KENTECH
Korea Institute of Energy Technology

Advanced Computer Vision
Week 04

Sep. 20, 2022
Seokju Lee

KENTECH
Korea Institute of Energy Technology

Camera Calibration

Camera Calibration: Review

- ✓ Process of estimating the **parameters** of a pinhole camera model.
- ✓ Determines which **incoming light (3D)** is **associated with** each pixel on the resulting **image (2D)**.
- ✓ In an ideal pinhole camera, **a simple projection matrix** is enough to do this.
- ✓ The camera projection matrix is derived from the **intrinsic** and **extrinsic** parameters of the camera.

$$s\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & \mathrm{skew\_c}f_x & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$= K[R|t]\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- $(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z})$: 3D point in the world coordinate
- $[\boldsymbol{R}|\boldsymbol{t}]$: extrinsic parameters to convert the world coordinate into the camera coordinate
- $\boldsymbol{K}$: intrinsic parameters to represent the camera characteristics
- $\boldsymbol{K}[\boldsymbol{R}|\boldsymbol{t}]$: camera projection matrix
- $(\boldsymbol{x}, \boldsymbol{y})$: 2D pixel location in the image plane
- $\boldsymbol{s}$: scale factor

Camera Calibration: Optimization

$$\mathbf{x} = K[R|t]\mathbf{X} = M\mathbf{X}$$

→ For each corresponding point i in the image:

$$\underbrace{\begin{bmatrix} u^{(i)} \\ v^{(i)} \\ 1 \end{bmatrix}}_{\text{Known}} \equiv \underbrace{\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}}_{\text{Unknown}} \underbrace{\begin{bmatrix} x_w^{(i)} \\ y_w^{(i)} \\ z_w^{(i)} \\ 1 \end{bmatrix}}_{\text{Known}}$$

$$u^{(i)} = \frac{p_{11}x_w^{(i)} + p_{12}y_w^{(i)} + p_{13}z_w^{(i)} + p_{14}}{p_{31}x_w^{(i)} + p_{32}y_w^{(i)} + p_{33}z_w^{(i)} + p_{34}}$$

$$v^{(i)} = \frac{p_{21}x_w^{(i)} + p_{22}y_w^{(i)} + p_{23}z_w^{(i)} + p_{24}}{p_{31}x_w^{(i)} + p_{32}y_w^{(i)} + p_{33}z_w^{(i)} + p_{34}}$$

→ Expanding the matrix as linear equations

Camera Calibration: Optimization

→ Rearranging the terms:

