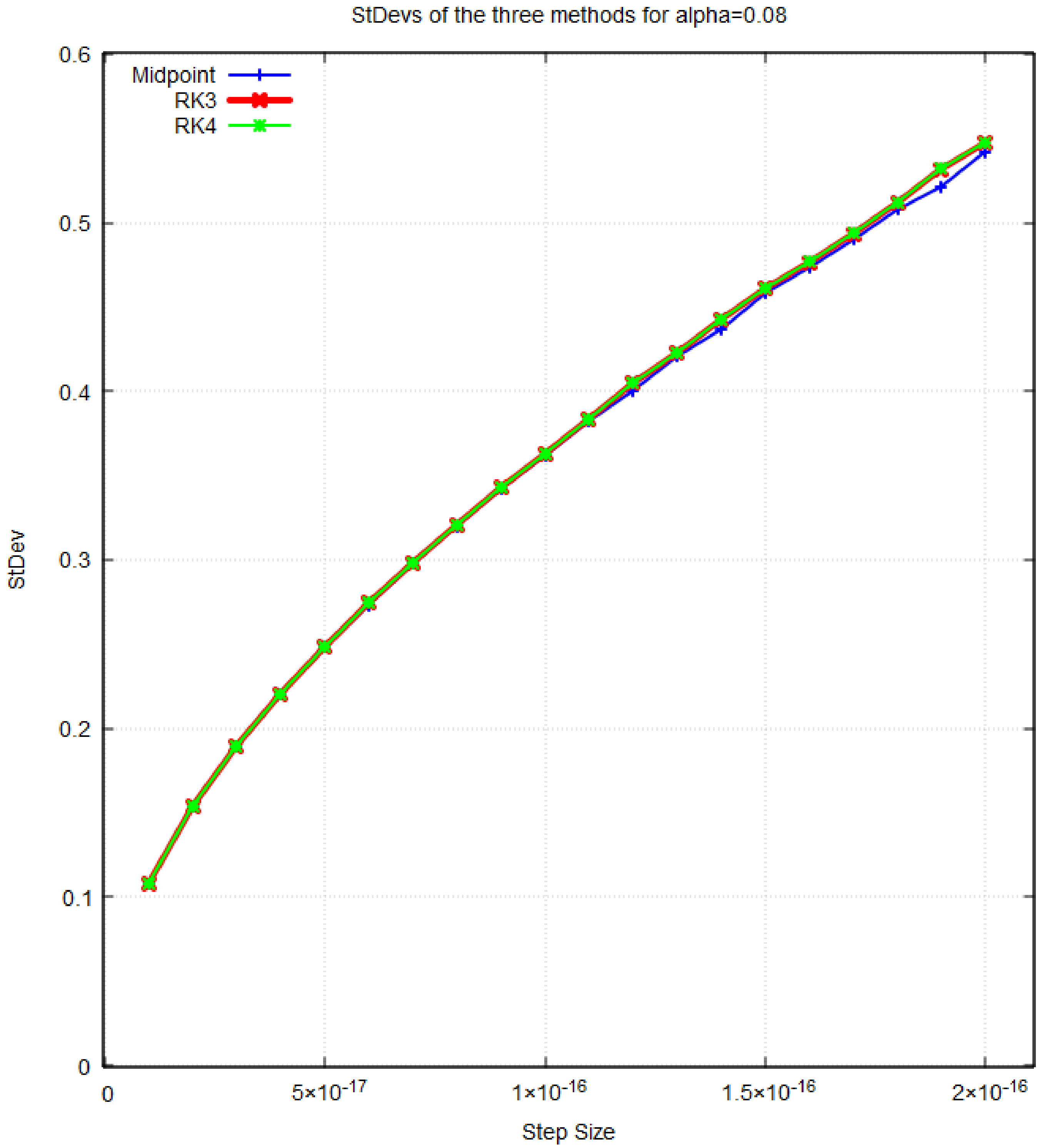


Here, the maximum no. of iterations (852) corresponds to step size 10^{-17} and the minimum no. of iterations (43) corresponds to step size $2 * 10^{-16}$.

