Chapter 9

9.1. The next-state and output expressions for the circuit in Figure P9.1 are

$$
\begin{aligned}
Y_1 &= \overline{w}_1 + y_1\overline{y}_2 \\
Y_2 &= \overline{w}_2 + \overline{y}_1 + w_1 y_2 \\
z_1 &= \overline{y}_1 \\
z_2 &= \overline{y}_2
\end{aligned}
$$

This gives the excitation table

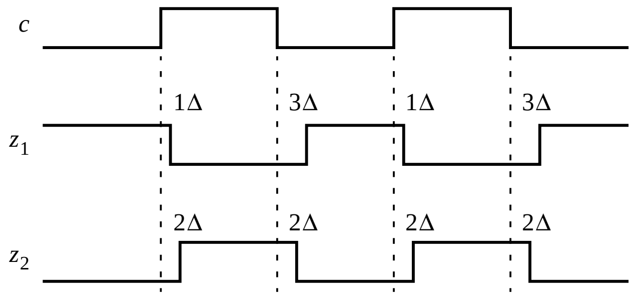
	Present state y_2y_1	Next state $w_2w_1 = 00$	01	10	11	z_2z_1
		Y_2Y_1				
A	00	11	10	11	10	11
B	01	11	11	01	01	10
C	10	11	10	11	10	01
D	11	11	10	01	10	00

The resulting flow table is

Present state	Next state $w_2w_1 = 00$	01	10	11	z_2z_1
A	D	C	D	C	11
B	D	D	B	B	10
C	D	C	D	C	01
D	D	C	B	C	00

The behavior is the same as described in the flow table in Figure 9.21a, if the state interchanges A $\leftrightarrow$ D and B $\leftrightarrow$ C are made.

9.2. The waveforms are



The flow table is

Present state	Next state $C = 0$	1	Outputs, z_2z_1 0	1
0	1	0	00	10
1	1	0	01	00

The circuit generates a non-overlapping 2-phase clock.

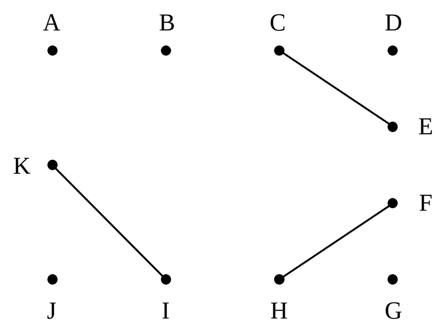
9.3. The partitioning procedure gives

$$\begin{aligned} P_1 &= (ADGJMPT)(BEHR)(CF)(ILOSV)(KNU) \\ P_2 &= (AD)(GP)(JMT)(B)(E)(HR)(C)(F)(ILOSV)(KNU) \\ P_3 &= (A)(D)(GP)(JMT)(B)(E)(HR)(C)(F)(ILOSV)(KNU) \\ P_4 &= P_3 \end{aligned}$$

This gives the flow table

Present state	Next state $w_2w_1 = 00$	01	10	11	z
A	A	B	C	–	0
B	D	B	–	–	0
C	G	–	C	–	0
D	D	E	F	–	0
E	G	E	–	–	0
F	J	–	F	–	0
G	G	H	I	–	0
H	J	H	–	–	0
I	A	–	I	–	1
J	J	K	I	–	0
K	A	K	–	–	1

The corresponding merger diagram is



This leads to the reduced flow table

Present state	Next state $w_2w_1 = 00$	01	10	11	z
A	A	B	C	–	0
B	D	B	–	–	0
C	G	C	C	–	0
D	D	C	F	–	0
F	J	F	F	–	0
G	G	F	I	–	0
I	A	I	I	–	1
J	J	I	I	–	0

