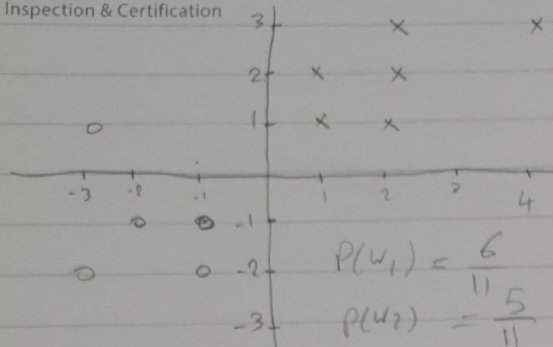


سؤال 1) کلاس 1 = کلاس قرمز

کلاس 2 = کلاس آبی



$$P(x|w_1) = N(x; \mu_1, \Sigma_1)$$

$$P(x) = \frac{1}{\sqrt{2\pi} \Sigma} \exp\left(-\frac{1}{2} (x - \mu)^T \Sigma^{-1} (x - \mu)\right)$$

$$P(x|w_2) = N(x; \mu_2, \Sigma_2)$$

$$P(w_1) = \frac{6}{11}$$

$$P(w_2) = \frac{5}{11}$$

$$\mu_1 = \begin{bmatrix} 1.5 \\ 1.5 \end{bmatrix}$$

$$\mu_2 = \begin{bmatrix} -1.5 \\ -1.5 \end{bmatrix}$$

$$\Sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0.66 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 0.66 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{P(w_1|x)}{P(w_2|x)} \geq \frac{P(w_1)}{P(w_2)} \xrightarrow{\log} \log(P(x|w_1)) - \log(P(x|w_2)) = \log\left(\frac{P(w_2)}{P(w_1)}\right)$$

$$\Rightarrow -\frac{1}{2} (x - \mu_1)^T \Sigma_1^{-1} (x - \mu_1) + \frac{1}{2} (x - \mu_2)^T \Sigma_2^{-1} (x - \mu_2) + \frac{1}{2} \log\left(\frac{|\Sigma_2|}{|\Sigma_1|}\right) = \log\left(\frac{P(w_2)}{P(w_1)}\right)$$

$$\Rightarrow x^T (\Sigma_1^{-1} - \Sigma_2^{-1}) x + 2 (\Sigma_2^{-1} \mu_2 - \Sigma_1^{-1} \mu_1)^T x + \mu_1^T \Sigma_1^{-1} \mu_1 - \mu_2^T \Sigma_2^{-1} \mu_2 - \log\left(\frac{|\Sigma_2|}{|\Sigma_1|}\right) - 2 \log\left(\frac{P(w_2)}{P(w_1)}\right) = 0$$

$$\Rightarrow x^T A x + b^T x + c = 0 \quad \Sigma_1^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1.5 \end{bmatrix} \quad \Sigma_2^{-1} = \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\xrightarrow{\text{حساب}} [x, y] \begin{bmatrix} 0.33 & 0 \\ 0 & -0.33 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + 2 \left(\begin{bmatrix} -2 \\ -0.66 \end{bmatrix} - \begin{bmatrix} 2 \\ 1.32 \end{bmatrix} \right)^T \begin{bmatrix} x \\ y \end{bmatrix}$$

$$+ 6.64 - 4.66 - \log 1 - 2 \log \frac{5}{6} = 0$$

$$\Rightarrow 0.33x^2 - 0.33y^2 - 8x - 4y + 2.13 = 0$$

$$R(\alpha_i | \underline{x}) = \sum_{j=1}^c \lambda_j P(w_j | \underline{x}) = \lambda_s \sum_{j=1}^c P(w_j | \underline{x})$$

سؤال 3: الف)

$$R(\alpha_{c+1} | \underline{x}) = \lambda_r$$

$$\Rightarrow \sum_{j=1}^c \lambda_s (1 - P(w_j | \underline{x})) \leq \lambda_s (1 - P(w_j | \underline{x}))$$

$$\Rightarrow P(w_i | \underline{x}) \geq P(w_j | \underline{x}) \quad (1)$$

2

$$\text{if } j=c \Rightarrow \lambda_s (1 - P(w_i | \underline{x})) \leq \lambda_r \Rightarrow P(w_i | \underline{x}) \geq 1 - \frac{\lambda_r}{\lambda_s} \quad (2)$$

$$(1), (2) \Rightarrow \text{Prof} \checkmark$$

$$\text{Discriminative Func: } P(w_i | \underline{x}) \geq P(w_j | \underline{x})$$