We can see from the graph that the error understandably decreases with increase in the number of iterations.

We can also see that the increase in accuracy is diminishing as we keep increasing the number of iterations.

The three methods of solving the ODE have negligible differences in error for our specific ODE, as we can see that the 3 graphs are nearly overlapping each other.



We notice that the differences in StDev (standard deviation) are more noticeable for larger step-sizes.

We see that RK3 and RK4 have almost the same performance, while Midpoint seems to perform slightly better. This observation is specific to the given ODE, and solving technique and not generalizable, as we know that both the local and the global errors produced in the Midpoint method are of higher order than those in RK3 or RK4. Perhaps this is because the method employed fixed the radius of the sphere to a constant value, eliminating much of the error that would be given by midpoint method?

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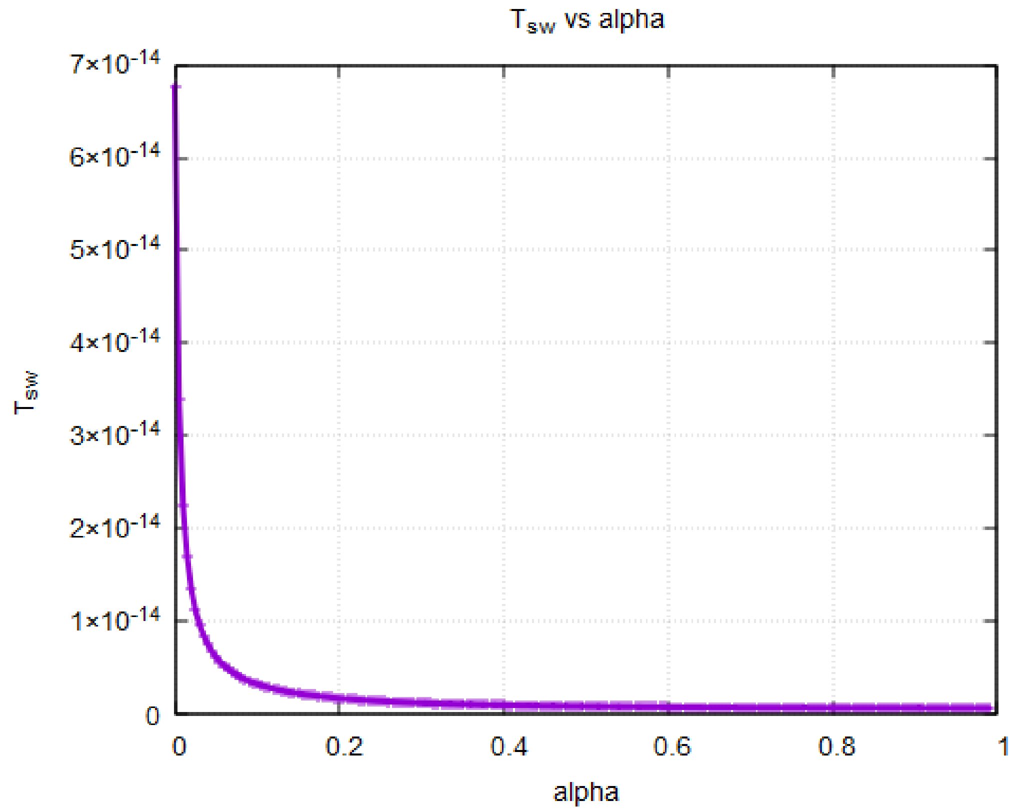
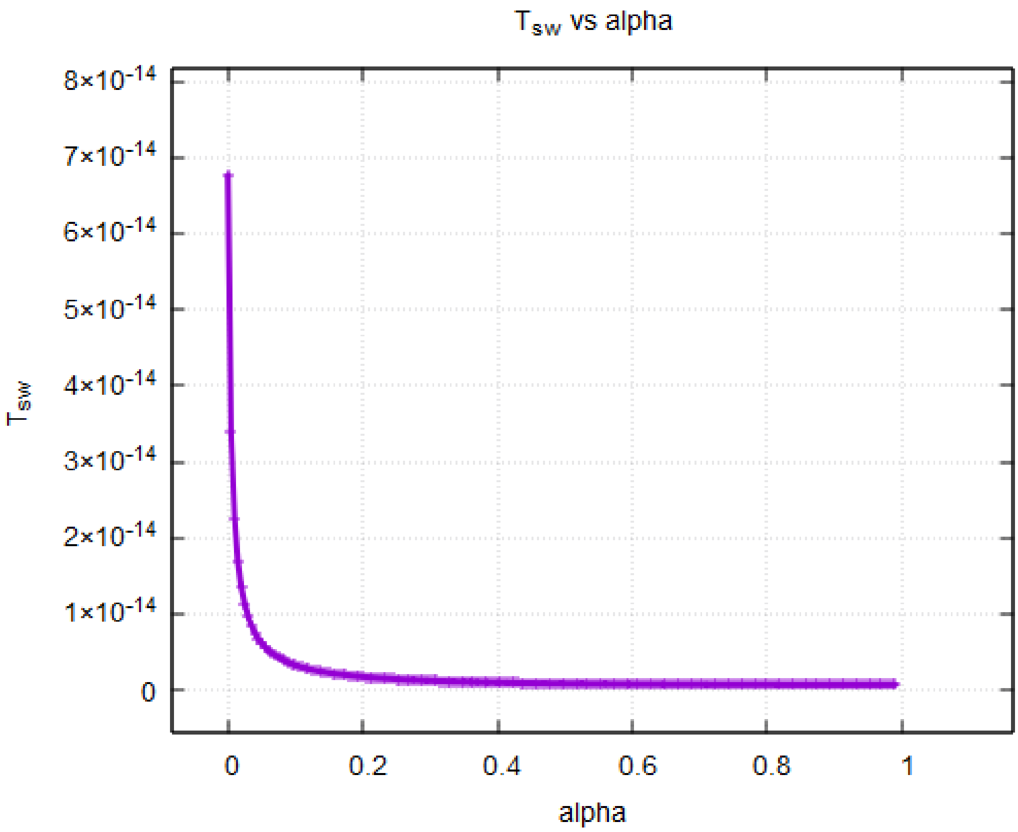