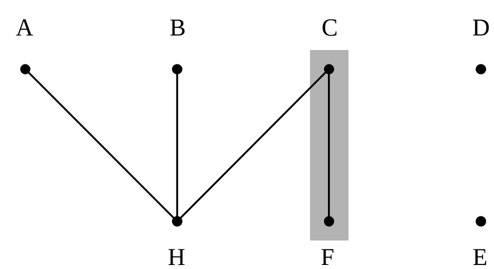
9.4. The partitioning procedure gives

$$
\begin{aligned}
P_1 &= (AF)(BEGL)(CJ)(DK)(HM) \\
P_2 &= (AF)(BG)(EL)(CJ)(DK)(HM) \\
P_3 &= (A)(F)(BG)(EL)(CJ)(DK)(HM) \\
P_4 &= P_3
\end{aligned}
$$

Replacing states B and G, E and L, C and J, D and K, and H and M with new states B, E, C, D, and H, respectively, produces the following flow table:

Present state	Next state				Output
	$w_2w_1 = 00$	01	10	11	z
A	A	B	C	–	0
B	D	B	–	H	0
C	F	–	C	H	0
D	D	E	C	–	1
E	A	E	–	H	0
F	F	E	C	–	0
H	–	B	C	H	1

The merger diagram is



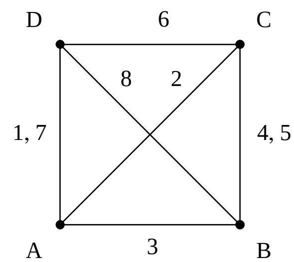
Only C and F can be merged if the Moore model is to be preserved. Therefore, the reduced flow table is

Present state	Next state				z
	$w_2w_1 = 00$	01	10	11	
A	A	B	C	–	0
B	D	B	–	H	0
C	C	E	C	H	0
D	D	E	C	–	1
E	A	E	–	H	0
H	–	B	C	H	1

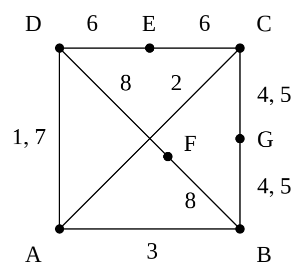
9.5. Relabel the flow table as

Present state	Next state				Output z			
	$w_2w_1 = 00$	01	10	11	00	01	10	11
A	1	3	7	2	0	–	1	1
B	5	3	4	8	0	1	0	0
C	5	6	4	2	0	1	0	1
D	1	6	7	8	–	–	1	0

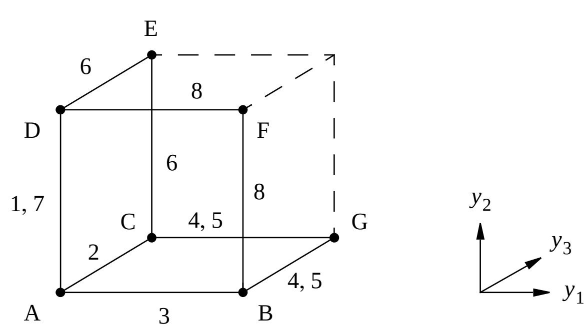
The transition diagram is



The diagonal transitions cannot be avoided without introducing additional states. A possible modification is



This transition diagram can be embedded onto a 3-cube as follows:



Then the modified flow table is

Present state	Next state				Output z			
	$w_2w_1 = 00$	01	10	11	00	01	10	11
A	A	B	D	A	0	–	1	1
B	G	B	B	F	0	1	0	0
C	C	C	G	A	0	1	0	1
D	A	E	D	D	–	–	1	0
E	–	C	–	–	–	1	–	–
F	–	–	–	D	–	–	–	0
G	C	–	B	–	0	–	0	–

The excitation table is

	Present state $y_3y_2y_1$	Next state $Y_3Y_2Y_1$				Output z			
		$w_2w_1 = 00$	01	10	11	00	01	10	11
A	000	000	001	010	000	0	–	1	1
B	001	101	001	001	011	0	1	0	0
C	100	100	100	101	000	0	1	0	1
D	010	000	110	010	010	–	–	1	0
E	110	–	100	–	–	–	1	–	–
F	011	–	–	–	010	–	–	–	0
G	101	100	–	001	–	0	–	0	–

