



$$a = R \cos(90^\circ - \theta) = R \sin \theta$$

$$h = R - R \sin(90^\circ - \theta) = R(1 - \cos \theta)$$

$$V = \frac{1}{3} \pi h^2 (3R - h)$$

$$= \frac{1}{3} \pi R^3 (1 - \cos \theta)^2 (2 + \cos \theta)$$

where  $V$  is a volume of a spherical cap

$$\text{If } V \text{ is constant, } R(\theta) = \left( \frac{3V}{\pi(1 - \cos \theta)^2 (2 + \cos \theta)} \right)^{1/3}$$

$$\text{Liquid vapour interface } A_{LV} = 2\pi R h = 2\pi R^2 (1 - \cos \theta)$$

$$\text{Solid liquid interface } A_{SL} = \pi a^2 = \pi R^2 \sin^2 \theta$$

total solid surface area

$$\begin{aligned} \text{Total interfacial energy } E &= \gamma_{LV} A_{LV} + \gamma_{SL} A_{SL} + \gamma_{SV} (A_s - A_{SL}) \\ &= \gamma_{LV} A_{LV} + (\gamma_{SL} - \gamma_{SV}) A_{SL} + \text{constant} \end{aligned}$$

$$\text{Ignore constant, } E' = \pi R^2 [2\gamma_{LV}(1 - \cos \theta) + (\gamma_{SL} - \gamma_{SV}) \sin^2 \theta]$$

$$\text{Use the principle of energy minimization } \left( \frac{dE'}{d\theta} \right)_V = 0$$

\*  $\gamma_{LV} = 72.8 \text{ mN/m}$  for water at room temp (see Wiki)  
 $\gamma_{SV} \approx 18 \text{ mN/m}$  for hydrophobic surface (Teflon/PTFE)