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Korea Institute of Energy Technology

Advanced Computer Vision
Week 04

Sep. 23, 2022
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Discussion: Dolly Zoom Effect

✓ How to film this video?

rag & bone films

Discussion: Dolly Zoom Effect

✓ A visual effect that zooms out of an object while moving towards it.

[Q] What happens to the focal length?

[Q] How to remove the blurry effects?

[Q] How to control the depth-of-field?

Focal Length: 300mm

Focal Length: 14mm

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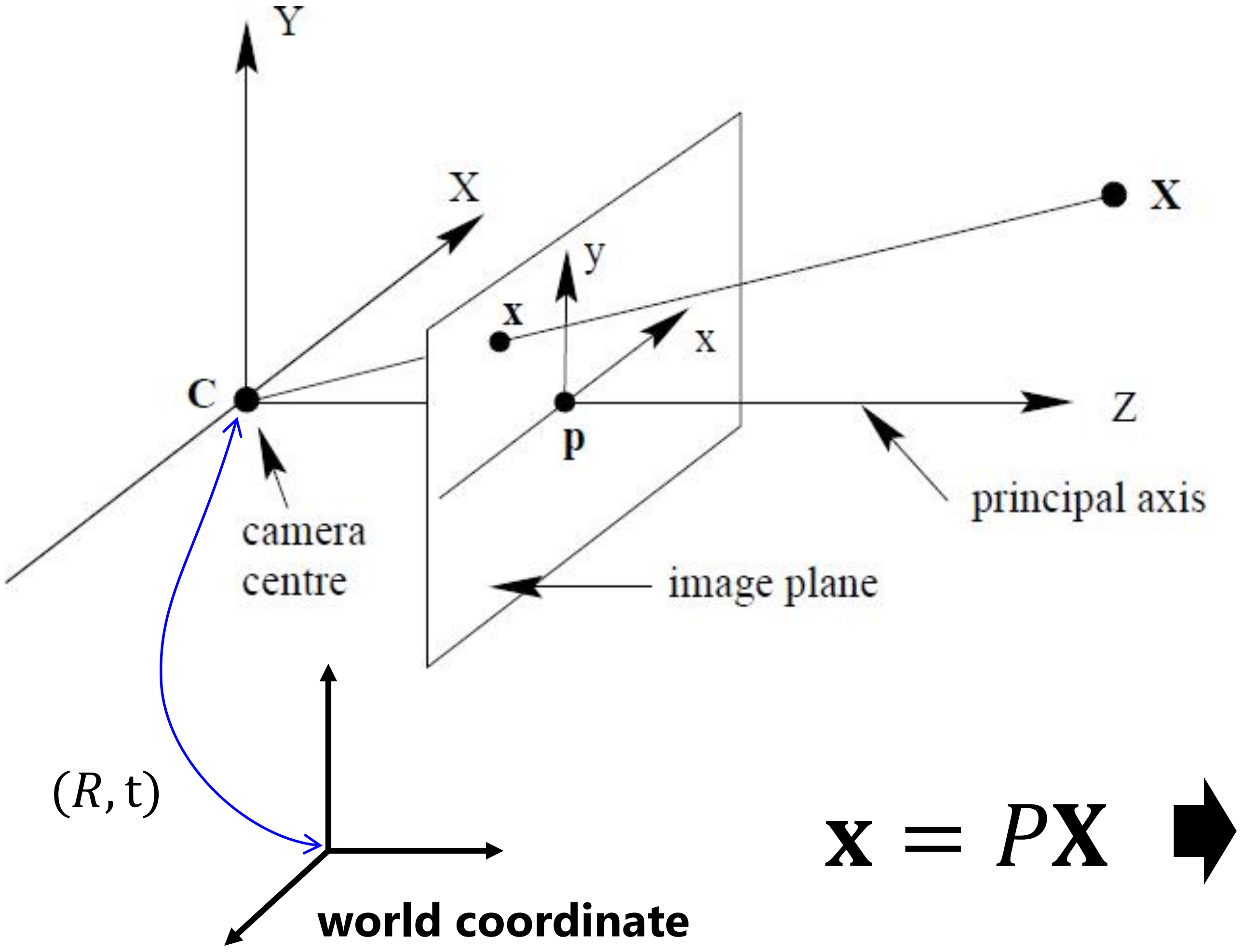
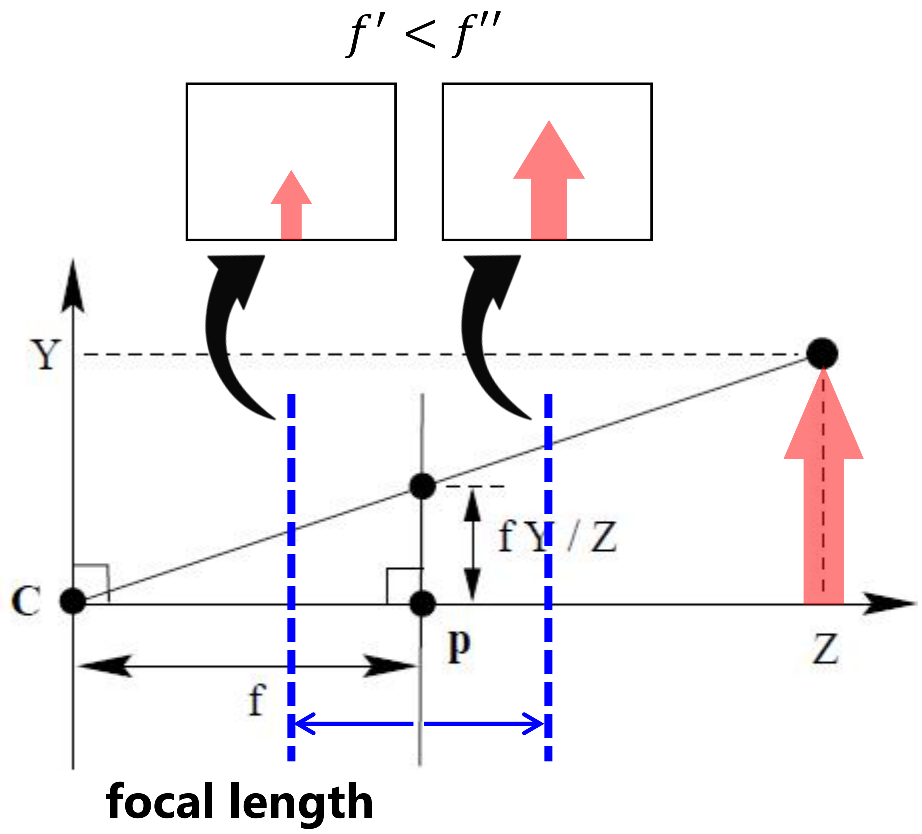
Projective Geometry

