% Parameters

nx = 41;

ny = 81;

Lx = 1.0;

Ly = 1.0;

dx = Lx / (nx - 1);

dy = Ly / (ny - 1);

tol = 1e-6; % Convergence tolerance

max\_iter = 1000; % Maximum number of iterations

% Grid setup

x = linspace(0, Lx, nx);

y = linspace(0, Ly, ny);

% Initialize solution (phi) and source term

phi = zeros(nx, ny);

source = zeros(nx, ny);

% Boundary conditions

for j = 1:ny

% Left boundary: phi(0, y)

phi(1, j) = 500 \* exp(-50 \* (1 + y(j)^2));

% Right boundary: phi(1, y)

phi(nx, j) = 100 \* (1 - y(j)) + 500 \* exp(-50 \* y(j)^2);

end

for i = 1:nx

% Bottom boundary: phi(x, 0)

phi(i, 1) = 100 \* x(i) + 500 \* exp(-50 \* (1 - x(i))^2);

% Top boundary: phi(x, 1)

phi(i, ny) = 500 \* exp(-50 \* ((1 - x(i))^2 + 1));

end

% Source term

for i = 1:nx

for j = 1:ny

dist\_squared = (1 - x(i))^2 + y(j)^2;

source(i, j) = 50000 \* exp(-50 \* dist\_squared) \* (100 \* dist\_squared - 2);

end

end

% ADI method

residuals = zeros(max\_iter, 1);

for it = 1:max\_iter

phi\_old = phi;

% Row-wise sweep (implicit in x)

for j = 2:ny-1

for i = 2:nx-1

phi(i, j) = (1/2) \* (source(i, j) \* dx^2 \* dy^2 / (dx^2 + dy^2) + ...

(phi(i+1, j) + phi(i-1, j)) \* dy^2 / (dx^2 + dy^2) + ...

(phi(i, j+1) + phi(i, j-1)) \* dx^2 / (dx^2 + dy^2));

end

end

% Column-wise sweep (implicit in y)

for i = 2:nx-1

for j = 2:ny-1

phi(i, j) = (1/2) \* (source(i, j) \* dx^2 \* dy^2 / (dx^2 + dy^2) + ...

(phi(i+1, j) + phi(i-1, j)) \* dy^2 / (dx^2 + dy^2) + ...

(phi(i, j+1) + phi(i, j-1)) \* dx^2 / (dx^2 + dy^2));

end

end

% Compute residual (L2 norm)

residual = sqrt(sum((phi(:) - phi\_old(:)).^2));

residuals(it) = residual;

% Check for convergence

if residual < tol

fprintf('Converged in %d iterations.\n', it);

residuals = residuals(1:it); % Trim the residual array

break;

end

end

% Plot residuals vs iterations

figure;

semilogy(1:length(residuals), residuals, 'LineWidth', 2);

xlabel('Iteration');

ylabel('Residual');

title('Residual vs Iterations (ADI Method)');

grid on;

% Contour plot of the computed phi field

figure;

contourf(x, y, phi', 50, 'LineColor', 'none'); % Smooth contours with 50 levels

xlabel('x');

ylabel('y');

title('\phi Contour Plot for 41x81 Grid (ADI Method)');

colorbar;

colormap('jet'); % Smooth color transition (blue -> yellow)