MATH230: Homework 4 (due Oct. 9)

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1 Chapter 5 #5

1.1 Solution for (a)

Let X be the number of pipework failure in the chemical plant. Assuming that the failures of pipework are independent, X follows a $b(x; 20, 0.3)$. Then the probability we are looking for is $P(X \geq 10)$.

$$P(X \geq 10) = 1 - P(X < 10) = 1 - \sum_{x=0}^{9} b(x; 20, 0.3) = 1 - 0.95204 = 0.04796$$

1.2 Solution for (b)

The probability we are interested in is $P(X \leq 4)$.

$$P(X \leq 4) = \sum_{x=0}^{4} b(x; 20, 0.3) = 0.23751$$

1.3 Solution for (c)

The probability of five pipes failing due to operator error is $P(X = 5)$, and we can write

$$P(X = 5) = \binom{20}{5} \left(\frac{3}{10}\right)^5 \left(\frac{7}{10}\right)^{15} = 0.17886$$

The probability is not that small, so $p = 0.3$ is reasonable.

2 Chapter 5 #22

Using multinomial distribution, we can write

$$\binom{8}{5, 2, 1} \left(\frac{1}{2}\right)^5 \left(\frac{1}{4}\right)^2 \left(\frac{1}{4}\right)^1 = \frac{21}{256}$$

3 Chapter 5 #32

Let X be the number of missiles in the selection which are not defective. Then, X follows a $h(x; 10, 3, 6)$.

3.1 Solution for (a)

The probability we are interested in is $P(X = 3)$.

$$P(X = 3) = h(3; 10, 3, 6) = \frac{1}{6}$$

3.2 Solution for (b)

The probability we are interested in is $P(X \geq 1)$.

$$P(X \geq 1) = 1 - P(X = 0) = 1 - h(1; 10, 3, 6) = \frac{29}{30}$$

4 Chapter 5 #51

Let X be the number of coin tosses required before the person to buy coffeee is determined. There are $2^3 = 8$ possible outcomes of three coins, and two of the outcome is all heads or all tails. Thus, the probability of determining the buyer in a trial is $p = 1 - 2/8 = 3/4$. As all coin tosses are independent, X follows a $g(x; p)$. The probability we are interested in is $P(X < 3)$.

$$P(X < 3) = \sum_{n=1}^{2} pq^{n-1} = \sum_{n=1}^{2} \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{n-1} = \frac{15}{16}$$

5 Chapter 5 #57

Let X be the number of word-processing errors made in the next page. Then, as the author makes two word-processing errors per page, X follows a $p(x; 2)$.

5.1 Solution for (a)

The probability we are interested in is $P(X \geq 4)$.

$$P(X \geq 4) = \sum_{x=4}^{\infty} p(x; 2) = \sum_{x=4}^{\infty} \frac{e^{-2}2^x}{x!} = e^{-2}\left[e^2 - \left(1 + \frac{2}{1} + \frac{2^2}{2!} + \frac{2^3}{3!}\right)\right] = 0.14288$$

5.2 Solution for (b)

The probability we are interested in is $P(X = 0)$.

$$P(X = 0) = p(0; 2) = e^{-2}$$

6 Chapter 5 #75

Let X be the first shift that the robot fails. Then X follows a $g(x; 0.1)$, and the probability we are interested in is $P(X \leq 6)$.

$$P(X \leq 6) = \sum_{n=1}^{6} g(x; 0.1) = \sum_{n=1}^{6} pq^{n-1} = \sum_{n=1}^{6} \left(\frac{1}{10}\right)\left(\frac{9}{10}\right)^{n-1} = 0.468559$$