



# A hybrid model approach for forecasting future residential electricity consumption



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## ABSTRACT

Urban energy management nowadays has put more focus on residential houses energy consumption. Lots of machine learning based data-driven approaches have the abilities to characterize and forecast total energy consumption of commercial data. However, a paucity of research applying data-driven methods have been tested on the hour ahead energy consumption forecast for typical single family houses in the US. With the advances in smart metering, sub meter usages forecast in household-level is also getting more and more popular on smart building control and demand response program. The situation here inspires us to develop a hybrid model to address the problem of residential hour and day ahead load forecasting through the integration of data-driven techniques with a physics-based model. In this article, we report on the evaluations of five different machine learning algorithms, artificial neural network (ANN), support vector regression (SVR), least-square support vector machine (LS-SVM), Gaussian process regression (GPR) and Gaussian mixture model (GMM), applied to four residential data set that contains smart meters. Both total and non-air conditioning (AC) power consumption are forecasted for hour ahead and 24-h ahead. The variation of patterns captured from non-AC part is further input as internal heat gain to a physics-based model. The model uses a 2R-1C thermal network and an AC regression model. By utilizing this hybrid approach, we get AC load prediction and non-AC energy forecast simultaneously. Total energy consumption is further produced by summing up the two sub meters forecast for hybrid model. The results from new modeling are compared with those from pure data-driven techniques. The final result showing improvements of coefficient of variance between the best data-driven model and hybrid model are 6–10% and 2–15% for hour ahead and 24-h ahead, respectively.

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## 1. Introduction

The US residential sector is currently responsible for an estimated 22% of the country's primary energy consumption [1]. Although the energy efficient technologies are developing fast, from 1990 to 2009, energy consumption in residential buildings still increased by 24% [2]. By 2040, residential energy consumption is projected to grow to 19% with all new energy efficient equipment, appliances and technologies [1]. State of the art building energy modeling tools play a key role during the decision making process regarding the implementation of energy conservations measures, for example, determining cost-effective retrofit equipment and optimizing energy systems for a building. Currently, there are

two main categories for methods according to ASHRAE: "Forward" and "Data-Driven" [3]. The "Forward" modeling approach takes input from weather data, building geometry, and building envelop, construction materials, equipment and operation schedules, and estimates the building heating and cooling load, and total building energy consumption based on those inputs. There are more than 100 building simulation programs based on this approach including DOE-2, TRNSYS, EnergyPlus, and most recently Modelica [4]. The fundamental principles of the "Forward" models are based on the theory of heat transfer and thermodynamics, and have been developed for many years while "Data-Driven" models rely more on the data, assuming there is already a mathematical relationship between inputs (e.g. weather parameters) and outputs (e.g. total building energy consumptions). This type of model includes, for example change-point models, thermal network models and various statistical models [3]. To further categorize the "Data-Driven" approach, there are "black-box" and "grey-box" models. The "black-box" model is purely machine learning driven and derived by

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### List of symbols

$S_i$	solar radiation absorbed by the $i$ th wall [J]
$S_f$	solar radiation absorbed by floor [J]
$C_z$	heat capacity of zone [J/°C]
$C_g$	heat capacity of ground floor [J/°C]
$C_w$	heat capacity of wall [J/°C]
$T_o$	ambient air temperature [°C]
$T_z$	zone air temperature [°C]
$T_f$	floor center temperature [°C]
$T_d$	deep soil temperature [°C]
$T_i$	the $i$ th wall center temperature [°C]
$\varepsilon_f$	solar absorptivity of the floor
$Q_{\text{int}}$	Internal convective heat gain [W]
$Q_{\text{ac}}$	HVAC system cooling or heating load [W]
$F_r$	Fraction of rated full load flow [W]
$L_t$	Load at time $t$ [W]
$R_{\text{in}}$	Half of wall thermal resistance with the convection resistance along the inside surface [k/W]
$R_{\text{ex}}$	Half of wall thermal resistance with the convection resistance along the outside surface [k/W]
$R_f$	Half of floor thermal resistance [k/W]
$R_s$	Deep soil thermal resistance [k/W]
$A$	wall or window surface area [m <sup>2</sup> ]

measured data. Recently, a lot of commercial energy consumption forecasting studies based on existing sensor data and those data mining technologies focus on the development of efficient and effective black-box models [5]. Previous studies on energy consumption forecasting using “black-box” models are limited in one type of methods such as neural networks. The predictions made are usually at one time step ahead, e.g. hourly. Commercial buildings are used as cases where energy patterns are smoother. However, there are very few studies on residential house energy consumption forecasting. The “Grey-box” model is built upon physical relationships, while the parameters of the physical model are usually unknown or uncertain. The measured data is used to identify those parameters and model tuning for better accuracy. The “Grey-box” models have been applied widely to estimate commercial building heating and cooling load [6], again, not many on the residential building sector.

The research gap between commercial and residential building comes from two main sources, which are (a) the lack of hourly or more granular data, and (b) the growing percentage of miscellaneous electrical load (MEL) in residential building, respectively. While MEL is approximately 15–25% of a typical home’s energy use, it is projected to increase by 52% by 2040 [1]. Unfortunately, for residential buildings none of the above models can work as good as, if not worse than commercial building because MEL is not weather related, and occupies a large portion of total building energy consumption. In this paper we try to close this gap by developing a new hybrid modeling approach through integrating “forward” and “data-driven” models to forecast hour ahead and 24 h ahead residential electrical load. In the first place, we conduct a comprehensive literature review on current state of the art load forecasting in residential buildings. Furthermore, we provide a technical review of five different modeling methods, following by a proposed new approach. We described our experimental test-bed, training and testing periods, and compared results from six models. Finally, we conclude this paper by discussions, limitation of this study and our future research directions.

## 2. Current state of the art

### 2.1. Data-driven models

Data-driven models using machine learning algorithms are explored a lot by researchers for building energy consumptions forecast. One common approach called artificial neural networks (ANN) is heuristic inspired by the structure of biological neural networks. For ANN, it is applied in industrial and commercial building energy models such as estimating the space heating energy consumptions from surveys, forecasting annual energy consumption of industrial sectors, predicting office building energy consumption in subtropical climates, and predicting cooling load of public buildings [7–10]. Most successful ANN types are feedback neural network and feedforward neural network in forecasting [5,11,12]. The success of ANN provides strong incentive for exploring how effective ANN is when hybridized with other techniques to improve forecast ability. Our use of this method is discussed in more details in Section 3.1.