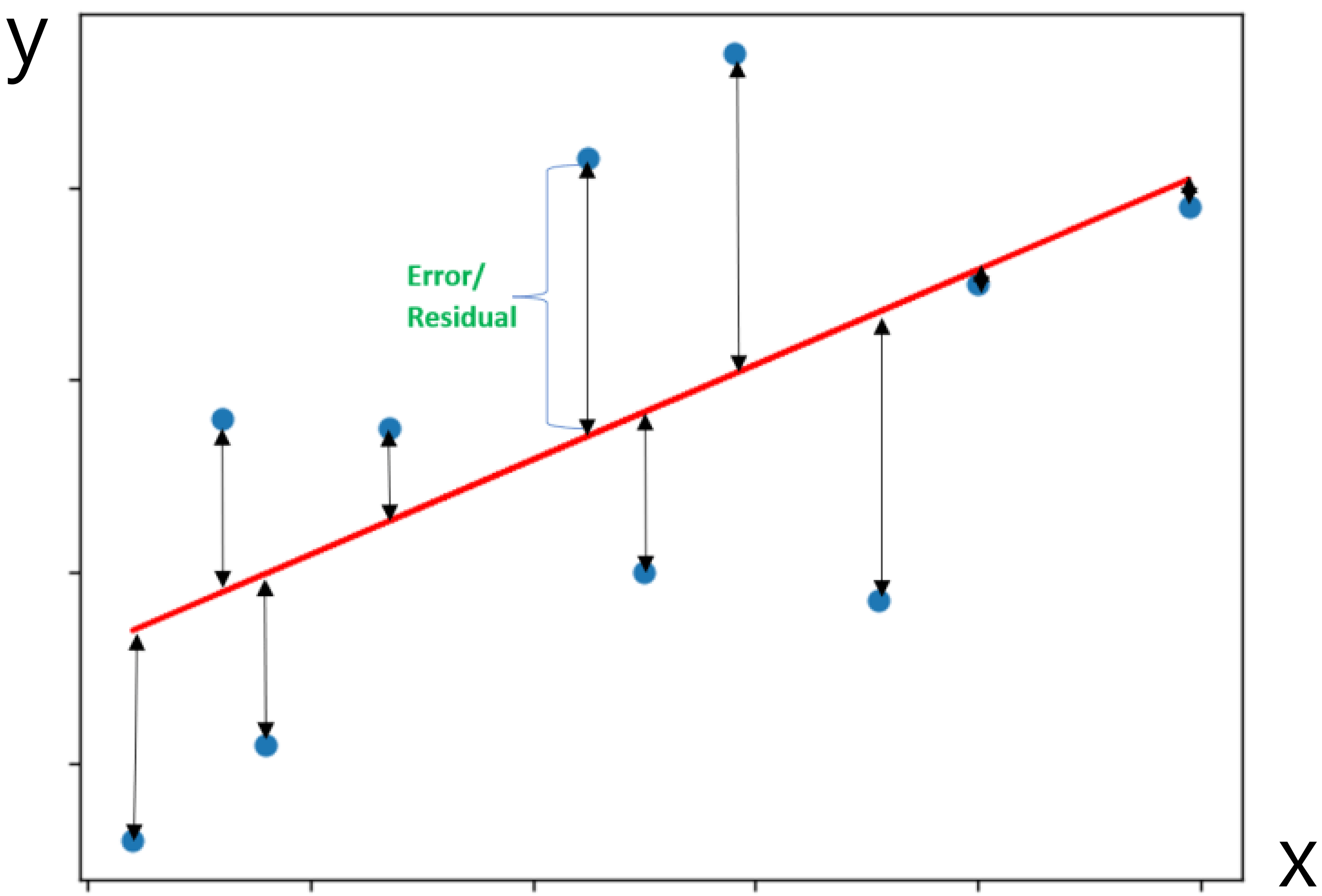
$$\underset{\underset{\text{Known}}{\underline{A}}}{\begin{bmatrix} x_w^{(1)} & y_w^{(1)} & z_w^{(1)} & 1 & 0 & 0 & 0 & 0 & -u_1 x_w^{(1)} & -u_1 y_w^{(1)} & -u_1 z_w^{(1)} & -u_1 \\ 0 & 0 & 0 & 0 & x_w^{(1)} & y_w^{(1)} & z_w^{(1)} & 1 & -v_1 x_w^{(1)} & -v_1 y_w^{(1)} & -v_1 z_w^{(1)} & -v_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_w^{(i)} & y_w^{(i)} & z_w^{(i)} & 1 & 0 & 0 & 0 & 0 & -u_i x_w^{(i)} & -u_i y_w^{(i)} & -u_i z_w^{(i)} & -u_i \\ 0 & 0 & 0 & 0 & x_w^{(i)} & y_w^{(i)} & z_w^{(i)} & 1 & -v_i x_w^{(i)} & -v_i y_w^{(i)} & -v_i z_w^{(i)} & -v_i \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_w^{(n)} & y_w^{(n)} & z_w^{(n)} & 1 & 0 & 0 & 0 & 0 & -u_n x_w^{(n)} & -u_n y_w^{(n)} & -u_n z_w^{(n)} & -u_n \\ 0 & 0 & 0 & 0 & x_w^{(n)} & y_w^{(n)} & z_w^{(n)} & 1 & -v_n x_w^{(n)} & -v_n y_w^{(n)} & -v_n z_w^{(n)} & -v_n \end{bmatrix}} \underset{\underset{\text{Unknown}}{\underline{\mathbf{p}}}}{\begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

→ Solve for $\mathbf{p}$:

$$A\mathbf{p} = 0$$

Camera Calibration: Least Squares Method

$$\mathbf{x} = K[R|t]\mathbf{X} = M\mathbf{X} \rightarrow \mathrm{y} = mx + b \qquad (\mathbf{X_i}, \mathbf{x_i}) \rightarrow (x_i, y_i)$$