Camera Anatomy

✓ So far, we learned how to capture the image.





$$\mathbf{x} = P\mathbf{X} \;\Rightarrow\; s\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & skew\_cf_x & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Deep inside of Mathematical model

- ✓ **Properties of projection:**
  - Points project to points, and lines project to lines
- ✓ **Perspective projection:**
  - 3D scene **points** are projected to **pixels** in the 2D retina plane at $\boldsymbol{f'}$
  - The **collinearity** of a scene point $\boldsymbol{P}$, its image $\boldsymbol{P'}$, and the poinhole $\boldsymbol{O}$

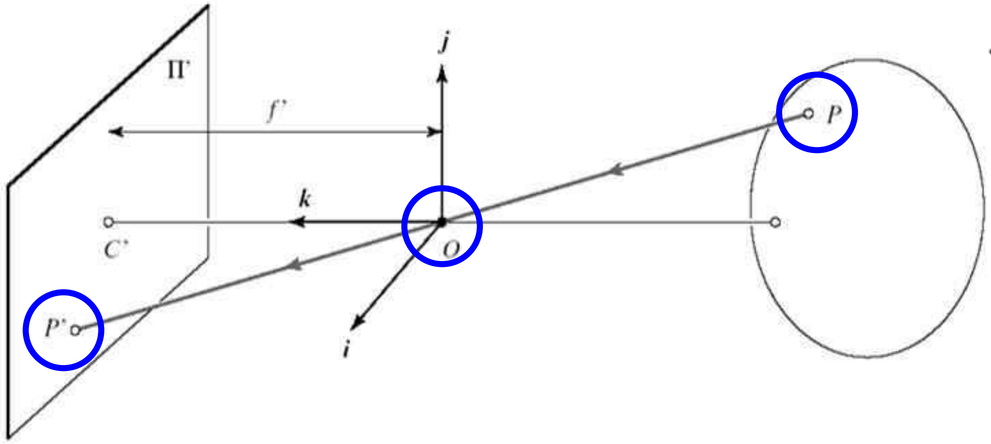
Collinearity?
A condition in which some of the independent variables are highly correlated.

$$\overrightarrow{OP'} = \lambda \overrightarrow{OP}$$

$$x' = \lambda x \qquad y' = \lambda y \qquad f' = \lambda z$$

$$x' = f' \frac{x}{z} \qquad y' = f' \frac{y}{z}$$

Depending on camera properties



"Homogeneous notations"

$$\begin{bmatrix} sx' \\ sy' \\ s \end{bmatrix} = \begin{bmatrix} f' & 0 & 0 & 0 \\ 0 & f' & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Homogeneous Notations

