MATH 2568

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homework #1

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Contents

2.3: Exercise 1

In Exercise 1 determine whether the given matrix is in reduced echelon form.

(1)
$$\begin{pmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Solution:

(1)

2.3: Exercise 8

(a) Consider the 2×2 matrix
$$\begin{pmatrix} a & b \\ c & 1 \end{pmatrix}$$
where $a, b, c \in \mathbb{R}$ and $a \neq 0$. Show that the above matrix is row equivalent to the following matrix
$$\begin{pmatrix} 1 & \frac{b}{a} \\ 0 & \frac{a-bc}{a} \end{pmatrix}.$$

(b) Show that the first matrix is the row equivalent to the identity matrix if and only if $a \neq bc$

Solution:

(a) $\boxed{n = 4}$ Each row of the matrix A has 4 elements: $(2, -1, 0, 1)$, $(3, 4, -7, 10)$, and $(6, -3, 4, 2)$. These elements are all real numbers, and they are ordered. Therefore, they satisfy the definition of a vector in $\mathbb{R}^4$.

(b) $\boxed{\begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}}$

(c) $a_{23} - a_{31} = \boxed{-13}$

1.2: Exercise 3

In Exercise 3, let $x = (1.2, 1.4, -2.45)$ and $y = (-2.6, 1.1, 0.65)$. Use MATLAB to compute the given expression.

(3) $3.27x - 7.4y$

Solution:

(3) $3.27x - 7.4y = \boxed{(23.1640, -3.5620, -12.8215)}$

Listing 1: MATLAB code

```
x = [1.2 1.4 -2.45];
y = [-2.6 1.1 0.65];

result = 3.27 * x -7.4 * y

>> ex3hw1

result =

    23.1640    -3.5620    -12.8215
```

1.3: Exercise 18

Every diagonal matrix is a scalar multiple of the identity matrix.

(18) True or False

Solution:

(18) $\boxed{False}$
Consider the following diagonal matrix:
$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$
This matrix is not a scalar multiple of the identity matrix because there is no scalar that can be multiplied by the identity matrix to produce this matrix.

1.4: Exercise 9

Find a real number a so that the vectors

$$x = (1, 3, 2) \text{ and } y = (2, a, -6)$$

are perpendicular.

Solution:

(9) Dot product:

$$\begin{aligned} x \cdot y &= (1, 3, 2) \cdot (2, a, -6) \\ &= 1 \cdot 2 + 3 \cdot a + 2 \cdot (-6) \\ &= 2 + 3a - 12 \\ &= 3a - 10 \end{aligned}$$

Setting the dot product to zero:

$$\begin{aligned} 3a - 10 &= 0 \\ 3a &= 10 \\ a &= \boxed{\frac{10}{3}} \end{aligned}$$

2.1: Exercise 7

Find a quadratic polynomial $p(x) = ax^2 + bx + c$ satisfying $p(0) = 1$, $p(1) = 5$, and $p(-1) = -5$.
Solution:

(a)

Using the given information, we set up a system of equations:

1. $p(0) = a(0)^2 + b(0) + c = 1,$ which gives $c = 1$.
2. $p(1) = a(1)^2 + b(1) + 1 = 5,$ which simplifies to $a + b = 4$.
3. $p(-1) = a(-1)^2 + b(-1) + 1 = -5,$ which simplifies to $a - b = -6$.

Solving the system of equations:
Adding equations 2 and 3, we obtain $2a = -2,$ so $a = -1$.
Substituting $a = -1$ into equation 2, we get $-1 + b = 4,$ so $b = 5$.
Therefore, the quadratic polynomial satisfying the given conditions is:

$$\boxed{p(x) = -x^2 + 5x + 1}$$

(b)

$$\begin{aligned} p(0) = L &\implies c = L \\ p(1) = M &\implies a + b + c = M \implies a + b = M - L \\ p(-1) = N &\implies a - b + c = N \implies a - b = N - L \end{aligned}$$

$$\begin{aligned} &2a = (a+b) + (a-b) = M - L + N - L = M + N - 2L \\ \implies &a = \frac{M + N - 2L}{2} \\ &2b = (a+b) - (a-b) = M - L - (N - L) = M - N \\ \implies &b = \frac{M - N}{2} \end{aligned}$$

$$\boxed{(a, b, c) = (\frac{M + N - 2L}{2}, \frac{M - N}{2}, L)}$$

(c)

$$\begin{aligned} ax_1^2 + bx_1 + c &= A_1 \\ ax_2^2 + bx_2 + c &= A_2 \\ ax_3^2 + bx_3 + c &= A_3 \end{aligned}$$

$$\boxed{\begin{aligned} q(x_1) &= A_1 \implies ax_1^2 + bx_1 + c = A_1 \\ q(x_2) &= A_2 \implies ax_2^2 + bx_2 + c = A_2 \\ q(x_3) &= A_3 \implies ax_3^2 + bx_3 + c = A_3 \end{aligned}}$$

2.2: Exercise 5

Find the cosine of the angle between the normal vectors to the planes

$$2x - 2y + z = 14 \qquad \text{and} \qquad x + y - 2z = -10$$

Solution:

(5)

$$\cos\theta = \frac{\begin{pmatrix}2 & -2 & 1\end{pmatrix} \cdot \begin{pmatrix}1 & 1 & -2\end{pmatrix}}{\left|\begin{pmatrix}2 & -2 & 1\end{pmatrix}\right| \left|\begin{pmatrix}1 & 1 & -2\end{pmatrix}\right|} = \frac{(2)(1) + (-2)(1) + (1)(-2)}{\sqrt{2^2 + (-2)^2 + 1^2}\sqrt{1^2 + 1^2 + (-2)^2}} = \frac{-2}{\sqrt{9}\sqrt{6}} = \boxed{-\frac{2}{3\sqrt{6}}}.$$