Differential Equations

The population dynamics of human civilians (H), human military (M_H) , alien civilians (A), alien military (M_A) , resources (R), and the war state (W) are governed by the following equations:

$$\begin{split} \frac{dH}{dt} &= r_H H - \text{recruitment}_H - \text{death}_H^{\text{scarcity}} - \text{combat}_H^{\text{loss}} \\ & \frac{dM_H}{dt} = \text{recruitment}_H - \text{combat}_H^{\text{loss}} \\ & \frac{dA}{dt} = r_A A - \text{recruitment}_A - \text{death}_A^{\text{scarcity}} - \text{combat}_A^{\text{loss}} \\ & \frac{dM_A}{dt} = \text{recruitment}_A - \text{combat}_A^{\text{loss}} \\ & \frac{dR}{dt} = \theta R \left(1 - \frac{\text{totalPopulation}}{\text{maxPopulation}} \right) - \sigma \cdot \text{totalPopulation} \end{split}$$

Where the total population is:

totalPopulation =
$$H + M_H + A + M_A$$

The war state (W) evolves as:

$$W = \begin{cases} 1 & \text{if } (R < R_{\text{threshold}}) \text{ or (totalPopulation} > \text{maxPopulation}) \\ 0 & \text{if } (R \ge R_{\text{threshold}}) \text{ and (totalPopulation} \le \text{maxPopulation}) \end{cases}$$

Parameters and Their Meanings

- $r_H = 0.0003$: Human civilian growth rate.
- $r_A = 0.0001$: Alien civilian growth rate.
- $k_{H_{\text{peace}}} = 0.001$, $k_{H_{\text{war}}} = 0.02$: Human military recruitment rates during peace and war.
- $k_{A_{\text{peace}}}=0.001,\ k_{A_{\text{war}}}=0.02$: Alien military recruitment rates during peace and war.
- $\alpha = 0.0005$: Alien military death rate due to human military.
- $\beta = 0.0005$: Human military death rate due to alien military.
- $\gamma_H = 0.002$, $\gamma_A = 0.002$: Death rates of civilians from resource scarcity.
- $\sigma = 0.05$: Resource consumption rate proportional to total population.
- $\theta = 0.05$: Resource replenishment rate.
- $R_{\text{threshold}} = 500$: Resource threshold for triggering scarcity and war.
- maxPopulation = 3000: Maximum carrying capacity of the environment.

Initial Conditions and Simulation Details

- H(0) = 1000: Initial human civilian population.
- $M_H(0) = 0$: Initial human military population.
- A(0) = 800: Initial alien civilian population.
- $M_A(0) = 0$: Initial alien military population.
- R(0) = 2000: Initial resources.
- W(0) = 0: Peace at the start (0 = peace, 1 = war).

Key Terms in the Equations

- recruitment_H, recruitment_A: Recruitment rates of human and alien military, adjusted for peace/war states.
- \bullet death scarcity, death scarcity: Deaths due to resource scarcity, proportional to the shortfall from $R_{\rm threshold}.$
- \bullet combat $_H^{\rm loss}, {\rm combat}_A^{\rm loss} \colon$ Deaths due to combat, active only during war.