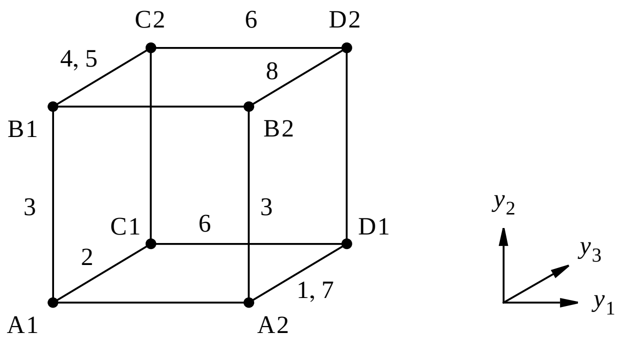
The next-state and output expressions are

$$
\begin{aligned}
Y_3 &= \overline{w}_2 y_3 + \overline{w}_2\overline{w}_1 y_1 + \overline{w}_1 y_3 y_1 \\
Y_2 &= w_2\overline{w}_1\overline{y}_3\overline{y}_1 + w_1\overline{y}_3 y_2\overline{y}_1 + w_2\overline{y}_3 y_2\overline{y}_1 + w_2 w_1\overline{y}_3\overline{y}_2 y_1 \\
Y_2 &= \overline{y}_3\overline{y}_2 y_1 + \overline{w}_2 w_1\overline{y}_3\overline{y}_2 + w_2\overline{w}_1 y_3\overline{y}_2 + w_2\overline{w}_1\overline{y}_2 y_1 \\
z &= \overline{w}_2 w_1 + w_1\overline{y}_2\overline{y}_1 + w_2\overline{w}_1\overline{y}_3\overline{y}_1
\end{aligned}
$$

9.6. Relabel the flow table in Figure 9.42 as

Present state	Next state				Output z			
	$w_2w_1 = 00$	01	10	11	00	01	10	11
A	1	3	7	2	0	–	1	1
B	5	3	4	8	0	1	0	0
C	5	6	4	2	0	1	0	1
D	1	6	7	8	–	–	1	0

Using pairs of equivalent states gives the following transition diagram:



Therefore, the modified flow table is

Present state	Next state				Output z			
	$w_2w_1 = 00$	01	10	11	00	01	10	11
A1	A1	B1	A2	A1	0	–	1	1
A2	A2	B2	D1	A2	0	–	1	1
B1	C2	B1	B1	B2	0	1	0	0
B2	B1	B2	B2	D2	0	1	0	0
C1	C1	C1	C2	A1	0	1	0	1
C2	C2	C2	B1	C1	0	1	0	1
D1	A2	C1	D1	D1	–	–	1	0
D2	D1	C2	D2	D2	–	–	1	0

The excitation table is

	Present state $y_3y_2y_1$	Next state $Y_3Y_2Y_1$ $w_2w_1 = 00$	01	10	11	Output z 00	01	10	11
A1	000	000	010	001	000	0	–	1	1
A2	001	001	011	101	001	0	–	1	1
B1	010	110	010	010	011	0	1	0	0
B2	011	010	011	011	111	0	1	0	0
C1	100	100	100	110	000	0	1	0	1
C2	110	110	110	010	100	0	1	0	1
D1	101	001	100	101	101	–	–	1	0
D2	111	101	110	111	111	–	–	1	0

The next-state and output expressions are

$$
\begin{aligned}
Y_3 &= \overline{w}_2\overline{w}_1y_2\overline{y}_1 + \overline{w}_1y_3\overline{y}_2\overline{y}_1 + w_2\overline{w}_1\overline{y}_2y_1 + w_2w_1y_2y_1 + \\
&\quad y_3y_2y_1 + \overline{w}_2w_1y_3 + w_1y_3y_2 + w_1y_3y_1 \\
Y_2 &= \overline{y}_3y_2 + \overline{w}_2w_1y_3 + \overline{w}_1y_2\overline{y}_1 + \overline{w}_2w_1y_2 + w_2y_2y_1 + w_2\overline{w}_1y_3\overline{y}_2 \\
Y_1 &= w_2y_1 + \overline{w}_1\overline{y}_2y_1 + w_1\overline{y}_3y_1 + w_2\overline{w}_1\overline{y}_3\overline{y}_2 \\
z &= \overline{w}_2y_1 + w_2\overline{y}_3\overline{y}_2 + w_1y_3\overline{y}_2 + \overline{w}_1y_3y_1
\end{aligned}
$$

