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DEPENDENCE OF THE DISSIPATION RANGE SPECTRUM OF INTERPLANETARY MAGNETIC FLUCTUATIONS ON THE RATE OF ENERGY CASCADE

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ABSTRACT

We investigate the nature of turbulent magnetic dissipation in the solar wind. We employ a database describing the spectra of over 800 intervals of interplanetary magnetic field and solar wind measurements recorded by the *ACE* spacecraft at 1 AU. We focus on the spectral properties of the dissipation range that forms at spacecraft frequencies ≥0.3 Hz and show that while the inertial range at lower frequencies displays a tightly constrained range of spectral indexes, the dissipation range exhibits a broad range of power-law indexes. We show that the explanation for this variation lies with the dependence of the dissipation range spectrum on the rate of energy cascade through the inertial range such that steeper spectral forms result from greater cascade rates.

Subject headings: MHD — solar wind — turbulence — waves

1. INTRODUCTION

Although the interplanetary magnetic field (IMF) has been studied since the first flights of spacecraft beyond the Earth's bow shock, most studies have focused either on the large-scale structure of the IMF (see Ness & Wilcox 1965; Smith et al. 2001b) or the nature of IMF fluctuations in the so-called inertial range (see Belcher & Davis 1971; Barnes 1979). This range, which extends from spacecraft-frame periods of a few hours to a few seconds at 1 AU, has been described as either propagating Alfvén waves undergoing relatively little interaction or evolution, or as fully developed turbulence exhibiting a self-organized Kolmogorov spectrum (Kolmogorov 1941). It is this association with the hydrodynamic turbulence spectrum created by an energy-conserving spectral cascade that has led to the name "inertial range." For the most part these two views of noninteracting waves and strongly interacting turbulence cannot be fully reconciled.

At higher frequencies (typically >0.3 Hz at 1 AU), the power spectrum of IMF fluctuations steepens to form the "dissipation range." It is called this in analogy to hydrodynamics where the steepening of the inertial range corresponds to the onset of dissipation. Early studies by Behannon (1975), Denskat et al. (1983), Smith et al. (1990), and Goldstein et al. (1994) show that the steepening occurs in close association with the local proton cyclotron frequency, and that there is often a degree of polarization within the dissipation range that is consistent with the removal of outward-propagating Alfvén (ion cyclotron) waves. Polarization signatures are absent from the inertial range except when wave-particle effects are strong, such as in the vicinity of shocks (Russell & Hoppe 1983; Gary et al. 1984) and comets (Tsurutani 1991). For these reasons it was generally regarded that the dissipation process responsible for the formation of the dissipation range is cyclotron damping. Leamon et al. (1998b) reveal that cyclotron damping of parallel-propagating Alfvén waves seems to offer a poor description of the onset of the dissipation spectrum once the variability of the IMF and plasma parameters is considered. Leamon et al. (1998a) quantify the limited applicability of cyclotron damping while Leamon et al. (1998b) and Leamon et al. (1999) argue in favor of the preferential dissipation of energy associated with quasi-perpendicular wavevectors via several mechanisms including the Landau resonance and possible current sheet formation. Leamon et al. (2000) and Smith et al. (2001a) demonstrate that the dissipation range scales approximately with the ion inertial scale, where ion kinetics become important.

The ion inertial scale marks the breakdown of the single fluid theory. Stawicki et al. (2001) assert that this results in a spectral form determined by whistler wave propagation beyond the cyclotron resonance (within what we here call the dissipation range). The Stawicki formalism allows for cyclotron damping of Alfvén waves, but then permits dissipation-free propagation of dispersive whistler waves to continue at higher frequencies. If this view is true, then the analysis shown here still constrains their so-called dispersion range.

The parallels between hydrodynamic (HD) and magnetohydrodynamic (MHD) turbulence (Kraichnan & Montgomery 1980; Biskamp 2003) have allowed HD turbulence theory to be a major influence in our understanding of the interplanetary medium (Frisch et al. 1975; Matthaeus & Goldstein 1982). However, Batchelor (1953) argues that in hydrodynamics, dissipation via viscosity is the fate of energy that cascades through the inertial range and that this dynamics lies entirely within the fluid approximation. Under these conditions the dissipation range can be described by a universal function, and the onset of dissipation depends not on the dissipation processes alone, but also on the rate at which energy cascades through the inertial range. If the dissipation range results from the onset of dissipative processes associated with kinetics that lie outside the fluid approximation, then any application of the hydrodynamic view of the dissipation range must be suspect.