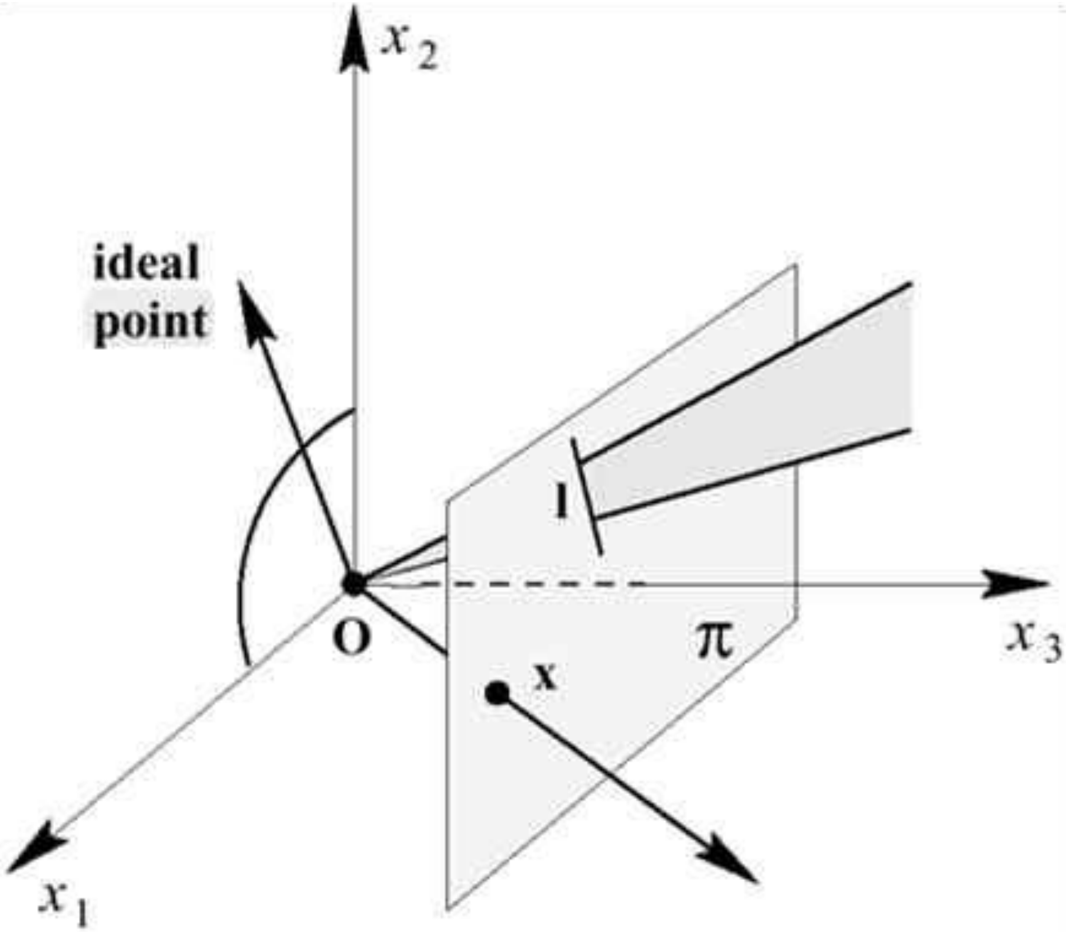
- ✓ A line in the plane $\boldsymbol{\pi}$: $(a,b,c)^T \xleftrightarrow{\text{the same line}} k(a,b,c)^T$
  - An equivalence class of vectors = homogeneous vectors

$$ax_1 + bx_2 + c = 0 \xleftrightarrow{\text{the same}} kax_1 + kbx_2 + kc = 0$$

- ✓ A line in the plane $\boldsymbol{\pi}$: $(x_1, x_2)^T$ in $\Re^2$
  - Identification of the plane with $\Re^2$
- ✓ The projective plane $\mathbf{P}^2$ as a set of rays in $\mathbf{R}^3$
  - The point $\mathbf{x}$ as a ray through the origin:

$$k(x_1, x_2, x_3)^T \Longleftrightarrow (x_1/x_3, x_2/x_3)^T$$

  - What happens if $x_3 = 0$?
    - Points at *Infinity* or *ideal* points
    - The *lines* as *planes* passing through the origin
    - x_1x_2-plane, parallel to image plane



Why Homogeneous Notations?

$$\begin{bmatrix} sx' \\ sy' \\ s \end{bmatrix} = \begin{bmatrix} f' & 0 & 0 & 0 \\ 0 & f' & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- ✓ Perspective projection:
  - Matrix representation by homogenous notations
  - Preserving the collinearity
- ✓ Nice mathematical representations for:
  - The points at infinity: $(x_1, x_2, 0)^T$
    - Points lying on the plane at infinity: $x_1 x_2$-plane
  - Mathematically, any finite points can be projected to the image plane.
- ✓ Any projective transformation can be represented by the linear transformation of homogeneous coordinates.