Another method, support vector machines (SVM) was first introduced based on statistic learning theory and structural risk minimization [14,16]. SVM can be easily extended to perform regression analysis with careful chosen approximation function. Support Vector Regression (SVR) is effectively used in forecasting building electricity consumption, cooling load for HVAC system [17,18], etc. An advantage of SVR is its insensitivity to outliers [15]. Although one drawback of SVR was its computational inefficiency, various optimization techniques such as genetic algorithm, self-organized map, and ant colony optimization [16–18] are attempted to improve the computation speed. However, the difficulty of integration with these tools with SVM model itself prohibited a lot of potential users. In Section 3.2, SVR is briefly described and discussed.

A modified SVM using least square optimization (LS-SVM) shows a simple way to reduce the computation burden. LS-SVM solves a linear system under least square cost function with equality constraints [19]. Although LS-SVM itself already outruns the traditional SVM, it can be further integrated with the advanced optimization techniques such as quantum-behaved particle swarm optimization and Bayesian framework [20,21]. Section 3.3 provides more details on LS-SVM application in our study.

For ANN and SVM, uncertainty analysis needs to be additionally conducted. Alternatively, forecast models developed from Gaussian distribution can solve the problem. For Gaussian family models, such as Gaussian Process Regression (GPR) and Gaussian Mixture Model (GMM), they have Bayesian estimation or similar uncertainty formulation component inherited from models themselves [22]. GPR is capable of forecasting weekly and annual load of domestic, commercial and industrial buildings [23]. Some studies also concluded that the GPR method can outperform SVR for one-step ahead daily maximum load forecast [24]. However, the matrix inversion of large dimension training data set during forecasting makes this model computationally expensive. Section 3.4 discusses GPR in greater detail.

Like GPR, GMM also provides a localized confidence interval calculation in addition to energy consumption prediction. For GMM, prediction and uncertainty is calculated simultaneously from conditional probability density of GMM joint likelihood function. GMM has been applied to commercial buildings for hourly and daily load prediction. One such study is on the DOE reference model (supermarket synthetic load) and field load for a retail store located in California. Authors had shown the accurate training and prediction at baseline load [25]. For residential houses, GMM was tested by an intra-hour resolution estimation with multiple residential properties [26]. In larger scale, a similar cluster study produced the daily

load profile using 500 households in a 2 years period [27]. Further implementations in our study are explained in Section 3.5.

## 2.2. Forward models

Forward models use laws of thermodynamics. There are two main parts of a model: building thermal zone envelopes and air conditioning modeling. For simplicity, most forward models integrate all physical phenomena (convection, conduction, and radiation) to one layer, and all layers (walls, floor, roof, etc.) to a thermal envelope zone. Thus, the envelope of the thermal zone in a building can be constructed as thermal resistance-capacity (RC) networks. For state space expression, RC networks are a set of linear ordinary differential equations (ODEs). A complete dynamic model of a multi-zone building can be expanded by linking all the linear ODEs representing high order RC networks.

Current dominant energy simulation tools are mostly software such as EnergyPlus for design purpose, retrofit demonstration, and energy estimation. They provide house templates developed on high order RC networks which meet various standards such as ASHRAE 90.1. However, huge efforts are required to manually calibrate the models for onsite building samples. Usually, lower order RC networks categorized by 3R-2C and 2R-1C are more popular in real building energy forecast and control [28].

The lowest order network, such as 1R-1C, is limited to a well-insulated room with fewer heat sources [29]. The higher order network, such as 4R-3C, does not provide significant improvement on temperature response prediction [30]. 3R-2C model is suitable for estimation and prediction of air conditioning consumption in a retail store, campus buildings, and a skyscraper [6,31,32]. Though many argue 2R-1C is less accurate in heating or cooling load prediction, it is not necessarily true for standard tests or small buildings [33,34]. However, it is rarely studied how this will affect the total building energy consumption forecasting when a physics model is integrated with other techniques. The application of 2R-1C thermal network in the hybrid approach is described in Section 3.7.1.

In energy forecast applications, forward model suffers from a lack of building information. Some parameters of RC network, such as thermal resistances and capacities, require estimations from onsite data. In time domain, parameters can be estimated using regression models [35,36]. More recently, extended Kalman Filter (EKF) extracting information directly from input/output data of the state space form of thermal networks is carried out to specify the lumped parameters. It has been successfully utilized in small office buildings by self-adaptive EKF to identify parameters in 1R-1C, 2R-1C, and 3R-2C network [37]. The solutions for these concerns in hybrid approach are discussed in Section 3.7.2.

## 2.3. Gaps and limitations

Most forward and data-driven models focus on non-residential buildings but rarely on residential dwellings. Being aware of the situation, some attempts to forecast residential houses energy consumptions using traditional ways results in noncompetitive outcomes [5]. Others improve the forecast by examining the impact of temporal and spatial granularity. Meanwhile, such kinds of investigation lead to lots of on-site sensors which are normally impossible to get for utility companies [38]. Furthermore, many tests involved in data-driven methods are only conducted on total building energy consumption forecast. They lack capability to predict important meter such as air conditioning that occupies a large portion in residential usage. In contrast, forward models are limited to thermal usage prediction such as air conditioning. For onsite buildings, they are restricted either by the availability of building information or by measurements such as internal load profile.

Our research proposes a hybrid approach, making use of forward and data-driven models to forecast both total and air-conditioning (AC) energy consumptions. By isolating the sub-meter AC usage, abnormal changes in energy consumption pattern is recognized. Improvement of accuracy can be analyzed by comparing pure data-driven model results. This method combines the advantage of the black box approach using data-driven methods. It overcomes the limits in forward model by adaptation of internal heat gain with data-driven non-air conditioning forecast.

## 3. Methodology and approach

Our primary objectives are to assess whether hybrid modeling approach for energy consumption forecasting is feasible and comparable to traditional data-driven models. Five data-driven models (ANN, SVR, LS-SVM, GPR, and GMM) are first built for forecasting the total and non-AC energy consumptions. One method, LS-SVM, is further selected to hybridize with a forward model to produce new total energy consumption prediction. Results are compared to previous pure data-driven forecasts using our residential data collected in San Antonio. The section is organized as follows. Section 3.1 provides test housing information and performance metrics utilized to assess the model's accuracy. Data-driven model structures, parameter selection, and software modules used are discussed from Sections 3.2–3.6. Section 3.7.1 demonstrates how to predict AC power consumption by the forward model. Section 3.7.2 demonstrates a hybrid diagram linking data-driven method and forward model for total building energy consumption forecast.

