Weighted Finite-State Transducers

Important operations and practical work

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Motivations

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Several Components are finite-state

1. Language models (e.g. *n-grams*)
2. Lexicons
3. Phonological rules
4. HMM tri-phones

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1. Language models (e.g. *n-grams*)
2. Lexicons
3. Phonological rules
4. HMM tri-phones

Use same representation and algorithms for all

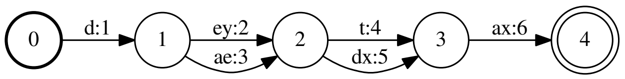
1. Consistency
2. Powerful algorithms available at all levels
3. Flexibility to combine or factor in unforeseen ways

[Mohri, 1997]

Transducers and Acceptors

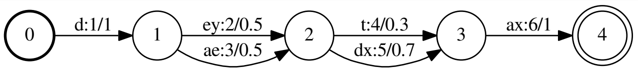
Finite State Acceptors

- **Finite State Automaton**
- Example I: **Pronunciation model acceptor:**
- 'data' accepted pronunciations



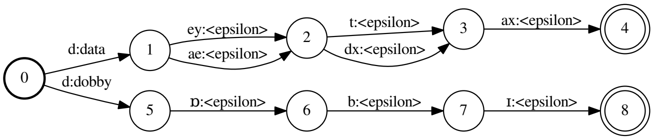
Finite State Acceptors

- Finite automaton with weights: **Weighted Finite State Automaton**
- Example II: **Pronunciation model acceptor with weights**
- 'data' weighted accepted pronunciations



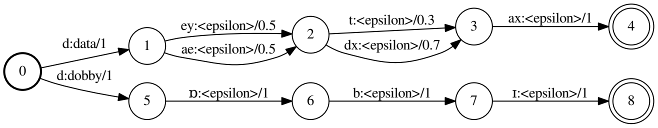
Finite State Transducers

- Finite State Transducer
- Example III: **Pronunciation lexicon dictionary**
- Dictionary of pronunciations transformation from sequence of phones to words in {data, dobby}



Weighted Finite State Transducers

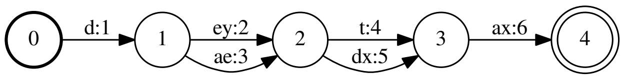
- Weighted Finite State Transducer
- Example IV: **Pronunciation lexicon dictionary with weights**
- Weighted Dictionary of pronunciations transformation from sequence of phones to words in {data, dobby}



Basic operations on WFSTs

Closure

A :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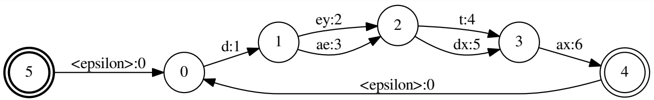
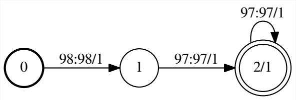
d:1
1
ey:2
ae:3
2
t:4
dx:5
3
ax:6
4

A*:

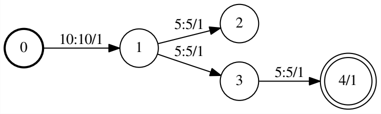

5
<epsilon>:0
0
d:1
1
ey:2
ae:3
2
t:4
dx:5
3
ax:6
4
<epsilon>:0

Union

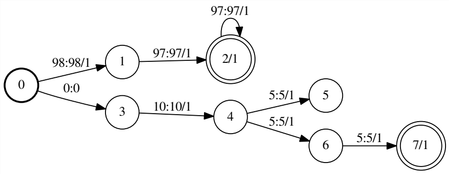
'Parallel combination' of A and B:



(a)



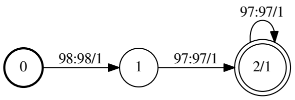
(b)



(c)

Concatenation

'Serial combination' of A and B:



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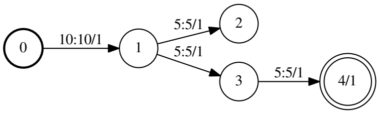
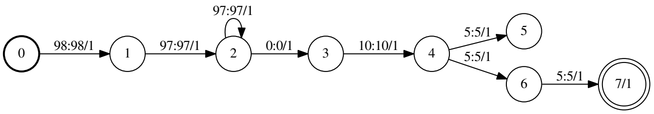


