HW1 - Logic.

~ 2022.03.10. 3pm

$\boxed{1}$ i) $p \vee (q \wedge r)$ is equivalent to $(p \vee q) \wedge (p \vee r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

ii) $(p \to q) \wedge (q \to r)$ is equivalent to $(p \to r)$

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$$(p \to q) \wedge (q \to r) \to (p \to r)$$

$$= \neg((\neg p \vee q) \wedge (\neg q \vee r)) \vee (\neg p \vee r)$$

$$= \neg(\neg p \vee q) \vee \neg(\neg q \vee r) \vee \neg p \vee r$$

$$= (p \wedge (\neg q)) \vee (q \wedge \neg r) \vee \neg p \vee r$$

$$= ((\underbrace{p \vee \neg p}_{=T}) \wedge (\neg q \vee (\neg p))) \vee ((q \vee r) \wedge (\underbrace{\neg r \vee r}_{=T}))$$

$$= (\neg q \vee (\neg p)) \vee (q \vee r) = (\neg q \vee q) \vee (\neg p) \vee r$$

$$= T \vee (\neg p) \vee r = T$$

iii) Replace "exclusive OR" operator using the 3 connectives. $(\wedge \vee \neg)$

$$p \oplus q = (\text{not}(p) \text{ and } q) \text{ or } (p \text{ and } \text{not}(q))$$
$$= (\neg p \wedge q) \vee (p \wedge (\neg q))$$

$\boxed{1}$ iv) Loves(p1, p2) $\vee$ Loves(p2, p1) : False
(add $P_1 \to P_2$ or $P_2 \to P_1$)

v) $\underbrace{\text{Loves}(p_1, p_1)}_{T} \to \underbrace{\text{Loves}(p_5, p_5)}_{T}$: True

vi) $\underbrace{\text{Loves}(p_1, p_2)}_{F} \to \underbrace{\text{Loves}(p_4, p_3)}_{T}$: True

vii) $\underbrace{\text{Loves}(p_1, p_3)}_{T} \to \underbrace{\text{Loves}(p_3, p_6)}_{F}$: False (add $P_3 \to P_6$)

viii) $\underbrace{\text{Loves}(p_1, p_4)}_{F} \to \underbrace{\text{Loves}(p_4, p_5)}_{F}$: True

ix) $\underbrace{\text{Loves}(p_1, p_4)}_{F} \leftrightarrow \underbrace{\text{Loves}(p_2, p_3)}_{F}$: True

x) $\underbrace{\text{Loves}(p_1, p_3)}_{T} \leftrightarrow \underbrace{\text{Loves}(p_5, p_5)}_{T}$: True

xi) $\forall x. \exists y. \text{Loves}(x, y)$: False (add $P_6 \to$ any P, $P_2 \to$ any P)
: 모든 x에 대해 Loves(x, y)를 만족하는 y가 존재한다.

xii) $\forall x. \exists y. \text{Loves}(y, x)$: False (add any $P \to P_4$, any $P \to P_6$)
: 모든 x에 대해 loves(y, x)를 만족하는 y가 존재한다.

xiii) $\forall x. \exists y. (x \neq y \wedge \text{Loves}(x, y))$: False
(add $P_6 \to$ any P except P_6, $P_2 \to$ any P except P_2, $P_5 \to$ any P except P_5)

xiv) $\forall x. \exists y. (x \neq y \wedge \text{Loves}(y, x))$: False
(add any P except $P_4 \to P_4$, any P except $P_6 \to P_6$, any P except $P_5 \to P_5$)

xv) $\exists x. \forall y. \text{Loves}(x, y)$: False (add $P_3 \to P_3$, $P_3 \to P_4$, $P_3 \to P_5$, $P_3 \to P_6$)
: 어떤 x에 대해 모든 y가 loves(x, y)를 만족하는 x가 존재한다.

xvi) $\exists x, \forall y. (x \neq y \to Loves(x, y))$: False (add $P_3 \to P_4$, $P_3 \to P_5$, $P_3 \to P_6$)

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i) $\bot = \neg T$

ii) $p \to q = \neg p \vee q$

iii) $p \leftrightarrow q = (p \to q) \wedge (q \to p) = (\neg p \vee q) \wedge (\neg q \vee p)$

iv) $p \vee q = \neg(\neg p \wedge \neg q)$

v) $T = \bot \to \bot$

vi) $\neg p = p \to \bot$

p	$\neg p$	$p \to \bot$
T	F	F
F	T	T

vii) $p \wedge q = \neg(\neg p \vee \neg q) = \neg(p \to \neg q) = (p \to \neg q) \to \bot$