### 3.1. Test bed and performance metrics

The residential data in our research was collected from four residential houses in San Antonio as shown in Fig. 1. Those four samples are single family dwelling around 110 m<sup>2</sup> each. Houses are named according to construction materials: SIP (Structure Insulated Panel), ACC (Autoclaved Aerated Concrete), Container, and Stick. For material information, calibrated thermal resistances and capacities are provided by manufacturers (e.g. 119 mm SIP U-value is 0.22 W/m<sup>2</sup>K). Energy consumptions are monitored at 5 min intervals for all the rooms including kitchen, bathroom, living and bedroom areas. Sub-metered data such as plugs, lighting, water heater and AC are also available. Outdoor air temperature and global horizontal solar radiation are collected from a weather station near the position. Solar radiations on the surface of each house at the same period are measured from sensors installed on site.

Hour ahead and 24-h ahead models are evaluated using performance metrics: mean absolute percentage error (MAPE), and coefficient of variation (CV) of root mean square error. These criteria are extremely popular and regarded as the standard measure for comparing model performance [5]. We consider MAPE as the primary measure for comparing the models. The MAPE measure determines the percentage of error per prediction

$$\text{MAPE}(\%) = \frac{1}{N} \sum_{i=1}^N \frac{|y_i - y_p|}{y_i} \times 100 \quad (1)$$

where  $y_i$  is the actual energy consumption,  $y_p$  is the predicted energy consumption,  $N$  is the total number of predictions. The smaller the MAPE to be calculated, the better the forecast is.



Fig. 1. Four test houses.

The second measure, CV, determines the variation of overall prediction with respect to the target's mean:

$$CV(\%) = \frac{\sqrt{\frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{y})^2}}{\bar{y}} \times 100 \quad (2)$$

where  $\bar{y}$  is average actual energy consumption. The smaller CV became, the more similar dispersions are between the forecast and the true value.

### 3.2. Artificial neural networks

A specific type of ANN, feed forward neural network (FFNN), is used in our study [7]. We explored both single hidden layer and double hidden layer FFNN. However, the structure of double hidden layer is abandoned for overfitting problem. This observation matches conclusion regarding hidden layer selection by Schenker [12]. A single layer model to approximate the nonlinear function in FFNN is expressed by the following equation:

$$f(x) = \sum_{j=1}^N w_j \phi_j \left[ \sum_{i=1}^M w_{ij} x_i + w_{io} \right] + w_{jo} \quad (3)$$

where  $w$  is the weights for input, hidden, and output layer,  $x$  is the training input,  $N$  represents the total number of hidden units,  $M$  represents the total number of inputs, and  $\phi$  represents the learning function for each hidden unit.

We applied sigmoid and linear transfer functions for the input layer and the output layer, respectively. Hidden layer function weights in Eq. (1) is learned from Levenberg–Marquardt back-propagation algorithm [39]. To select a suitable number of neurons, a hold-out test is performed for parameter tuning based on 75% of input data for training and rest of 25% for testing. The parameter tuning process stops when the hold-out set's mean square error is minimized. We discover a single hidden layer with more than 15 neurons are capable enough to properly forecast for all samples. After parameter tuning, forecasting is made by moving window techniques with a fixed size training input [13], where a fixed previous 1 month data is used.

### 3.3. Support vector regression

For SVR, a data set is firstly transformed into a high dimension space. Predictions are discriminated from training data as a “tube” enclosed with a desired pattern curve with certain tolerances. The support vectors are the points which mark the margins

of the tube. They are learned through the quadratic minimization of the equivalent Lagrangian form. The optimization function is:

$$\begin{aligned} \min & \frac{1}{2} w^T \cdot w + C \sum_{i=1}^n (\xi_i + \xi_i^*) \\ \text{s.t. } & y_i - w^T \varphi(x_i) - b \leq \varepsilon + \xi_i \\ & w^T \varphi(x_i) = b - y_j \leq \varepsilon + \xi_i^* \end{aligned} \quad (4)$$

where  $w$  is the weights for regression,  $\xi$  is the error slacks guaranteeing the solutions,  $C$  is the regularized penalty,  $\varepsilon$  defines the desired tolerance range of the “tube” and  $\varphi$  is the kernel function. The kernel function implicitly computes the high dimension mapping according to the Mercer's theorem [19]. The commonly used choices are linear, polynomial, and radial basis function (RBF) [16,40]. By literature review, RBF with weight parameter  $\gamma$  tends to give good performance by smoothness. Hence, LIBSVM [41] using RBF is implemented in this study. We find the SVR model is relatively insensitive to the value of  $\varepsilon$  smaller than 0.01 whereas both  $C$  and  $\gamma$  necessitate independent tuning. These parameters are determined via grid search with 10 fold cross-validation based on mean square error. The grid search scale for  $C$  and  $\gamma$  is maintained between  $10^{-3}$  and  $10^3$ .

### 3.4. Least squares support vector machines

LSSVM, like SVR, is modeled on a high dimension space. The main difference is the constraints are changed from inequality to equality. Then, the simplified cost function can be optimized by least square method:

$$\begin{aligned} \min & \frac{1}{2} w^T \cdot w + C \sum_{i=1}^n e_i^2 \\ \text{s.t. } & y_i = w^T \varphi(x_i) + b + e^i \end{aligned} \quad (5)$$

Here, we have fewer parameters ( $C$  and  $\gamma$ ) to be optimized than SVR. LS-SVMlab Toolbox [42] is applied. The parameters are tuned by Section 3.3's grid search algorithm. Though LS-SVM find solution much faster than quadratic form of SVM, it has forecast drawback countering large input [43]. Since LS-SVM uses full set of input, the sparsity of pattern is loosed. Hence, for LS-SVM, hour ahead forecast uses at most 1 week long training data while the 24-h ahead forecast uses at most 1 month of training input.

### 3.5. Gaussian process regression

GPR uses mean and covariance function enumerated by a set of hyper-parameters to regress training data. A popular choice of



covariance function is the squared exponential function [23,24]. As long as training data are from same measurement, GPR treats each observation as an equivalent Gaussian noise model. By imposing noise models to a squared exponential function, the covariance function is defined as:

$$f(x, x') = \sigma_f^2 \exp \left[ \frac{-(x - x')^2}{l^2} \right] + \sigma_n^2 \delta(x, x') \quad (6)$$

where  $\sigma_f^2$  is the maximum allowable covariance,  $\sigma_n^2$  is the deviation defined in Gaussian noise model,  $l$  is the covariance weight. The reliability of the forecast depends on how well the covariance hyper-parameters  $l$ ,  $\sigma_f^2$  and  $\sigma_n^2$  are selected.

The optimization of hyper-parameters is a non-convex problem. Exact inference for a GP with Gaussian likelihood is used in this study. The algorithm minimizes negative log marginal likelihood and its derivatives with respect to the hyper-parameters. By utilizing GPML Toolbox [44], parameter evaluations are iterated 100 times with initialization 0.1 for all hyper parameters. One disadvantage of GPR is that the dimension of the covariance function is dependent on the training data points used. Because the covariance matrix must be inverted in order to make a prediction, it makes GPR models more computationally expensive than other models [27].

