Uncertainty management in rule based expert system

Mr. R. S. Mirajkar

DYPCET, Kolhapur

Bayes' theorem in Artificial intelligence

- Bayes' theorem is also known as Bayes' rule, Bayes' law, or Bayesian reasoning, which determines the **probability of an event with uncertain knowledge.**
- **Bayes' theorem is a way to figure out conditional probability.**
- Conditional probability is the probability of an event happening, given that it has some relationship to one or more other events.
- It is a way to calculate the value of P(B|A) with the knowledge of P(A|B).

- **Example: If cancer corresponds to one's age then by using Bayes' theorem, we can determine the probability of cancer more accurately with the help of age.**
- Bayes' theorem –
- Conditional probability of event A with known event B:
  - $P(A \cap B) = P(A|B)\,P(B)$ or
- Similarly, the probability of event B with known event A:
  - $P(A \cap B) = P(B|A)\,P(A)$
- Equating right hand side of both the equations, we will get:

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)} \qquad \text{....(a)}$$

- The above equation (a) is called as **Bayes' rule** or **Bayes' theorem**. This equation is basic of most modern AI systems for **probabilistic inference**.

- P(A|B) is known as **posterior**, which we need to calculate, and it will be read as Probability of hypothesis A when we have occurred an evidence B.

- P(B|A) is called the **likelihood**, in which we consider that hypothesis is true, then we calculate the probability of evidence.

- P(A) is called the **prior probability**, probability of hypothesis before considering the evidence

- P(B) is called **marginal probability**, pure probability of an evidence.

Bayes' Theorem Example

- You might be interested in finding out a **patient's probability of having liver disease if they are an alcoholic**. "Being an alcoholic" is the test (kind of like a litmus test) for liver disease.

- **A** could mean the event "Patient has liver disease." Past data tells you that 10% of patients entering your clinic have liver disease. P(A) = 0.10.
- **B** could mean the litmus test that "Patient is an alcoholic." Five percent of the clinic's patients are alcoholics. P(B) = 0.05.
- Among those patients diagnosed with liver disease, 7% are alcoholics.
- This is your **B|A:** the probability that a patient is alcoholic, given that they have liver disease, is 7%.
- Bayes' theorem tells you: **P(A|B)** = (0.07 * 0.1)/0.05 = 0.14
- In other words, if the patient is an alcoholic, their chances of having liver disease is 0.14 (14%).

Application of Bayes' theorem in Artificial Intelligence:

- It is used to calculate the next step of the robot when the already executed step is given.
- Bayes' theorem is helpful in weather forecasting.

Bayesian Belief Network in artificial intelligence

- Bayesian belief network is key computer technology for dealing with probabilistic events and to solve a problem which has uncertainty.

- A Bayesian network is a probabilistic graphical model which represents a set of variables and their conditional dependencies using a directed acyclic graph.

- It is also called a Bayes network, belief network, decision network, or Bayesian model.

- Bayesian networks are probabilistic, because these networks are built from a probability distribution, and also use probability theory for prediction and anomaly detection.

- Real world applications are probabilistic in nature, and to represent the relationship between multiple events, we need a Bayesian network.

- It can also be used in various tasks including prediction, anomaly detection, diagnostics, automated insight, reasoning, time series prediction, and decision making under uncertainty.

- Bayesian Network can be used for building models from data and experts opinions, and it consists of two parts:

  - **Directed Acyclic Graph**
  - **Table of conditional probabilities.**

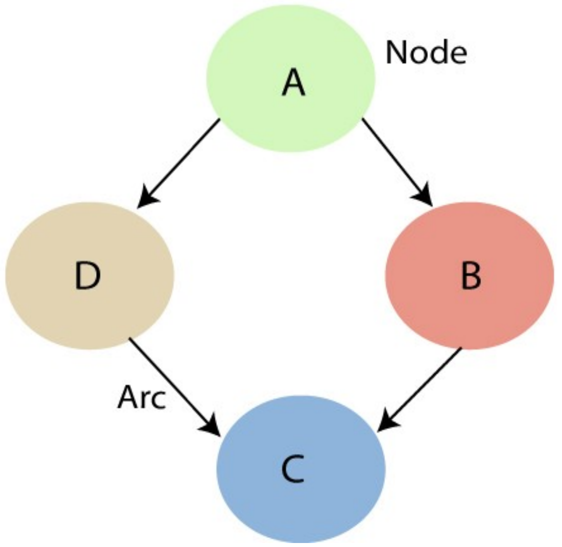
- The generalized form of Bayesian network that represents and solve decision **problems under uncertain knowledge is known as an Influence diagram.**

