# Spatial Transformer Networks

Jaderberg, Max, Karen Simonyan, and Andrew Zisserman.
Advances in neural information processing systems 28 (2015).

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Advanced Computer Vision

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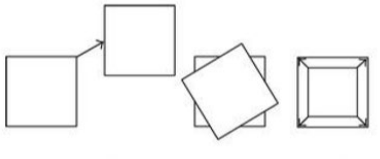
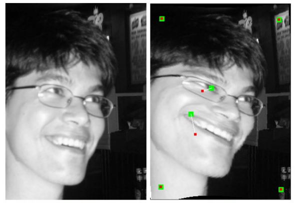
Jaderberg, Max, Karen Simonyan, and Andrew Zisserman. "Spatial transformer networks." Advances in neural information processing systems 28 (2015).




# ◆ Introduction




- The Convolutional Neural Network (CNN) define an exceptionally powerful class of models,
  but has Limitation : **lack of ability to be <u>Spatially invariant</u> to the input data**
  == invariance of the model towards spatial transformations of images (rotation, translation, scaling)
- <u>Max-pooling layer</u> – satisfy this, but 2x2 pixel-wise operation is difficult to cope with various spatial variability
  ⇒ **Spatial transformer module** – include in to standard neural network


**Spatial transformer** module

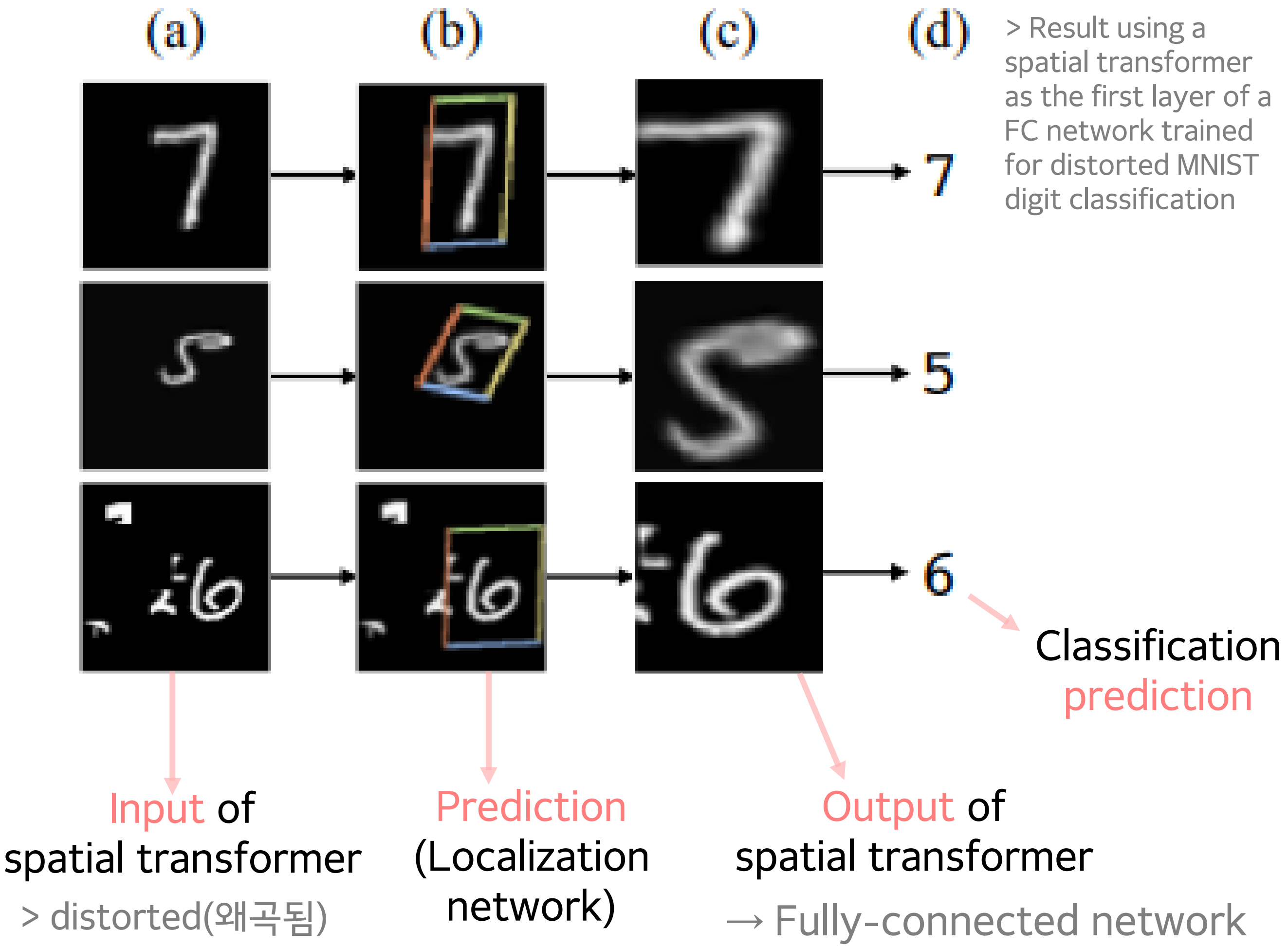
- A learnable module that can be placed in a CNN, to increase the spatial invariance in an efficient manner
- A dynamic mechanism – unlike pooling layer (receptive fields are fixed and local)
- Input image → transformation is performed on the entire feature map (can include scaling, cropping, rotations, non-rigid deformations)
- Networks select regions that are most relevant(attention), transform those regions to a canonical(일반적인) pose

