MATH230: Homework 13 (due Dec. 4)

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1 Chapter 10 #4

Let's state the null hypothesis H_0 as $p = 0.6$, and the alternative hypothesis H_1 as $p < 0.6$.

1.1 Solution for (a)

Let X be the number of orders that arrived late. Then, X follows $b(10; p)$. The problem statement suggests that the H_0 is rejected if $X \leq 3$. We can write the type I error probability $\alpha(p)$ as

$$\alpha(p) = \sum_{x=0}^{3} \binom{10}{x} p^x (1-p)^{10-x}$$

Then we obtain $\alpha(0.6) = 0.0547618816$.

1.2 Solution for (b)

H_0 is accepted if $X > 3$. We can write the type II error probability $\beta(p)$ as

$$\beta(p) = \sum_{x=4}^{10} \binom{10}{x} p^x (1-p)^{10-x}$$

Then we obtain $\beta(0.3) = 0.3503892816, \beta(0.4) = 0.6177193984, \beta(0.5) = 0.828125$.

2 Chapter 10 #15

Let's state the null hypothesis H_0 as $\mu = 200$, and the alternative hypothesis H_1 as $\mu \neq 200$.

2.1 Solution for (a)

Let $\bar{X}$ be a sample mean of nine samples. Then H_0 is rejected if $|\bar{X} - 200| \geq 9$. Let $Z = \sqrt{9}(\bar{X} - \mu)/15$, then $Z \sim N(0, 1)$. Thus, H_0 is rejected if $|Z| \geq 9/5$ and the probability we are looking for is $P(|Z| \geq 9/5) = 0.07186064$.

2.2 Solution for (b)

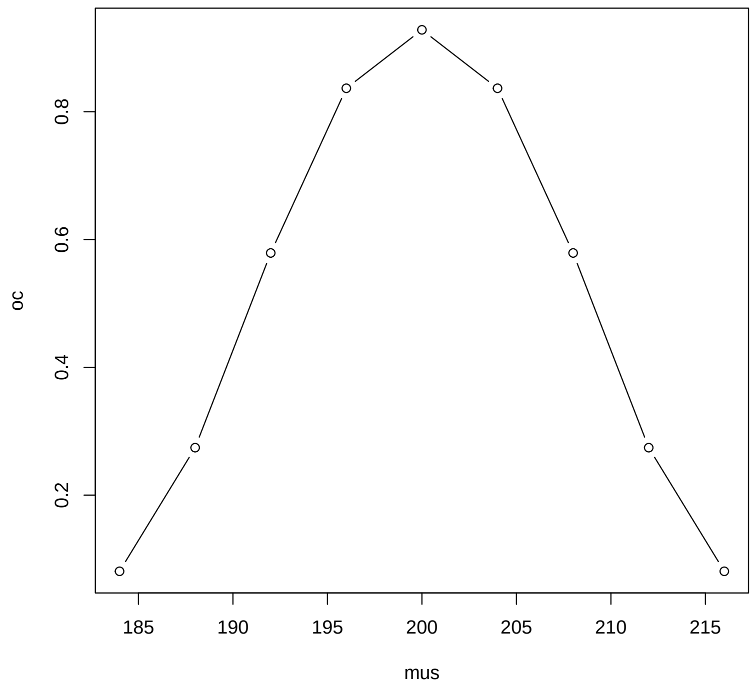
When $\mu = 215$, then H_0 is accepted if $|\bar{X} - 200| < 9$, which is equivalent to $-24/5 < Z < -6/5$. The probability we are looking for is $P(-24/5 < Z < -6/5) = 0.1150689$.

3 Chapter 10 #18

For given value of μ, H_0 is accepted if $|\bar{X} - \mu| < 0$, which is equivalent to

$$\frac{\mu - 209}{5} < Z < \frac{\mu - 191}{5}$$

Now, the probabilities of failing to reject H_0 for given values of μ can be calculated with the formula above. They are 0.08075637, 0.27423977, 0.57892278, 0.83668356, 0.92813936, 0.83668356, 0.57892278, 0.27423977, and 0.08075637 respectively. The OC curve can be plotted as below:



4 Chapter 10 #20

Let $n = 49, \bar{x} = 970, \sigma = 70, \alpha = 0.05, \mu_0 = 1000$ and the null hypothesis H_0 and the alternative hypothesis H_1 as $\mu = \mu_0, \mu < \mu_0$, respectively. We can write

$$\frac{\sqrt{n}(\bar{x} - \mu_0)}{\sigma} = -3 < -z_\alpha = -1.644854$$

so we can reject H_0 at 0.05 level of significance.

5 Chapter 10 #24

Let $n = 54, \bar{x} = 7.2, \sigma = 1.4, \mu_0 = 8.4$, and the null hypothesis H_0 and the alternative hypothesis H_1 as $\mu = \mu_0$ and $\mu \neq \mu_0$, respectively. We can write the P-value as

$$P = 2P\left(Z > \left|\frac{\sqrt{n}(\bar{x} - \mu_0)}{\sigma}\right|\right) = 3.001757 \times 10^{-10} < 0.05$$

where $Z \sim N(0,1)$. Thus, we can reject H_0 and conclude that the average distance changed.