Bayesian network graph - nodes and Arcs (directed links)

- Node - Random variables, and a variable can be continuous or discrete.
- Arc or directed arrows - Causal relationship or conditional probabilities between random variables. These directed links or arrows connect the pair of nodes in the graph.
- These links represent that one node directly influence the other node, and if there is no directed link that means that nodes are independent with each other
- A, B, C, and D are random variables
- If we consider node B, which is connected with node A

by directed arrow then node A is called parent of Node B.

- Node C is independent of node A.



Markov Decision Process (MDP)

- **Used to formalize the reinforcement learning problems**.
- If the environment is completely observable, then its dynamic can be modeled as a **Markov Process.**

- A Markov chain or Markov process describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event.

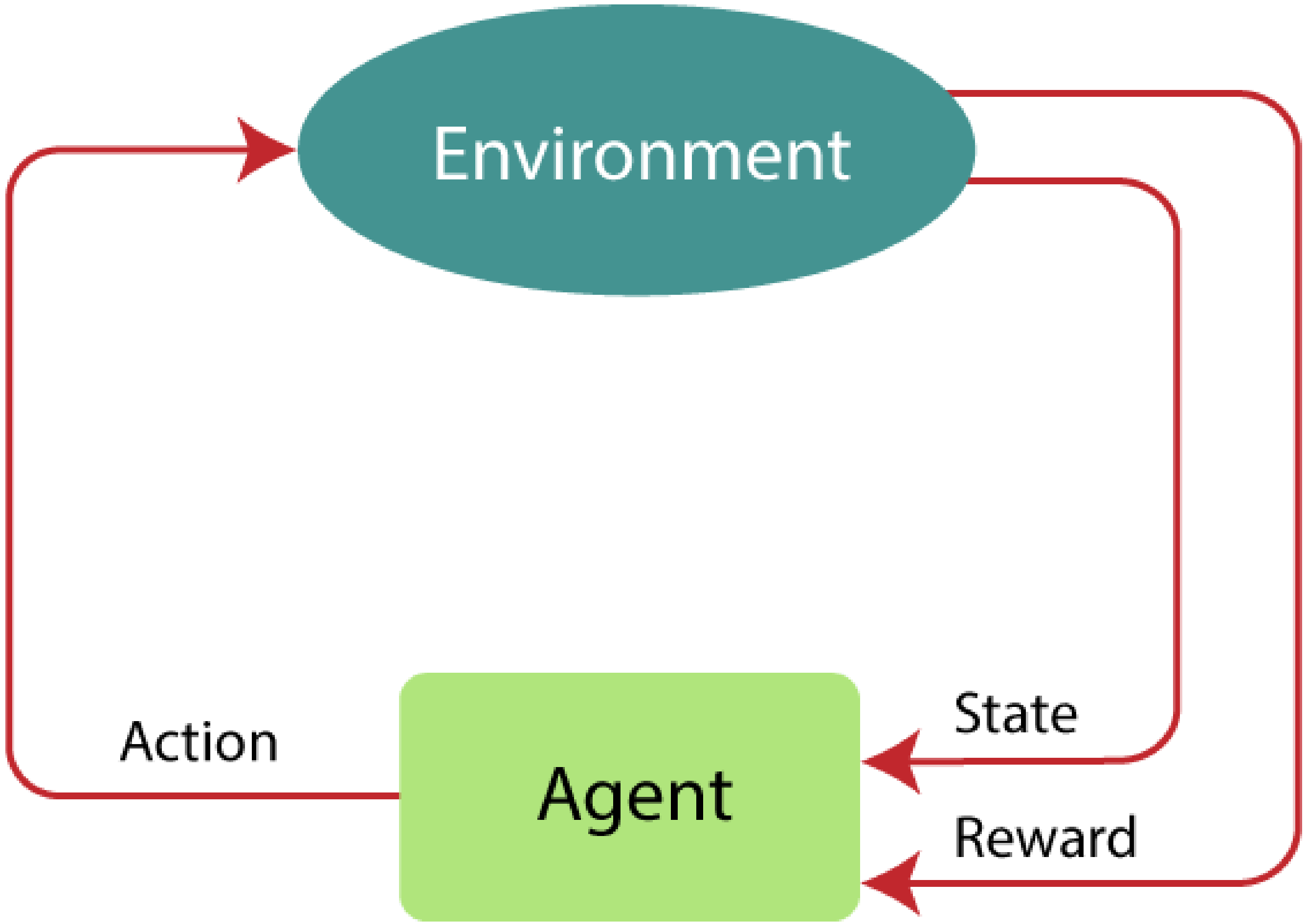
- It is a process for which predictions can be made regarding future outcomes based solely on its present state