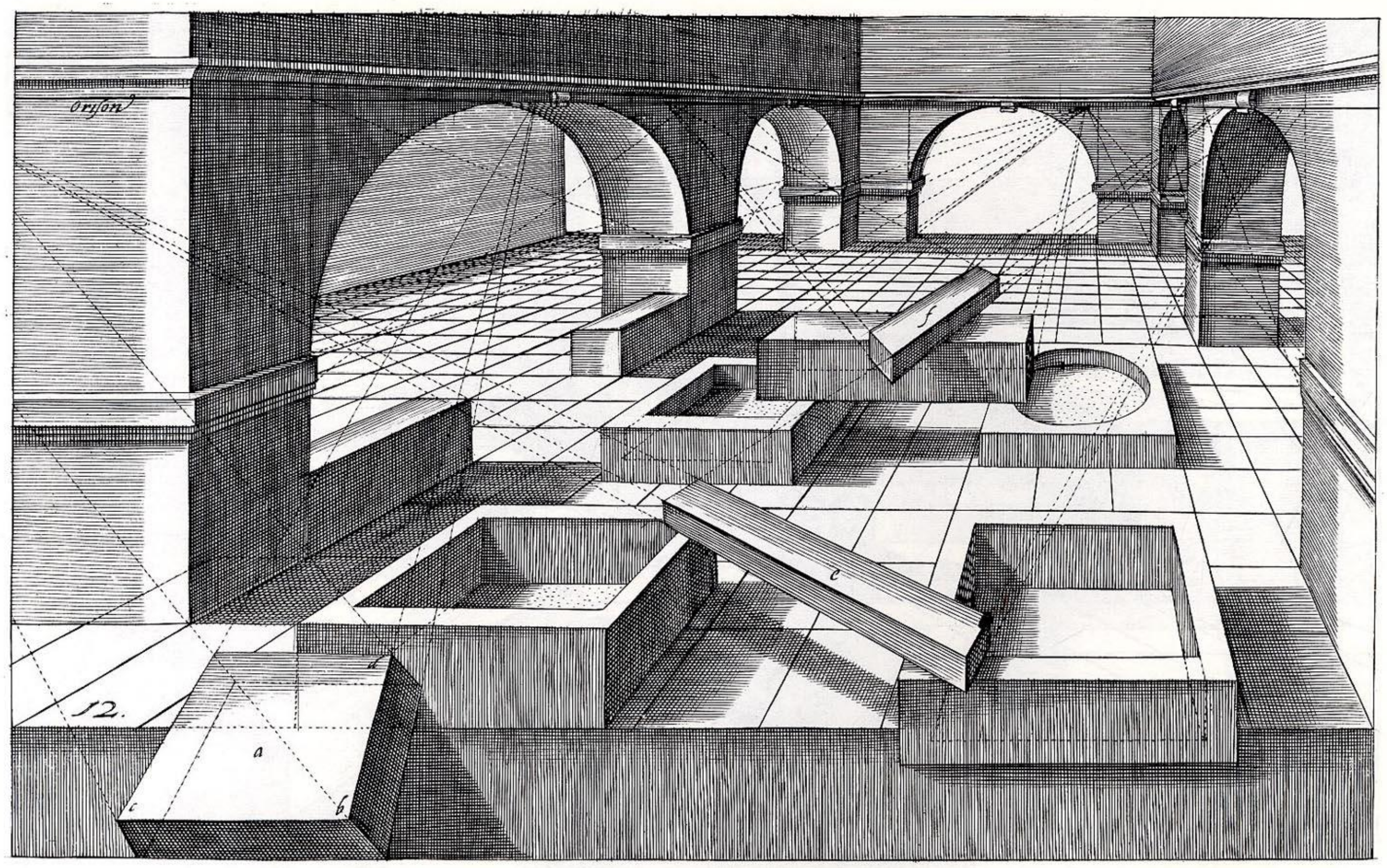
$$\mathbf{x}' = \mathbf{H}\mathbf{x}$$, where $\mathbf{H}$ is homogenous matrix   "Projectively equivalent point"

Projective Geometry: Vanishing Point

- ✓ Parallel lines cannot meet in Euclidean space, but meet at infinity in projective space.