We demonstrate here that the power-law index of the dissipation range (the steepness of the dissipation range spectrum) depends on the rate at which energy cascades through the inertial range. We demonstrate this in stark contrast to the predictions of hydrodynamic turbulence and its most immediate MHD analogs.

2. OBSERVATIONS

We use magnetic field and thermal proton data recorded by the *ACE* spacecraft (Smith et al. 1998; McComas et al. 1998)

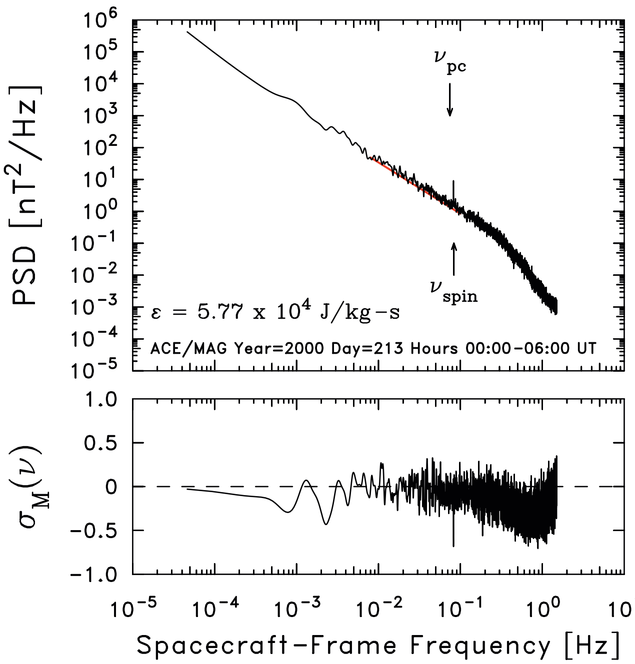

FIG. 1.—Example spectrum taken from the study. Note steepening at frequencies >0.3 Hz (*top*) and first signs of significant magnetic helicity (*bottom*) in association with the local value of the proton cyclotron frequency.

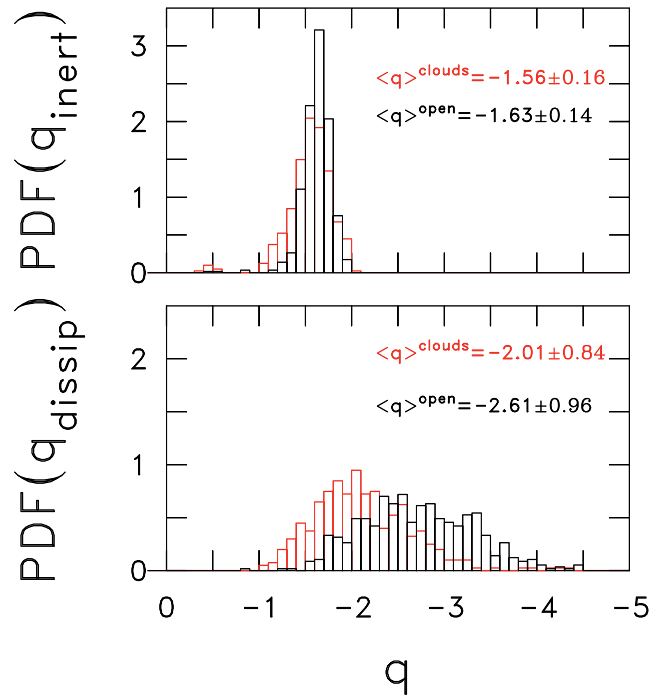

FIG. 2.—Distribution of inertial range power-law indexes (*top*) and dissipation range indexes (*bottom*) for intervals used in this study. Note the tight distribution evident in the inertial range while the dissipation range values vary over a far greater extent.

while at L1 from 1998 through 2002. We have built a database using the spectra of magnetic fluctuations from 960 intervals of data that were chosen with the intent of spanning the largest possible range of plasma conditions in the solar wind at 1 AU. No attempt has been made to characterize the underlying distribution of 1 AU observations.

We work with samples that are one to several hours in duration. We include magnetic cloud observations (Burlaga 1995; 393 intervals taken from 28 distinct clouds), along with rarefaction intervals, observations behind shocks, high- and low-speed wind conditions, etc., that make up the open field line contribution (567 intervals). When employing thermal proton data in this analysis the database is reduced to 352 (526) intervals for clouds (open field lines) due to SWEPAM data availability.