9.7. Using the one-hot encoding, the FSM in Figure 9.42 can be implemented as

State assignment	Present state	Next state $w_2w_1 = 00$	01	10	11	Output z 00	01	10	11
0001	A	A	E	F	A	0	–	1	1
0010	B	G	B	B	H	0	1	0	0
0100	C	C	C	G	I	0	1	0	1
1000	D	F	J	D	D	–	–	1	0
0011	E	–	B	–	–	–	1	–	–
1001	F	A	–	D	–	0	–	1	–
0110	G	C	–	B	–	0	–	0	–
1010	H	–	–	–	D	–	–	–	0
0101	I	–	–	–	A	–	–	–	1
1100	J	–	C	–	–	–	1	–	–

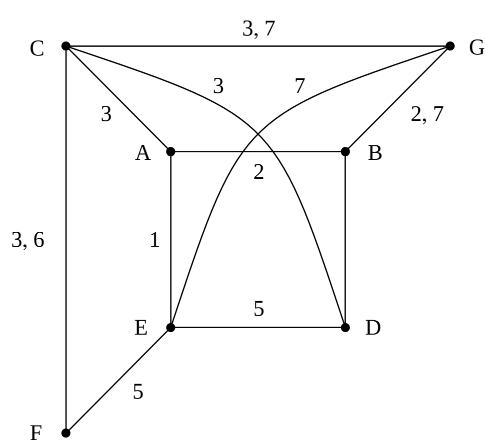
9.8. Using the merger diagram in Figure 9.40a, the FSM in Figure 9.39 becomes

Present state	Next state $w_2w_1 = 00$	01	10	11	Output z
A	A	G	E	–	0
B	B	C	B	D	0
C	B	C	E	C	1
D	–	C	E	D	0
E	A	–	E	D	1
G	B	G	–	D	1

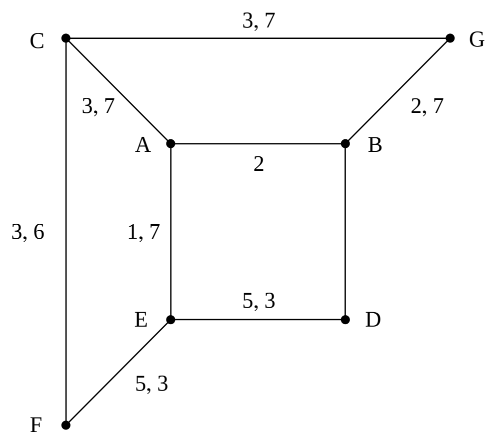
9.9. The relabeled flow table is

Present state	Next state $w_2w_1 = 00$	01	10	11	Output z
A	1	2	3	–	
B	4	2	–	7	
C	6	–	3	7	
D	4	5	3	–	
E	1	5	–	7	
F	6	5	3	–	
G	–	2	3	7	

The corresponding transition diagram is as follows. Note that the diagonal transitions are shown only when they involve a transition to a stable state.



The diagonal transition $D \rightarrow C$ labeled 3 can be removed by using the unspecified entry in row E, such that the required transition is performed as $D \rightarrow E \rightarrow F \rightarrow C$; this involves placing a label 3 on the paths from D to E and E to F. Similarly, the diagonal transition $E \rightarrow G$ labeled 7 can be removed by using the unspecified entry in row A, such that the required transition is performed as $E \rightarrow A \rightarrow C \rightarrow G$. These modifications produce the following transition diagram:



Then the modified flow table is

Present state	Next state $w_2w_1 = 00$	01	10	11	Output z
A	A	B	C	C	0
B	D	B	–	G	0
C	F	–	C	G	0
D	C	E	E	–	1
E	A	E	F	A	0
F	F	E	C	–	0
G	–	B	C	G	1

Thus, a possible state assignment is: $A = 000$, $B = 001$, $C = 100$, $D = 011$, $E = 010$, $F = 110$, and $G = 101$. Then, the state-assigned table is