- ✓ Projection of a point at infinity

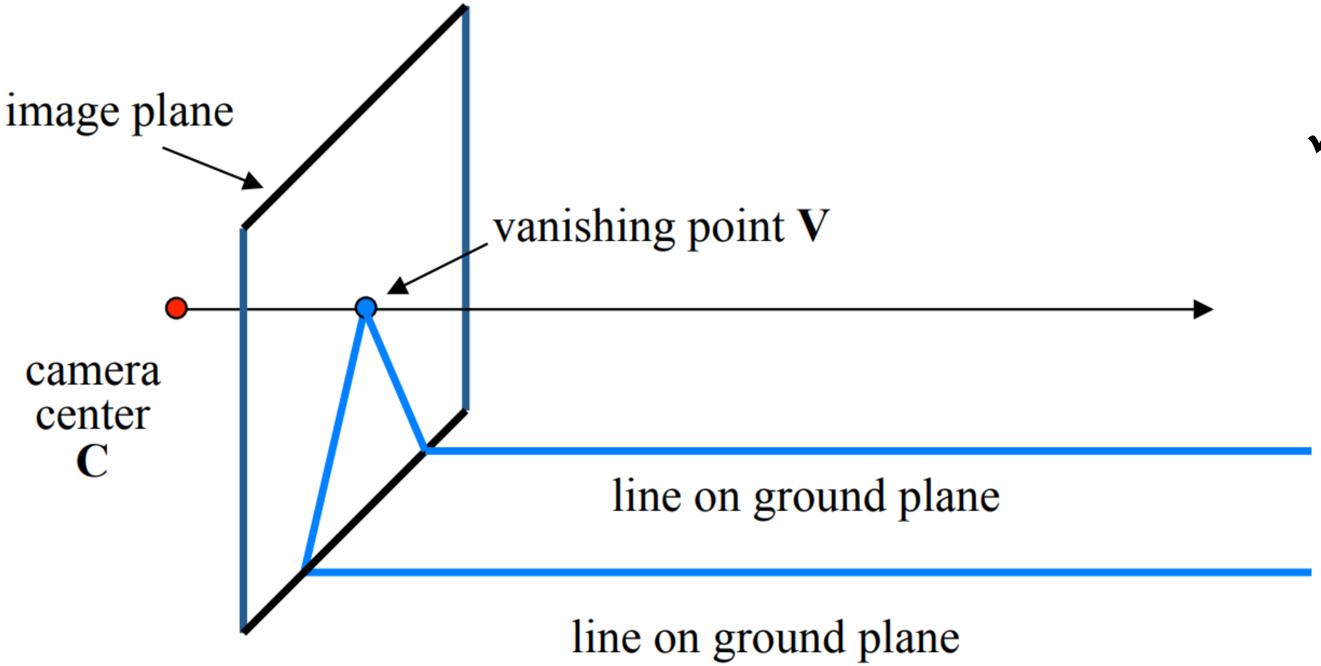
- ✓ Every image point is a potential vanishing point



Vanishing Point

✓ Any two parallel lines in 3D have the same vanishing point $\mathbf{v}$.

✓ The ray from $\mathbf{C}$ through $\mathbf{v}$ is parallel to the lines.



✓ Nevertheless, what are the parallel straight lines that still do not meet?

→ Lines parallel to the image plane

→ Lines parallel to the $x_1 x_2$-plane

→ Cannot converge to the image plane

Proof: Two Parallel Lines Can Intersect

- Consider the following linear system in Euclidean space:

$$\begin{cases} Ax + By + C = 0 \\ Ax + By + D = 0 \end{cases}$$

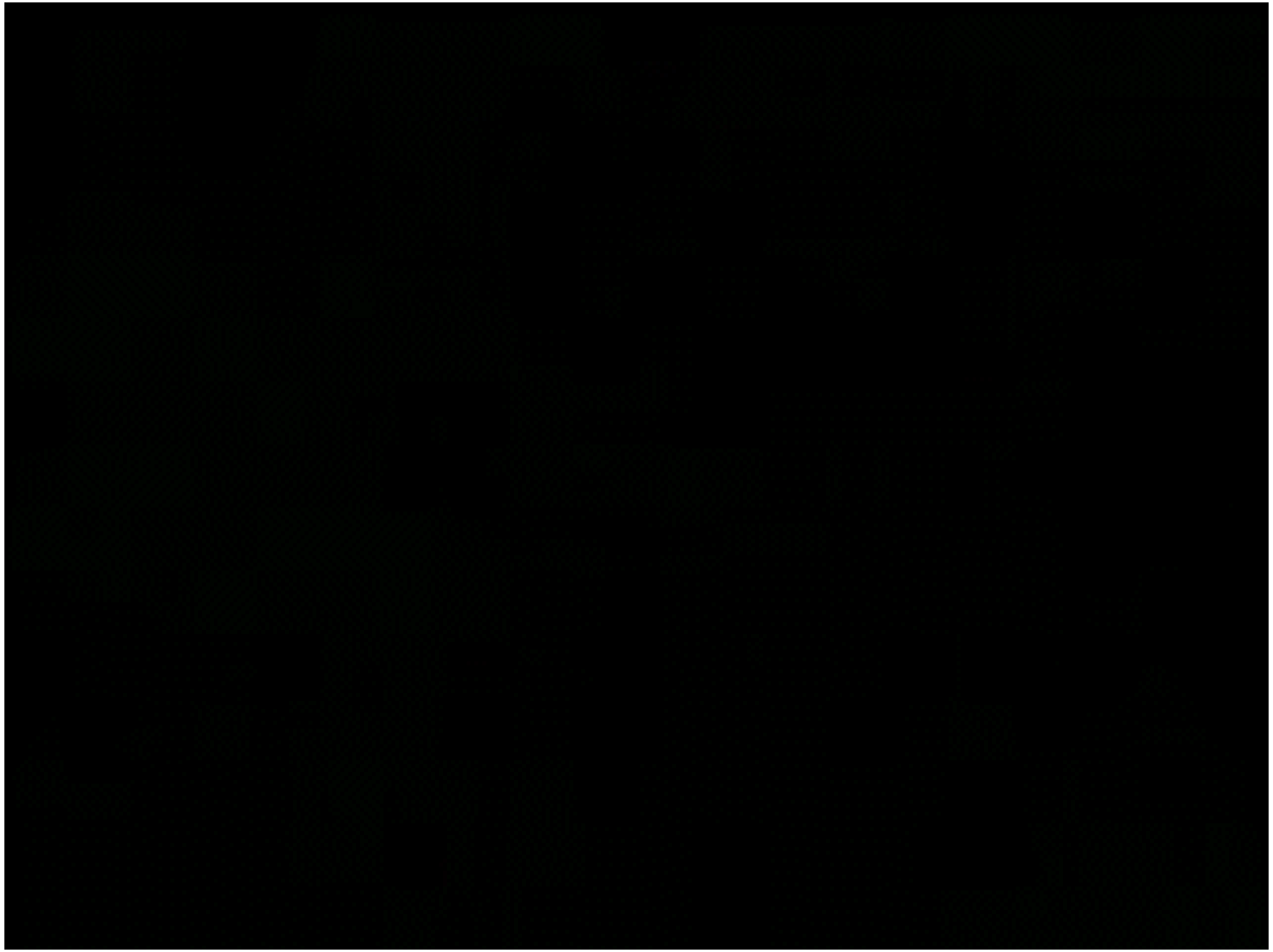
- And we know that there is no solution for above equations because of C ≠ D.
- If C = D, then two lines are identical (overlapped).
- Let's rewrite the equations for projective space by replacing x and y to x/w, y/w respectively.