- **When an agent can determine the state of the system at all times, it is called fully observable.**
- For example, in a chess game, the state of the system, that is, the position of all the players on the chess board, is available the whole time so the player can make an optimal decision.
- Agent constantly interacts with the environment and performs actions; at each action, the environment responds and generates a new state.

Reinforcement learning (RL)

- Reinforcement learning (RL) is an area of machine learning concerned with how intelligent agents ought to take actions in an environment in order to maximize the notion of cumulative reward.
- Reinforcement learning is one of three basic machine learning paradigms, alongside supervised learning and unsupervised learning.
- Feedback based Learning

- A Markov process is **a random process in which the future is independent of the past, given the present**


Environment
Action
Agent
State
Reward

- MDP contains a tuple of four elements (S, A, Pa, Ra):

  - A set of finite States S
  - A set of finite Actions A
  - Rewards received after transitioning from state S to state S', due to action a.
  - Probability P_a.

- MDP uses **Markov property**

Markov Property:

- If the agent is present in the **current state s1, performs an action a1 and move to the state s2,**
- Then the state transition from **s1 to s2** only depends on the current state and future action.
- States do not depend on past actions, rewards, or states.

The future is independent of the past given the present

- As per Markov Property, the current state transition does not depend on any past action or state.
- Hence, MDP is an RL problem that **satisfies the Markov property.**
- In a Chess game, the players only focus on the current state and do not need to remember past actions or states.

MDP is:

- MDP=⟨S,A,P,R⟩ where **S** are the states, **A** the actions, **P** the transition probabilities (to go from one state to another given an action), **R** the rewards (given a certain state, and possibly action),

- **States:** **E**xample - grid maps in robotics, or for example door open and door closed.
- **Actions:** A fixed set of actions, such as for example going north, south, east, for a robot, or opening and closing a door.
- **Transition probabilities:** The probability of going from one state to another given an action.

For Example:

- What is the probability of an open door if the action is open?.
- In a perfect world - 1.0, but if it is a robot, it could have failed in handling the doorknob correctly.
- In the case of a moving robot would be the action north, which in most cases would bring it in the grid cell north of it, but in some cases could have moved too much and reached the next cell.

Rewards:

- These are used to guide the planning.
- In the case of the grid example, we might want to go to a certain cell, and the reward will be higher if we get closer.
- In the case of the door example, an open door might give a high reward.

Hidden Markov Models

- Hidden Markov Models (HMMs) are a class of probabilistic graphical model that allow us to **predict a sequence of unknown (hidden) variables** from a **set of observed variables.**