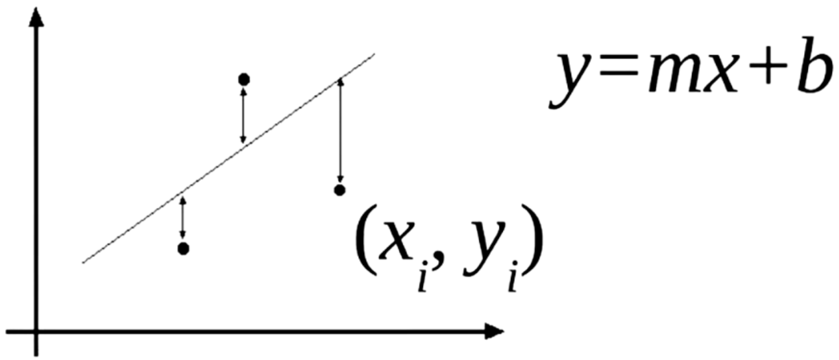


- ✓ Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- ✓ Find (m, b) to minimize!

$$E = \sum_{i=1}^{n}(y_i - mx_i - b)^2$$

Camera Calibration: Least Squares Method

- ✓ Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- ✓ Find (m, b) to minimize!



$$E = \sum_{i=1}^{n} (y_i - mx_i - b)^2$$

$$E = \sum_{i=1}^{n} \left(\begin{bmatrix} x_i & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - y_i \right)^2 = \left\| \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \right\|^2 \quad = \left\| \mathbf{Ap} - \mathbf{y} \right\|^2$$

$$= \mathbf{y}^T \mathbf{y} - 2(\mathbf{Ap})^T \mathbf{y} + (\mathbf{Ap})^T (\mathbf{Ap})$$

$$\frac{dE}{dp} = 2\mathbf{A}^T \mathbf{Ap} - 2\mathbf{A}^T \mathbf{y} = 0$$

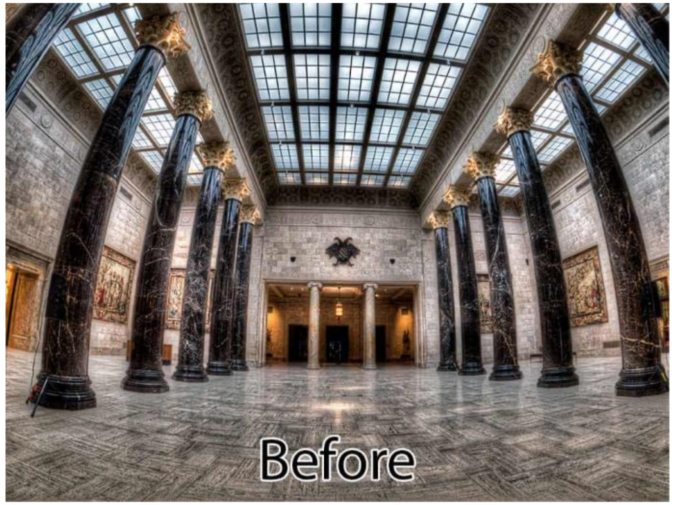
“Least Squares Method (LSM)”

→ Closed form solution!

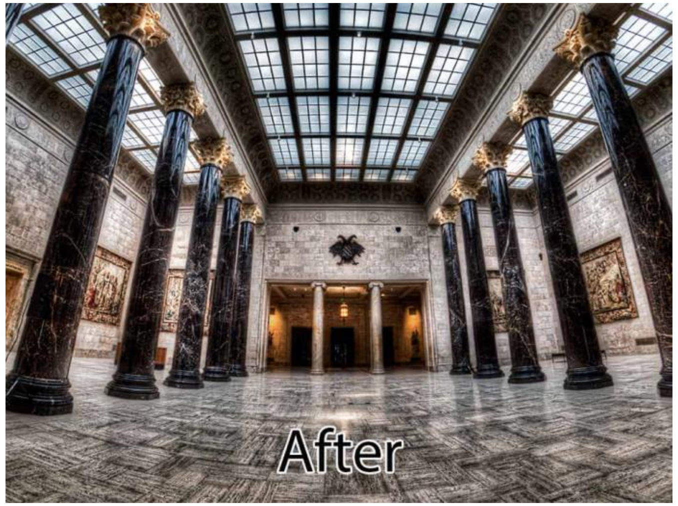
$$\mathbf{A}^T \mathbf{Ap} = \mathbf{A}^T \mathbf{y} \Rightarrow \mathbf{p} = \left(\mathbf{A}^T \mathbf{A} \right)^{-1} \mathbf{A}^T \mathbf{y}$$

Camera Lens Distortion → Undistortion

- ✓ Correction of the image distortion.