# ◆ Introduction

❖ **Image classification**

**Using Spatial Transformer**

- The action of the spatial transformer is conditioned on individual data samples, trained well without extra supervision
- Can be trained with standard back-propagation (allowing **end-to-end** training of models)
- Can be added into CNNs to help with a variety of tasks (image classification, co-localization, spatial attention)
- The framework they present in this paper can be seen as a generalisation of differentiable attention to any spatial transformation



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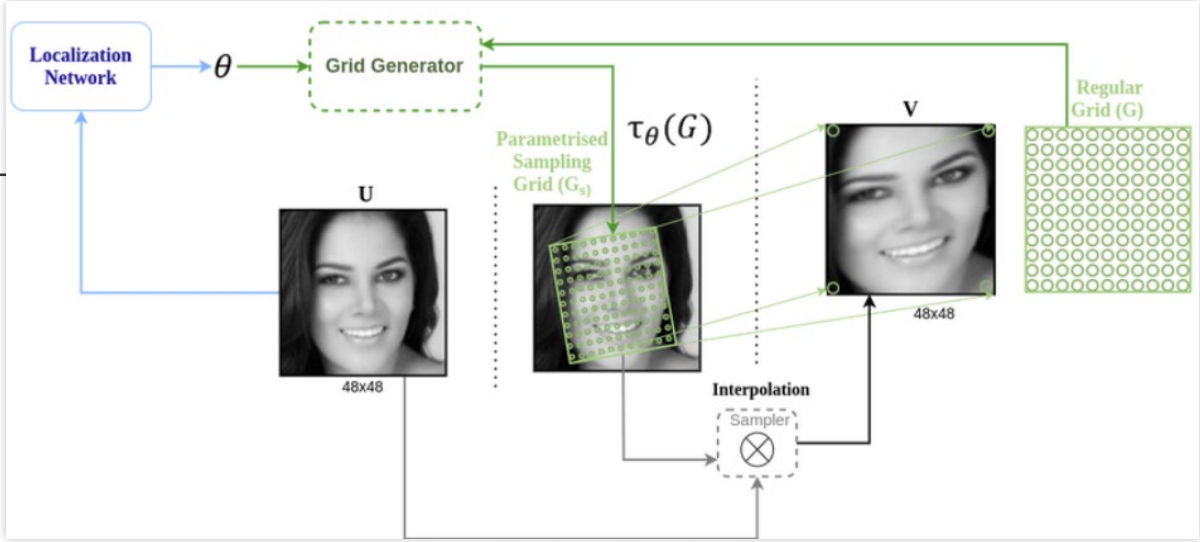
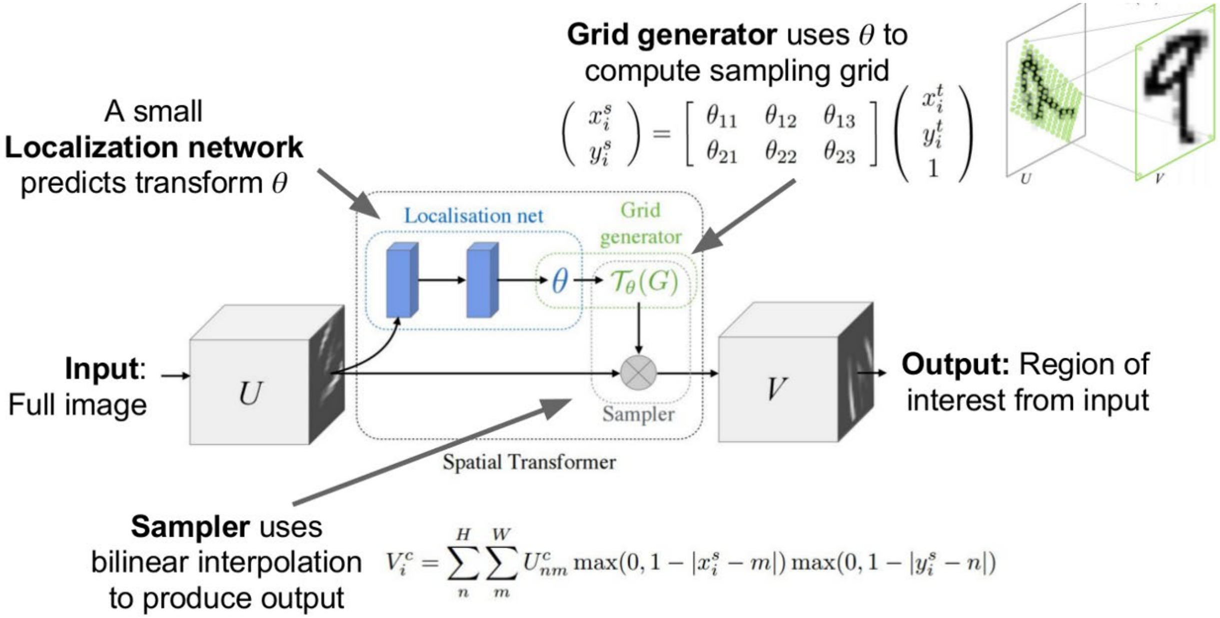
# ◆ Spatial Transformers




## ❖ Spatial Transformer

1. Localisation network
2. Grid generator
3. Sampler




(W x H x Ch x Batch size)

1. Input feature map U ⇒ **localisation net**

   ⇒ Transformation parameter $\theta$

2. $\theta$ ⇒ **Grid generator** ⇒ Sampling grid $T_\theta(G)$

   (include sampling point location)