### 3.6. Gaussian mixture models

GMM uses mixed joint Gaussian distributions to train the model. It is defined as a weighted sum of  $M$  Gaussian distributions:

$$p(x; \theta) = \sum_{m=1}^M \pi_m \phi(x; \mu_m, \sigma_m^2) \quad (7)$$

$$s.t. \sum_{m=1}^M \pi_m = 1, \pi_m > 0$$

where  $\pi_m$  is mixture weight,  $\phi$  is a Gaussian distribution function,  $\mu_m$  and  $\sigma_m^2$  are the mean and variance.

There are two steps to obtain the GMM with unknown  $\mu_m$ ,  $\sigma_m^2$ , and  $M$ : parameter estimation and component number selection. The parameters estimation of  $\mu_m$  and  $\sigma_m^2$  are obtained by the maximum likelihood method, maximizing the joint likelihood of observations from a training data. The algorithm used is Expectation-Maximization (EM) [45]. The EM iterative update stops at threshold at  $10^{-5}$ . For the component number selection, parameters learned from a batch of models with different numbers of components are compared using Bayesian Information Criteria (BIC) in Eq. (8). Then the optimal one is chosen by penalizing over complexity for a data population of size  $N$ .

$$\text{BIC}(x, \theta) = -\sum_{n=1}^N \log P(x_n | \theta) + \frac{L}{2} \log N \quad (8)$$

where  $L$  is the number of free parameters determined by number of component and dimension of repressors.

### 3.7. Hybrid model

#### 3.7.1. Forward model

The thermodynamic modeling of a residential building can be divided into a system part (AC model) and a building part (RC thermal networks of zone model). Consider the house shown in Fig. 1, it is separated by 2R-1C wall model from the outside environment, denoted by  $T_o$ . as shown in Fig. 2. Heat transfer from infiltration and window are expressed as single  $R$  connecting zone temperature node. Direct heat sources are heat exchange  $Q_{ac}$  from AC, the solar irradiance on wall and floor ( $S_i$  and  $S_f$ ), and internal heat gain

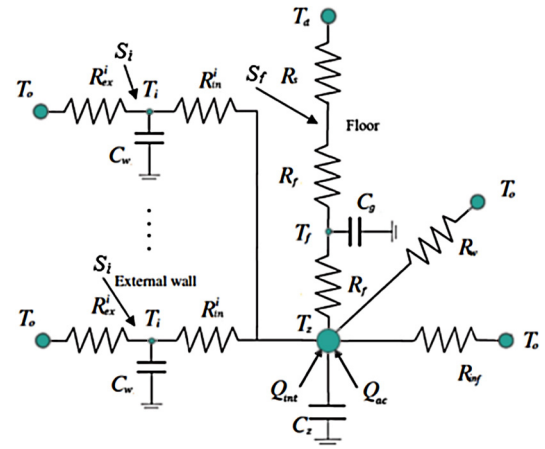


Fig. 2. A RC-network model for thermal zone.

$Q_{int}$ . For ease of description, zones interactions between each other are not included in the figure example since a reduced order model with “super-zone” is further developed [46]. Residential houses typically have fewer rooms and only one thermostat. So the parameters and key temperature nodes of the original RC network can be preserved using one aggregated super zone.

For a wall's 2R-1C thermal network, the heat balance equation is given by:

$$C_w T_i' = - \left( \frac{1}{R_{in}^i} + \frac{1}{R_{ex}^i} \right) T_i + \frac{T_o}{R_{in}^i} + \frac{T_z}{R_{ex}^i} + S_i \quad (9)$$

where  $C_w$  is the total thermal capacity of the  $i$ th wall,  $R_{in}^i$  and  $R_{ex}^i$  is half of the conduction resistance of the  $i$ th wall with the convective resistance along the surface side (internal and external surface convective resistance, respectively),  $T_i$  is center wall temperature of the  $i$ th wall,  $T_o$  is the outdoor air temperature,  $T_z$  is the thermal zone temperature, and  $S_i$  is the total solar heat gain absorbed by the external and internal surfaces of the  $i$ th wall.

For  $S_i$ , solar radiation absorbed by external surfaces can be measured from solar sensors installed on site. For internal surfaces, an assumption is made that total solar radiation transmitting through windows first hits the floor and then is reflected to all other surfaces without multiple further reflections. The solar distribution factors are area-weighted. Each surface has the same solar absorptivity except floor. Using the assumption, the solar heat flux on the given interior  $i$ th surface is computed as:

$$S_{int}^i = (1 - \varepsilon_f) \cdot S_{ext} \cdot \frac{A_i}{\sum_{i=1}^n A_i} \quad (10)$$

where  $S_{int}^i$  is the internal solar heat flux absorbed by the  $i$ th surface,  $\varepsilon_f$  is the solar absorptivity of the floor,  $S_{ext}$  is the total solar radiation transmitted from the exterior to the thermal zone through windows, and  $A_i$  is the  $i$ th surface area.

For the roof, the same calculation is applied. However, the floor model is slightly different. It is treated as a more complex model in ASHRAE Standard 140 [47].

$$C_g T_f' = - \left( \frac{1}{R_f} + \frac{1}{R_f + R_s} \right) T_f + \frac{T_z}{R_f} + \frac{T_d}{R_f + R_s} + S_f \quad (11)$$

where  $C_g$  is the thermal capacity of the ground floor and the soil thermal mass,  $R_f$  is half of the thermal resistance of the ground floor,  $R_s$  is the deep soil thermal resistance where an assumption of ground depth should be made,  $T_f$  is floor temperature in the center,

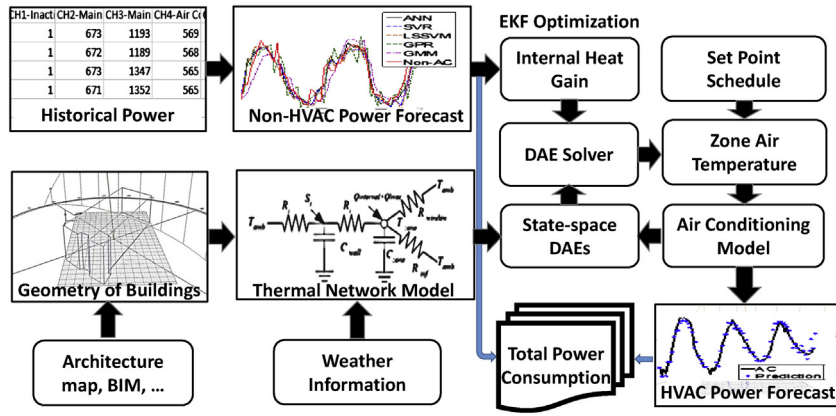


Fig. 3. Hybrid approach.