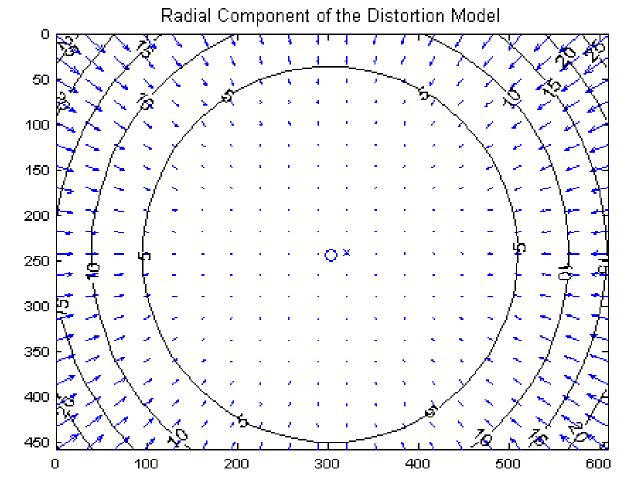


Undistortion



Radial distortion model

$$x_{\text{corrected}} = x\left(1 + k_1 r^2 + k_2 r^4 + k_3 r^6\right)$$
$$y_{\text{corrected}} = y\left(1 + k_1 r^2 + k_2 r^4 + k_3 r^6\right)$$

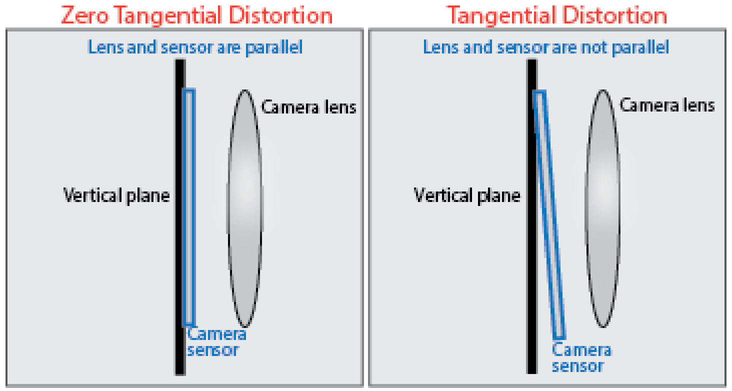


```
Pixel error              = [0.1174, 0.1159]
Focal Length             = (657.303, 657.744)                  +/- [0.2849, 0.2894]
Principal Point          = (302.717, 242.334)                  +/- [0.5912, 0.5571]
Skew                     = 0.0004198                                +/- 0.0001905
Radial coefficients      = (-0.2535, 0.1187, 0)                +/- [0.00231, 0.009418, 0]
Tangential coefficients  = (-0.0002789, 5.174e-005)         +/- [0.0001217, 0.0001208]
```

Tangential distortion model

→ caused by **unparallel** *lens* and *image plane*!

$$x_{\text{corrected}} = x + \left[2p_1xy + p_2\left(r^2 + 2x^2\right)\right]$$
$$y_{\text{corrected}} = y + \left[p_1\left(r^2 + 2y^2\right) + 2p_2xy\right]$$