3. U + $T_\theta(G)$ ⇒ **Sampler**

   → $T_\theta(G)$ has sampling point – apply to U

   ⇒ Output feature map V

https://manjunathbhat9920.medium.com/spatial-transformer-network-82666f184299

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# ◆ Spatial Transformers

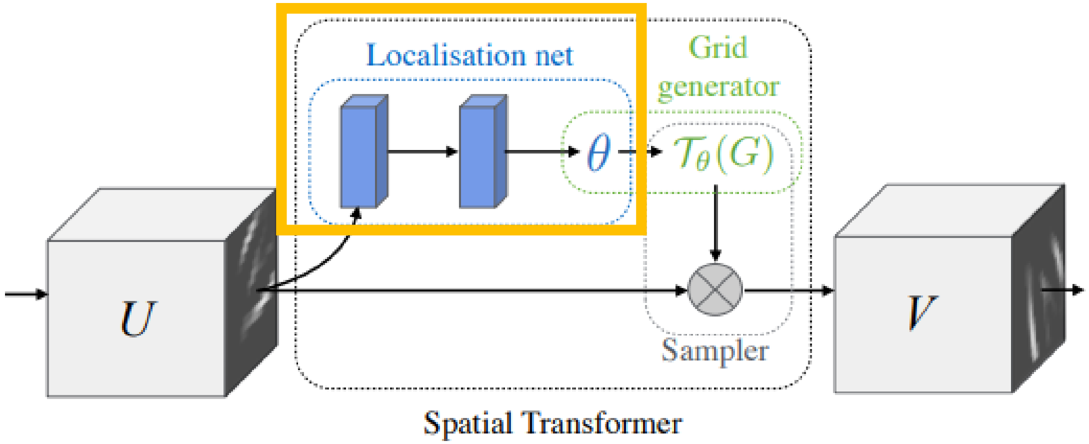
## 1. Localisation network

$$A_\theta = \begin{bmatrix} s & 0 & t_x \\ 0 & s & t_y \end{bmatrix}$$

> Only scaling + translation

- The localisation network is composed of fully-connected layers or convolution layers
- Predict **Transformation parameter matrix $\theta$** as output
- Input : $U \in R^{H \times W \times C}$ (width W, height H and C channels) => $\theta = f_{loc}(U)$
- The size of $\theta$ vary depending on transformation type (ex. affine=6-dimensional)
- Localisation network function $f_{loc}(U)$ can take any form (Convolution layer, FC layer) but Must include a final regression layer to produce the transform parameters.

$$\begin{pmatrix} x_i^s \\ y_i^s \end{pmatrix} = \mathcal{T}_\theta(G_i) = A_\theta \begin{pmatrix} x_i^t \\ y_i^t \\ 1 \end{pmatrix} = \underbrace{\begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \end{bmatrix}}_{\text{Transformation parameter matrix}} \begin{pmatrix} x_i^t \\ y_i^t \\ 1 \end{pmatrix}$$



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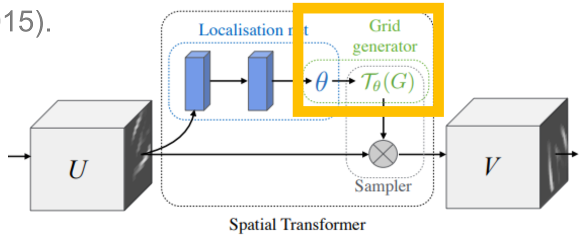


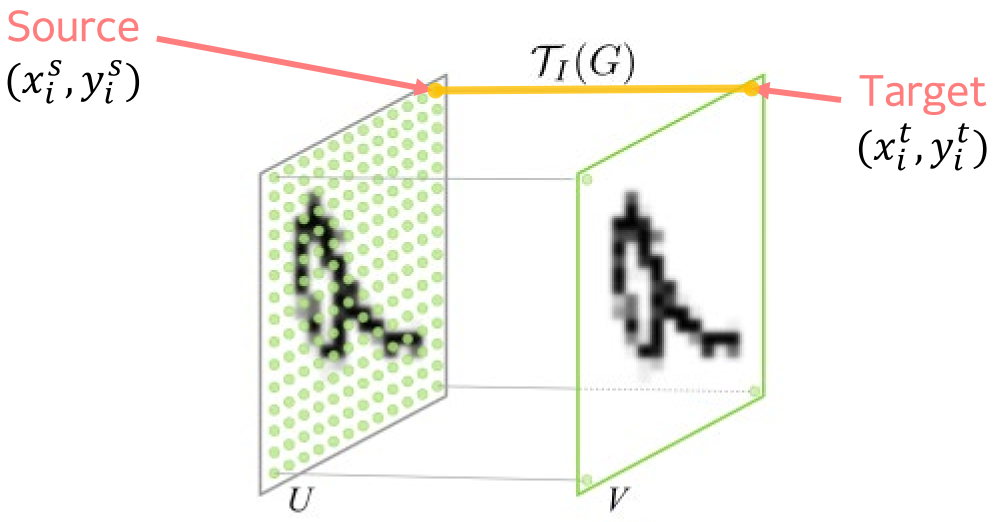

# ◆ Spatial Transformers

## 2. Parameterised Sampling Grid

$$\begin{pmatrix} x_i^s \\ y_i^s \end{pmatrix} = \boxed{\mathcal{T}_\theta(G_i)} = A_\theta \begin{pmatrix} x_i^t \\ y_i^t \\ 1 \end{pmatrix} = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \end{bmatrix} \begin{pmatrix} x_i^t \\ y_i^t \\ 1 \end{pmatrix}$$