We use the prewhitened Blackman-Tukey method employed in Leamon et al. (1998b) and Smith et al. (2006). Figure 1 shows a typical IMF spectrum taken from the database. The top panel shows the inertial range spectrum that spans several decades in spacecraft frequency and ends at ~0.3 Hz where the IMF spectrum steepens to form the dissipation range. Both the proton cyclotron frequency, ν_{pc}, and the spacecraft spin frequency, ν_{spin}, are ~0.08 Hz. A very slight spin tone is evident at this frequency, indicative of a small error in the instrument calibration. The inertial range power-law index in the frequency range 0.008–0.1 Hz (marked by the red line on the figure) is $q_{inert} = -1.57 \pm 0.01$. An inferred energy cascade rate through the inertial range, $\epsilon = 5.77 \times 10^4$ J kg^{-1} s^{-1}, is obtained using the method outlined below. The dissipation range is significantly steeper with $q_{dissip} = -3.36 \pm 0.01$ over the range 0.4–0.8 Hz. The bottom panel shows the energy-normalized magnetic helicity spectrum, $\sigma_M(\nu)$, which is statistically equivalent to zero within the inertial range (Matthaeus & Smith 1981; Matthaeus et al. 1982; Matthaeus & Goldstein 1982) and possesses a significant bias within the dissipation range consistent with the onset of cyclotron damping (Goldstein et al. 1994; Leamon et al. 1998b). Leamon et al. (1998a) demonstrate that cyclotron damping accounts for approximately two-thirds of the energy dissipation.

Figure 2 shows the distribution of power-law indexes found in this study. While we make no attempt to characterize the underlying statistics of the solar wind, we note that inertial range values (*top*) fall almost entirely within the range $-\frac{3}{2} > q_{inert} > -\frac{5}{3}$ while the dissipation range values (*bottom*) are significantly more broadly distributed over the range $-1 > q_{dissip} > -4$. Magnetic clouds, represented in red, demonstrate a less steep dissipation range form, on average, than open field lines (*black*). In only a few cases does the dissipation range appear flatter than the inertial range. These events are almost always magnetic clouds with low fluctuation levels and low spectral cascade rates (see below). Having a dissipation range that is flatter than the inertial range is almost certainly an error due to instrument noise or interval selection and could be excluded from the study. However, we elect to include these intervals to avoid any criticism of bias.

The underlying question is "What accounts for the broad distribution in dissipation range spectral forms?" This is made more pressing by the realization that the theory of a turbulent inertial range describes the inertial range spectrum very well. In HD turbulence, the dissipation range has a universal spectral form based on the collisional viscosity ν and the rate of energy cascade ϵ while the dissipation range sets in when dissipation successfully competes with the spectral cascade (Kolmogorov 1941). In this formalism the inertial range extends down to the "Kolmogorov dissipation scale" $\eta = (\nu_{HD}^3/\epsilon)^{1/4}$ (Frisch 1995) where ν_{HD} is the hydrodynamic viscosity. If the cascade rate increases, the dissipation spectrum shifts to smaller spatial scales, but is argued to maintain a universal form $F(\eta k)$. That form $F(\eta k) \rightarrow$ a constant in the inertial range, but provides the

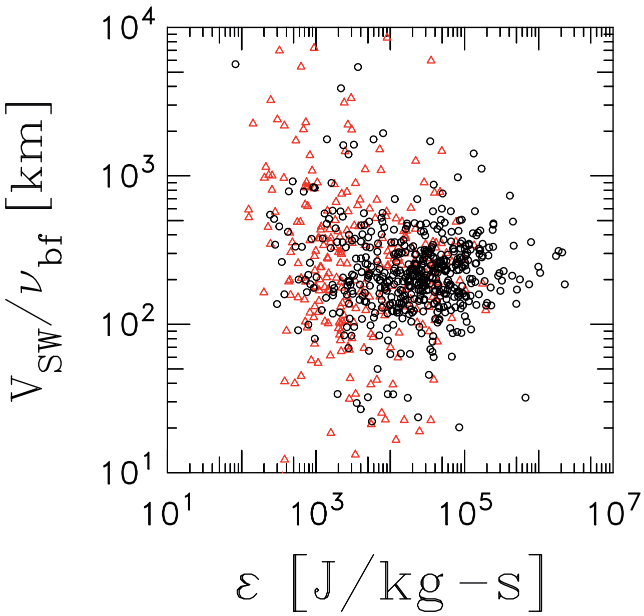

FIG. 3.—Plot of length scale of spectral break V_{SW}/ν_{bf} vs. Kolmogorov energy cascade rate ϵ. Open field line results are black circles and magnetic clouds are represented by red triangles. In traditional hydrodynamics the rate of energy cascade through the inertial range determines the onset of dissipation. In the intervals studied here, the cascade rate spans a very large range of values, but there is no evident dependence of dissipation onset on ϵ.

