SSW215-A Individual Software Engineering
Fall 2022
Assignment #6
Due November 9, 10:00 am

There are 3 problems in this assignment. Solve the models using Python and Gurobi optimizer. Please submit three Python files including the full working code of problems. Use the comment method in your python file (e.g., #) for answering explanatory questions.

Problem 1

Consider the following Linear Programming model:

$$\text{Maximize} \quad Z = 15x_1 + 20x_2,$$

subject to

$$\begin{aligned} x_1 + 2x_2 &\geq 10 \\ 2x_1 - 3x_2 &\leq 6 \\ x_1 + x_2 &\geq 6 \end{aligned}$$

and

$$x_1 \geq 0, \qquad x_2 \geq 0.$$

a) Find the optimal solution by using Python and Gurobi optimizer
b) Define a function named problem_one including the Python program for solving model such that (i) the function inputs right hand side values of constraints (e.g., 10, 6, and 6) and (ii) the function returns the values of decision variables and the objective function

Problem 2

The SOUTHERN CONFEDERATION OF KIBBUTZIM is a group of three kibbutzim (Communal farming communities) in Israel. Overall planning for this group is done in its Coordinating Technical Office. This office currently is planning agricultural production for the coming year. The agricultural output of each kibbutz is limited by both the amount of available irrigable land and the quantity of water allocated for irrigation by the Water Commissioner (a national government official). These data are given in Table 1.

Table 1. Resource data for the Southern Confederation of Kibbutzim.

Kibbutz	Usable Land (Acres)	Water Allocation (Acre Feet)
1	400	600
2	600	800
3	300	375

The crops suited for this region include sugar beets, cotton, and sorghum, and these are the three being considered for the upcoming season. These crops differ primarily in their expected net return per acre and their consumption of water. In addition, the Ministry of Agriculture has set a maximum quota for the total acreage that can be devoted to each of these crops by the Southern Confederation of Kibbutzim, as shown in Table 2.

Table 2. Crop data for the Southern Confederation of Kibbutzim.

Crop	Maximum Quota (Acres)	Water Consumption (Acre Feet/Acre)	Net Return ($/Acre)
Sugar beets	600	3	1,000
Cotton	500	2	750
Sorghum	325	1	250

Because of the limited water available for irrigation, the Southern Confederation of Kibbutzim will not be able to use all its irrigable land for planting crops in the upcoming season. To ensure equity between the three kibbutzim, it has been agreed that every kibbutz will plant the same proportion of its available irrigable land. For example, if kibbutz 1 plants 200 of its available 400 acres, then kibbutz 2 must plant 300 of its 600 acres, while kibbutz 3 plants 150 acres of its 300 acres. However, any combination of the crops may be grown at any of the kibbutzim. The job facing the Coordinating Technical Office is to plan how many acres to devote to each crop at the respective kibbutzim while satisfying the given restrictions. The objective is to maximize the total net return to the Southern Confederation of Kibbutzim as a whole.
The formulation as a Linear Programming problem is as follows.

a) Define and describe the set of decision variables for the Southern Confederation of Kibbutzim problem
b) Define the type of decision variables (e.g., Integer, continuous, binary)
c) Find the optimal solution by using Python and Gurobi optimizer

Maximize $Z = 1{,}000(x_1 + x_2 + x_3) + 750(x_4 + x_5 + x_6) + 250(x_7 + x_8 + x_9)$,

subject to the following constraints:

1. Usable land for each kibbutz:

$$x_1 + x_4 + x_7 \le 400$$
$$x_2 + x_5 + x_8 \le 600$$
$$x_3 + x_6 + x_9 \le 300$$

2. Water allocation for each kibbutz:

$$3x_1 + 2x_4 + x_7 \le 600$$
$$3x_2 + 2x_5 + x_8 \le 800$$
$$3x_3 + 2x_6 + x_9 \le 375$$

3. Total acreage for each crop:

$$x_1 + x_2 + x_3 \le 600$$
$$x_4 + x_5 + x_6 \le 500$$
$$x_7 + x_8 + x_9 \le 325$$

4. Equal proportion of land planted:

$$\frac{x_1 + x_4 + x_7}{400} = \frac{x_2 + x_5 + x_8}{600}$$
$$\frac{x_2 + x_5 + x_8}{600} = \frac{x_3 + x_6 + x_9}{300}$$
$$\frac{x_3 + x_6 + x_9}{300} = \frac{x_1 + x_4 + x_7}{400}$$

5. Nonnegativity:

$$x_j \ge 0, \qquad \text{for } j = 1, 2, \ldots, 9.$$

Problem 3

The Atlantic Coast Conference has four basketball games on a night. The conference office wants to assign four teams of officials to the four games in a way that Minimizes the total distance traveled by the officials. The supply is always one team of officials, and the demand is for only one team of officials at each game. The distances in miles for each team of officials to each game location are shown in Table 3. The problem is formulated as follows.

Table 3. The distances in miles for each team of officials to each game location.

	Game Sites			
Officials	RALEIGH	ATLANTA	DURHAM	CLEMSON
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

$$\text{Minimize } Z = 210x_{AR} + 90x_{AA} + 180x_{AD} + 160x_{AC} + 100x_{BR} + 70x_{BA} + 130x_{BD} + 200x_{BC} + 175x_{CR} + 105x_{CA} + 140x_{CD} + 170x_{CC} + 80x_{DR} + 65x_{DA} + 105x_{DD} + 120x_{DC}$$

subject to:

$$x_{AR} + x_{AA} + x_{AD} + x_{AC} = 1$$
$$x_{BR} + x_{BA} + x_{BD} + x_{BC} = 1$$
$$x_{CR} + x_{CA} + x_{CD} + x_{CC} = 1$$
$$x_{DR} + x_{DA} + x_{DD} + x_{DC} = 1$$
$$x_{AR} + x_{BR} + x_{CR} + x_{DR} = 1$$
$$x_{AA} + x_{BA} + x_{CA} + x_{DA} = 1$$
$$x_{AD} + x_{BD} + x_{CD} + x_{DD} = 1$$
$$x_{AC} + x_{BC} + x_{CC} + x_{DC} = 1$$

$$x_{ij} \geq 0$$

a) Define and describe the set of decision variables for this problem
b) Define the type of decision variables (e.g., Integer, continuous, binary)
c) What is the optimal assignment of teams of officials to games?
d) What are the total miles traveled for the teams of officials to reach all of the games?