$$\begin{cases} A\frac{x}{w} + B\frac{y}{w} + C = 0 \\ A\frac{x}{w} + B\frac{y}{w} + D = 0 \end{cases} \Rightarrow \begin{cases} Ax + By + Cw = 0 \\ Ax + By + Dw = 0 \end{cases}$$

- Now, we have a solution, (x, y, 0) since (C - D)w = 0, ∴ w = 0. Therefore, two parallel lines meet at (x, y, 0), which is the point at infinity.

✓ **Homogeneous coordinates** are very useful and fundamental concept in computer graphics, such as projecting a 3D scene onto a 2D plane.

Optical Illusion by False Linear Perspective



[1] YouTube, “Ames Room (Philip Zimbardo)”

Back to Camera Intrinsic Parameters

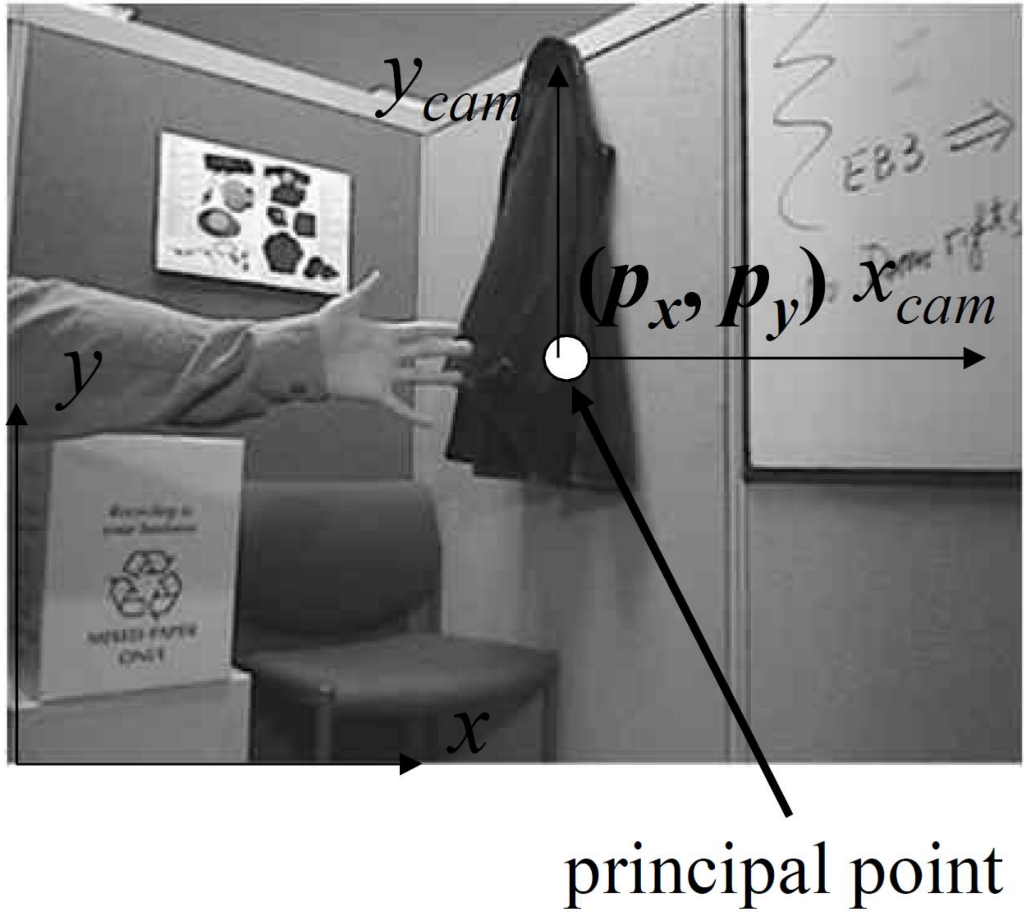
- ✓ Principal point offset:

$$(X_C, Y_C, Z_C)^T \mapsto \left(\frac{fX_C}{Z_C} + p_x, \frac{fY_C}{Z_C} + p_y\right)^T$$