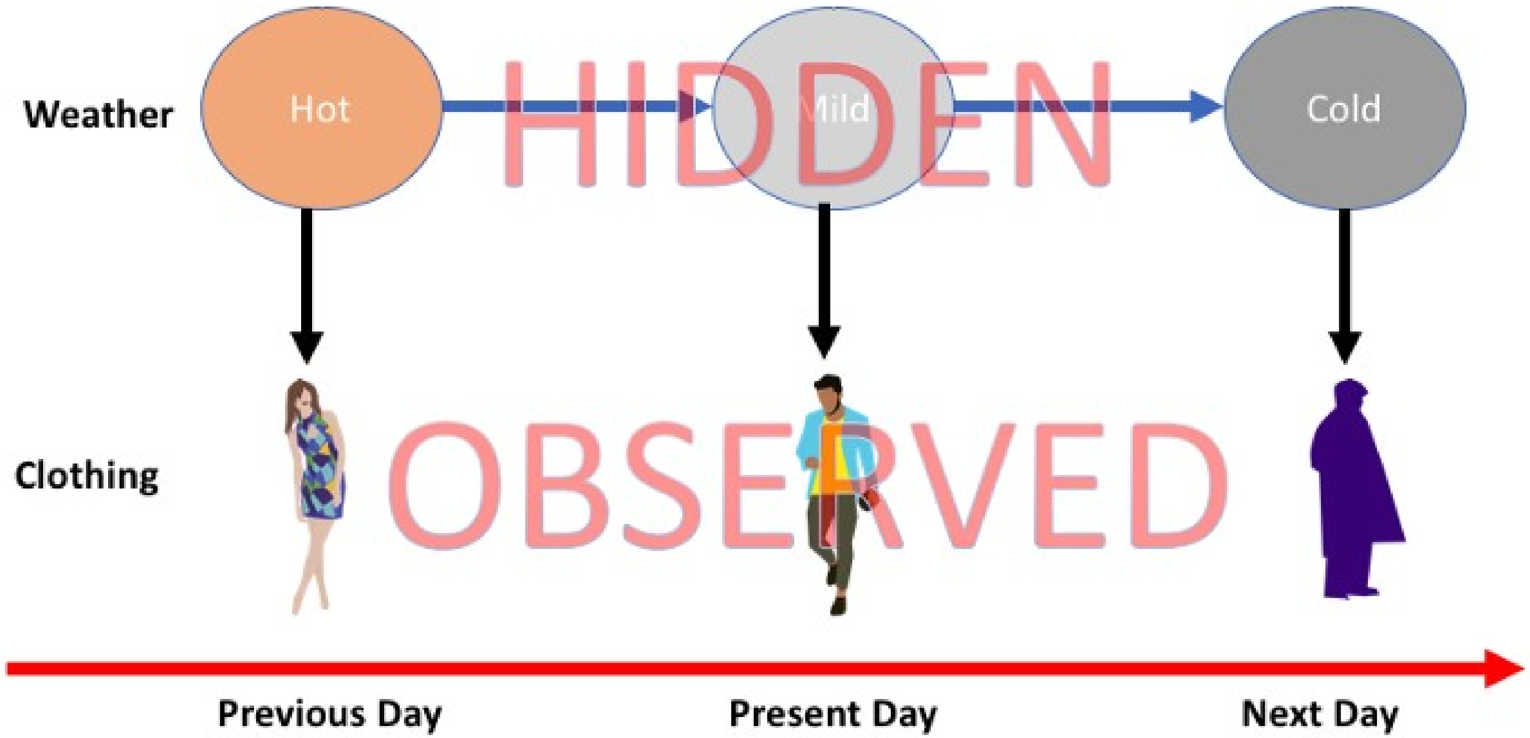
- It is used to describe the **evolution of observable events that depend on internal factors, which are not directly observable.**

- A simple example of an HMM is predicting the weather (hidden variable) based on the **type of clothes that someone wears (observed).**

Why Hidden, Markov Model?

- The reason it is called a Hidden Markov Model is because we are constructing an **inference model based on the assumptions of a Markov process.**
- The Markov process assumption is simply that the "**future is independent of the past given the present**".
- In other words, assuming we know our present state, we do not need any other historical information to predict the future state.

- The **weather, the hidden variable**, can be hot, mild or cold
- Observed variables are the type of clothing worn.
- The arrows represent transitions from a hidden state to another hidden state or from a hidden state to an observed variable
- Notice that, true to the Markov assumption, each state only depends on the previous state and not on any other prior states.



Perception behind HMMs

- HMMs are **probabilistic models**.
- They allow us to compute the joint probability of a set of hidden states given a set of observed states.
- Once we know the **joint probability of a sequence of hidden states, we determine the best possible sequence**.
- i.e. the sequence with the highest probability and choose that sequence as the best sequence of hidden states.

Partially Observable Markov Decision Process (POMDP)

- A partially observable Markov decision process (POMPD) is a Markov decision process in which the agent cannot directly observe the underlying states in the model.
- The Markov decision process (MDP) is a mathematical framework for modeling decisions showing a system with a series of states and providing actions to the decision maker based on those states.

- A partially observable Markov decision process (POMDP) is a combination of an regular Markov Decision Process to model system dynamics with a hidden Markov model that connects unobservable system states probabilistically to observations.

Examples:

- **Chess –** The board is fully observable, and so are the opponent's moves.

- **Driving –** The environment is partially observable because what's around the corner is not known

Utility Theory

- Utility is the **level of satisfaction** a person derives from consuming a good or service.
- Utility theory explains individuals' choices and measures their level of satisfaction from consuming a good or service.
- The level of satisfaction is measured in units called 'utils.'

