MATH230: Homework 9 (due Nov. 6)

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1 Chapter 8 #8

1.1 Solution for (a)

Adding up the numbers and dividing them with sample size, 12, we get $428/12 = 35.67$ grams.

1.2 Solution for (b)

After sorting the samples in increasing order, the 6th and 7th samples are 31 and 34, respectively. Thus, the median is $(31 + 34)/2 = 32.5$ grams.

1.3 Solution for (c)

Of all the sample, 29 grams appeared the most frequently. Thus, the mode is 29.

2 Chapter 8 #23

2.1 Solution for (a)

We can write

$$\begin{aligned}
\mu &= \sum_{x\in\{4,5,6,7\}} xP(X = x) = 4(0.2) + 5(0.4) + 6(0.3) + 7(0.1) = 5.3 \\
\sigma^2 &= \sum_{x\in\{4,5,6,7\}} (x - \mu)^2 P(X = x) = \frac{81}{100}
\end{aligned}$$

2.2 Solution for (b)

As sample distribution's mean is same as original distribution's mean, $\mu_X = \mu = 5.3$. Also, since $n = 36$, the variance is $\sigma_X^2 = \sigma^2/n = 9/400$.

2.3 Solution for (c)

We can write

$$P(\bar{X} \geq 5.5) = P(\bar{X} - 5.3 \geq 0.2) = P\left(\frac{\bar{X} - 5.3}{0.9/\sqrt{36}} \geq \frac{4}{3}\right)$$

As the sample size is sufficient, by applying the central limit theorem, for $Z \sim N(0,1)$ we can write

$$P\left(\frac{\bar{X}-5.3}{0.9/\sqrt{36}} \geq \frac{4}{3}\right) \simeq P\left(Z \geq \frac{4}{3}\right) = 1 - P\left(X < \frac{4}{3}\right) = 0.09121122$$

Thus, the probability we are looking for is

$$P(\bar{X} < 5.5) = 1 - P(\bar{X} \geq 5.5) = 1 - 0.09121122 = 0.9087888$$

3 Chapter 8 #25

3.1 Solution for (a)

Let X_1, X_2, X_3, X_4 be the random samples. They follow normal distribution of mean $\mu = 18$, and variance $\sigma^2 = 3^2$. Let $\bar{X}$ be the sample mean. Then, $\bar{X}$ has mean $\mu_X = \mu$ and variance $\sigma_X^2 = \sigma^2/4$. Moreover, since $\bar{X}$ is linear combination of normally distributed random variables, it is also normally distributed. Then we can write

$$P(16 < \bar{X} < 19) = \Phi\left(\frac{19-\mu}{\sigma}\right) - \Phi\left(\frac{16-\mu}{\sigma}\right) = 0.6562962$$

where Φ is probability mass function of standard normal distribution.

3.2 Solution for (b)

Let $\bar{X}'$ be the sample mean of 5 samples. Then, $P(\bar{X}' > \bar{x}) = 0.2$ holds. As $\bar{X}'$ is normally distributed with mean μ and variance $\sigma^2/5$, $\Phi((\bar{x}-\mu)/(\sigma/\sqrt{5})) = 0.8$ should hold. Using a calculator, we can conclude that the value of $\bar{x}$ is about 19.12915.

4 Chapter 8 #41

Let $n = 30$. Then we can write

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

4.1 Solution for (a)

Using a calculator's help,

$$P(S^2 > 7.338) = P\left(\frac{(n-1)S^2}{\sigma^2} > 42.5604\right) = 0.04996394$$

4.2 Solution for (b)

Using a calculator's help,

$$\begin{aligned} P(2.766 < S^2 < 7.883) &= P\left(\frac{(n-1)S^2}{\sigma^2} < 42.5604\right) - P\left(\frac{(n-1)S^2}{\sigma^2} < 16.0428\right) \\ &= 0.9250851 \end{aligned}$$