	Present state $y_3y_2y_1$	Next state $Y_3Y_2Y_1$ $w_2w_1 = 00$	01	10	11	Output z
A	000	000	001	100	100	0
B	001	011	001	–	101	0
C	100	110	–	100	101	0
D	011	011	010	010	–	1
E	010	000	010	110	000	0
F	110	110	010	100	–	0
G	101	–	001	100	101	1

The next-state and output expressions are

$$\begin{aligned} Y_3 &= w_2\overline{y}_2 + \overline{w}_1 y_3 + w_2\overline{w}_1\overline{y}_1 \\ Y_2 &= \overline{w}_1\overline{y}_3 y_1 + \overline{w}_2 y_3\overline{y}_1 + \overline{w}_2 w_1 y_2 + w_2\overline{w}_1\overline{y}_3 y_2 + y_2 y_1 \\ Y_1 &= w_1 y_3\overline{y}_2 + \overline{w}_2 w_1\overline{y}_2 + \overline{w}_2\overline{w}_1 y_1 + \overline{y}_3 y_2 y_1 \\ z &= y_2 y_1 + y_3 y_1 \end{aligned}$$

9.10. The minimum-cost hazard-free implementation is

$$f = \overline{x}_1\overline{x}_3\overline{x}_4 + x_1x_2x_4 + x_1x_3x_4$$

9.11. The minimum-cost hazard-free implementation is

$$f = \overline{x}_1\overline{x}_2\overline{x}_4\overline{x}_5 + \overline{x}_1\overline{x}_2x_3\overline{x}_4 + x_1x_2\overline{x}_3\overline{x}_4 + x_1x_2\overline{x}_4x_5$$

9.12. The minimum-cost hazard-free POS implementation is

$$f = (x_1 + x_2 + x_4)(x_1 + x_2 + \overline{x}_3)(x_1 + \overline{x}_3 + \overline{x}_4)(x_2 + \overline{x}_3 + x_4)$$

9.13. The minimum-cost hazard-free POS implementation is

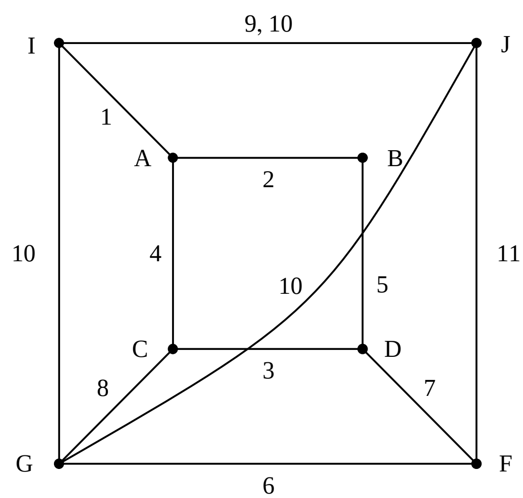
$$f = (x_1 + x_2 + \overline{x}_4 + x_5)(x_1 + x_2 + \overline{x}_3 + \overline{x}_4)(\overline{x}_1 + \overline{x}_2 + x_4)$$

9.14. If $A = B = D = E = 1$ and C changes from 0 to 1, then f changes $0 \rightarrow 1 \rightarrow 0$ and g changes $0 \rightarrow 1 \rightarrow 0 \rightarrow 1$. Therefore, there is a static hazard on f and a dynamic hazard on g.

9.16. The flow diagram in Figure P9.3 meets the vending machine specification if $w_2 = D$ and $w_1 = N$. Therefore, the reduced flow table is the same as the same as the answer to Problem 9.3. The relabeled flow table is

Present state	Next state DN=00	01	10	11	Output z
A	1	2	4	–	0
B	5	2	–	–	0
C	8	3	4	–	0
D	5	3	7	–	0
F	11	6	7	–	0
G	8	6	10	–	0
I	1	9	10	–	1
J	11	9	10	–	0

The transition diagram is



A suitable state assignment is: $A = 000$, $B = 001$, $C = 010$, $D = 011$, $F = 111$, $G = 110$, $I = 100$, and $J = 101$. Then the state-assigned table is

