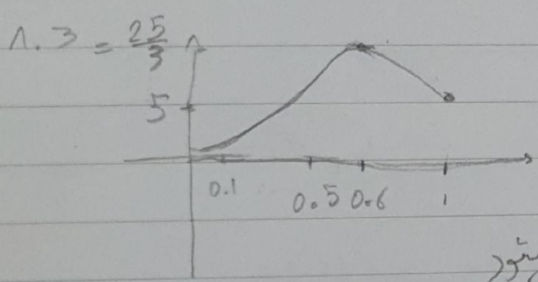
سوال 2 الف

$$L(\theta) = \arg\max_{\theta} P(x|\theta) = \prod_{1}^{n} \begin{cases} \frac{1}{\theta} & 0 \le x_n \le \theta \\ 0 & o.w \end{cases}$$

$$\Rightarrow \log P(x|\theta) = \sum_{1}^{n} \frac{1}{\theta} \quad 0 \le x_n \le \theta \Rightarrow$$ مشتق : $\sum_{1}^{n} -\frac{1}{\theta^2} \Rightarrow$ جواب ندارد

$\Rightarrow$ بیشترین حالت : $\sum_{1}^{n} \frac{1}{\theta} \Rightarrow \frac{n}{\theta} \quad 0 \le x_n \le \theta \Rightarrow$ تمام xها دراین بازه باید باشند $\Rightarrow \theta = \max[D]$

ب)



نیاز به داشتن بقیه مقادیر نداریم چرا که همان طور که در سوال قبل
پیدا کردیم $\max \theta = \max[D]$ و می دانیم با فاصله از آن $P(x|\theta)$ کم می شود

if $\theta \ge 0.6 \Rightarrow P(x|\theta) = \sum_{1}^{5} x_n$

سوال 3:

$$P(x|\theta) = \prod_{i=1}^{d} \theta_i^{x_i} (1-\theta_i)^{1-x_i}$$

$$L(\theta) = \arg\max_{\theta} P(x|\theta) \rightarrow \log L(\theta) = \sum_{i=1}^{d} \log \theta_i^{x_i} (1-\theta_i)^{1-x_i}$$

$$= \sum_{i=1}^{d} x_i \log(\theta_i) + (1-x_i)\log(1-\theta_i) \xrightarrow{\sum_{i=1}^{n} x_i = Y} Y \log\theta_i + (n-Y)\log(1-\theta_i)$$

$$\Rightarrow \frac{\partial L(\theta)}{\partial \theta_i} = Y\frac{1}{\theta_i} + (\theta - Y)\frac{-1}{1-\theta_i} = 0 \Rightarrow \theta_i = \frac{Y}{n} = \frac{\sum_{i=1}^{d} x_i}{n}$$ = میانگین کردها

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سوال 4:

$n_0 = \frac{\sigma^2}{\sigma_0^2}$, μ_0 = avg $k = -n_0+1, \dots, 0$

الف) $\mu_n = \frac{1}{n+n_0} \sum_{k=-n_0+1}^{n} x_k$, $\sigma_n^2 = \frac{\sigma}{n+n_0} \rightarrow \mu_0 = \frac{1}{n_0} \sum_{i=-n_0+1}^{0} x_i$

$$\mu_n = \frac{\sum_{i=1}^{n} x_i}{n + \frac{\sigma^2}{\sigma_0^2}} + \frac{\frac{\sigma^2}{\sigma_0^2}}{n_0\left(\frac{\sigma^2}{\sigma_0^2}+n\right)} \sum_{k=-n_0+1}^{0} x_k = \frac{\sum_{i=1}^{n} x_i}{n+n_0} + \frac{\sum_{k=-n_0+1}^{0} x_k}{n_0+n} = \frac{\sum_{k=-n_0+1}^{n} x_k}{n_0+n}$$

$$\sigma_n^2 = \frac{\sigma^2 \sigma_0^2}{n\sigma_0^2+\sigma^2} = \frac{\sigma^2}{n+\frac{\sigma^2}{\sigma_0^2}} = \frac{\sigma^2}{n+n_0}$$