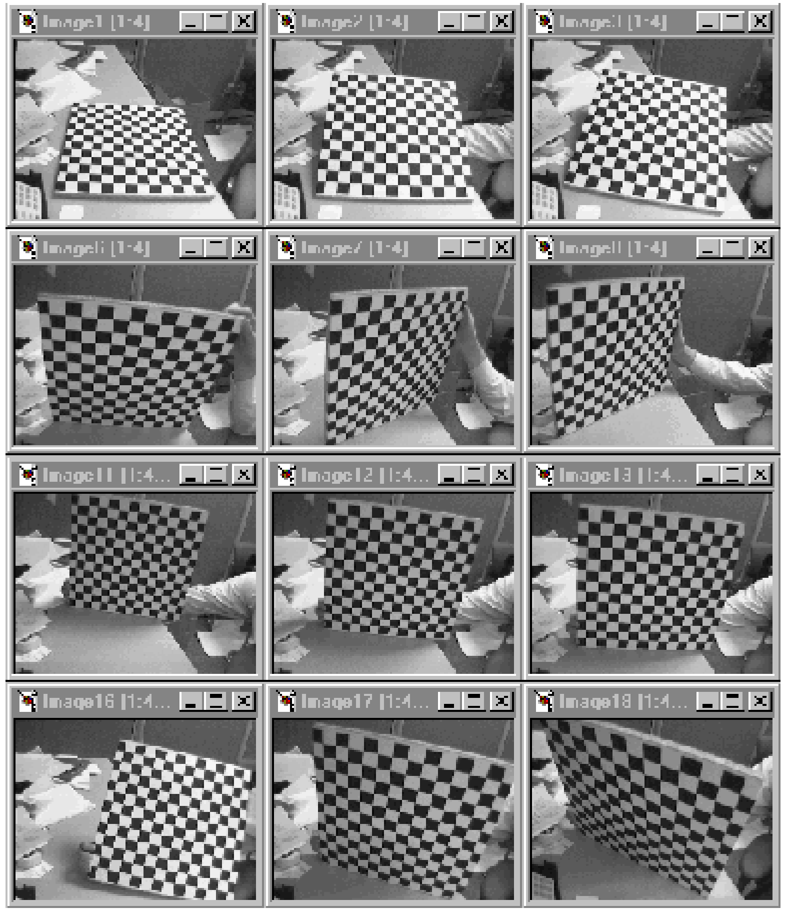


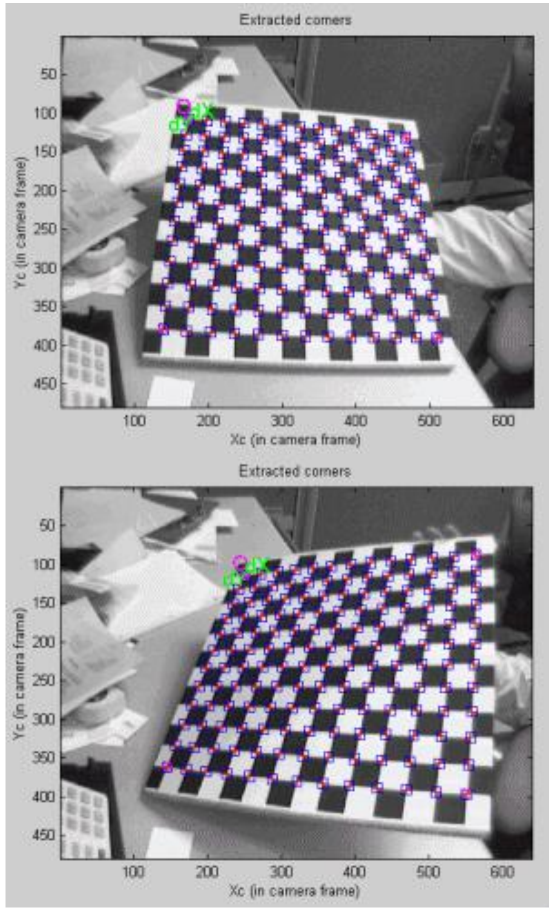
$$\begin{bmatrix} x_d \\ y_d \end{bmatrix} = \begin{bmatrix} x_u \\ y_u \end{bmatrix} + \left(k_1r^2 + k_2r^4 + k_3r^6 + \cdots\right)\begin{bmatrix} x_u \\ y_u \end{bmatrix} + \begin{bmatrix} 2p_1x_uy_u + p_2\left(r^2 + 2x_u^2\right) \\ p_1\left(r^2 + 2y_u^2\right) + 2p_2x_uy_u \end{bmatrix},$$

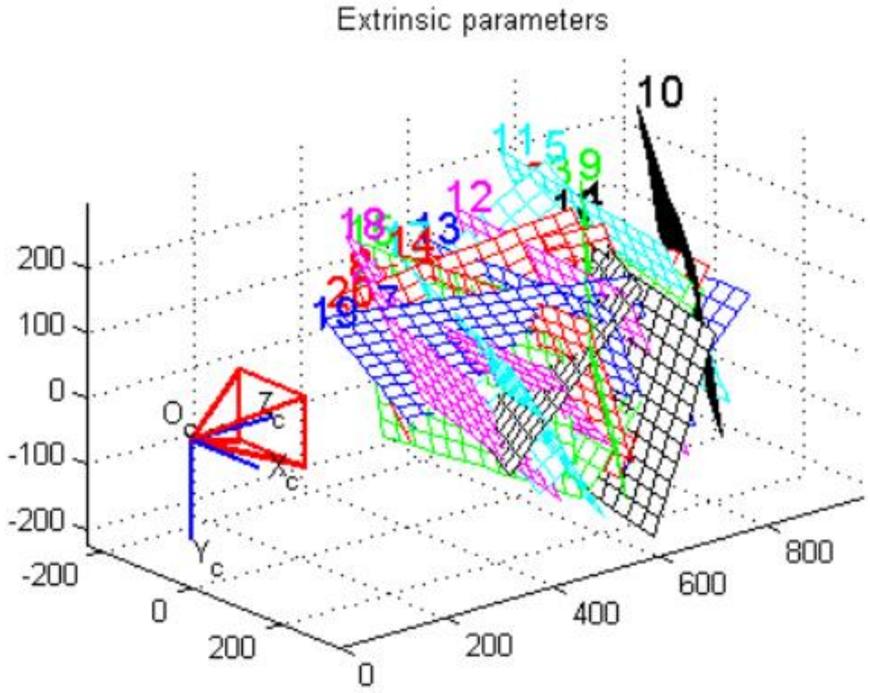
Let's Try Camera Calibration!