	Present state	Next state				Output
		DN=00	01	10	11	
	$y_3y_2y_1$	$Y_3Y_2Y_1$				z
A	000	000	001	010	–	0
B	001	011	001	–	–	0
C	010	110	010	010	–	0
D	011	011	010	111	–	0
F	111	101	111	111	–	0
G	110	110	111	100	–	0
I	100	000	100	100	–	1
J	101	101	100	100	–	0

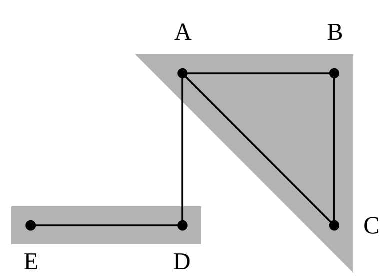
The next-state and output expressions are

$$
\begin{aligned}
Y_3 &= Dy_3 + Ny_3 + y_3y_1 + y_3y_2 + Dy_1 + \overline{D}y_2\overline{y}_1\overline{N} \\
Y_2 &= D\overline{y}_3 + \overline{y}_3y_2 + \overline{N}\overline{y}_3y_1 + Ny_2 + Dy_2y_1 + \overline{D}y_2\overline{y}_1 \\
Y_1 &= \overline{N}\overline{y}_3y_1 + \overline{D}y_1\overline{N} + N\overline{y}_3\overline{y}_2 + Ny_3y_2 + \overline{y}_3\overline{y}_2y_1 + y_3y_2y_1 \\
z &= y_3\overline{y}_2\overline{y}_1
\end{aligned}
$$

9.17. A possible Moore-type flow table is

Present state	Next state				Output
	$wc = 00$	01	10	11	z
A	A	B	D	–	0
B	A	B	–	C	0
C	–	B	D	C	0
D	A	–	D	E	0
E	A	E	D	E	1

A merger diagram for this flow table is



Merging states A, B and C into a new state A, and states D and E into a new state E, gives the Mealy-type flow table

Present state	Next state				Output z			
	$wc = 00$	01	10	11	00	01	10	11
A	A	A	D	A	0	0	0	0
D	A	D	D	D	0	1	0	1

Then, the excitation table is

Present state y	Next state				Output			
	$wc = 00$	01	10	11	00	01	10	11
	Y				z			
0	0	0	1	0	0	0	0	0
1	0	1	1	1	0	1	0	1

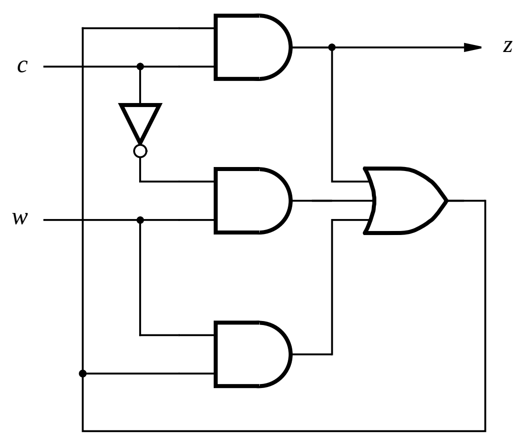
The next-state expression is

$$Y = w\overline{c} + cy + wy$$

Note that the term wy is included to prevent a static hazard. The output expression is

$$z = cy$$

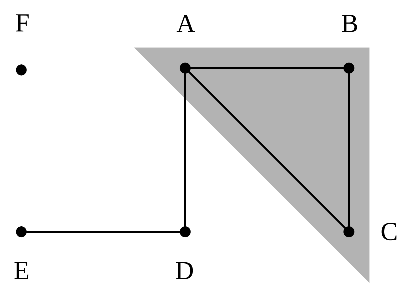
The resulting circuit is



9.18. A possible Moore-type flow table is

Present state	Next state				Output
	$wc = 00$	01	10	11	z
A	A	B	D	–	0
B	A	B	–	C	0
C	–	B	D	C	0
D	A	–	D	E	0
E	A	E	F	E	1
F	A	B	F	F	0