This is the graph of Switching Time (T_{sw}) vs α. Switching Time has been taken as the time required for θ to go from 179° to 90°, and α is taken in intervals of 0.005. Below is the same graph with different axes for better representation.



Thus, Switching Time (T_{sw}) decreases rapidly with increase in α

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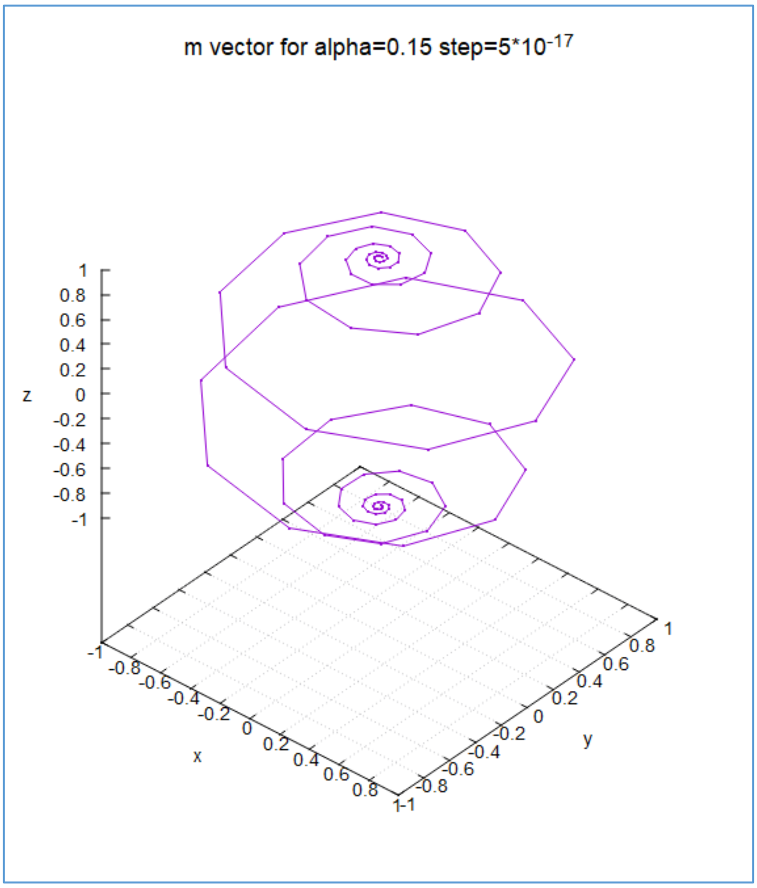