$T_d$  is the deep soil temperature, and  $S_f$  is the solar absorbed by the ground floor.

By calculating all needed inputs, the zone temperature governing equation is:

$$C_z T'_z = - \left( \sum_{i=1}^n \frac{1}{R_{in}^i} + \frac{1}{R_w} + \frac{1}{R_{inf}} + \frac{1}{R_f} \right) T_z + \sum_{i=1}^n \frac{T_i}{R_{in}^i} + \frac{T_o}{R_w} + \frac{T_o}{R_{inf}} + \frac{T_g}{R_f} + Q_{int} + Q_{ax} \quad (12)$$

where  $R_w$  is the total window resistance,  $R_{inf}$  is the infiltration resistance,  $Q_{int}$  is the internal heat gain due to occupancy (body heat, appliances usages, light, etc),  $Q_{ac}$  is AC cooling load.

For  $Q_{ac}$ , a regression model describes AC “on” stage performance. Two performance curves are formulated: total cooling capacity temperature curve and total cooling capacity air flow curve. The temperature curve is a biquadratic function with two independent variables: outside air temperature and return zone air temperature change. The flow curve is a quadratic function with single variable: air flow rate (fraction of rated full load flow). AC load is calculated by:

$$Q_{AC} = (a + b \cdot T_o + c \cdot T_o^2 + d \cdot dT_{zone} + e \cdot dT_{zone}^2 + f \cdot T_o \cdot dT_{zone}) \cdot (g + h \cdot F_r + i \cdot F_r^2) \quad (13)$$

where  $T_o$  is the outside air temperature,  $dT_{zone}$  is the change of return zone air temperature,  $F_r$  is the rate of actual flow comparing to full load flow, and parameters  $a, \dots, i$  need to be estimated. To obtain the parameters, two stages “on” and “off” of AC load are recognized at 5 min resolution after low pass filter eliminates the transition period. Then, the ordinary least squares are used to fit only “on” stage data.

### 3.7.2. Hybrid modeling approach

As shown in Fig. 3, there are two major steps in the hybrid modeling approach.

First, we forecast non-AC electricity consumption from historical non-AC consumption information. As internal heat gain is highly correlated with non-AC electricity consumption, it can be directly estimated from the non-AC prediction using heat convection and conduction. Secondly, predicted weather together with the internal heat gain forecast are inputted to the thermal network differential algebraic equations (DAEs) to simulate zone temperature. Then, the calculated zone temperature change is updated to an AC regression model with a set point schedule. Afterward, the AC cooling power consumption is further predicted. By summing up the AC and non-AC predictions, we have a total electricity consumption.

For the non-AC forecast, the target is the non-AC energy consumption standing for plug load, lighting, water heater etc. The hour ahead input set, defined as H1, is used to forecast the non-AC load  $L_t$  at hour. The features including the time information and the historical non-AC electricity consumption are:

$$H1 : f(t, L_{t-1}, L_{t-2}, \dots, L_{t-5}) \quad (14)$$

where  $L_{t-1}$  represents the historical non-AC electricity consumption for the previous 1 h,  $\dots$ , and  $L_{t-5}$  represents for the previous 5 h. The 24-h ahead input set, defined as H2, is used to forecast the next 24-h load  $L_t$  to  $L_{t+23}$ . For one future time load  $L_t$  of the next 24-h window, the required input features are:

$$H2 : f(t, L_{t-24}, \dots, L_{t-31}, \text{avg}(L_{t-24}, L_{t-25}), \text{avg}(L_{t-24}, L_{t-25}, L_{t-26}), \dots, \text{avg}(L_{t-24}, \dots, L_{t-31})) \quad (15)$$

For the hour ahead forecast case, input shows that the features need previous 1–5 h historical load information. By comparison, the 24-h ahead case needs not only previous 24–31 h historical load but also moving averages defined in Eq. (15). The input feature time window is limited to these ranges, since the entropies of the information will increase when using too much historical information for one future load prediction. That will cause a well trained model to have a poor forecast performance.

For the AC forecast, a close loop simulation using the feedback of updated zone temperature from Eq. (12) calculates the cooling load. Given initial conditions of three unknown set  $T_i$ ,  $T_{zone}$  and  $Q_{ac}$  in Eq. (9) and Eq. (14), the change of  $T_{zone}$  is updated with the following predicted inputs: (i) internal heat gain, (ii) solar radiation, and (iii) outdoor air temperature.  $Q_{ac}$  is calculated from the AC model described in Eq. (13). For each iteration, the load input  $Q_{ac}$  will be substituted by previous forecasted value. During the simulation,  $T_z$  should be maintained at the set point temperature after the transient state. An optimization process, extended Kalman filter (EKF), is utilized to optimize the total heat transfer coefficients if the steady state  $T_z$  does not follow the observed track. How to model and solve the dual problem (optimizing parameter while using estimation of internal heat gain) in EKF has been previously studied [48]. After the calibrated cooling and heating load calculations, the electricity consumption of the AC system can be found, as expressed by:

$$P = \frac{m \sum_{i=1}^n q_i}{\text{COP}} \quad (16)$$

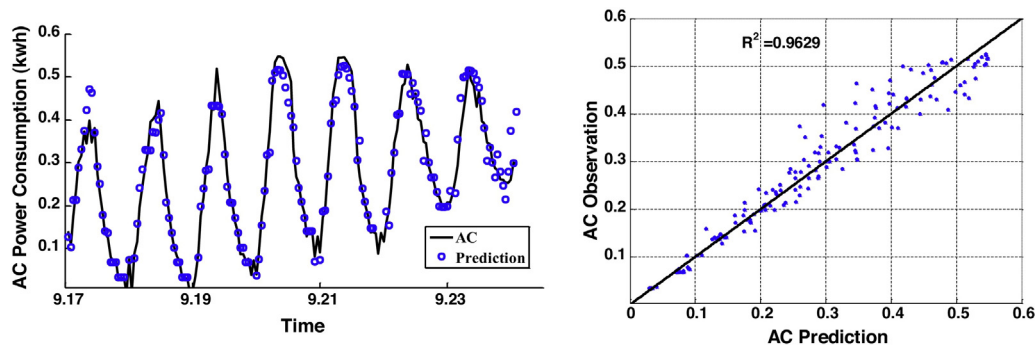


Fig. 4. a) AC power consumption forecasting of SIP house.

b) MAPE.

where  $P$  is the electricity consumption forecast,  $q_i$  is the minutes resolution AC cooling or heating load in the forecast time window, COP is the coefficient of performance, and  $m$  is the time scale (if  $q$  is 5 min temporal load,  $m$  is 1/12).

