CMSC 27100 - Problem Set 1

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A	$\neg A$	$A \vee \neg A$
T	F	T
F	T	T

A	B	$A \Rightarrow B$	$B \Rightarrow A$	$A \iff B$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

A	B	C	$\neg A$	$\neg B$	$\neg C$	$A \vee \neg B$	$A \Rightarrow \neg C$	$(A \vee \neg B) \wedge (A \Rightarrow \neg C)$
T	T	T	F	F	F	T	F	F
T	T	F	F	F	T	T	T	T
T	F	T	F	T	F	T	F	F
T	F	F	F	T	T	T	T	T
F	T	T	T	F	F	F	T	F
F	T	F	T	F	T	F	T	F
F	F	T	T	T	F	T	T	T
F	F	F	T	T	T	T	T	T

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We must demonstrate the existence and uniqueness of x when $R(x)$ is true. We can write existence as $\exists x \epsilon A, R(x)$. To write uniqueness, we must show that there is no y where $y \neq x$ and $R(y)$ is true. This is equivalent to $\neg(\exists y \epsilon A, \neg(y = x) \wedge R(y))$. Combining the existence and uniqueness conditions, we get that $\boxed{\exists x \epsilon A, R(x) \wedge \neg(\exists y \epsilon A, \neg(y = x) \wedge R(y))}$.

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- $\neg(P \Rightarrow Q) = \neg(\neg P \vee Q) = \boxed{P \wedge \neg Q}$
- $\neg((P \vee \neg Q) \wedge (R \vee \neg S)) = \neg(P \vee \neg Q) \vee \neg(R \vee \neg S) = \boxed{(\neg P \wedge Q) \vee (\neg R \wedge S)}$
- $\neg((P \Rightarrow Q) \Rightarrow \neg R) = \neg((\neg P \vee Q) \Rightarrow \neg R) = \neg(\neg(\neg P \vee Q) \vee \neg R) = \neg((P \vee \neg Q) \vee \neg R) = \neg(P \vee \neg Q) \wedge R = \boxed{(\neg P \vee Q) \wedge R}$

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- $S_2 \cup S_3 = \{1, 2, 3\}$, so $S_1 \times (S_2 \cup S_3) = \boxed{\{(Red, 1), (Blue, 1), (Red, 2), (Blue, 2), (Red, 3), (Blue, 3)\}}$
- $S_1 \cap S_2 = \boxed{\emptyset}$
- $S_2 \cap S_3 = \{2\}$, so $\{orange\} \times \{S_2 \cap S_3\} = \boxed{\{(orange, 2)\}}$

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To show that $(S_1 \setminus S_2) \cup (S_2 \setminus S_3) \subseteq (S_1 \cup S_2) \setminus (S_1 \cap S_2 \cap S_3)$, we need to show that if $x\epsilon(S_1 \setminus S_2) \cup (S_2 \setminus S_3)$, then $x\epsilon(S_1 \cup S_2) \setminus (S_1 \cap S_2 \cap S_3)$.

Assume that $x\epsilon(S_1 \setminus S_2) \cup (S_2 \setminus S_3)$. This is equivalent to $x\epsilon(S_1 \setminus S_2)$ or $x\epsilon(S_2 \setminus S_3)$. We can break this into two cases.

Case 1: If $x\epsilon(S_1 \setminus S_2)$, then $x\epsilon S_1$ and $x \notin S_2$. If this is true, $x\epsilon(S_1 \cup S_2)$ because if $x\epsilon S_1$, then it must be an element of the union of S_1 and S_2. Thus, if $x\epsilon(S_1 \setminus S_2)$, then $x\epsilon(S_1 \cup S_2)$. Similarly, if $x \notin S_2$, then $x \notin (S_1 \cap S_2 \cap S_3)$ because the intersection of S_1 and S_2 and S_3 must also contain elements from S_2. Thus, if $x \notin S_2$, then $x \notin (S_1 \cap S_2 \cap S_3)$. We have shown that if $x\epsilon(S_1 \setminus S_2)$, then $x\epsilon(S_1 \cup S_2)$ and $x \notin (S_1 \cap S_2 \cap S_3)$, so if $x\epsilon(S_1 \setminus S_2)$, then $x\epsilon(S_1 \cup S_2) \setminus (S_1 \cap S_2 \cap S_3)$.

Case 2: If $x\epsilon(S_2 \setminus S_3)$, then $x\epsilon S_2$ and $x \notin S_3$. If this is true, $x\epsilon(S_1 \cup S_2)$ because if $x\epsilon S_2$, then it must be an element of the union of S_1 and S_2. Thus, if $x\epsilon(S_2 \setminus S_3)$, then $x\epsilon(S_1 \cup S_2)$. Similarly, if $x \notin S_3$, then $x \notin (S_1 \cap S_2 \cap S_3)$ because the intersection of S_1 and S_2 and S_3 must also contain elements from S_3. Thus, if $x \notin S_3$, then $x \notin (S_1 \cap S_2 \cap S_3)$. We have shown that if $x\epsilon(S_2 \setminus S_3)$, then $x\epsilon(S_1 \cup S_2)$ and $x \notin (S_1 \cap S_2 \cap S_3)$, so if $x\epsilon(S_2 \setminus S_3)$, then $x\epsilon(S_1 \cup S_2) \setminus (S_1 \cap S_2 \cap S_3)$.

Through both cases, we have shown that if $x\epsilon(S_1 \setminus S_2) \cup (S_2 \setminus S_3)$, then $x\epsilon(S_1 \cup S_2) \setminus (S_1 \cap S_2 \cap S_3)$. This proves that $(S_1 \setminus S_2) \cup (S_2 \setminus S_3) \subseteq (S_1 \cup S_2) \setminus (S_1 \cap S_2 \cap S_3)$.

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6.1

A bijective function is both injective and surjective. Surjectivity means that for every $b \epsilon B$, there is an $a \epsilon A$ such that the function maps a to b, and injectivity means that this a is unique. As given in the problem, we have a bijective function and the sets contain n elements. If we enumerate the elements in $A = \{a_1, a_2, ..., a_n\}$ and $B = \{b_1, b_2, ..., b_n\}$, the first element a_1 can map to any b_i element in B, a total of n elements. The next arbitrary element in A, say a_2, can map to the remaining $n-1$ elements in B since injectivity means each b has a unique a, so a_2 cannot map to the element in B that a_1 has already mapped to. Continuing this pattern, the third arbitrary element has $n-2$ elements in B to map to, so the total number of bijections between sets A and B is $\boxed{n!}$.

6.2

There is no condition that restricts what elements of A can map to in B. So, for every element in A, it has the option of mapping to any of the m elements in B. We do this for each of the n elements in A, so there are $\boxed{m^n}$ functions that map from A to B.