

MATH 2568
Dr. Krishnan
homework #1

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1.1: Exercise 2

In Exercise 2, let $x = (2, 1, 3)$ and $y = (1, 1, -1)$ and compute the given expression.

$$(2) \quad 2x - 3y$$

Solution:

$$(2) \quad 2x - 3y = (4, 2, 6) - (3, 3, -3) = \boxed{(1, -1, 9)}$$

1.1: Exercise 4

Let A be the 3×4 matrix

$$A = \begin{pmatrix} 2 & -1 & 0 & 1 \\ 3 & 4 & -7 & 10 \\ 6 & -3 & 4 & 2 \end{pmatrix}.$$

- (a) For which n is a row of A a vector in \mathbb{R}^n ?
- (b) What is the 2^{nd} column of A ?
- (c) Let a_{ij} be the entry of A in the i^{th} row and the j^{th} column. What is $a_{23} - a_{31}$?

Solution:

- (a) $\boxed{n = 4}$ Each row of the matrix A has 4 elements: $(2, -1, 0, 1)$, $(3, 4, -7, 10)$, and $(6, -3, 4, 2)$. These elements are all real numbers, and they are ordered. Therefore, they satisfy the definition of a vector in \mathbb{R}^4 .

$$(b) \quad \boxed{\begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}}$$

$$(c) \quad a_{23} - a_{31} = \boxed{-13}$$

1.2: Exercise 3

In Exercise 3, let $x = (1.2, 1.4, -2.45)$ and $y = (-2.6, 1.1, 0.65)$. Use MATLAB to compute the given expression.

$$(3) \quad 3.27x - 7.4y$$

Solution:

$$(3) \quad 3.27x - 7.4y = \boxed{(23.1640, -3.5620, -12.8215)}$$

Listing 1: MATLAB code

```
x = [1.2 1.4 -2.45];
y = [-2.6 1.1 0.65];

result = 3.27 * x - 7.4 * y

>> ex3hw1

result =

    23.1640    -3.5620   -12.8215
```

1.3: Exercise 18

Every diagonal matrix is a scalar multiple of the identity matrix.

(18) True or False

Solution:

(18) False

Consider the following diagonal matrix:

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

This matrix is not a scalar multiple of the identity matrix because there is no scalar that can be multiplied by the identity matrix to produce this matrix.

1.4: Exercise 9

Find a real number a so that the vectors

$$x = (1, 3, 2) \text{ and } y = (2, a, -6)$$

are perpendicular.

Solution:

(9) Dot product:

$$\begin{aligned} x \cdot y &= (1, 3, 2) \cdot (2, a, -6) \\ &= 1 \cdot 2 + 3 \cdot a + 2 \cdot (-6) \\ &= 2 + 3a - 12 \\ &= 3a - 10 \end{aligned}$$

Setting the dot product to zero:

$$\begin{aligned} 3a - 10 &= 0 \\ 3a &= 10 \\ a &= \frac{10}{3} \end{aligned}$$

2.1: Exercise 7

Find a quadratic polynomial $p(x) = ax^2 + bx + c$ satisfying $p(0) = 1$, $p(1) = 5$, and $p(-1) = -5$.

Solution:

(a)

Using the given information, we set up a system of equations:

1. $p(0) = a(0)^2 + b(0) + c = 1$, which gives $c = 1$.
2. $p(1) = a(1)^2 + b(1) + 1 = 5$, which simplifies to $a + b = 4$.
3. $p(-1) = a(-1)^2 + b(-1) + 1 = -5$, which simplifies to $a - b = -6$.

Solving the system of equations:

Adding equations 2 and 3, we obtain $2a = -2$, so $a = -1$.

Substituting $a = -1$ into equation 2, we get $-1 + b = 4$, so $b = 5$.

Therefore, the quadratic polynomial satisfying the given conditions is:

$$\boxed{p(x) = -x^2 + 5x + 1}$$

(b)

$$\begin{aligned}
 p(0) = L &\implies c = L \\
 p(1) = M &\implies a + b + c = M \implies a + b = M - L \\
 p(-1) = N &\implies a - b + c = N \implies a - b = N - L
 \end{aligned}$$

$$\begin{aligned}
 2a &= (a + b) + (a - b) = M - L + N - L = M + N - 2L \\
 \implies a &= \frac{M + N - 2L}{2} \\
 2b &= (a + b) - (a - b) = M - L - (N - L) = M - N \\
 \implies b &= \frac{M - N}{2}
 \end{aligned}$$

$$(a, b, c) = \left(\frac{M + N - 2L}{2}, \frac{M - N}{2}, L \right)$$

(c)

$$\begin{aligned}
 ax_1^2 + bx_1 + c &= A_1 \\
 ax_2^2 + bx_2 + c &= A_2 \\
 ax_3^2 + bx_3 + c &= A_3
 \end{aligned}$$

$$\begin{aligned}
 q(x_1) &= A_1 \implies ax_1^2 + bx_1 + c = A_1 \\
 q(x_2) &= A_2 \implies ax_2^2 + bx_2 + c = A_2 \\
 q(x_3) &= A_3 \implies ax_3^2 + bx_3 + c = A_3
 \end{aligned}$$

2.2: Exercise 5

Find the cosine of the angle between the normal vectors to the planes

$$2x - 2y + z = 14 \quad \text{and} \quad x + y - 2z = -10$$

Solution:

(5)

$$\cos \theta = \frac{(2 \ -2 \ 1) \cdot (1 \ 1 \ -2)}{|(2 \ -2 \ 1)| |(1 \ 1 \ -2)|} = \frac{(2)(1) + (-2)(1) + (1)(-2)}{\sqrt{2^2 + (-2)^2 + 1^2} \sqrt{1^2 + 1^2 + (-2)^2}} = \frac{-2}{\sqrt{9}\sqrt{6}} = \boxed{-\frac{2}{3\sqrt{6}}}$$