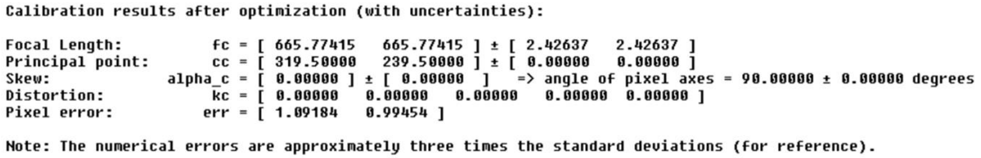
Camera calibration code is available at:
https://view.kentech.ac.kr/1d2a3167-672f-4456-a5b8-b8c56c15eefe

- ✓ Camera Calibration Toolbox [1]







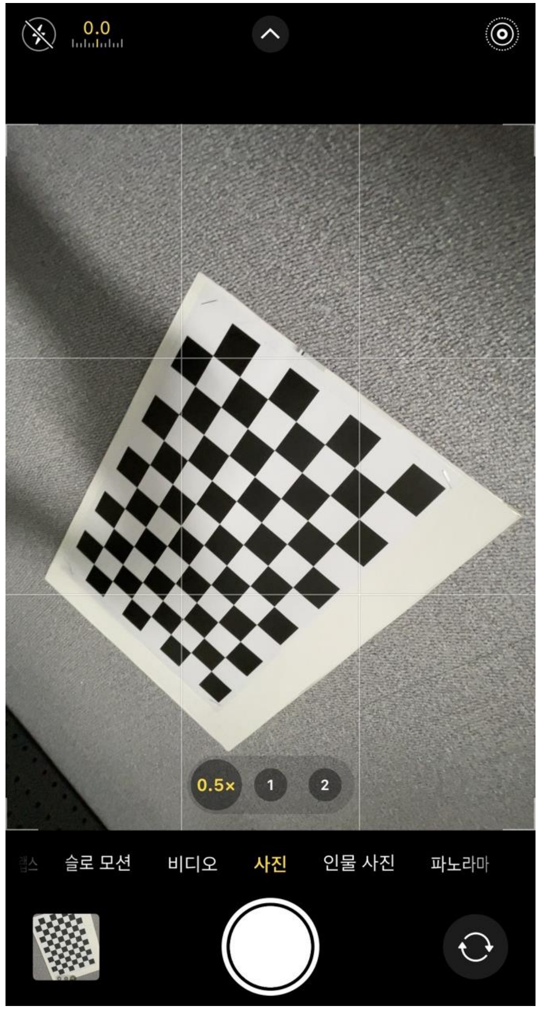


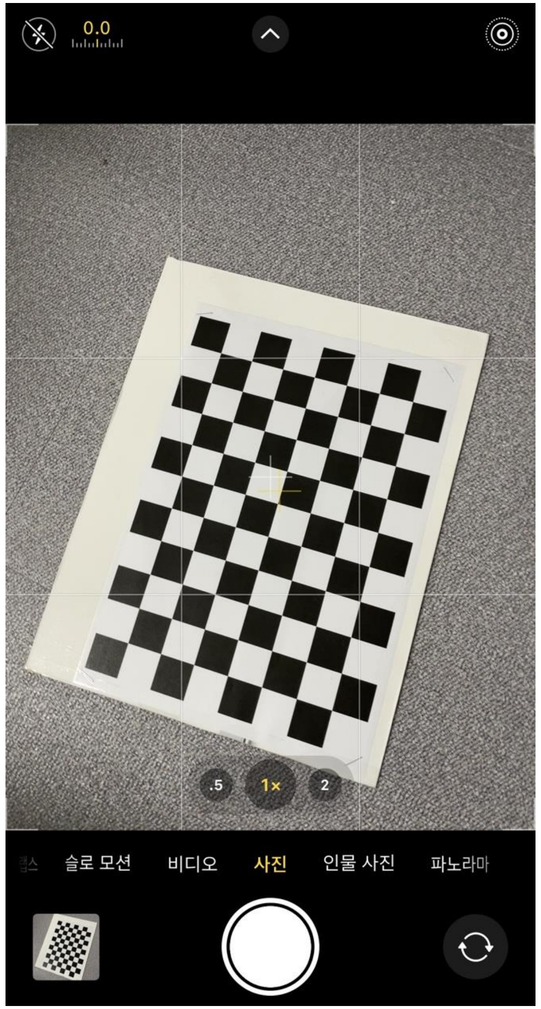
```
Calibration results after optimization (with uncertainties):

Focal Length:          fc = [ 665.77415   665.77415 ] ± [ 2.42637   2.42637 ]
Principal point:       cc = [ 319.50000   239.50000 ] ± [ 0.00000   0.00000 ]
Skew:             alpha_c = [ 0.00000 ] ± [ 0.00000  ]   => angle of pixel axes = 90.00000 ± 0.00000 degrees