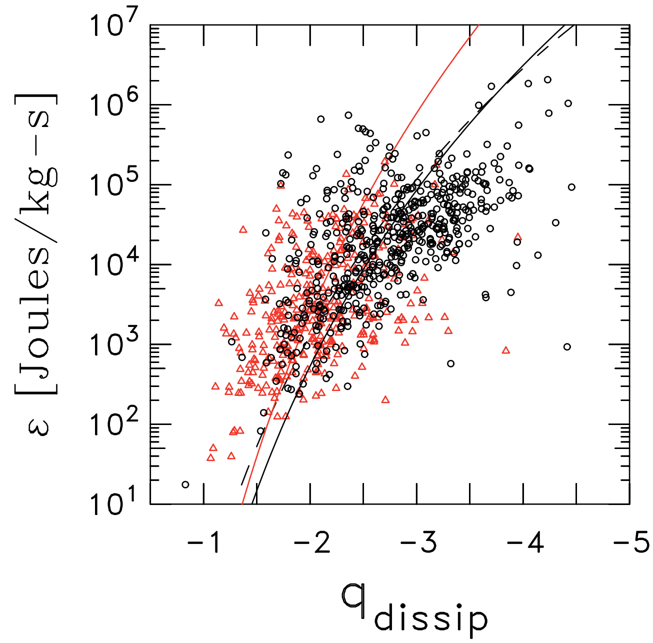

FIG. 4.—Plot of ϵ vs. the dissipation range spectral index q_{dissip}. Again, open field line results are black circles and magnetic clouds are represented by red triangles. Steepness of the dissipation range spectrum is clearly dependent on the rate of energy cascade through the inertial range. For steady-state spectra where the cascade rate equals the dissipation rate, the dissipation range spectral form is dependent on the rate of energy dissipation.

steepening seen with dissipation. There is some controversy over the form and universality of the dissipation range spectrum (Frisch & Morf 1981; Frisch 1995).

We can test the applicability of this idea to solar wind observations. It is possible to estimate the energy cascade rate using the theory of Kolmogorov (1941) where the omnidirectional power spectrum for velocity fluctuations, $P_V(k)$, in HD inertial range turbulence is

$$P_V(k) = C_K \epsilon^{2/3} k^{-5/3}, \quad (1)$$

where $C_K \simeq 1.6$ [$F(\eta k) \rightarrow C_K$ as $\eta k \rightarrow 0$], ϵ is the energy cascade rate through the inertial range, and k is the wavenumber. We can then use the unidirectional spectrum of IMF fluctuations as measured by spacecraft in a supersonic and super-Alfvénic solar wind, $P_B(\nu)$, to estimate ϵ (Leamon et al. 1999):

$$\epsilon = (2\pi/V_{SW})\nu^{5/2}[(1 + R_A)(5/3)P_B(\nu)(V_A/B_0)^2/C_K]^{3/2}, \quad (2)$$

where V_{SW} is the solar wind speed, ν is the measured frequency in the spacecraft frame, R_A is the Alfvén ratio, V_A is the Alfvén speed, and B_0 is the mean magnetic field strength. We use the average V_{SW}, V_A, and B_0 for each interval and assume R_A = 1/2 (Roberts et al. 1987).

For a Kolmogorov (1941) cascade and a −5/3 power law spectrum, the above expression for ϵ has the same value at every frequency in the inertial range provided R_A is a constant. Our results show some small variation in the inertial range power-law index from slightly above the −5/3 value down to the Kraichnan (1965) value of −3/2 and slightly below. With few options, we elect to use equation (2) and apply it to the measured spectrum at ν = 0.01 Hz under the assumption that the Kolmogorov (1941) cascade is a good description of the underlying physics, but that various in situ processes have distorted the spectrum. While this is not perfect and it may obscure interesting systematic effects that depend on the inertial range spectrum, it is a start and the only way to proceed at this point in the analysis.