Utility Theory

- **Marginal utility** is the satisfaction that a person receives from consuming an additional unit of the same good or service.
- **Total utility** is the aggregate satisfaction a person receives from consuming all the units of the same good or service.
- As the number of units increases, marginal utility decreases, and total utility increases.
- When the total utility reaches its maximum level, the marginal utility is zero.

Utility Theory

- Expected utility, subjective utility, marginal utility, and total utility.

- **Marginal utility** is the satisfaction that a person receives from consuming an additional unit of the same good or service.

- **Expected utility** is the utility that an economic agent is expected to reach in the future given several probable outcomes.
- **Expected utility** value is a probability concept that is used when several future outcomes are possible.

Utility Theory

- **The subjective utility is utility based on an individual's perceived personal level of satisfaction that they obtain from consuming a good or service.**

- What does a utility function represent?
- A utility function is a representation to define individual preferences for goods or services beyond the explicit monetary value of those goods or services.

Expected Utility Example

- if a consumer has a choice between two investments, one that is safe and offers a guaranteed return of 5%, and one that is risky but offers a potential return of 10%, the consumer would need to consider the probability of each investment achieving its expected return to make an informed decision.

Subjective utility Example

- **Subjective utility** is the personal satisfaction that a consumer derives from consuming a good or service.
- Vary from person to person
- It is important to note that subjective utility is individual and cannot be compared between different people.
- For example, one person might derive a lot of utility from eating ice cream, while another person might not enjoy it at all.

- **Marginal utility** is the change in utility that a consumer receives from consuming one additional unit of a good or service.
- The law of marginal utility states that the marginal utility of a good or service will decrease as the consumer consumes more of it.
- For example, the **first slice of pizza** that a consumer eats might give them a lot of utility, but the second slice will give them less utility, and so on.

- **Total utility** is the sum of the utility that a consumer receives from consuming all units of a good or service.
- For example, if a consumer consumes three slices of pizza, their total utility will be the sum of the marginal utility that they received from each slice.
- **Example:** If you eat five slices of pizza, your total utility for those five slices would be the sum of the marginal utilities for each slice.
- If the marginal utilities for the slices are 10, 8, 6, 4, and 2, your total utility for the five slices would be 10 + 8 + 6 + 4 + 2 = 30.
- This total utility measures the overall satisfaction you received from consuming all five slices.

What is utility?

- **Utility** is an economic theory that **measures the value, happiness, or satisfaction** that someone gets from consuming a product or service.
- People tend to purchase things because they want or need those things. Utility measures how much value those purchases provide.
- Utility is a significant concept in economics because it helps explain many aspects of **supply, demand, and pricing**.

Utility Theory helps in Reinforcement Learning

- In reinforcement learning, agents use utility functions to evaluate the desirability of different actions and outcomes.
- Agents aim to maximize the expected cumulative utility (reward) over time.
- The utility function helps the AI agent assess the long-term consequences of its actions and make decisions accordingly.
- For example, in a **game-playing AI, the utility function can be used to evaluate different moves and select the one that maximizes the expected score.**

Resource Allocation

- In AI systems managing resources, utility theory can be used to allocate resources efficiently.
- For instance, in autonomous vehicles, the **utility of different routes or actions can be considered to optimize navigation decisions**, such as selecting the fastest route while considering factors like traffic, fuel consumption, and passenger comfort.

Decision Support Systems

- Utility theory is employed in decision support systems to assist human decision-makers.
- These systems use utility functions to quantify the preferences and priorities of users or decision-makers, helping them make informed choices in complex decision-making scenarios.
- For example, in healthcare, utility theory can be used to assist doctors in treatment decision-making by considering the expected quality of life for different treatment options.

Search Algorithms

- In AI search algorithms like A* search, heuristic functions can be seen as approximate utility functions.
- These functions estimate the cost or utility of reaching a goal state from a given state, guiding the search towards more promising paths and improving efficiency.

Recommendation Systems

- Utility theory can be applied in recommendation systems to personalize content or product recommendations based on user preferences.
- **By modeling the utility or satisfaction of users for different items or content**, recommendation algorithms can suggest items that are likely to maximize user satisfaction.

Tutorial 3

1. Write a note on Bayes Theorem
2. Define Bayesian networks and explain how they model probabilistic relationships among variables.
3. Explain Naïve Bayes Classifier with Example
4. Define Markov Decision Processes (MDPs) and explain their use in modeling decision-making under uncertainty.
5. Explain Hidden Markov Model
6. Write a note on Utility Theory

Thank You