ب)

$$P \sim N(\mu_0, \sigma_0^2) \rightarrow \mu_0 = \frac{1}{n_0} \sum_{i=-n_0+1}^{0} x_i \quad , \quad \sigma_0^2 = \frac{\sigma^2}{n}$$

در این صورت MLE و BayesE برای حالتی که ML تمامی داده‌ها و Bayes $n+n_0$ داده داریم برابر است

سوال 5:

$$MAP \rightarrow \arg\max_{\mu} \frac{P(x|\mu)P(\mu)}{P(x)} = \arg\max P(x|\mu)P(\mu) = \hat{\mu}$$

$$\Rightarrow \hat{\mu} = \prod_{i=1}^{N} P(x_i|\mu)P(\mu) \rightarrow \log \rightarrow \sum_{i=1}^{N}\left(\log P(x_i|\mu) + \log P(\mu)\right)$$

$$= \sum_{i=1}^{N}\left[-\frac{1}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}(x_i-\mu)^2 + \log\left(\frac{\mu}{\sigma_\mu^2}\right) - \frac{\mu^2}{2\sigma_\mu^2}\right] \rightarrow \frac{\delta L}{\delta \mu} = 0$$

$$\Rightarrow \sum_{i=1}^{N}\left[\frac{-1}{2\sigma^2}(-2)(x_i-\mu) + \frac{1}{\mu} - \frac{2\mu}{2\sigma_\mu^2}\right] = 0 \rightarrow \sum_{i=1}^{N}\left[\frac{1}{\sigma^2}x_i - \mu\left(\frac{1}{\sigma^2}+\frac{1}{\sigma_\mu^2}\right) + \frac{1}{\mu}\right] = 0$$

درجه 2 بر حسب μ $\rightarrow$ $\mu = \frac{\frac{\sum x_i}{\sigma^2} \pm \sqrt{\left(\frac{\sum x_i}{\sigma^2}\right)^2 + 4N^2\left(\frac{1}{\sigma^2}+\frac{1}{\sigma_\mu}\right)}}{2N\left(\frac{1}{\sigma^2}+\frac{1}{\sigma_\mu^2}\right)}$

سئوال ۵:

$$p(z) = \frac{\lambda^z e^{-\lambda}}{z!} \quad , \quad \log p(z) = z \log \lambda - \lambda - \log z! \qquad ②$$

$$Q(\Theta, \Theta^{(g)}) = \sum_{j=1}^{m} \sum_{i=1}^{n} \log\left(\alpha_j p_j(x_i \mid \theta_j)\right) p(j \mid x_i, \Theta^{(g)})$$

$$= \sum_{j=1}^{m} \sum_{i=1}^{n} \log(\alpha_j)\, p(j \mid x_i, \Theta^{(g)}) + \sum_{j=1}^{m} \sum_{i=1}^{n} \log\left(p_j(x_i \mid \theta_j)\right) p(j \mid x_i, \Theta^{(g)})$$

$$① \rightarrow \hat{\alpha}_j = \frac{1}{n} \sum_{i=1}^{n} p(j \mid x_i, \Theta^{(g)}) \quad , \quad p(j \mid x_i, \Theta^{(g)}) = \frac{\alpha_j^{(g)} p_j(x_i \mid \theta_j^{(g)})}{\sum_{j=1}^{m} \alpha_j^{(g)} p_j(x_i \mid \theta_j^{(g)})}$$

$$② \rightarrow \frac{\delta LL}{\delta \lambda} = 0 \Rightarrow \frac{x_i}{\lambda_j} - 1 = 0 \Rightarrow \lambda_j = x_i \Rightarrow \hat{\lambda}_j = \frac{\sum_{i=1}^{n} p(j \mid x_i, \Theta^{(g)})\, x_i}{\sum_{i=1}^{n} p(j \mid x_i, \Theta^{(g)})}$$