(e)



(f)

Composition

Motivations to decompose the problem

1. Hard to build everything from scratch
2. Combine in unexpected ways and compact representation
3. Simplification of construction and modelling

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Several WFSTs building blocks

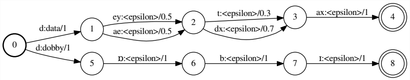
1. HMM tri-phones from acoustic modelling
2. Lexicon pronunciation dictionary from linguists
3. Use extra dataset to build the Language models

Dive into the algorithm details:
https://www.youtube.com/watch?v=DyY69sX7RGk

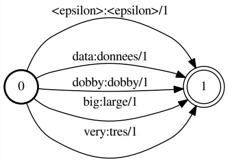
Composition

- Example V: **Composition of English Pronunciation and English to French**
- One WFST going from **English Phones** to **English Words**
- One WFST going from **English words** to **French Words**
- Composition outputs an WFST going from **English phones** to **French Words**

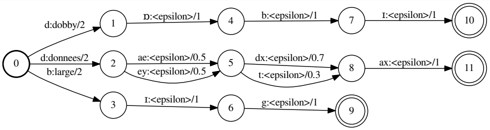
Composition



(g)



(h)



(i)

Practical work

Summary

- Virtual Machine to avoid any installation issues
- Download it and try it **as soon as possible**!
- Additional Jupyter notebook for the figures of this presentation
- OpenFST and Kaldi libraries
- 3 Exercises from Basic commands to Mispell detector

Questions?

Semirings I

WFSTs and their algorithms rely on algebraic structures called **semirings**.

Definition

A **semiring** is an algebraic structure $(\mathbb{K}, \oplus, \otimes, \overline{0}, \overline{1})$:

1. *Associative and Commutative* operation $\oplus$, with the identity $\overline{0}$
2. *Associative* operation $\otimes$, with the identity $\overline{1}$
3. $\oplus$ distributes over $\otimes$:
   $\forall a, b, c \in \mathbb{K}$
   $(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$
   $c \otimes (a \oplus b) = (c \otimes a) \oplus (c \otimes b)$
4. $\overline{0}$ is an annihilator for $\otimes$: $\forall a \in \mathbb{K}, a \otimes \overline{0} = \overline{0} \otimes a = \overline{0}$

Implications for optimisation, search, and combinations algorithms

Semirings II

Seminring	Set	$\oplus$	$\otimes$	$\bar{0}$	$\bar{1}$
Boolean	0,1	$\vee$	$\wedge$	0	1
Probability	$\mathbb{R}_+ \cup \{+\infty\}$	$+$	$\times$	0	1
Log	$\mathbb{R}_+ \cup \{-\infty, +\infty\}$	$\oplus_{log}$	$+$	∞	0
Tropical	$\mathbb{R}_+ \cup \{+\infty\}$	min	$+$	$+\infty$	0

Table 1: Example of Semirings: $x \oplus_{log} y = -log(e^{-x} + e^{-y})$

[Mohri et al., 2002]

Finite State Automaton and Finite State Transducer

Definition

Definition A WFST is defined as an 8-tuple, $T = (\Sigma, \Delta, Q, I, F, E, \lambda, \rho)$. Here Σ represents the finite alphabet of input symbols, Δ represents the finite output alphabet, Q represents the finite set of states, $I \subseteq Q$ the set of initial states, $F \subseteq Q$ the set of final states, $E \subseteq Q \times (\Delta \cup \{\epsilon\}) \times (\Sigma \cup \{\epsilon\}) \times \mathbb{K} \times Q$ a finite set of state-to-state transitions, $\lambda : I \rightarrow K$ the initial weight function, and $: F \rightarrow \mathbb{K}$ the final weight function mapping F to $\mathbb{K}$.

- A WFSA is simply a WFST where the output labels have been omitted
- Similarly, FSAs and FSTs lack weights on the arcs or states

References i

Mohri, M. (1997).
Finite-state transducers in language and speech processing.
Computational linguistics, 23(2):269–311.

Mohri, M., Pereira, F., and Riley, M. (2002).
Weighted finite-state transducers in speech recognition.
Computer Speech & Language, 16(1):69–88.