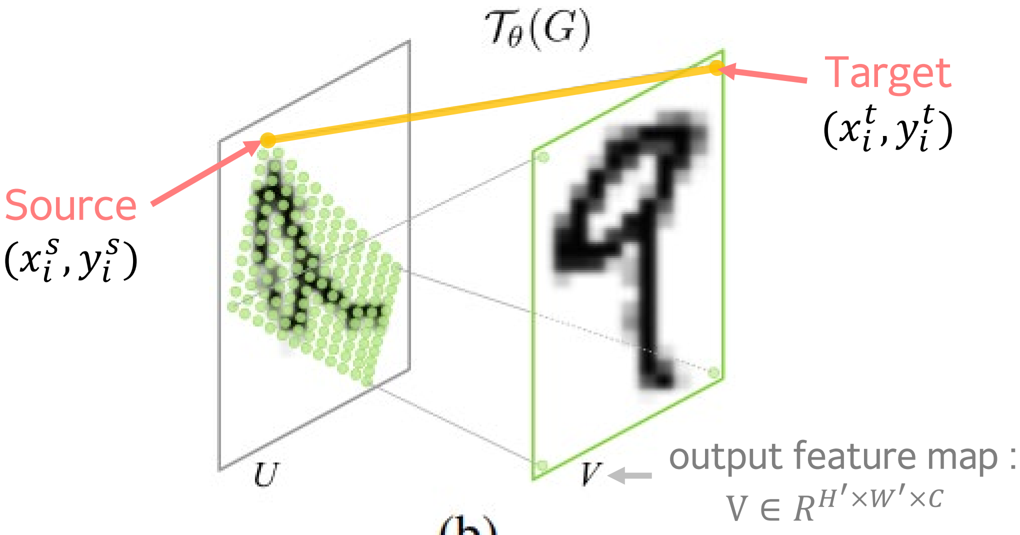
- Regular grid G = $(x_i^t, y_i^t)$ ⇒ Apply transformation $T_\theta$ on G ⇒ $\boldsymbol{T_\theta(G)}$
- **Sampling Grid $\boldsymbol{T_\theta(G)}$** = result of warping the regular grid G with transformation $T_\theta$

✓ Grid generator – **Identity transformation**




(a) Sampling Grid :
Regular grid $G = T_I(G)$

✓ Grid generator – **Affine transformation**




(b)

Sampling Grid : $G = T_\theta(G)$

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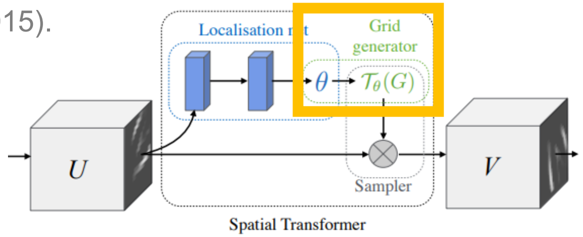



# ◆ Spatial Transformers

## 2. Parameterised Sampling Grid

✓ **Grid generator Examples**

➢ The transformation can have **any parameterised form**, provided that it is **differentiable** with respect to the parameters

### Affine transform

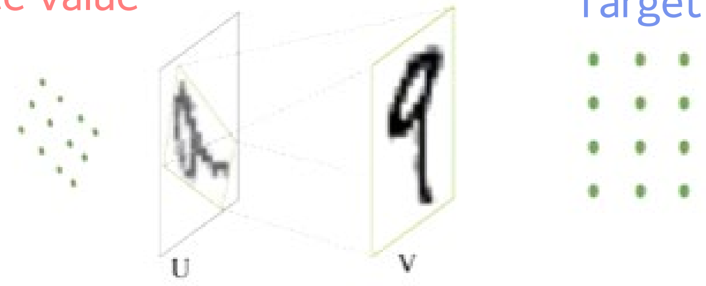
scale, rotation, translation, skew, cropping

Affine Transformation

$$\begin{pmatrix} x_i^s \\ y_i^s \end{pmatrix} = \mathcal{T}_\theta(G_i) = \mathtt{A}_\theta \begin{pmatrix} x_i^t \\ y_i^t \\ 1 \end{pmatrix} = \boxed{\begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \end{bmatrix}} \begin{pmatrix} x_i^t \\ y_i^t \\ 1 \end{pmatrix}$$

Source value

Target value

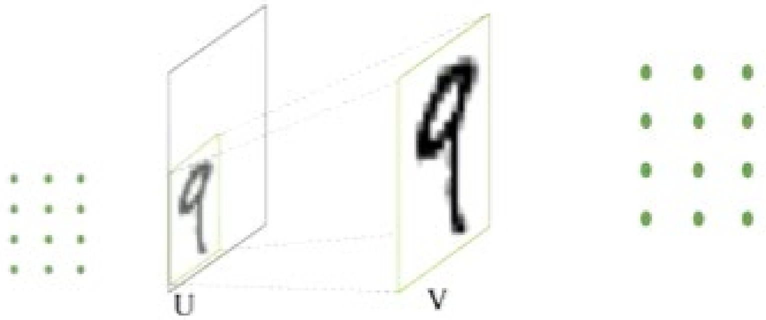



### Attention model

isotropic scale, translation, cropping

Attention Transformation

$$\begin{pmatrix} x_i^s \\ y_i^s \end{pmatrix} = \mathcal{T}_\theta(G_i) = \mathtt{A}_\theta \begin{pmatrix} x_i^t \\ y_i^t \\ 1 \end{pmatrix} = \boxed{\begin{bmatrix} s & 0 & t_x \\ 0 & s & t_y \end{bmatrix}} \begin{pmatrix} x_i^t \\ y_i^t \\ 1 \end{pmatrix}$$



