

MATH230: Homework 6 (due Oct. 16)

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1 Chapter 6 #1

1.1 Solution for (a)

We can write

$$\mu = \int_{-\infty}^{\infty} x f(x; A, B) dx = \int_A^B \frac{x}{B-A} dx = \left[\frac{x^2}{2(B-A)} \right]_A^B = \frac{A+B}{2}$$

1.2 Solution for (b)

We can write

$$\begin{aligned} \sigma^2 &= \int_{-\infty}^{\infty} (x - \mu)^2 f(x; A, B) dx = \int_A^B \left(x - \frac{A+B}{2} \right)^2 \frac{1}{B-A} dx \\ &= \frac{1}{B-A} \left[\frac{1}{3} \left(x - \frac{A+B}{2} \right)^3 \right]_A^B = \frac{1}{3(B-A)} \cdot 2 \left(\frac{B-A}{2} \right)^3 = \frac{(B-A)^2}{12} \end{aligned}$$

2 Chapter 6 #2

We can write

$$P(X < 3.5 \mid X \geq 1) = \frac{P(1 \leq X < 3.5)}{P(X \geq 1)} = \frac{\int_1^{3.5} f(x; 0, 5) dx}{\int_1^{\infty} f(x; 0, 5) dx} = \frac{\int_1^{3.5} \frac{1}{5} dx}{\int_1^{\infty} \frac{1}{5} dx} = \frac{5}{8}$$

3 Chapter 6 #10

Let $\Phi(z)$ be distribution function of standard normal distribution. Then we can write

$$P(\mu - 2\sigma < X < \mu + 2\sigma) = \Phi(2) - \Phi(-2)$$

According to the table A.3 in the textbook, we can conclude that the quantity in question is 0.9544.

4 Chapter 6 #49

4.1 Solution for (a)

We can write

$$\mu = \frac{\alpha}{\alpha + \beta} = \frac{1}{1 + 3} = \frac{1}{4}$$

For median, we can calculate the integral from 0 to t , where $0 < t < 1$.

$$\begin{aligned} F(t) &= \int_0^t \frac{1}{B(1, 3)} (1-x)^2 dx = \frac{\Gamma(4)}{\Gamma(1)\Gamma(3)} \int_0^t (1-x)^2 dx = 3 \left[-\frac{1}{3}(1-x)^3 \right]_0^t \\ &= 1 - (1-t)^3 \end{aligned}$$

Then, the solution of $F(t) = 1/2$ is $t = 1 - \sqrt[3]{1/2}$ and it is the median.

4.2 Solution for (b)

According to the formula of variance of gamma distribution,

$$\sigma^2 = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} = \frac{3}{4^2 \cdot 5} = \frac{3}{80}$$

5 Chapter 6 #53

5.1 Solution for (a)

According to the formula of mean of gamma distribution,

$$\mu = \frac{\alpha}{\alpha + \beta} = \frac{5}{5 + 10} = \frac{1}{3}$$

5.2 Solution for (b)

According to the formula of variance of gamma distribution,

$$\sigma^2 = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} = \frac{50}{15^2 \cdot 16} = \frac{1}{72}$$

6 Chapter 6 #58

6.1 Solution for (a)

Let $N(t)$ be the number of cars passed in first t minutes. Then $N(t)$ follows a $p(x; 5)$. The probability we are interested in is $P(N(t) > 10)$.

$$P(N(t) > 10) = 1 - \sum_{x=0}^{10} P(N(t) = x) = 1 - \sum_{x=0}^{10} \frac{e^{-5} \cdot 5^x}{x!} = 0.01370$$

6.2 Solution for (b)

Let X be the arrival time of the tenth car. Then the probability we are interested in is $P(X > 2)$. Using the relation of exponential distribution and Poisson distribution,

$$P(X > 2) = P(N(2) < 10) = \sum_{k=0}^9 e^{-10} \cdot \frac{10^k}{k!} = 0.45793$$

7 Chapter 6 #59

7.1 Solution for (a)

By definition of Poisson process, the time T between events follows an exponential distribution with mean $1/5$. The probability we are interested in is $P(T > 1)$.

$$P(T > 1) = \frac{e^{-5} \cdot (5)^0}{0!} = e^{-5} = 0.00674$$

7.2 Solution for (b)

As stated earlier, T follows an exponential distribution with mean $1/5$.