Thiessen Polygon Formulas

The Thiessen polygons method, also known as Voronoi polygons or Dirichlet polygons, is a geometric technique used to partition a plane into regions based on proximity to a given set of points. Named after Alfred Thiessen, this method is commonly used in spatial analysis, geography, and other fields where the concept of proximity zones is relevant.

Here are the key steps involved in constructing Thiessen polygons:

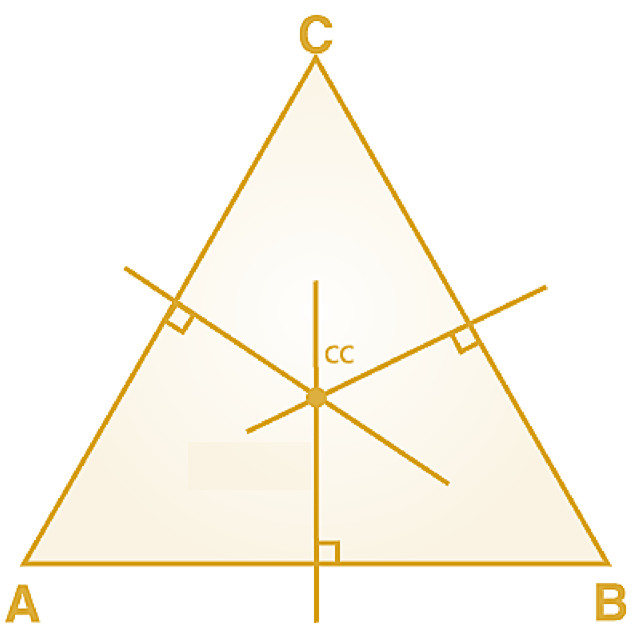
Input a Set of Points or Sites:

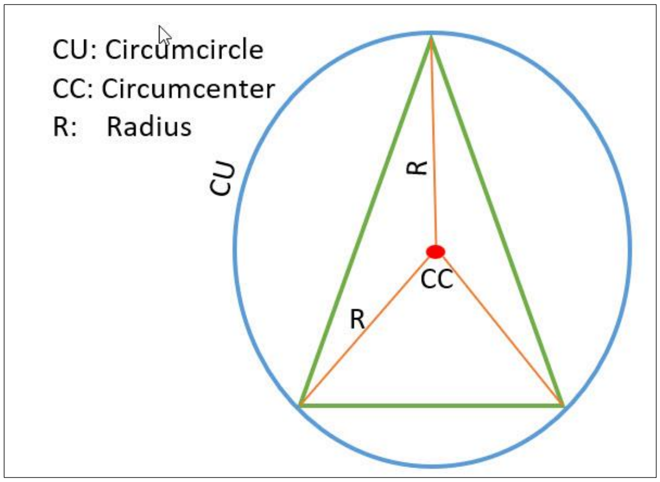
- Begin with a set of discrete points scattered across the plane. These points represent measurement locations or observation sites.

Calculate Voronoi Diagram:

- The Delaunay triangulation is a method to connect these points in such a way that no point is inside the circumcircle of any triangle formed by the points. Connect the points to create Delaunay triangles, forming a triangulation of the given set.
- Determine the circumcenter of each Delaunay triangle, as it will be a key point in constructing the Voronoi diagram. Illustrate circumcircles for each triangle.

- Connect the circumcenters to form polygons and edges. These edges, along with the original points, create the Voronoi diagram. Each Voronoi polygon corresponds to an area that is closer to its associated point (station) than to any other point in the set.
- Determine the area of individual polygons by applying the Shoelace formula, a mathematical technique for computing the area of a polygon given the coordinates of its vertices. This method involves systematically summing the products of coordinates in a specific pattern.




