MATH230: Homework 12 (due Nov. 27)

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1 Chapter 9 #43

Let $n_1 = n_2 = 12, \bar{x}_1 = 36300, s_1 = 5000\bar{x}_2 = 38100, s_2 = 6100, \alpha = 0.05$. Using the formula for approximate confidence interval, we can write

$$\nu = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{(s_1^2/n_1)^2/(n_1-1) + (s_2^2/n_2)^2/(n_2-1)}$$

and

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\nu,\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = -1800 \pm 4732.522$$

This result can be written as $(-6532.522, 2932.522)$.

2 Chapter 9 #56

2.1 Solution for (a)

Let $\hat{p} = 90/120, \hat{q} = 1 - \hat{p}, n = 120, \alpha = 0.05$. Using the formula for approximate confidence interval, we can write

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.75 \pm 0.07747438$$

The result can be written as $(0.6725256, 0.8274744)$.

2.2 Solution for (b)

If $\hat{p}$ is used as an estimate of the proportion, we are $100(1-\alpha)\%$ confident that the error will not exceed the following value:

$$z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.07747438$$

3 Chapter 9 #67

3.1 Solution for (a)

Let $\hat{p} = 2/3, \hat{q} = 1 - \hat{p}, n = 1600, \alpha = 0.05$. Using the formula for approximate confidence interval, we can write

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.6666667 \pm 0.0230984$$

The result can be written as $(0.6435683, 0.6897651)$.

3.2 Solution for (b)

If $\hat{p}$ is used as an estimate of the proportion, we are $100(1-\alpha)\%$ confident that the error will not exceed the following value:

$$z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.0230984$$

4 Chapter 9 #72

Let $n = 24, s^2 = 14, \alpha = 0.05$. Using the formula for confidence interval, we can write

$$\left(\frac{(n-1)s^2}{\chi^2_{(n-1),\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{(n-1),1-\alpha/2}}\right) = (8.456853, 27.54832)$$

5 Chapter 9 #78

Let $n_1 = 5, n_2 = 7, \alpha = 0.1$. We can calculate sample mean and sample variance as

$$\begin{aligned}
\bar{x}_1 &= \frac{103 + 94 + 110 + 87 + 98}{5} = 98.4 \\
\bar{x}_2 &= \frac{97 + 82 + 123 + 92 + 175 + 88 + 118}{7} = 110.7143 \\
s_1^2 &= \frac{(103 - \bar{x}_1)^2 + \cdots + (98 - \bar{x}_1)^2}{5 - 1} = 76.3 \\
s_2^2 &= \frac{(97 - \bar{x}_2)^2 + \cdots + (118 - \bar{x}_2)^2}{7 - 1} = 1035.905
\end{aligned}$$

Using the formula for confidence interval, we can write

$$\left(\frac{1}{f_{(n_1-1,n_2-1),\alpha/2}}\frac{s_1^2}{s_2^2}, \frac{1}{f_{(n_1-1,n_2-1),1-\alpha/2}}\frac{s_1^2}{s_2^2}\right) = (0.01624629, 0.4539481)$$

Since the confidence interval doesn't contain 1, it is reasonable to assume that $\sigma_1^2 \neq \sigma_2^2$.