$= (p \to (q \to \bot)) \to \bot$

$p \wedge q$	p	q	$q \to \bot$	$p \to (q \to \bot)$	$(p \to (q \to \bot)) \to \bot$
T	T	T	F	F	T
F	T	F	T	T	F
F	F	T	F	T	F
T	F	F	T	T	F

3 negation of the original statement

i) $\neg \forall p. (Person(p) \to \exists c. (Cat(c) \wedge Loves(p, c) \wedge \forall r. (Robot(r) \to \neg Loves(c, r))))$

$\to \exists p. \neg(\to)$

$= \exists p. (person(p) \wedge \neg \exists c. (\quad))$

$= \exists p. (person(p) \wedge \forall c (\neg Cat(c) \vee \neg Loves(p, c) \vee \neg \forall r. (Robot(r) \to \neg Loves(c, r))))$

$= \exists p. (person(p) \wedge \forall c (\neg Cat(c) \vee \neg Loves(p, c) \vee \exists r. (Robot(r) \wedge Loves(c, r))))$

$\forall r.(Robot(r) \wedge Loves(c,r)).$

ii) $\left(\forall x.\left(Person(x) \leftrightarrow \exists r.(Robot(r) \wedge Loves(x,r))\right)\right)$
$\rightarrow \left(\forall r. \forall c.(Robot(r) \wedge Cat(c) \rightarrow Loves(r,c))\right)$

$\not\rightarrow \forall x.(Person(x) \leftrightarrow \exists r.(Robot(r) \wedge Loves(x,r)))$
$\wedge \exists r. \exists c. \neg \left(\left(Robot(r) \wedge Cat(c)\right) \rightarrow Loves(r,c)\right)$
$Robot(r) \wedge Cat(c) \wedge \neg Loves(r,c)$

iii) $\forall c.(Cat(c) \rightarrow \exists r.(Robot(r) \wedge \forall x.(Loves(c,x) \leftrightarrow r = x)))$
$\not\rightarrow \exists c.(Cat(c) \wedge \forall r. \neg(Robot(r) \wedge \forall x.(\quad)))$
$= \exists c.(Cat(c) \wedge \forall r.(\neg Robot(r) \vee \exists x.(Loves(c,x) \leftrightarrow r \neq x)))$

$\boxed{4}$ Use the predicates Person, Robot, Cat and Loves

i) robots do not love

$\forall r.$(r is a robot $\rightarrow$ robot doesn't love every thing)

$= \forall r.$(Robot(r) $\rightarrow$ ¬(robot loves at least one person/cat))

$= \forall r.\left(Robot(r) \rightarrow \neg\left(\exists p.(Person(p) \wedge Loves(r,p)) \vee \exists c.(Cat(c) \wedge Loves(r,c))\right)\right)$

$\Rightarrow \forall r.\left(Robot(r) \rightarrow \left(\forall p.(Person(p) \wedge \neg Loves(r,p)) \wedge \forall c.(Cat(c) \wedge \neg Loves(r,c))\right)\right)$

ii) each robots loves every cat, but no cat loves any person.

all robots loves all cat and all cat not love all person

$\forall r.\left(Robot(r) \rightarrow (\forall c.(Cat(c) \wedge Loves(r,c))\right)$
$\wedge \forall c.\left(Cat(c) \rightarrow (\forall p.(Person(p) \wedge \neg Loves(c,p)))\right)$

iii) each cat only loves itself. = all cats love itself.

$\forall c.(Cats(c) \wedge Loves(c,c))$

iv) if you pick a person, you'll find that they love a cat if and if only they also love a robot.

$\leftrightarrow$

= all person love cat $\leftrightarrow$ love robot

$\forall p. (Person(p) \rightarrow (\exists c. (Cat(c) \wedge Loves(p,c)) \leftrightarrow \exists r. (Robot(r) \wedge Loves(p,r))))$

v) each person loves exactly two cats and nothing else. (can be different cat)

c_1, c_2

$$\forall p. \Big(Person(p) \rightarrow \exists c1. \exists c2. (Cat(c_1) \wedge Cat(c_2) \wedge Loves(p,c_1) \wedge Loves(p,c_2) \wedge (c_1 \neq c_2))$$
$$\wedge (\forall x. Cat(x) \wedge Loves(p,x) \rightarrow (c_1 = x \vee c_2 = x)$$
$$\wedge (\forall r (Robot(r) \wedge \neg Loves(p,r)))$$

vi) no two robots love exactly the same set of cats

$$\neg \Big(\exists r1. \exists r2 (Robot(r1) \wedge Robot(r_2) \wedge (r_1 \neq r_2))$$
$$\rightarrow \exists c. (Cat(c) \wedge Loves(r_1, c) \wedge Loves(r_2, c)) \Big)$$