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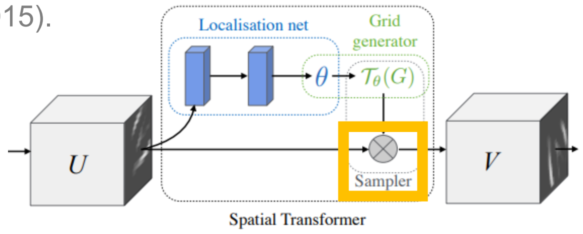
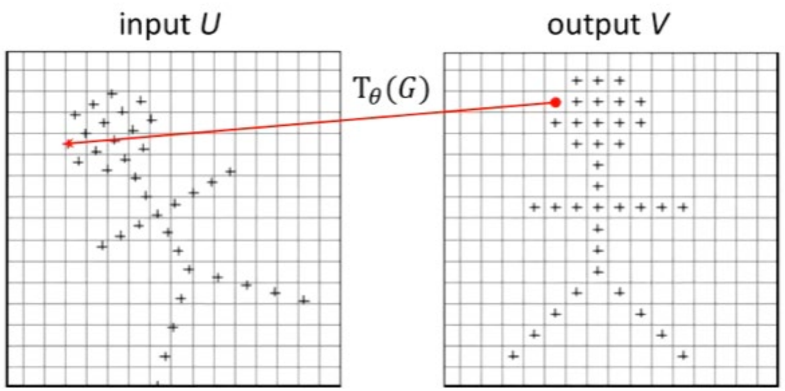



## ◆ Spatial Transformers

### 3. Differentiable Image Sampling

- $Sample(T_\theta(G), U) \rightarrow \boldsymbol{V}$
- Sampler : the set of sampling points $\boldsymbol{T_\theta(G)}$ + input feature map U ⇒ produce sampled **output feature map** V
- Each $(x_i^s, y_i^s)$ coordinate in $T_\theta(G)$ defines the spatial location in the input U where sampling kernel is applied to get the value at a particular pixel in the output V




- With high probability, location points are not an exact integer (like (1, 2))
  → need **interpolation** of surrounding values
- Any sampling kernel can be used.
- Interpolation function: Sampling kernel k()

$$V_i^c = \sum_n^H \sum_m^W U_{nm}^c k(x_i^s - m; \Phi_x) k(y_i^s - n; \Phi_y) \ \forall i \in [1 \dots H'W'] \ \forall c \in [1 \dots C]$$

Parameters of k()

Output value $(x_i^t, y_i^t)$
Output feature map channel c

Input value
(n,m) value

Sampling grid coordinate

> General form

https://jamiekang.github.io/2017/05/27/spatial-transformer-networks/

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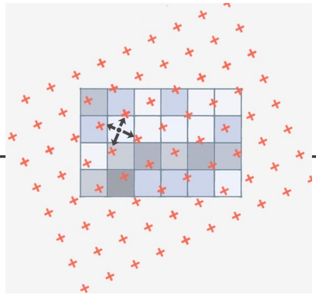
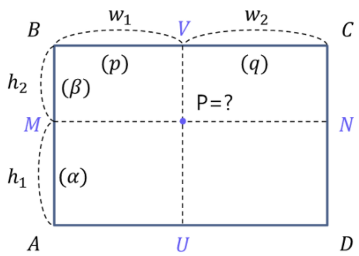





# ◆ Spatial Transformers

## 3. Differentiable Image Sampling

### ▪ Nearest Integer Sampling

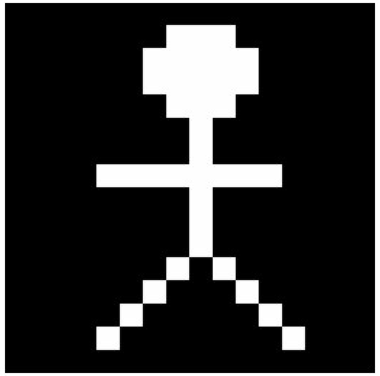
$$V_i^c = \sum_n^H \sum_m^W U_{nm}^c \delta(\lfloor x_i^s + 0.5 \rfloor - m)\delta(\lfloor y_i^s + 0.5 \rfloor - n)$$

Target value ($V_i^c$)

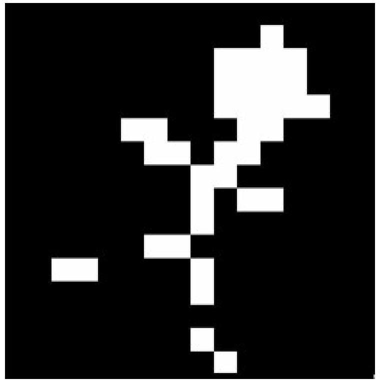
Source value ($U_{nm}^c$)

[ ] : rounding to the nearset integer

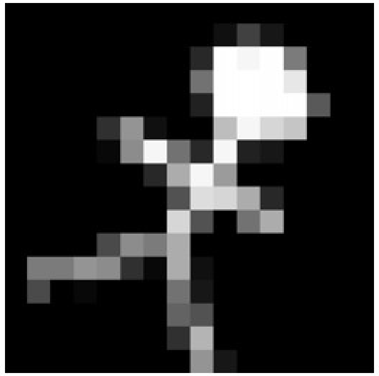
Original Image



Nearest Neighbor



Bilinear Interpolation



### ▪ Bilinear Sampling → linear interpolation on the x, y axes respectively

$$V_i^c = \sum_n^H \sum_m^W U_{nm}^c \max(0, 1 - |x_i^s - m|) \max(0, 1 - |y_i^s - n|)$$

$$\frac{\partial V_i^c}{\partial U_{nm}^c} = \sum_n^H \sum_m^W \max(0, 1 - |x_i^s - m|) \max(0, 1 - |y_i^s - n|)$$

$$\frac{\partial V_i^c}{\partial x_i^s} = \sum_n^H \sum_m^W U_{nm}^c \max(0, 1 - |y_i^s - n|) \begin{cases} 0 & \text{if } |m - x_i^s| \geq 1 \\ 1 & \text{if } m \geq x_i^s \\ -1 & \text{if } m < x_i^s \end{cases}$$

- ➢ It is a (sub-)**differentiable** sampling mechanism so that it is convenient for backpropagation
- ➢ If non-differentiable, backpropagation can be calculated by dividing it by sections.

