Class 26

Sunday, July 25, 2021 10:13 PM

$fib(n) \% M$

$1 \leq n \leq 10^{15}$

$M = 10^9 + 7$

$f(n) = f(n-1) + f(n-2)$ $\longrightarrow$ $\begin{vmatrix} f(n) \\ f(n-1) \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} f(n-1) \\ f(n-2) \end{vmatrix}$

A 2×1 B 2×2 C 2×1

$f(n-1) = 1 \cdot f(n-1) + 0 \cdot f(n-2)$

$x \cdot f(n-1) + y \cdot f(n-2)$

$$\begin{vmatrix} f(n) \\ f(n-1) \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \begin{vmatrix} f(n-1) \\ f(n-2) \end{vmatrix} \Longrightarrow \begin{vmatrix} f(n-1) \\ f(n-2) \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \begin{vmatrix} f(n-2) \\ f(n-3) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \begin{vmatrix} f(n-2) \\ f(n-3) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}^2 \begin{vmatrix} f(n-2) \\ f(n-3) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}^3 \begin{vmatrix} f(n-3) \\ f(n-4) \end{vmatrix} \cdots\cdots = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}^{n-1} \begin{vmatrix} f(1) \\ f(0) \end{vmatrix}$$

$$\begin{vmatrix} f(3) \\ f(2) \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}^2 \begin{vmatrix} f(1) \\ f(0) \end{vmatrix} \longrightarrow \begin{vmatrix} f(n) \\ f(n-1) \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}^{n-1} \begin{vmatrix} f(1) \\ f(0) \end{vmatrix}$$

$$\begin{vmatrix} f(3) \\ f(2) \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \begin{vmatrix} f(1) \\ f(0) \end{vmatrix} \rightarrow \begin{vmatrix} f(n) \\ f(n-1) \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \begin{vmatrix} f(1) \\ f(0) \end{vmatrix}$$

$$A^0 = I_A \qquad A^n = \begin{cases} A^{n/2} \times A^{n/2} & \text{if } n \text{ is even} \\ A^{n-1} \times A & \text{if } n \text{ is odd} \\ I_A & \text{if } n \text{ is } 0 \end{cases} \qquad A^n$$

$$\boxed{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}$$

$A^n \rightarrow A$ non-sq

$A_{n \times m} \times A_{n \times m}$

$n \neq m$

$C = A \times B$

$r_1 c_2 \quad r_1 \boxed{c_1 \quad r_2} c_2$

Given,

$a+b$

ab

$f(1) = a^1 + b^1 = a + b$

$f(2) = a^2 + b^2 = (a+b)^2 - 2ab$

$= (a+b)(a+b) - 2ab$

$f(3) = a^3 + b^3 = \underbrace{(a^2+b^2)}_{f(2)}(a+b) - \underbrace{(a+b)}_{f(1)}(ab)$

$f(0) = a^0 + b^0 = 2$

$f(1) = a + b = p$

$$\boxed{f(n) = f(n-1)\cdot(a+b) - f(n-2)\cdot(ab)}$$

$$\boxed{f(n) = f(n-1)\cdot(a+b) - f(n-2)\cdot(ab)}$$

$$\begin{vmatrix} f(n) \\ f(n-1) \end{vmatrix} = \begin{vmatrix} (a+b) & -ab \\ 1 & 0 \end{vmatrix} \begin{vmatrix} f(n-1) \\ f(n-2) \end{vmatrix}$$

$$\begin{vmatrix} f(n) \\ f(n-1) \\ f(n-2) \\ g(n) \\ g(n-1) \\ g(n-2) \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 0 & 0 & 0 & c_1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_2 & a_2 & b_2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{vmatrix} \begin{vmatrix} f(n-1) \\ f(n-2) \\ f(n-3) \\ g(n-1) \\ g(n-2) \\ g(n-3) \end{vmatrix}$$

6×6

f(u, rem)

{ u == des && rem == 0

res = 0/1 $\hookrightarrow$ 1

res $\cancel{+}$ f(v, rem - cst[u......v])

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