$$\text{for } j=1, \dots, c \Rightarrow P(\underline{x} | w_i) P(w_i) \geq P(\underline{x} | w_j) P(w_j) \quad (1)$$

$$\text{for } j=c+1 \Rightarrow P(w_i | \underline{x}) \geq 1 - \frac{\lambda_r}{\lambda_s} \Rightarrow P(\underline{x} | w_i) P(w_i) \geq (1 - \frac{\lambda_r}{\lambda_s}) P(\underline{x})$$

$$\Rightarrow P(\underline{x} | w_i) P(w_i) \geq (1 - \frac{\lambda_r}{\lambda_s}) \left(\sum_{j=1}^c P(\underline{x} | w_j) P(w_j) \right) \quad i=c+1 \quad (2)$$

$$(1), (2) \Rightarrow \text{Prof} \checkmark$$

$$\text{if } \frac{\lambda_r}{\lambda_s} = 1 \quad (2) \Rightarrow 1 - \frac{\lambda_r}{\lambda_s} = 0 \Rightarrow P(w_i | \underline{x}) \geq 0 \Rightarrow \text{همیشه درست است (احتمال } P > 0 \text{)}$$

یعنی در این حالت هیچ وقت عمل $c+1$ اتفاق نمی افتد

$$\text{if } \frac{\lambda_r}{\lambda_s} = 0 \Rightarrow 1 - \frac{\lambda_r}{\lambda_s} = 1 \Rightarrow P(w_i | \underline{x}) \geq 1 \Rightarrow \text{فقط } P(w_i | \underline{x}) = 1 \text{ چون احتمال}$$

یکی در هر شرایط کلاس داده ای مادر هیچ دسته ای قرار ندارد و همیشه عمل $c+1$ اتفاق می افتد

$$\text{if } 0 < \frac{\lambda_r}{\lambda_s} < 1 \Rightarrow \text{Risk برای عمل } c+1 \text{ به Risk بقیه نزدیک تر باشد احتمال}$$

آنکه داده در هیچ دسته ای نباشد بیشتر است

$$y_{red} = [(8, 3), (8, -3), (0, 5), (0, -5)]$$

$$y_{blue} = [(0, 0), (-5, 5), (-5, 0), (-5, -5)] \quad x = (5, 0)$$

① $d = \max |x_i, y_j|$

1) $(3, 3) / (3, 3) / (5, 5) / (5, 5)$

2) $(5, 0) / (10, 5) / (10, 0) / (10, 5) \rightarrow \min = 3 \Rightarrow \text{کلاس} = \text{Red}$

② $d = \sum_{i=1}^d |x_i - y_j|$

1) $6 / 6 / 10 / 10$

2) $5 / 15 / 10 / 15 \rightarrow \min = 5 \Rightarrow \text{کلاس} = \text{Blue}$

③ $d = \sum_{i=1}^d (x_i - y_j)^2$

1) $18 / 18 / 50 / 50$

2) $25 / 125 / 100 / 125 \rightarrow \min = 18 \Rightarrow \text{کلاس} = \text{Red}$

$d = \sqrt{(x - \eta)^T \Sigma^{-1} (x - \eta)}$

$r_1 = \sqrt{[1.5 \ 1.5] \cdot \frac{2}{3} \begin{bmatrix} 3.5 & 3 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 1.5 \\ 1.5 \end{bmatrix}} = 4.33$

$r_2 = \sqrt{[1.5 \ 1.5] \cdot \frac{2}{3} \begin{bmatrix} 3.5 & -3 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 1.5 \\ 1.5 \end{bmatrix}} = 0.866$

(c)

برای M مثبت و نیمه قطعی $M \Rightarrow$ positive - semi definite (7)

$$\Rightarrow \text{for any } z \Rightarrow z^T M z \geq 0 \Rightarrow \det(M) \geq 0 \Rightarrow 20 - z^2 \geq 0$$

$$\Rightarrow -\sqrt{20} \leq z \leq \sqrt{20}$$

$$\frac{P(X|w_1)}{P(X|w_2)} > \frac{\lambda_{12} - \lambda_{22}}{\lambda_{21} - \lambda_{11}} \frac{P(w_2)}{P(w_1)} \Rightarrow \frac{x + \frac{1}{2}}{\frac{3x^2 + 3}{4}} > \frac{2-1}{3-1} \cdot \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{8}$$

$$\Rightarrow 3x^2 - 24x - 9 < 0 \Rightarrow x^2 - 8x - 3 < 0 \Rightarrow \text{؟ وقت}$$