Distortion:            kc = [ 0.00000   0.00000   0.00000   0.00000  0.00000 ]
Pixel error:          err = [ 1.09184   0.99454 ]

Note: The numerical errors are approximately three times the standard deviations (for reference).
```

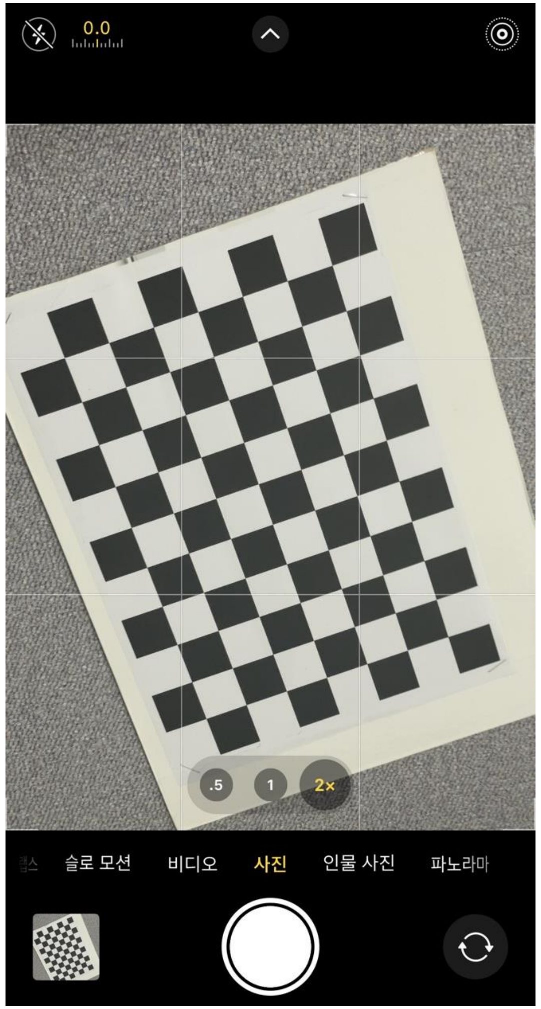
[1] http://robots.stanford.edu/cs223b04/JeanYvesCalib/htmls/example.html

Try with Your Mobile Phone

- ✓ Calibrate different camera modes (e.g., different lens)







(0.5x) Wide-angle lens

vs.

(2x) Telephoto lens

[Q] What happens to the focal length?