#### 4. Results and discussions

Our experimental results of forecasting are organized in the following order: performance of AC energy consumption, performance of non-AC energy consumption, performance of total building energy consumption and final discussion. For evaluation of the hybrid approach, 1 week from 2013.9.17 to 2013.9.24 is tested for all the residential houses. One week up to 1 month of non-AC power consumption data before the tested period is used for training. For all approaches, both hour ahead and 24-h ahead forecasting windows are presented and compared to the similar studies within the literature.

##### 4.1. Performance of air-conditioning energy consumption forecasting

As the first step of hybrid modeling approach, we need to forecast the AC energy usage (power consumption). To do this, a closed-loop thermostat control strategy developed from ASHRAE standard test-case [50] is used. For AC forecasting errors, MAPEs are 7.03%, 8.29%, 9.15%, and 8.03% for SIP, ACC, Container, Stick houses, respectively. An example is shown in Fig. 4 for the SIP house AC forecasting. Examining the results in Fig. 4a, most errors beyond the baseline are due to the insensitivity of the model to AC cooling peak or transitional stages (i.e. peak at Sept. 17th, and bottom transitional stage at Sept. 24th). With an  $R^2$  value of 0.9629 shown in Fig. 4b, the forecasts show fairly accurate predictions on AC power consumptions.

##### 4.2. Performance of non-air-conditioning energy consumption forecasting

For the non-AC energy forecasting, we conducted a comparative analysis of five data-driven methods (ANN, SVR, LSSVM, GPM and GMM). Figs. 5 and 6 show hour ahead and 24 h ahead forecasting results of SIP and ACC houses, respectively. They present two types of common energy consumption patterns observed: sinusoid shaped and irregular shaped. For the sinusoid patterns, traditional methods such as ANN, and SVR have good performances with MAPE below 10%. Other methods also show reliable performances for most of the days as shown in Fig. 5a. For the irregular patterns, it can be seen that some methods (MAPE around 11% for ANN) are still able to account for the larger variation that occurs, but forecasting performance decreased. Results of 24-h ahead forecasts are significantly different than hour ahead cases. Models who can catch up

the dynamics for both patterns at hour ahead cases cannot keep the same performance for 24-h ahead cases. The comparative energy consumption plots in Figs. Fig. 55b and Fig. 66b clearly illustrate that predictions are either over fitting (e.g. Sept. 21st on Fig. 5b) or insensitive (erratic spikes from Sept. 19th to 22nd on Fig. 6b) at certain periods of tested houses.

The reduced performance for 24-h ahead forecasts is due to model limitations and input constraints. Firstly, some techniques, especially the Gaussian family tools, tend to over fit (GPR) or smooth (GMM) the prediction, and thus lose more in the predictive accuracy for 24-h forecast cases shown in Figs. Fig. 55b and Fig. 66b. Secondly, constraints on input feature selections are not obvious in hour ahead cases since the training features include previous 1–5 h. However, for 24-h ahead forecast, models learned the patterns only from yesterday. Hence, the successfully prediction of this longer forecast window is based on one key assumption: daily usages are similar without large deviation, which may not be the case for all days. Fig. 7 shows an example for forecasting a longer period using representative techniques (SVR, ANN, and GRR) for the SIP house. Most large forecast errors (above 2.5 kWh level) reveal models' inability to capture the random spikes in energy consumption that occur for short periods of time (e.g. within 5 min). These spikes in consumption can be attributed to the on/off cycling of appliances such as a water heater [47]. Finally, the random behavior of residents such as operating any appliance, adjusting the thermostat, further creates the randomness of the energy usage patterns.

##### 4.3. Performance of total building energy consumption forecasting

MAPE and CV (RMSE) are calculated for all data-driven and hybrid models during a test period from 2013.9.17 to 2013.9.23 for four residential houses. Based on the previous study [51], hour ahead and 24-h ahead forecast are suggested to be integrated with LSSVM for hybrid modeling. Fig. 8 visualizes hour ahead forecast and 24-h ahead forecast performances based on MAPE and CV. It illustrates which method has the best and most consistent performance for all different houses.

The values of MAPE and CVs are provided in Tables 1 and 2 for hour ahead and 24-h ahead, respectively. The proposed hybrid model and commonly applied data-driven methods, ANN, SVR and LSSVM, maintain consistent performances, while GPM and GMM vary in terms of prediction accuracies. In terms of CVs, the best method for hour ahead electricity consumption forecasting is the hybrid model. Meanwhile, in terms of MAPE, hybrid model, ANN, SVR, and LSSVM are not significantly different. However, some improved results from the hybrid approach can be observed in Fig. 8. Table 1 also implies that the accuracy of the hybrid model can achieve at most 15% higher, in terms of MAPE and CV, than the purely data-driven approach.

