# CSED321: Inductive Proofs (due Mar. 20)

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## 1 Question #1

*Proof.* By rule induction on judgement $s$ lparen, we consider the following:
First, let's consider $\dfrac{}{\epsilon \text{ lparen}}\, Leps$ where $s = \epsilon$:

| | |
|---|---|
| $\epsilon$ mparen | by the rule $Meps$ |

Now, let's consider $\dfrac{s_1 \text{ lparen} \qquad s_2 \text{ lparen}}{(s_1)s_2 \text{ lparen}}\, Lseq$ where $s = (s_1)s_2$:

| | |
|---|---|
| $s_1$ mparen | by the induction hypothesis on $s_1$ lparen |
| $s_2$ mparen | by the induction hypothesis on $s_2$ lparen |
| $(s_1)$ mparen | by the rule $Mpar$ |
| $(s_1)s_2$ mparen | by the rule $Mseq$ |

□

## 2 Question #2

*Proof.* By rule induction on judgement $s$ tparen, we consider the following:
First, let's consider $\dfrac{}{\epsilon \text{ tparen}}\, Teps$ where $s' = \epsilon$.

| | |
|---|---|
| $s$ tparen | by assumption |
| $ss' = s\epsilon = s$ | by definition of $\epsilon$ |
| $ss'$ tparen | by $s$ tparen |

Now, let's consider $\dfrac{s_1 \text{ tparen} \qquad s_2 \text{ tparen}}{s_1(s_2) \text{ tparen}}\, Tseq$ where $s' = s_1(s_2)$.

| | |
|---|---|
| $s$ tparen | by assumption |
| $ss' = ss_1(s_2)$ | by the variable setting |
| "$s$ tparen implies $ss_1$ tparen" | by the induction hypothesis on $s_1$ tparen |
| $ss_1$ tparen | by assumption $s$ tparen |
| $ss_1(s_2)$ tparen | by the rule $Tseq$ with $ss_1$ tparen, $s_2$ tparen |

□

## 3 Question #3

*Proof.* By rule induction on judgement $s$ mparen, we consider the following:
First, let's consider $\frac{}{\epsilon \text{ mparen}}\, Meps$ where $s = \epsilon$.

| | |
|---|---|
| $\epsilon$ tparen | by the rule $Teps$ |

Now, let's consider $\frac{s \text{ mparen}}{(s') \text{ mparen}}\, Mpar$ where $s = (s')$.

| | |
|---|---|
| $s'$ tparen | by the induction hypothesis |
| $\epsilon$ tparen | by the rule $Teps$ |
| $\epsilon(s')$ tparen | by the rule $Tseq$ with $\epsilon$ tparen, $s'$ tparen |
| $\epsilon(s') = (s')$ | by the definition of $\epsilon$ |
| $(s')$ tparen | by $\epsilon(s')$ tparen and $\epsilon(s') = (s')$ |

Finally, let's consider $\frac{s_1 \text{ mparen} \qquad s_2 \text{ mparen}}{s_1 s_2 \text{ mparen}}\, Mseq$ where $s = s_1 s_2$.

| | |
|---|---|
| $s_1$ tparen | by the induction hypothesis on $s_1$ mparen |
| $s_2$ tparen | by the induction hypothesis on $s_2$ mparen |
| $s_1 s_2$ tparen | by Lemma 1.2 |

□