MATH230: Homework 14 (due Dec. 11)

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1 Chapter 10 #28

Let $n_1 := n_2 := 25, \alpha := 0.05, \bar{x}_1 := 20, \bar{x}_2 := 12, s_1 := 1.5, s_2 := 1.25$. Let μ_1 and μ_2 be mean percent absorbency of cotton fiber and acetate, respectively. Set the null hypothesis H_0 as $\mu_1 - \mu_2 = 0$, and the alternative hypothesis H_1 as $\mu_1 - \mu_2 > 0$. Let s_p be the square root of the value of pooled sample variance. Then we can write

$$\frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{n_1^{-1} + n_2^{-1}}} = 20.4859 > 1.677224 = t_{n_1+n_2-2,\alpha}$$

so we can reject H_0 since there is a strong evidence that $\mu_1 - \mu_2 > 0$ holds, and the answer for the question is yes.

2 Chapter 10 #30

We can write

$$z := \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = 3.66228$$

Then, we can write the p-value as

$$P(|Z| > z) = 2P(Z > z) = 0.0002499803$$

Since p-value is sufficiently small, we can reject the null hypothesis.

3 Chapter 10 #40

Using this R code,

```
x1 <- c(0.97, 1.16, 0.72, 0.86, 1.00, 0.85, 0.81, 0.58, 0.62, 0.57, 1.32,
    0.64, 1.24, 0.98, 0.99, 1.09, 0.90, 0.92, 0.74, 0.78, 0.88, 1.24, 0.94,
    1.18)
x2 <- c(0.48, 0.71, 0.98, 0.68, 1.18, 1.36, 0.78, 1.64)
t.test(x1, x2, alternative="two.sided")
```

We can conclude that the p-value is 0.6876, which is not sufficient to reject the null hypothesis. In other words, the evidence is not sufficient.

4 Chapter 10 #44

Using this R code,

```
x1 <- c(224, 270, 400, 444, 590, 660, 1400, 680)
x2 <- c(116, 96, 239, 329, 437, 597, 689, 576)
t.test(x1, x2, alternative="two.sided", paired=TRUE)
```

The p-value is 0.03186, which is sufficient to reject the null hypothesis. In other words, we can say that the length of storage influences the concentration in question.

5 Chapter 10 #62

Let $n := 48, x = 16, p_0 := 1/4, q_0 := 1 - p_0, \alpha := 0.05$, and p be the proportion of rats developing tumors. Set the null hypothesis H_0 as $p = p_0$, and the alternative hypothesis H_1 as $p > p_0$. Then we can write

$$z = \frac{x - np_0}{\sqrt{np_0q_0}} = 1.333333 < 1.644854 = z_\alpha$$

so we fail to reject H_0.

6 Chapter 10 #71

Let $n := 25, \sigma_0^2 := 1.15, s^2 := 2.03, \alpha := 0.05$, and σ^2 be the variance of the contents. Set the null hypothesis H_0 as $\sigma^2 = \sigma_0^2$, and the alternative hypothesis H_1 as $\sigma^2 > \sigma_0^2$. Then we can write

$$v := \frac{(n-1)s^2}{\sigma_0^2} = 42.36522 > 36.41503 = \chi^2_{n-1,\alpha}$$

so we can reject H_0.

7 Chapter 10 #80

Using this R code,

```
x <- c(14, 18, 32, 20, 16)
chisq.test(x, p=rep(1/5, 5))
```

The p-value is 0.04043, which is sufficient to reject the null hypothesis. Thus, the distribution of grades is not uniform.

8 Chapter 10 #86

Using this R code,

```
m <- matrix(c(21, 36, 30, 48, 26, 19), nrow=2, byrow=TRUE)
chisq.test(m)
```

The p-value is 0.0007232, which is sufficient to reject the null hypothesis. Thus, we know that hypertension is not independent of smoking habits.

9 Chapter 10 #92

Using this R code,

```
m <- matrix(c(11, 13, 9, 32, 28, 27, 7, 9, 14), nrow=3, byrow=TRUE)
chisq.test(m)
```

The p-value is 0.4323, which is not sufficient to reject the null hypothesis. Thus, we fail to reject the null hypothesis and know that the remedies are equally effective.