Finite Element Methods
EN. 560.730

Fall 2021

Computer Assignment #2
Professor S. Ghosh **Due December 3, 2021**

ALL CHANGES TO THE PROGRAMS MUST BE DOCUMENTED

PROGRAM MODIFICATION: The program QUAD4 in your book (uploaded) is for bilinear quadrilateral elements. You may use any version of the QUAD4 element code that has been provided to you.

(a) The program incorporates 2x2 integration points. Modify this program to incorporate *1x1-point and 1x2 Gaussian quadrature,* in addition to *2x2* -point Gaussian quadrature.

(b) Write a subroutine to calculate the determinant of the Jacobian matrix *J* at every node and print its value. Does *J* become negative at any point for the problem mesh?

(c) Write a subroutine to calculate nodal values of stresses from integration points by the method of weighted averaging.

$$f_{node} = \frac{\sum_{\#ofsurroundingIP} \frac{1}{r} * f_{IP}}{\sum_{\#ofsurroundingIP} \frac{1}{r}}$$

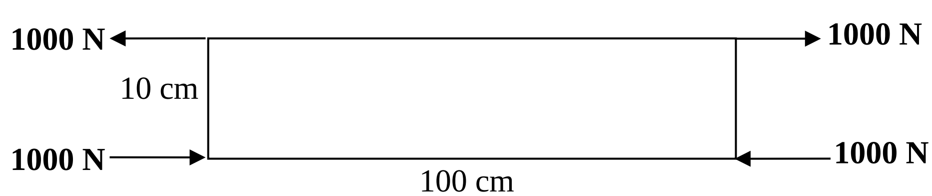
where *r* is the distance from the node to a Gauss quadrature (integration) point.

(d) Write a subroutine to calculate the average stress in each element. From this calculate the traction on every side of the element and find the traction difference on every edge from adjacent elements.

I. PROBLEM ON PATCH TEST AND ELEMENT LOCKING

(i) Use QUAD4 code with the *2x2, 1x1* and *1x2* integration points to verify whether the element passes the patch tests (1) and (2) given in class. Use the same *parametric displacement fields and stress field* provided in class.

(ii) Consider a beam of dimensions 100 cm x 10 cm x 1 cm is subjected to a loading that is equivalent to an applied bending moment as shown. The material Young's Modulus id **E**= 10^4 N/m^2 . Solve the problem for 2 Poisson ratios, viz. ν =0.3 and ν =0.49.

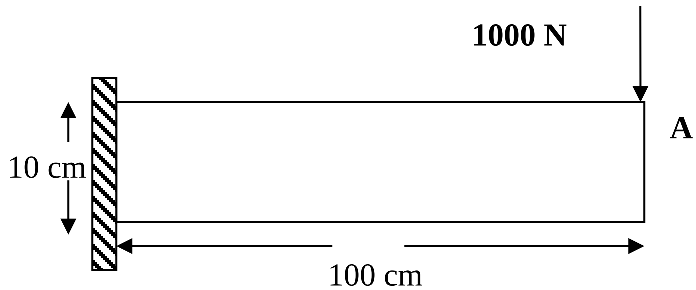


(a) Solve the problem using the 3 QUAD4 programs (with different Ips) for the 2 different values of ν. Use symmetry about the center line to create your FE model and mesh. Along a line near the center of the beam, plot the values of the stress σ_{xx} for both values of ν.

(b) Compare with the analytical solution from your *Strength of Materials* knowledge for a beam under a moment loading. Comment on your solution.

II. SOLVING A STRESS ANALYSIS PROBLEM

Consider the following **Plane Strain** problem. A **cantilever** beam of unit thickness (**t=1cm**) is loaded with a point load of 1000 N at the point A. Material properties: **E**= 10^4 N/m^2 and Poisson ratio ν =0.3



__Analytical Solution__ (ref. Theory of Elasticity by Timoshenko and Goodier).

$$\sigma_{xx} = -\frac{Pxy}{I}, \quad P = applied\ load, I = Moment\ of\ Inertia$$

$$\sigma_{yy} = 0$$

$$\sigma_{xy} = -\frac{P}{I}\frac{1}{2}(c^2 - y^2), \quad 2c = width\ \ and\ l = length$$

$$v_{x=l} = \frac{Pl^3}{3EI} \quad (displacement\ at\ free\ edge)$$

Solve this problem by your QUAD4 program!!

1. Discretize the beam domain into a mesh of 20+ elements for the QUAD4 code. *You may use the mesh-generator program in the book or generate by hand.*

2. Discuss methods of applying the fixed boundary condition.

__PROBLEM TO BE SOLVED__

For six cases including integration points 2x2, 1x1 and 1x 2 and Poisson ratio 0.3 and 0.499.

(a) Obtain the displacement of the point A from your computations. Compare this with analytical results at the same point?

(b) Plot the stress $\boldsymbol{\sigma}_{xx}$ along the vertical line through the middle of the beam.
Plot the analytical stress along this line and compare with the stress generated at the nodal points.

(c) Discuss your results