Figure 3 compares the energy cascade rate to the length scale of the spectral break V_{SW}/ν_{bf}, where ν_{bf} is the measured frequency of the spectral break, for each period studied. If dissipation onset occurs in analogy with hydrodynamics, $V_{SW}/\nu_{bf} \sim \eta$, and if $\nu_{HD} \simeq$ constant, $V_{SW}/\nu_{bf} \sim \epsilon^{-1/4}$. The figure is not supportive of the HD theory or extensions made exclusively within the MHD formalism since there is no obvious variation of the dissipation scale with changing cascade rate. If dissipation is due to kinetic processes outside the MHD description, no such dependence would be expected.

Although not strictly required, it is reasonable to assume that the dissipation range spectrum must show some sort of variation as both the spectral energy cascade rate and the resulting rate of energy dissipation change. This would be one means to accommodate the new input of energy into the dissipation process. If the length scale associated with dissipation does not change, then it is reasonable to think the spectral form may. Figure 4 plots the dissipation range spectral index q_{dissip} versus the energy cascade rate ϵ for the events studied. A clear correlation is evident with the steepest dissipation range spectra associated with the highest rate of cascade. The power-law fit parameters are listed in Table 1. In general $q_{dissip} \sim -1.05\epsilon^{0.09}$. Although this seems to be counterintuitive at first, the correlation is clear. There must

TABLE 1
FIGURE 4 LEAST-SQUARES FIT PARAMETERS

Population	Fit Function
Open IMF	$q = (-1.19 \pm 0.06)\epsilon^{0.082 \pm 0.005}$
Clouds	$q = (-1.16 \pm 0.06)\epsilon^{0.070 \pm 0.007}$
Combined	$q = (-1.02 \pm 0.03)\epsilon^{0.093 \pm 0.004}$

be some response of the dissipation process to increasing cascade rate that facilitates the increased rate of dissipation such that the spectrum steepens.

This conclusion is not altogether unexpected. Figure 2 of Leamon et al. (1998b) shows a correlation between the dissipation range power-law index and the thermal proton temperature in their 33 event study. Although this comparison fails to take into account the possible and likely variation of initial solar wind temperature in the acceleration region, it does show that steeper spectra are associated with hotter intervals of plasma that may result from more rapid heating. Based on their reported correlation, Leamon et al. (1998b) conclude "This suggests that steeper dissipation range spectra imply greater heating rates."

3. DISCUSSION AND SUMMARY

In traditional hydrodynamic turbulence theory the dissipation range, much like the inertial range, is described by a universal function. The onset of the dissipation spectrum is determined by the rate of energy cascade through the inertial range that drives dissipation. The corresponding adjustment of the dissipation scale is the means by which the spectrum adapts to changing cascade and dissipation rates.

It is becoming clear that this is not the case in the solar wind and most likely is not the case in most or any astrophysical plasmas. In the case of the solar wind, the fluid approximation fails at the same spatial scales where dissipation sets in. Therefore, the onset of dissipation represents the coupling of a turbulent fluid cascade to kinetic dissipation processes. Unless the fluid approximation extends to scales much smaller than those at which viscosity and resistivity become significant and kinetic dissipation via particle effects is negligible, dissipation in plasma-based MHD turbulence will always result from the breakdown of the fluid approximation and the ascendancy of kinetic particle effects such as cyclotron and Landau damping. It is not impossible to envision a collisional plasma in which the MHD turbulent cascade terminates in a fluidlike and universal dissipation range analogous to hydrodynamics, but it is likely that in a collisionless plasma kinetic effects will interrupt the cascade first. We have shown, using solar wind observations from 1 AU, that this results in a dissipation range spectrum that depends on the rate of energy cascade in a manner that is distinctly different from the hydrodynamic case. Because the cascade rate varies greatly with the overall fluctuation levels of the IMF, there is significant variation in the dissipation range power-law index at 1 AU.

We have studied 878 intervals of solar wind observations recorded by the *ACE* spacecraft at 1 AU. These intervals are distributed across a broad range of solar wind conditions. We observe that a dissipation range can be found in essentially all intervals and that it occurs for spacecraft frequencies $\simeq$0.3 Hz. Furthermore, we find that the onset of the dissipation spectrum has nothing to do with the traits of the inertial range spectrum as far as we have been able to discern. Most especially, the onset of dissipation does not depend on the rate of energy cascade through the inertial range as would be predicted by hydrodynamic turbulence.

We have also shown that the form of the dissipation range spectrum departs from the predictions of hydrodynamic turbulence and its immediate MHD analogs. In a counterintuitive finding, we conclude that the dissipation range spectrum steepens with increasing energy cascade rate. This result holds for both open field lines and magnetic clouds, with apparent unity between the two distinct populations.

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