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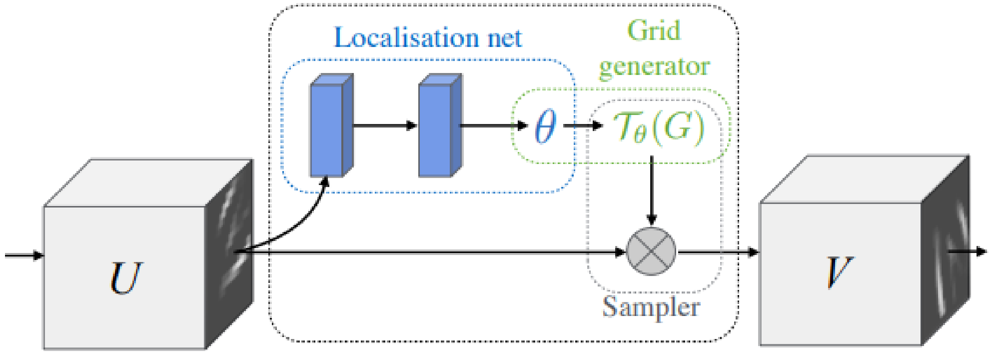
# ◆ Spatial Transformers

## ❖ Spatial Transformer Networks

- The combination of Localisation network, grid generator and sampler form spatial transformer module

  + CNN achitecture ⇒ **Spatial Transformer Network**
- Spatial transformer module can be dropped into a CNN architecture at any point, and in any number
- Learn during the training process to minimize the overall cost function of CNN ⇒ it has little effect on training speed
- The knowledge(how to transform) is compressed and cached in the weights of the localisation network during training
- It is possible to have multiple spatial transformers in a CNN.
  - To put the ST layer just before the CNN input : most efficient

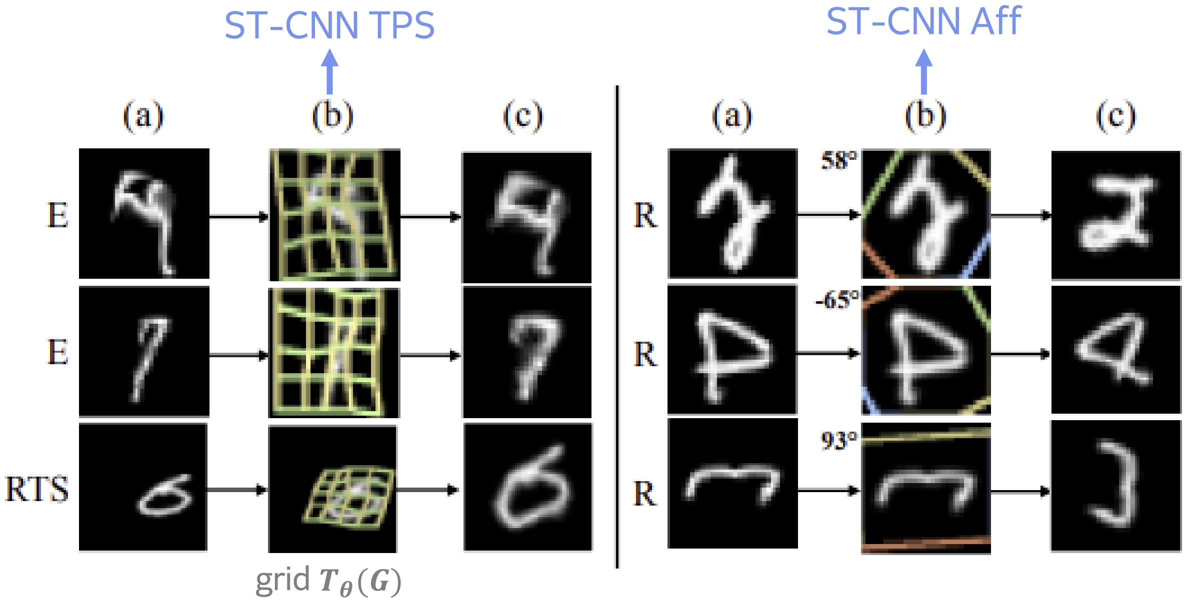


# ◆ Experiments

## 1. Distorted MNIST

✓ Model
- FCN, CNN
- ST + FCN
- ST + CNN

| Model | | MNIST Distortion R | RTS | P | E |
|---|---|---|---|---|---|
| FCN | | 2.1 | 5.2 | 3.1 | 3.2 |
| CNN | | 1.2 | 0.8 | 1.5 | 1.4 |
| ST-FCN | Aff | 1.2 | 0.8 | 1.5 | 2.7 |
| | Proj | 1.3 | 0.9 | 1.4 | 2.6 |
| | TPS | 1.1 | 0.8 | 1.4 | 2.4 |
| ST-CNN | Aff | 0.7 | 0.5 | 0.8 | 1.2 |
| | Proj | 0.8 | 0.6 | 0.8 | 1.3 |
| | TPS | 0.7 | 0.5 | 0.8 | 1.1 |




- MNIST dataset – distorted in various ways (4)
  - rotation (R), rotation-translation-scale (RTS), projective transformation (P), elastic warping (E)
- As we can see, ST-FCN outperforms FCN and ST-CNN outperforms CNN.
  - ST-CNN models consistently perform better than ST-FCN models
- CNN > FCN ⇒ The CNN models include two max-pooling layers → Maxpooling layer makes spatial invariance high
- Spatial transformation : affine (AFF), projective (Proj), thin plate spline transformation (TPS)
  - Between different classes of transformation, the TPS is the most powerful