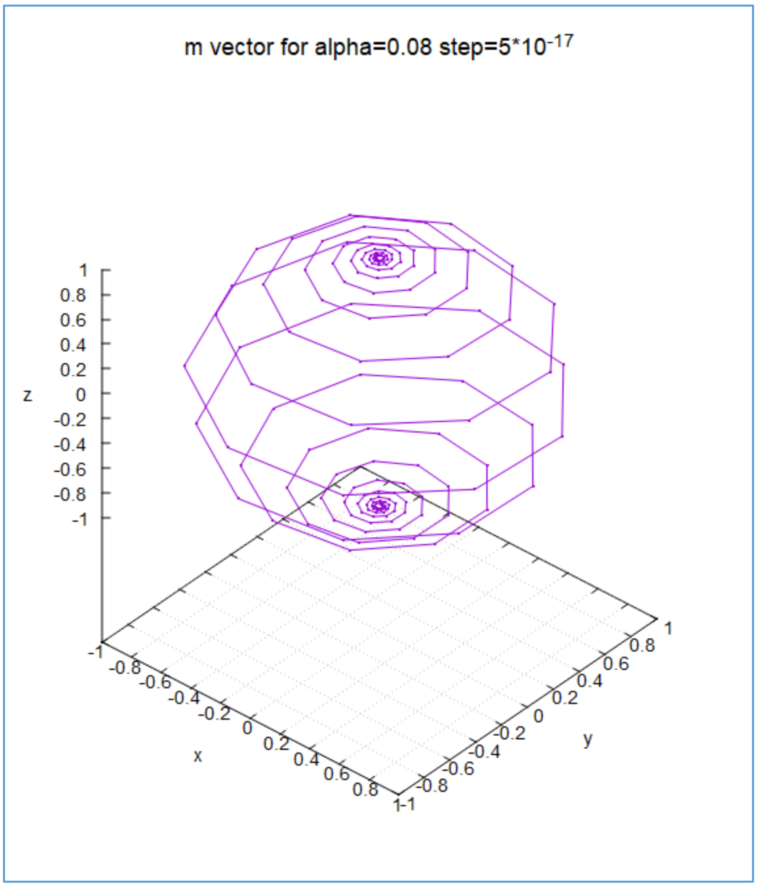
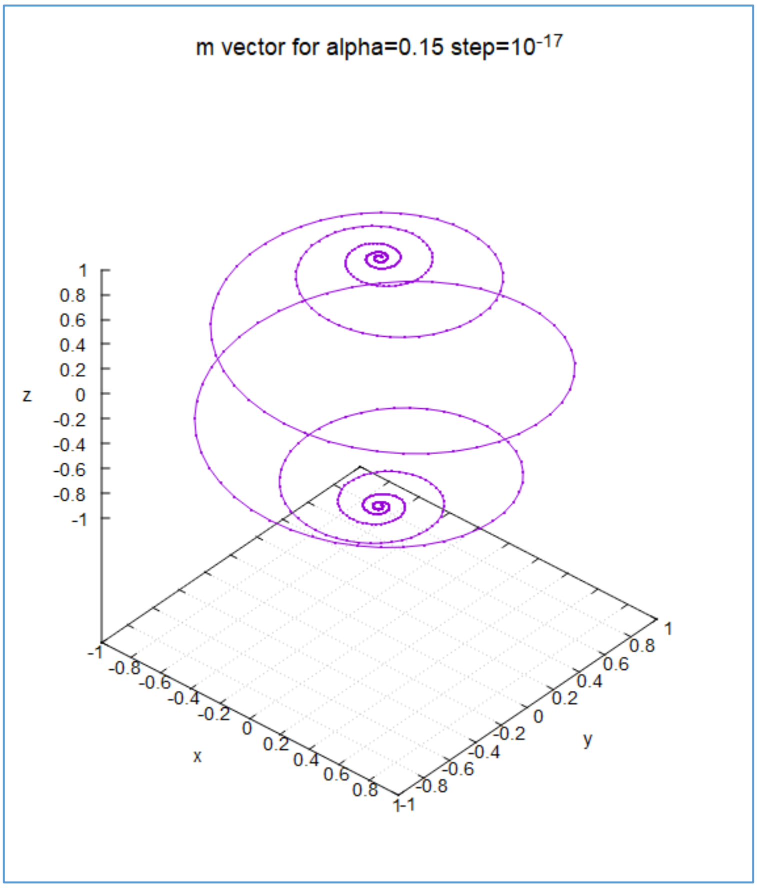
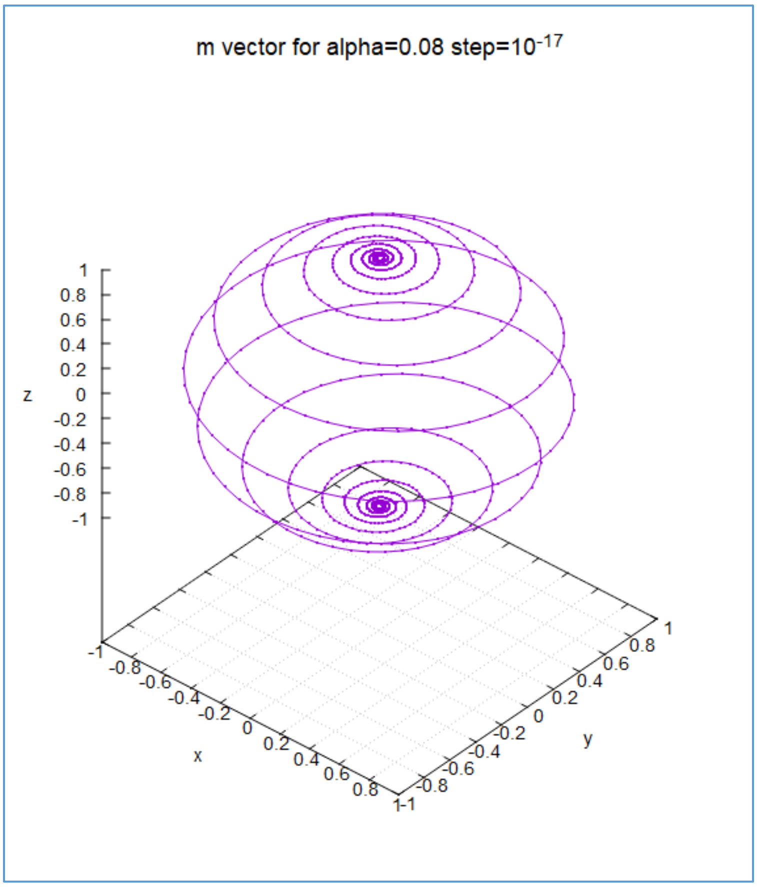
Plots of reduced magnetization vector $(\boldsymbol{m})$ in three dimensions with varying alpha and step









Decreasing step size gives us a smoother and more accurate curve, and decreasing α gives us higher switching time and thus more spirals.