- ✓ Magnification in pixel unit = focal length

- ✓ In homogeneous coordinates with a scale Z_c:

$$\begin{bmatrix} fX_c + Z_c p_x \\ fY_c + Z_c p_y \\ Z_c \end{bmatrix} = \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$



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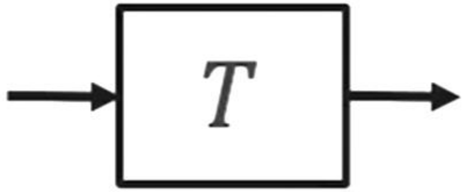
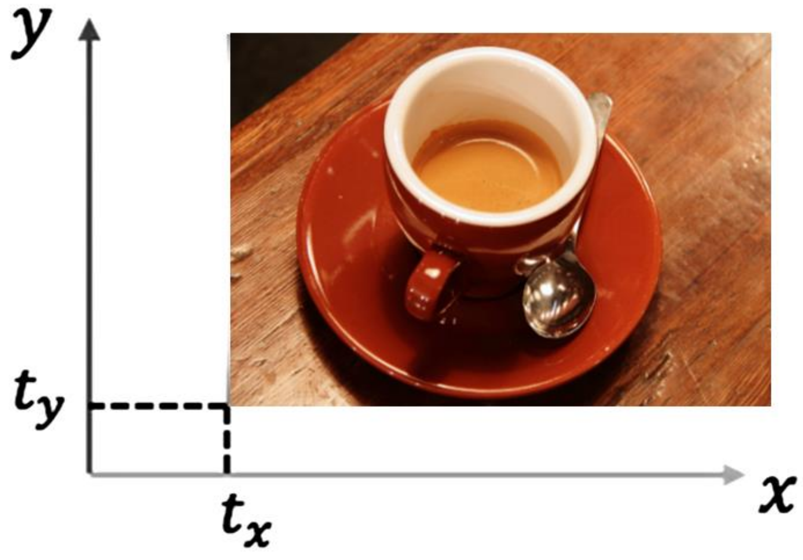
Geometric Transformations

Translation

$$\mathbf{x}' = \mathbf{H}\mathbf{x}$$





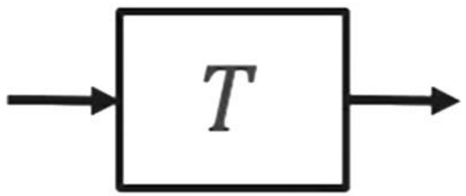
$$x_2 = x_1 + t_x \qquad y_2 = y_1 + t_y$$

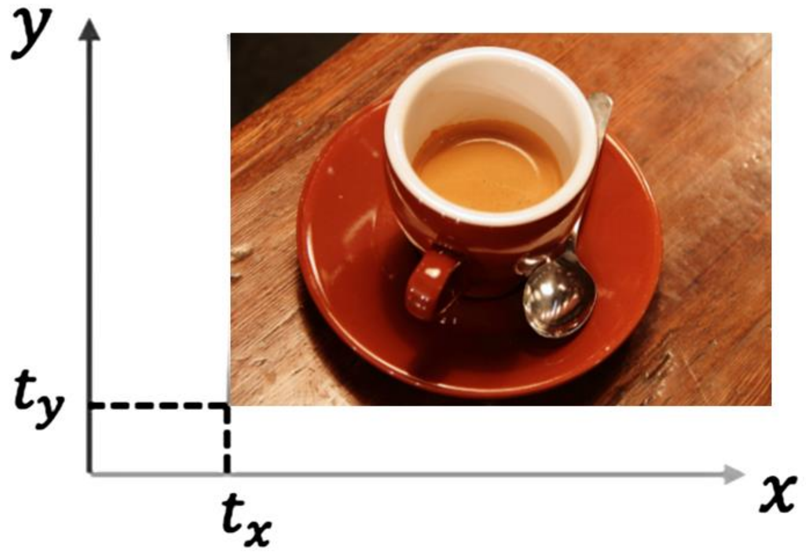
Can translation be expressed by 2x2 matrix? No

[1] Original slide by Shree Nayar

Translation in Homogeneous Coordinates

$$\mathbf{x}' = \mathbf{H}\mathbf{x}$$





$$x_2 = x_1 + t_x \qquad y_2 = y_1 + t_y$$

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

[1] Original slide by Shree Nayar

Scaling, Rotation, Skew, Translation

$$\begin{bmatrix}\tilde{x}_2\\ \tilde{y}_2\\ \tilde{z}_2\end{bmatrix} = \begin{bmatrix}s_x & 0 & 0\\ 0 & s_y & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ y_1\\ 1\end{bmatrix}$$