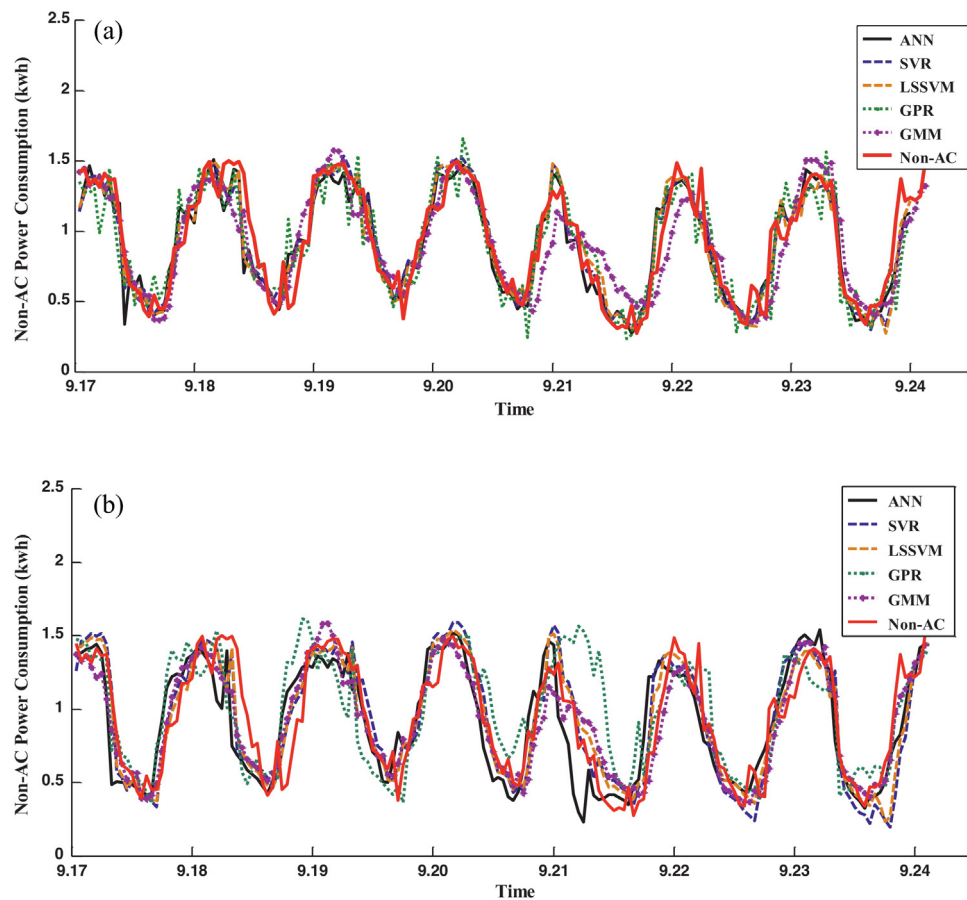


Fig. 5. Non-AC forecast (sinusoid shaped).

**Table 1**  
Hour ahead total load forecast.

Model	MAPE	CV	Model	MAPE	CV
House: SIP			House: container		
Hybrid	8.16%	11.21%	Hybrid	21.44%	25.58%
ANN	9.26%	12.49%	ANN	22.80%	27.54%
SVR	9.58%	12.76%	SVR	22.11%	26.07%
LSSVM	9.09%	13.18%	LSSVM	23.26%	28.70%
GMM	14.29%	19.50%	GMM	28.44%	31.91%
GPR	15.86%	21.39%	GPR	26.24%	29.75%
House: ACC			House: stick		
Hybrid	12.60%	14.14%	Hybrid	21.51%	26.17%
ANN	12.50%	15.08%	ANN	23.86%	28.19%
SVR	12.70%	15.72%	SVR	23.22%	27.28%
LSSVM	12.94%	15.47%	LSSVM	23.44%	27.42%
GMM	19.21%	23.50%	GMM	27.22%	31.60%
GPR	18.57%	22.62%	GPR	30.11%	32.27%

**Table 2**  
24-h ahead total load forecast.

Model	MAPE	CV	Model	MAPE	CV
House: SIP			House: container		
Hybrid	10.02%	11.28%	Hybrid	26.43%	32.82%
ANN	11.92%	12.59%	ANN	27.39%	33.35%
SVR	11.97%	12.42%	SVR	28.25%	34.91%
LSSVM	10.39%	11.83%	LSSVM	28.95%	34.19%
GMM	17.80%	20.24%	GMM	34.01%	37.81%
GPR	18.85%	22.69%	GPR	35.94%	38.57%
House: ACC			House: stick		
Hybrid	14.36%	17.06%	Hybrid	27.52%	29.26%
ANN	14.41%	18.85%	ANN	30.22%	34.40%
SVR	15.69%	19.55%	SVR	28.56%	31.87%
LSSVM	15.10%	19.72%	LSSVM	28.32%	31.36%
GMM	25.47%	28.85%	GMM	31.13%	35.15%
GPR	27.12%	30.03%	GPR	35.76%	39.34%

A comparison of the 24 h ahead forecasting results find similar patterns. The hybrid model tends to have lower MAPE and CV values. However, the forecasting error compared to hour ahead is statistically distinct (i.e., MAPE and CV values are higher for the same houses, particularly for Container and Stick houses). This implies that temporal variance (hour or 24 h ahead) in the forecasting window has a significant impact on the predictive capability of both data-driven and hybrid models. As discussed in the non-AC forecast section, models trained with historical information with more recent load information has higher chances to characterize the electricity consumption behavior at an hourly interval. For 24-h ahead prediction, models will account for energy usages both at

day and night times, and thus lose the ability to adapt to erratic changes.

#### 4.4. Discussions

In the residential building energy research sector, forecasting is difficult due to the large variation of energy patterns from stochastic occupancy behaviour. Our hybrid modeling approach is more reasonable to be compared with similar buildings. Three previous research studies for hour ahead total energy consumption forecasting of residential buildings are presented in Table 3. The forecasting error is considered reasonable if the model returns MAPE values



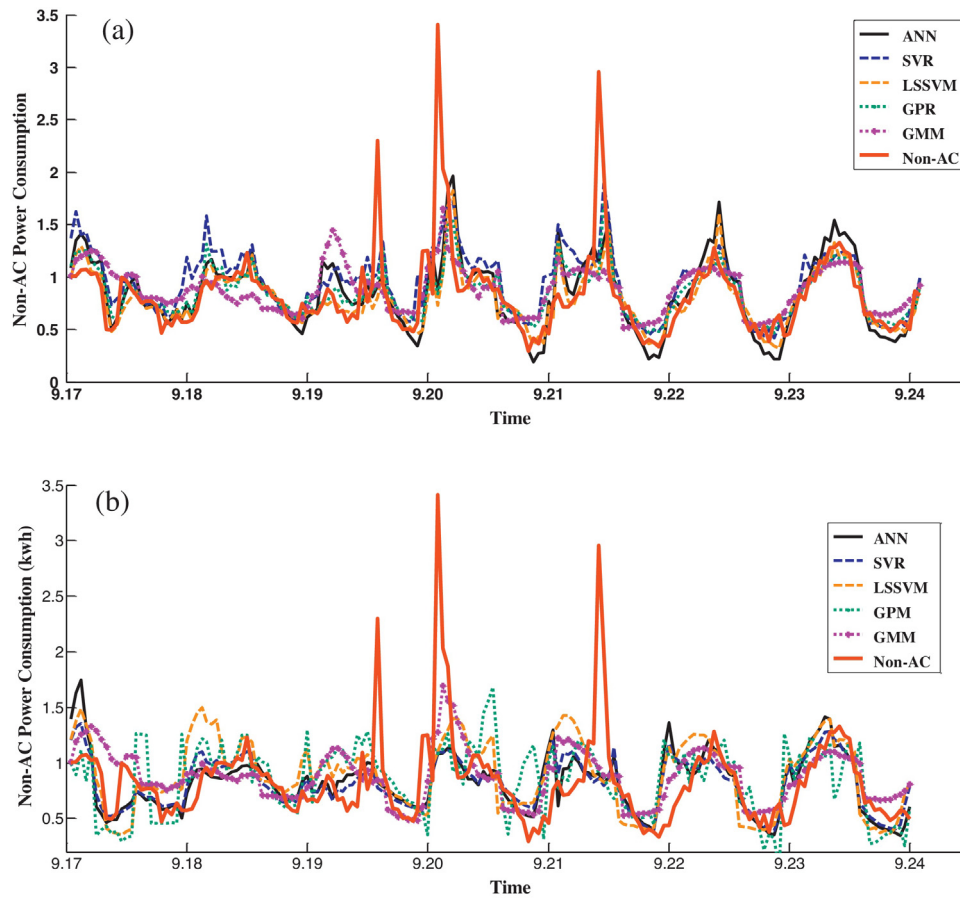


Fig. 6. Non-AC forecast (irregular shaped).

