Unit 3: Uncertainty Management in Rule-Based Expert Systems - Theory Answers

1. Write a note on Bayes Theorem

Bayes' Theorem is a fundamental concept in Artificial Intelligence for managing uncertainty. It is used to determine the **probability of a hypothesis given certain evidence**. It provides a principled way to update our belief in a hypothesis when new evidence is presented.

Mathematical Formula: $P(A|B) = (P(B|A) * P(A)) / P(B)$

Where:

- **P(A|B):** Posterior probability – the probability of hypothesis A given evidence B.
- **P(B|A):** Likelihood – the probability of evidence B given that A is true.
- **P(A):** Prior probability – the initial probability of A.
- **P(B):** Marginal probability – the total probability of the evidence.

Example:

- P(A) = Probability of having liver disease = 0.10
- P(B) = Probability of being an alcoholic = 0.05
- P(B|A) = 0.07 Then, P(A|B) = (0.07 * 0.10)/0.05 = 0.14 → 14%

Applications in AI:

- Prediction in medical diagnosis
- Weather forecasting
- Robotics navigation and decision making

(Refer to slides 2–6.)

2. Define Bayesian networks and explain how they model probabilistic relationships among variables.

A **Bayesian Network** is a **probabilistic graphical model** that represents a set of variables and their **conditional dependencies** via a **directed acyclic graph (DAG)**.

Components:

- **Nodes:** Represent random variables (e.g., symptoms, diseases).
- **Arcs (Directed Edges):** Represent direct influence of one variable on another.
- **Conditional Probability Tables (CPTs):** Quantify the relationships between connected variables.

Advantages:

- Models uncertainty effectively

- Supports reasoning, prediction, and learning from data

Applications:

- Medical diagnosis
- Sensor fusion
- Decision support systems

Bayesian networks combine **expert knowledge** and **data-driven evidence** to represent real-world probabilistic systems.

(Refer to slides 7–9.)

3. Explain Naïve Bayes Classifier with Example

Although not detailed in the current slides, the **Naïve Bayes Classifier** is closely related to Bayes' Theorem. It is a classification technique based on the assumption that the features are conditionally independent given the class label.

Formula: $P(C|X) \propto P(X1|C) * P(X2|C) * ... * P(Xn|C) * P(C)$

Example: To classify whether an email is spam based on words like “offer”, “free”, and “click”. Naïve Bayes calculates the probability of spam given the presence of these words using Bayes’ Theorem.

Despite its simplicity, it performs well in many complex real-world problems, especially in **text classification** and **spam detection**.

(Use related understanding from Slide 2–6.)

4. Define Markov Decision Processes (MDPs) and explain their use in modeling decision-making under uncertainty.

A **Markov Decision Process (MDP)** is a mathematical model used for decision-making in environments that exhibit randomness and sequential structure.

Components of MDP:

- **S:** Set of states
- **A:** Set of actions
- **P:** Transition probability function ($P(s'|s, a)$)
- **R:** Reward function ($R(s, a)$)

Markov Property:

- The probability of transitioning to the next state depends only on the current state and action, not on prior states (i.e., "memoryless").

Applications:

- Robotics

- Game playing
- Autonomous control systems

Example:

- A robot navigating a grid world where each cell is a state, and actions are movement directions. Transitions and rewards are defined accordingly.

(Refer to slides 10–18.)

5. Explain Hidden Markov Model

A **Hidden Markov Model (HMM)** is a statistical model where the system being modeled is assumed to be a Markov process with **hidden states**.

Key Concepts:

- **Hidden States:** Cannot be directly observed (e.g., weather conditions)
- **Observed States:** Directly measurable outcomes (e.g., clothing choice)
- **Transition Probabilities:** Probabilities of moving from one hidden state to another
- **Emission Probabilities:** Probability of observing a visible outcome given a hidden state

Applications:

- Speech recognition
- Bioinformatics (gene prediction)
- Part-of-speech tagging in NLP

HMMs help in predicting hidden factors based on observable data sequences.

(Refer to slides 19–22.)

6. Write a note on Utility Theory

Utility Theory is a framework for understanding how rational agents make decisions under uncertainty. It quantifies the satisfaction (utility) an agent derives from different outcomes.

Types of Utility:

- **Total Utility:** Sum of satisfaction from all consumed goods.
- **Marginal Utility:** Additional utility from consuming one more unit.
- **Expected Utility:** Weighted average utility based on probabilities.
- **Subjective Utility:** Personal perception of satisfaction.

Applications in AI:

- **Reinforcement Learning:** Agents use utility functions to choose actions that maximize expected rewards.

- **Resource Allocation:** Helps autonomous systems make optimal choices.
- **Recommendation Systems:** Personalize suggestions based on user satisfaction.

Example: Choosing between a risky and safe investment by comparing expected utilities.

Utility theory is foundational in rational agent behavior and decision theory in AI.

(Refer to slides 26–41.)