Scaling

$$\begin{bmatrix}\tilde{x}_2\\ \tilde{y}_2\\ \tilde{z}_2\end{bmatrix} = \begin{bmatrix}1 & m_x & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ y_1\\ 1\end{bmatrix}$$

Skew

$$\begin{bmatrix}\tilde{x}_2\\ \tilde{y}_2\\ \tilde{z}_2\end{bmatrix} = \begin{bmatrix}1 & 0 & t_x\\ 0 & 1 & t_y\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ y_1\\ 1\end{bmatrix}$$

Translation

$$\begin{bmatrix}\tilde{x}_2\\ \tilde{y}_2\\ \tilde{z}_2\end{bmatrix} = \begin{bmatrix}cos\theta & -sin\theta & 0\\ sin\theta & cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ y_1\\ 1\end{bmatrix}$$

Rotation

Composition of these transformations?

[1] Original slide by Shree Nayar

Affine Transformation

Any transformation of the form:

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

[1] Original slide by Shree Nayar

Affine Transformation

Any transformation of the form:

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

- Origin does not necessarily map to the origin
- Lines map to lines
- Parallel lines remain parallel
- Closed under composition
- *c.f.*, camera intrinsics

[1] Original slide by Shree Nayar

Projective Transformation

Any transformation of the form:

$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

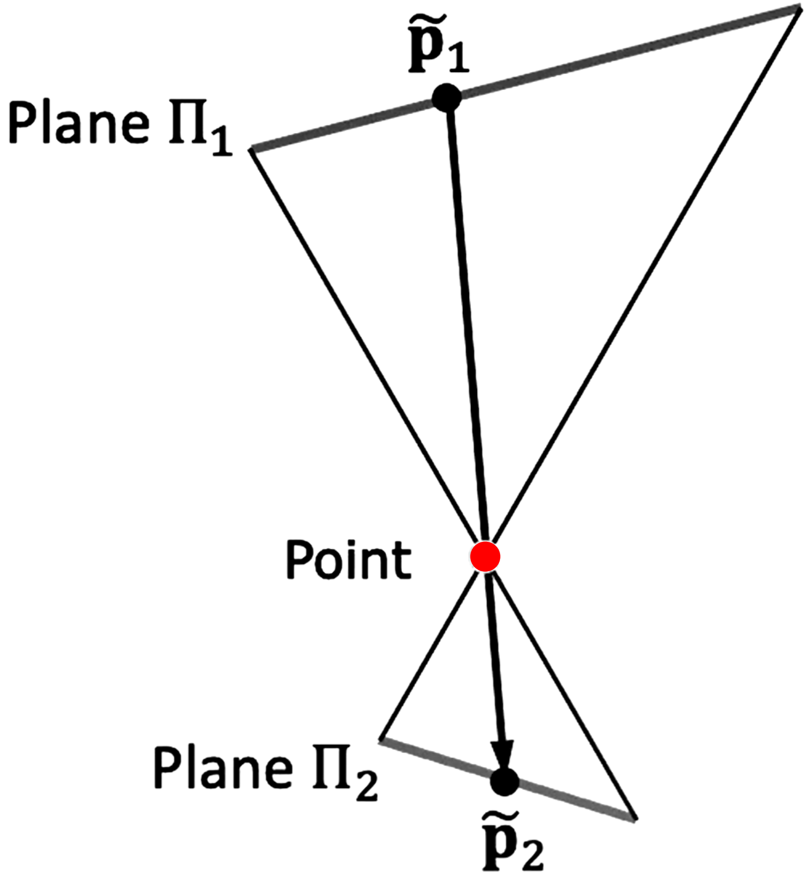
Also called **Homography**

$$\tilde{\mathbf{p}}_2 = H\tilde{\mathbf{p}}_1$$

[1] Original slide by Shree Nayar

Projective Transformation

Mapping of one plane to another through a point



$$\tilde{\mathbf{p}}_2 = H\tilde{\mathbf{p}}_1$$

$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

Same as **imaging** a **plane** through a **pinhole**

- Origin does not necessarily map to the origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Closed under composition

[1] Original slide by Shree Nayar

Let's Try Geometric Transformation

Codes are available at:

https://github.com/Leo-LiHao/OpenCV-Python-Tutorials

```
$ git clone https://github.com/Leo-LiHao/OpenCV-Python-Tutorials
$ cd OpenCV-Python-Tutorials/Src/ImageProcessing/GeometricTransform
```

Please try **four transformation codes** in the following order.

```
$ python GeometricTransform_rotateAndTrans.py
$ python GeometricTransform_resize.py
$ python GeometricTransform_affine.py
$ python GeometricTransform_perspective.py
```

Updated codes (for python3) are uploaded in https://view.kentech.ac.kr/f088fa7f-874e-44bc-bd6d-6084b42dfdf7