The corresponding merger diagram is



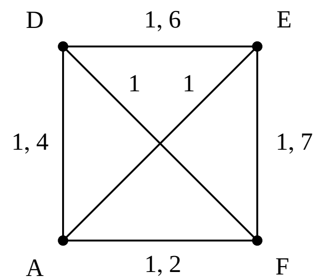
Merging rows A, B, and C into a new row A gives the reduced flow table

Present state	Next state				Output z
	$wc = 00$	01	10	11	z
A	A	A	D	A	0
D	A	–	D	E	0
E	A	E	F	E	1
F	A	A	F	F	0

To determine a suitable state assignment, relabel the flow table as follows:

Present state	Next state				Output
	$wc = 00$	01	10	11	z
A	1	2	4	3	0
D	1	–	4	6	0
E	1	5	7	6	1
F	1	2	7	8	0

The transition diagram is



The flow table is
Both diagonal transitions, under the label 1, can be omitted because there exist alternate paths along the edges for this label. Let the transition from E to A take place via D. Then, a possible state assignment is $A = 00$, $D = 01$, $E = 11$, and $F = 10$, which leads to the excitation table:

	Present state	Next state				Output
		$wc = 00$	01	10	11	
	y_2y_1	Y_2Y_1	Y_2Y_1	Y_2Y_1	Y_2Y_1	z
A	00	00	00	01	00	0
D	01	00	–	01	11	0
E	11	01	11	10	11	1
F	10	00	00	10	10	0

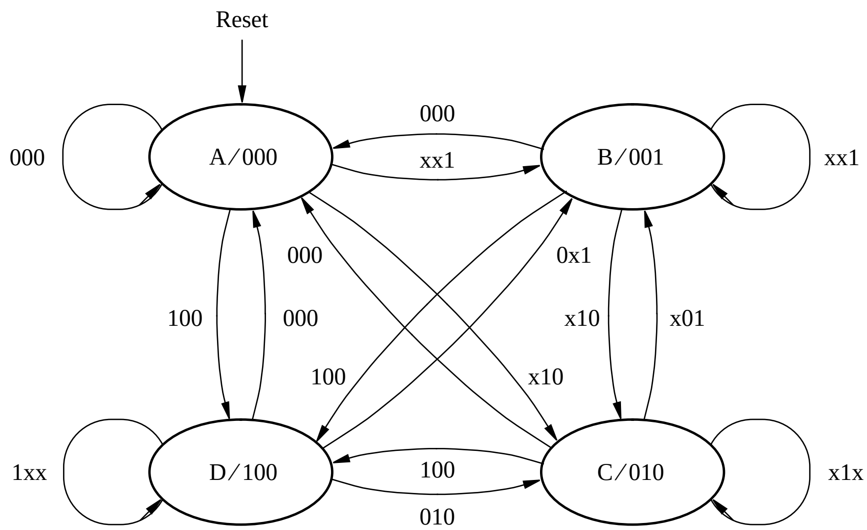
The resulting next-state expressions are

$$
\begin{aligned}
Y_2 &= wy_2 + cy_1 \\
Y_1 &= cy_1 + \overline{w}y_1y_2 + w\overline{y}_2\overline{c} + wy_1\overline{y}_2
\end{aligned}
$$

The product term $wy_1\overline{y}_2$ is included to avoid a static hazard.

The output expression is $z = y_1y_2$.

9.19. A possible state diagram for the three-input arbiter is



9.20. Using the mutual exclusion element, the input valuation $r_2r_1 = 11$ cannot occur. Hence, the flow table is

Present state	Next state				Output
	$r_2r_1 = 00$	01	10	11	g_2g_1
A	A	B	C	–	00
B	A	B	C	–	01
C	A	B	C	–	10

The excitation table is

	Present state	Next state				Output
		$r_2r_1 = 00$	01	10	11	
	y_2y_1	Y_2Y_1				g_2g_1
A	00	00	01	10	–	00
B	01	00	01	10	–	01
C	10	00	01	10	–	10
D	11	–	–	–	–	–

The resulting next state and output equations are

$$
\begin{aligned}
Y_1 &= r_1 \\
Y_2 &= r_2 \\
g_1 &= y_1 \\
g_2 &= y_2
\end{aligned}
$$