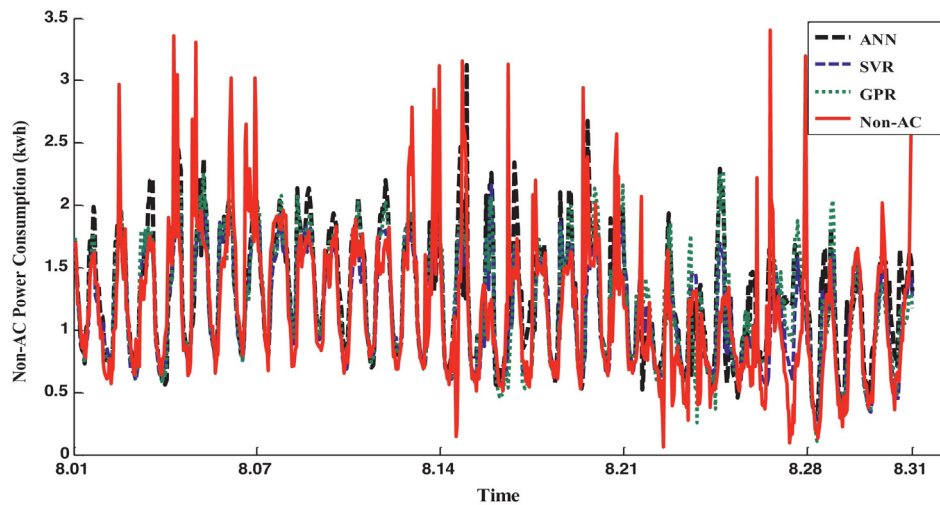


Fig. 7. Non-AC forecast for 1 month of SIP.

**Table 3**  
Hour ahead forecast performance of recent studies.

Reference	Year	Type	Location	Method	Forecast	Performance
Ghofrani et al. [52]	2011	Dwelling	Navada	Kalman filter	Hour ahead	MAPE: 12–30%
Edwards et al. [1]	2012	Two-floor house	Tennessee	Linear regression various types of ANN, SVR, LSSVM	Hour ahead	MAPE: 9–30%; CV: 11–38%
Jain et al. [38]	2014	Multi-family	New York	SVR	Hour ahead	CV: 10–11%

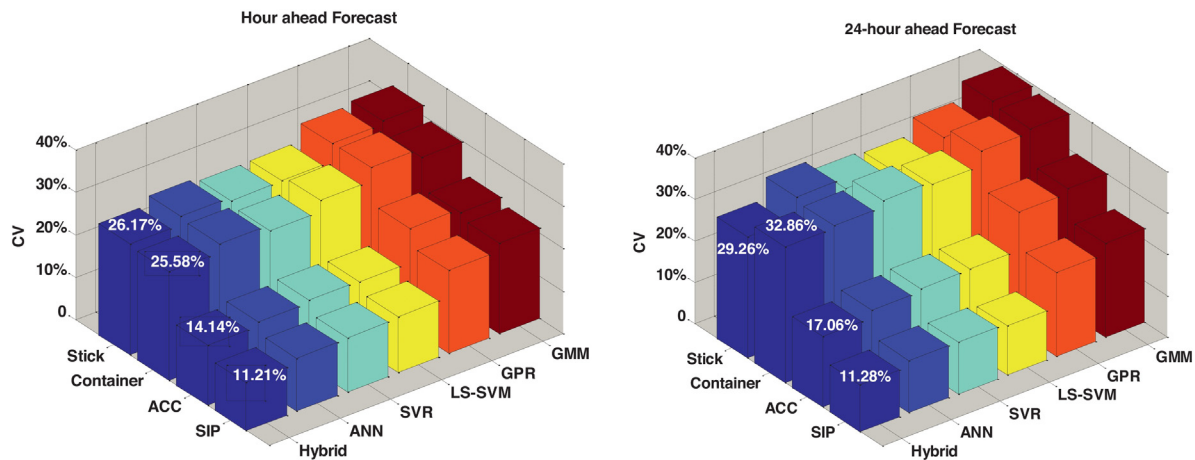


Fig. 8. Forecast performances based on CV.

between 9% and 30%, and CV values between 10% and 38% for hour ahead. From Table 1, we can conclude our hybrid approach is slightly better than other existing models.

For the hybrid approach, the improvements can be traced back to the decomposition of total building energy consumption. By isolating certain appliances such as air conditioning, electricity usages are filtered to a sub-meter level. However, due to the portion of air conditioning usage of sample houses (roughly from 20% to 35%), the large improvement of accuracies is not expected. Another capability of the hybrid approach is the simultaneous prediction on both total energy consumption and sub-meter appliances, such as AC. This will be a valuable feature for the application of HVAC optimization and advance control strategies such as model predictive control.

## 5. Conclusion

This paper aims to develop and demonstrate an innovative hybrid modeling approach for residential building energy consumption forecasting. This study demonstrated the capability of the proposed hybrid modeling approach, which integrates data-driven techniques with forward physics-based models, and applied to single-family residential houses. The proposed modeling approach was validated through 1 month measured data from four residential buildings. Five data-driven methods (ANN, SVM, LSSVM, GPM, and GMM) based on the same inputs were compared. The final data analysis shows that hybrid modeling approach is slightly better for the hour ahead forecasting in terms of CV. For the 24-h ahead forecasting, all results are similar. Though performance error matrixes are varied for the 24-h ahead predictions, trends of improvement using hybrid model can be detected.

The study presented here uses 5 min interval data to predict AC electricity usage. AC prediction is benefited from a fine level of data monitoring at intra-hour interval. However, such data is not typically available to home owners and utility companies. In the future, we will explore the model performance by using hourly data. Further study is also required to understand and improve 24 h ahead forecasting. Implementing a hybrid model coupled with occupancy behavior studies will be a solution for characterizing the large variation [49]. Accurate residential building forecasting in the energy consumption is critical to improve energy efficiency. Fully use of monitored data to understand and explore not only total energy usage but also appliances such as air conditioning will help promote demand response programs and develop energy benchmark models.

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