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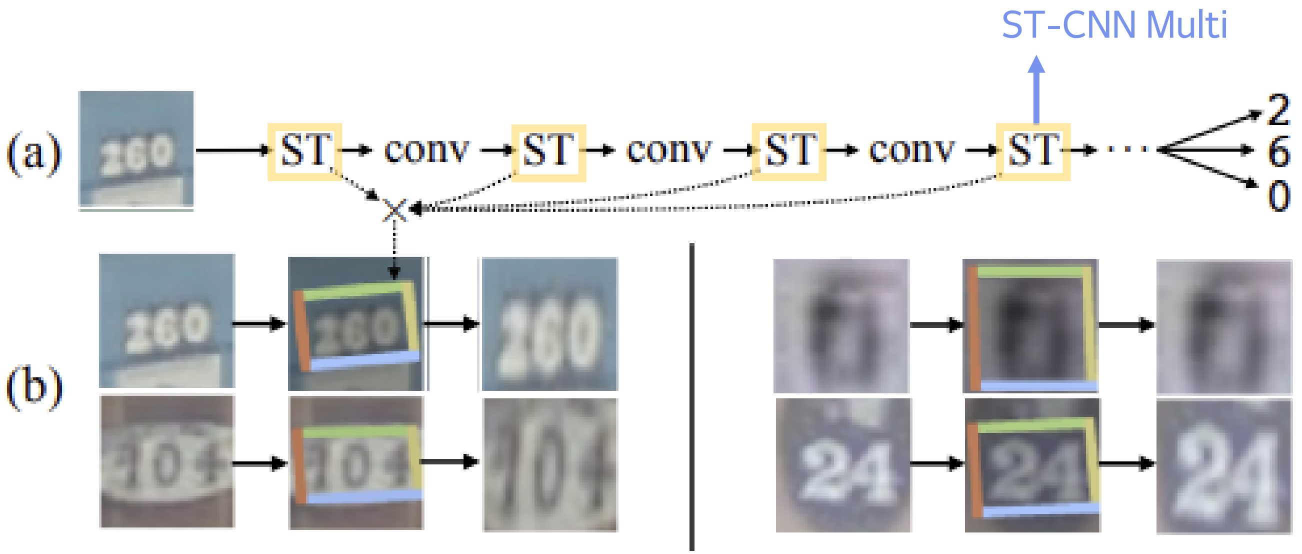
## ◆ Experiments

### 2. Street View House Numbers

| Model | | Size 64px | Size 128px |
|---|---|---|---|
| Maxout CNN [10] | | 4.0 | - |
| CNN (ours) | | 4.0 | 5.6 |
| DRAM* [1] | | 3.9 | 4.5 |
| ST-CNN | Single | 3.7 | **3.9** |
| ST-CNN | Multi | **3.6** | **3.9** |




- This dataset contains around 200k real world images of house numbers
- Put ST layer → CNN convolutional stack
  - ST-CNN Single : Only one ST at the beginning of network
  - ST-CNN Multi : One ST before each convolutional layer
- Affine transformation, bilinear sampler is used
- ST-CNN outperforms Maxout and CNN
  - ST-CNN Multi outperforms ST-CNN Single a bit– 속도 6% 느려짐

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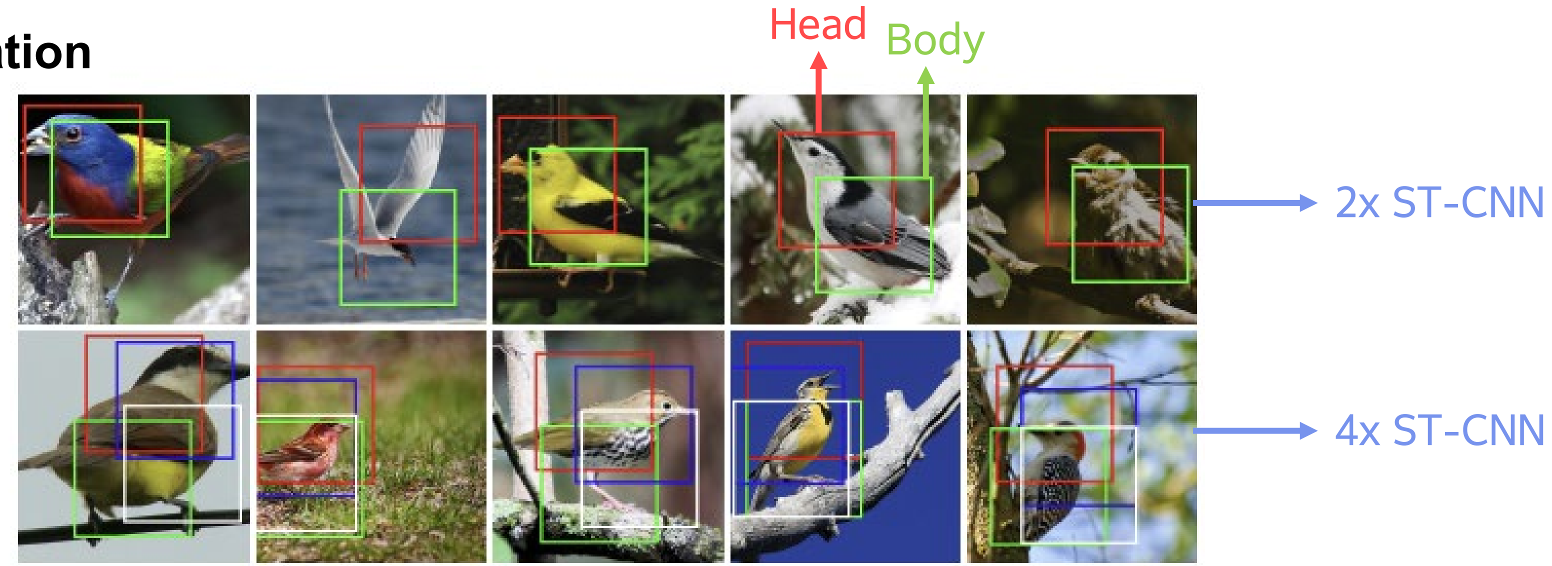
# ◆ Experiments

