# MATH230: Homework 14 (due Dec. 11)

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### 1 Chapter 10 #28

Let  $n_1 := n_2 := 25$ ,  $\alpha := 0.05$ ,  $\bar{x}_1 := 20$ ,  $\bar{x}_2 := 12$ ,  $s_1 := 1.5$ ,  $s_2 := 1.25$ . Let  $\mu_1$  and  $\mu_2$  be mean percent absorbency of cotton fiber and acetate, respectively. Set the null hypothesis  $H_0$  as  $\mu_1 - \mu_2 = 0$ , and the alternative hypothesis  $H_1$  as  $\mu_1 - \mu_2 > 0$ . Let  $s_p$  be the square root of the value of pooled sample variance. Then we can write

$$\frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{n_1^{-1} + n_2^{-1}}} = 20.4859 > 1.677224 = t_{n_1 + n_2 - 2, \alpha}$$

so we can reject  $H_0$  since there is a strong evidence that  $\mu_1 - \mu_2 > 0$  holds, and the answer for the question is yes.

### 2 Chapter 10 #30

We can write

$$z := \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = 3.66228$$

Then, we can write the p-value as

$$P(|Z| > z) = 2P(Z > z) = 0.0002499803$$

Since p-value is sufficiently small, we can reject the null hypothesis.

## 3 Chapter 10 #40

Using this R code,

We can conclude that the p-value is 0.6876, which is not sufficient to reject the null hypothesis. In other words, the evidence is not sufficient.

#### 4 Chapter 10 #44

Using this R code,

```
x1 <- c(224, 270, 400, 444, 590, 660, 1400, 680)
x2 <- c(116, 96, 239, 329, 437, 597, 689, 576)
t.test(x1, x2, alternative="two.sided", paired=TRUE)
```

The p-value is 0.03186, which is sufficient to reject the null hypothesis. In other words, we can say that the length of storage influences the concentration in question.

### 5 Chapter 10 #62

Let  $n := 48, x = 16, p_0 := 1/4, q_0 := 1 - p_0, \alpha := 0.05$ , and p be the proportion of rats developing tumors. Set the null hypothesis  $H_0$  as  $p = p_0$ , and the alternative hypothesis  $H_1$  as  $p > p_0$ . Then we can write

$$z = \frac{x - np_0}{\sqrt{np_0q_0}} = 1.3333333 < 1.644854 = z_{\alpha}$$

so we fail to reject  $H_0$ .

### 6 Chapter 10 #71

Let n := 25,  $\sigma_0^2 := 1.15$ ,  $s^2 := 2.03$ ,  $\alpha := 0.05$ , and  $\sigma^2$  be the variance of the contents. Set the null hypothesis  $H_0$  as  $\sigma^2 = \sigma_0^2$ , and the alternative hypothesis  $H_1$  as  $\sigma^2 > \sigma_0^2$ . Then we can write

$$v := \frac{(n-1)s^2}{\sigma_n^2} = 42.36522 > 36.41503 = \chi_{n-1,\alpha}^2$$

so we can reject  $H_0$ .

## 7 Chapter 10 #80

Using this R code,

```
x <- c(14, 18, 32, 20, 16)
chisq.test(x, p=rep(1/5, 5))
```

The p-value is 0.04043, which is sufficient to reject the null hypothesis. Thus, the distribution of grades is not uniform.

## 8 Chapter 10 #86

Using this R code,

```
x <- matrix(c(21, 36, 30, 48, 26, 19), nrow=2)
chisq.test(x)
```

The p-value is 0.06234, which is not sufficient to reject the null hypothesis. Thus, we fail to reject the null hypothesis and know that hypertension is independent of smoking habits.

## 9 Chapter 10 #92

Using this R code,

```
x <- matrix(c(11, 13, 9, 32, 28, 27, 7, 9, 14), nrow=3)
chisq.test(x)
```

The p-value is 0.4323, which is not sufficient to reject the null hypothesis. Thus, we fail to reject the null hypothesis and know that the remedies are equally effective.