# **Definitions**

# Automata Theory Definitions

#### Deterministic Finite Automaton (DFA)

A DFA is a 5-tuple:

$$M = (Q, \Sigma, \delta, q_0, F)$$

Where:

- Q: finite set of states
- Σ: finite input alphabet
- $\delta: Q \times \Sigma \rightarrow Q$ : transition function
- q₀ ∈ Q: start state
- F ⊆ Q: set of accepting (final) states
- For every state and symbol, exactly one transition exists.

## Nondeterministic Finite Automaton (NFA)

An NFA is also a 5-tuple:

$$M = (Q, \Sigma, \delta, q_0, F)$$

Where:

- $\delta: Q \times \Sigma \rightarrow 2^Q$
- Allows multiple transitions for the same symbol and state.

# • Epsilon-NFA (ε-NFA)

Similar to NFA but includes ε-transitions (moves without consuming input):

$$\delta \colon \mathbb{Q} \times (\Sigma \cup \{\epsilon\}) \to 2^{\mathbb{Q}}$$

Can change state without reading input.

### Mealy Machine

A Mealy machine is a 6-tuple:

$$M = (Q, \Sigma, \Gamma, \delta, \lambda, q_0)$$

Where:

- $\Gamma$ : output alphabet
- $\delta: Q \times \Sigma \rightarrow Q$ : transition function
- $\lambda: Q \times \Sigma \to \Gamma$ : output function
- Output depends on current state and input.

#### Moore Machine

A Moore machine is a 6-tuple:

$$M = (Q, \Sigma, \Gamma, \delta, \lambda, q_0)$$

Where:

- λ: Q → Γ: output depends only on current state
- Output is associated with states, not transitions.

# Context-Free Grammar (CFG)

A CFG is a 4-tuple:

$$G = (V, \Sigma, R, S)$$

Where:

- V: variables (non-terminals)
- Σ: terminals
- R: rules  $A \rightarrow \alpha$ , where  $\alpha \in (V \cup \Sigma)^*$
- S ∈ V: start symbol
- Generates Context-Free Languages (CFLs).

# Chomsky Normal Form (CNF)

A CFG is in CNF if every production is of the form:

- A → BC (two non-terminals)
- A → a (a terminal)
- $S \rightarrow \epsilon$  (optional, if empty string is in language)
- Useful for parsing algorithms like CYK.

#### Greibach Normal Form (GNF)

A CFG is in GNF if every production is of the form:

 $A \rightarrow a \alpha$ 

Where:

- $a \in \Sigma$  (a terminal)
- α ∈ V\* (zero or more non-terminals)
- Starts with a terminal.

#### Backus-Naur Form (BNF)

A notation used to write CFGs:

Example:

```
<expr> ::= <term> | <expr> + <term>
```

Where:

- ::= means "is defined as"
- (...> denotes non-terminals
- Common in language syntax specs.

# Pushdown Automaton (PDA)

A PDA is a 7-tuple:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

Where:

- Γ: stack alphabet
- $\delta: Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \rightarrow 2^{(Q \times \Gamma^*)}$ : transition function
- Z<sub>o</sub>: initial stack symbol

Recognizes context-free languages using a stack.

#### Nondeterministic PDA (N-PDA)

Same as PDA, but allows multiple choices:

- All PDAs are **nondeterministic** unless specified otherwise.
- A **DPDA** has at most one possible move per step.

#### Turing Machine (TM)

A TM is a 7-tuple:

```
M = (Q, \Sigma, \Gamma, \delta, q_0, q_accept, q_reject)
```

Where:

- $\Gamma$ : tape alphabet (includes blank symbol)
- $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ : transition function
- qo: start state
- q\_accept, q\_reject: halting states
- Recognizes recursively enumerable languages
- ☑ Has **infinite tape** and serves as a general model of computation.