CU: Circumcircle
CC: Circumcenter
R: Radius
CU
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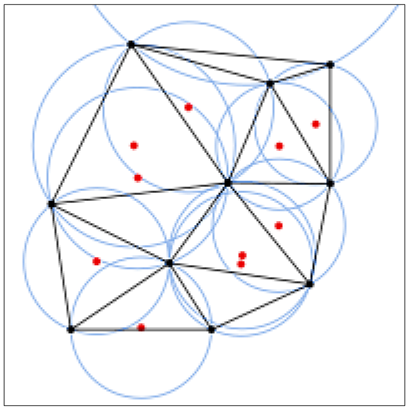
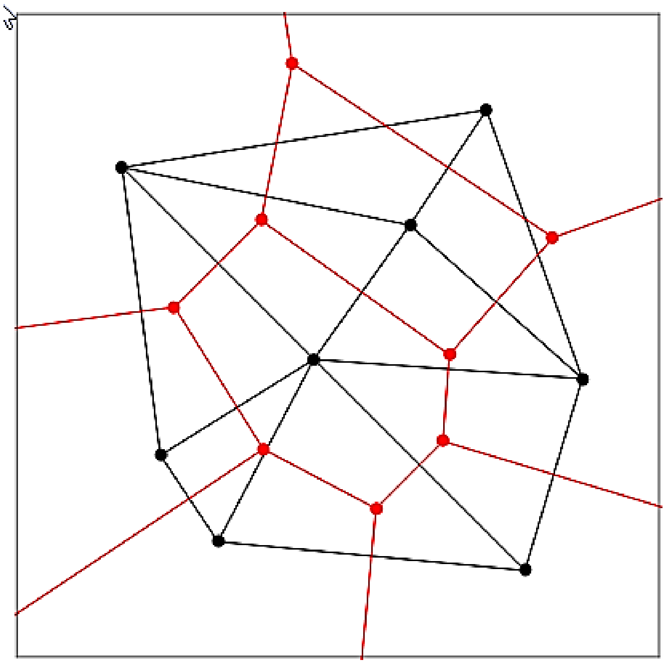




These are three steps to draw polygons. Black points are stations and red points are circumcenters.

Shoelace Formula

The general case for finding areas of polygons

$$\frac{1}{2}\left|\left(\sum_{i=1}^{n-1} x_i y_{i+1}\right) + x_n y_1 - \left(\sum_{i=1}^{n-1} x_{i+1} y_i\right) - x_1 y_n\right|$$

The general formula for the area of an n-sided polygon is given above.

For a triangle this gives:

$$\frac{1}{2}|x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3|$$

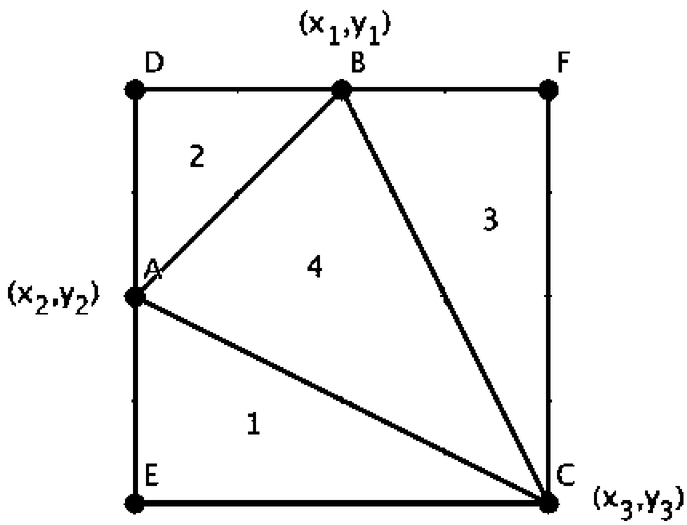
For a quadrilateral this gives:

$$\frac{1}{2}|x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1 - x_2y_1 - x_3y_2 - x_4y_3 - x_1y_4|$$

For a pentagon this gives:

$$\frac{1}{2}|x_1y_2 + x_2y_3 + x_3y_4 + x_4y_5 + x_5y_1 - x_2y_1 - x_3y_2 - x_4y_3 - x_5y_4 - x_1y_5|$$

You might notice a nice shoelace like pattern (hence the name) where x coordinates criss cross with the next y coordinate along. To finish off let's see if it works for an irregular pentagon.



This is a sample polygon with three vertices.

Signed area of a polygon on the Earth's surface

Firstly, we convert the coordinates from degree to radians as below:

$$Lat_{Rad} = \frac{Lat_{Deg} \times \pi}{180}$$

$$Lon_{Rad} = \frac{Lon_{Deg} \times \pi}{180}$$

Then we calculate area in square kilometers as below:

$$A = \frac{R^2}{10^6} \sum_{i=2}^{n} [2 \times Atan2(t \times \sin(\Delta\lambda), 1 + t \times \cos(\Delta\lambda))]$$

Atan2 is an angle, θ, measured in radians, such that tan(θ) = y / x, where (x, y) is a point in the Cartesian plane.

The R parameter represents the radius of the Earth and is utilized to scale the computed area to square kilometers. The Earth's average radius is approximately 6,371,009 meters.

$$t = tan_i - tan_{i-1}$$

$$tan_i = \tan\left(\frac{\pi - Lat_i^{Rad}}{2}\right)$$

$$\Delta\lambda = Lon_i^{Rad} - Lon_{i-1}^{Rad}$$

Average Area

The formula for calculating the average value of a region (polygon or rectangular region) is as follows:

$$\bar{V} = \sum_{i=1}^{n} V_i \times \frac{A_i}{\sum A_i}$$