## 3. Fine-Grained Classification

| Model | | |
|---|---|---|
| Cimpoi '15 [4] | | 66.7 |
| Zhang '14 [30] | | 74.9 |
| Branson '14 [2] | | 75.7 |
| Lin '15 [20] | | 80.9 |
| Simon '15 [24] | | 81.0 |
| CNN (ours) | 224px | 82.3 |
| 2×ST-CNN | 224px | 83.1 |
| 2×ST-CNN | 448px | 83.9 |
| 4×ST-CNN | 448px | **84.1** |




- Use a spatial transformer network with multiple transformers in parallel to perform fine-grained bird classification
- CUB 200-2011 birds dataset - 200 species of birds, 11,788 images
- Strong baseline CNN model : Inception-v2 (ImageNet pre-trained) for classifying 200 species (82.3% accuracy)
  - 2x ST-CNN , 4x ST-CNN : 2 or 4 parallel STs
- 4x ST-CNN achieves an accuracy of 84.1%, outperforms the baseline by 1.8%
- Interesting behaviour : Each box = ST found (without supervision)
  - 2x ST-CNN : red ST – head detector, green – detect central part of the body

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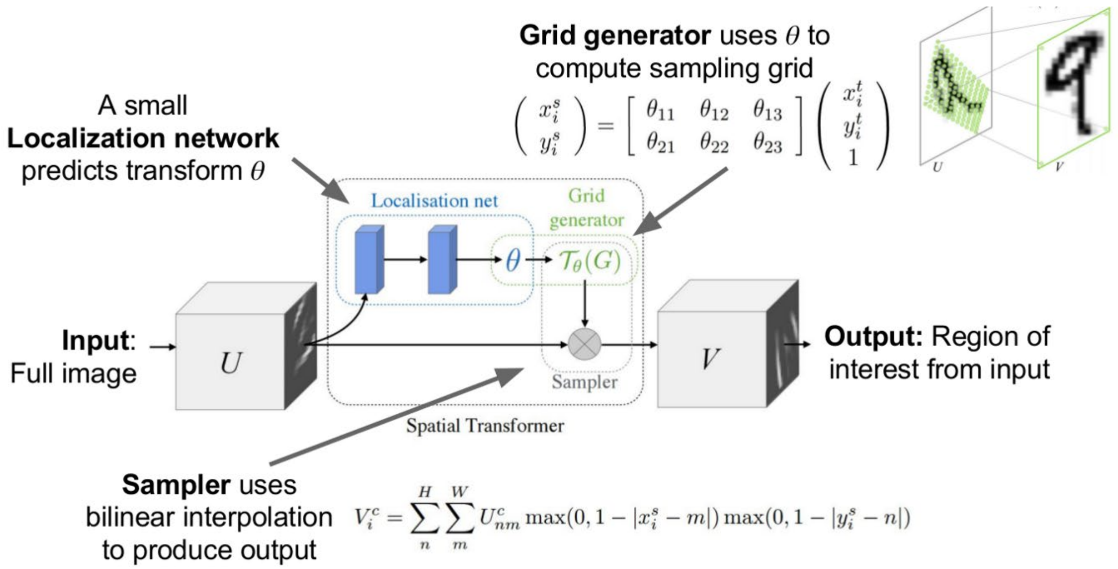
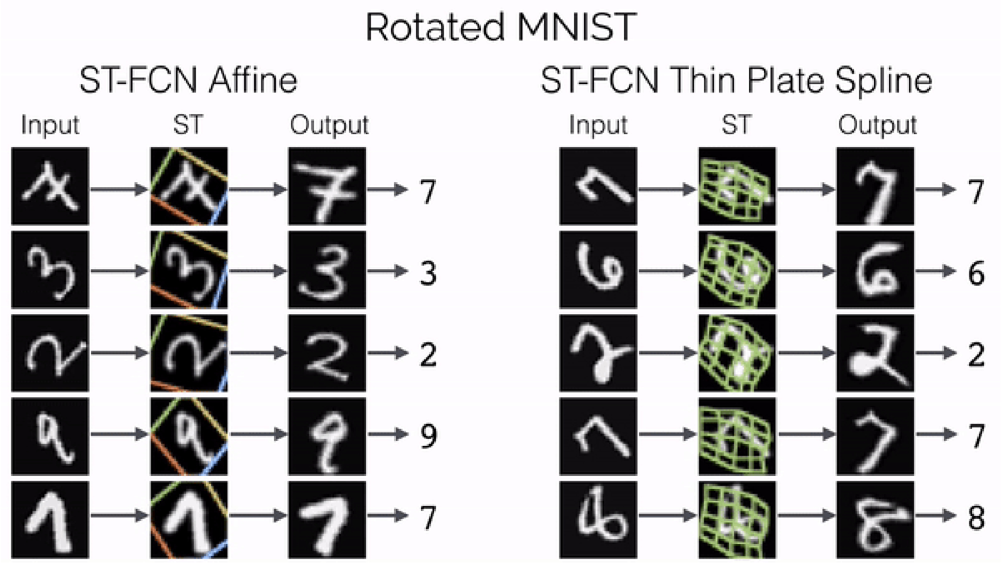
# ◆ Conclusion

- A new self-contained module for neural networks – the spatial transformer
- This module can be dropped into a network and perform explicit spatial transformations of features
- **Differentiable** and learnt in an end-to-end fashion – without making any changes to the loss function
- While CNNs provide an incredibly strong baseline, they see gains in accuracy using spatial transformers across multiple tasks, resulting in state-of-the-art performance.

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https://towardsdatascience.com/review-stn-spatial-transformer-network-image-classification-d3cbd98a70aa