Checker board from https://markhedleyjones.com/projects/calibration-checkerboard-collection

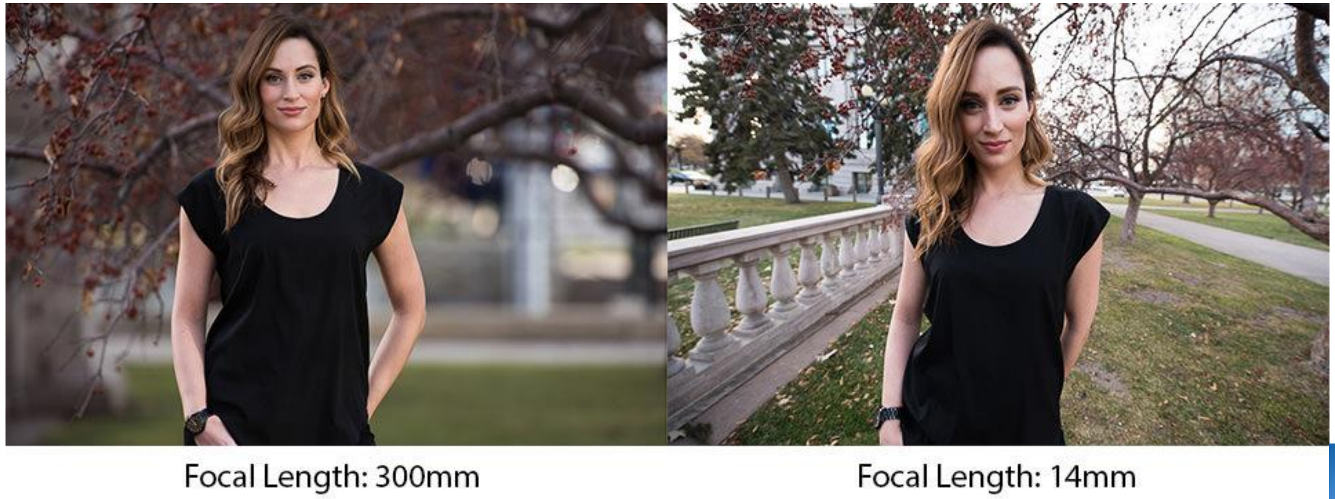
Discussion: Dolly Zoom Effect

- ✓ How to film this video?

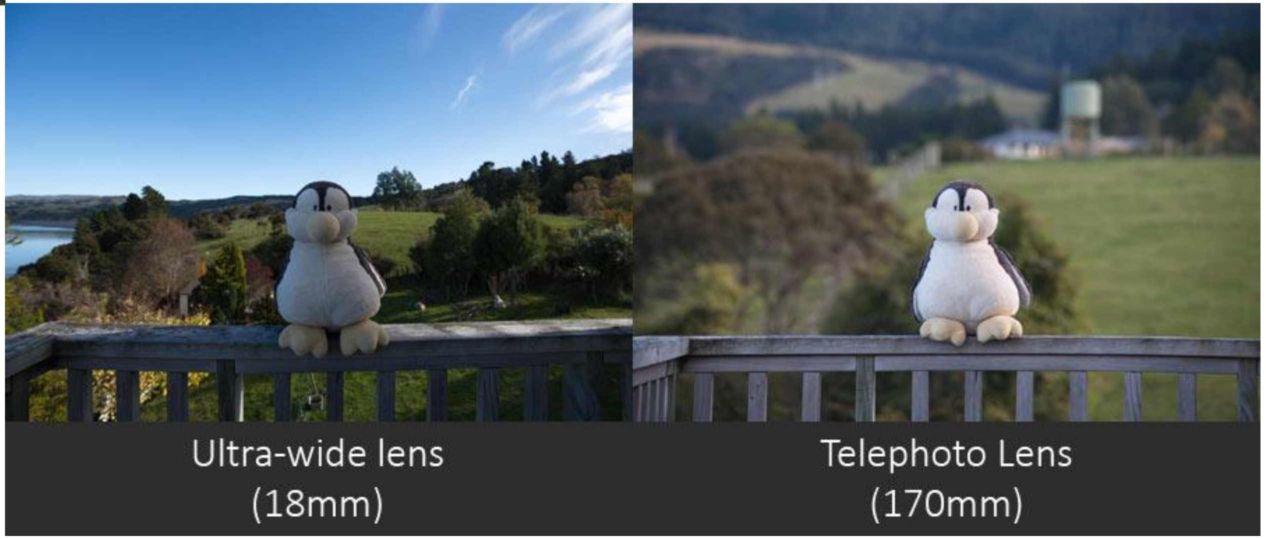
rag & bone films

Discussion: Dolly Zoom Effect

✓ A visual effect that zooms out of an object while moving towards it.



[Q] What happens to the focal length?



[Q] How to remove the blurry effects?

[Q] How to control the depth-of-field?