STUDY ON FIBONACCI NUMBERS AND IT'S APPLICATION

A Dissertation submitted to Department of Mathematics, Nowgong College, affiliated to Gauhati University for the partial fulfillment of B.Sc Degree in Mathematics, 2022

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CERTIFICATE

This is to certify that the dissertation entitled **"Study on Fibonacci Numbers and it's Application"** is submitted to Department of Mathematics, Nowgong College, Assam by Mr. Soidul Hussain for partial fulfillment of the requirements for the Bachelor degree of Science in Mathematics. The work is assigned for B.Sc . 6th Semester course in Mathematics for Paper Code **MAT-HE-6086(project)**. The research work was carried out by Mr. Soidul Hussain (Roll No- US-191-307-0185) under my supervision and has not been submitted for any other degree or diploma.

Date:.

(P .Bora)

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Saidul Hussain
16/06/22

PREFACE

A Dissertation is an academic explanatory writing based on original research of a selected topic. It is usually submitted as part of a degree programme such a part of a bachelor's degree that we are pursuing at Nowgong College.This dissertation is the longest piece of writing I have ever done.

A student's academic experience results in the writing of a dissertation on a topic of an academic investigation. Researching and then writing about the topic of interest with findings becomes the basis of student's academic resume including his/her professional proficiency. Therefore, a good dissertation could be the starting point for further study of a student. Moreover, it will be an important indicator of their aptitude for research. If students are not thinking of further study, this in-depth exploration of their chosen topic could be a considerable matter at job interviews.

I am taking the opportunity to do a dissertation work exit in the syllabus of Mathematics in B. SC degree course with an aim to utilize my academic aptitude throughout the work.

Soidul Hussain

Soidul Hussain.

Content

STUDY ON FIBONACCI NUMBERS AND IT'S APPLICATION

Abstract :

Fibonacci sequence of numbers and the associated "Golden Ratio" are manifested in nature and in certain works of art. We observe that many of the natural things follow the fibonacci-sequence. It appears in biological settings such as branching in trees, phyllotaxis (the arrangement of leaves on a stem), the fruit sprouts of a pineapple, the flowering of an artichoke, an uncurling fern and the arrangement of a pine cone's bract's etc. At present Fibonacci numbers plays very important role in coding theory.

Fibonacci numbers in different forms are widely applied in constructing security coding.

Keywords : Fibonacci Numbers, Golden Ratio, Coding Encryption, Decryption.

INTRODUCTION

The Fibonacci numbers were first discovered by an italian Mathematician Leonardo Pisano. He was known by his nick name, Fibonacci. The Fibonacci sequence is a sequence in which each term is the sum of the two numbers preceding it. The Fibonacci Numbers are defined by the recursive relation defined by the equations $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 3$ where $F_1 = 1$; $F_2 = 1$ where F_n represents the nth Fibonacci numbers. The Fibonacci Sequence Can elaborately written as $\{1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots\}$

One of the most common experiments dealing with the Fibonacci Sequence is his experiment with rabbits. Fibonacci put one male and one female rabbit in a field. Fibonacci supposed that the rabbits lived infinitly and every month a new pair of one male and one female was produced.

Fibonacci asked how many would be formed in a years. Following the Fibonacci Sequence perfectly the rabbits reproduction was determind ... 144 rabbits. Through unrealistic, the rabbit sequence allows people to attach a highly evolved series of complex numbers to an everyday, logical, comprehensible thought. Borthers and peterson (2016) elaboratly described the history and application of Fibonacci numbers.

CHAPTER : 1

FIBONACCI SEQUENCE IN NATURE

Fibonacci can be found in nature not only in the famous rabbit experiment, but also in beautiful flowers. On the head of a sunflower and the seeds are packed in a certain way so that they follow the pattern of the Fibonacci Sequence. This spiral prevents the seed of the sunflowers from growing themselves out, thus helping them with servial.

The petals of flowers and other plants may also be related to the Fibonacci in the way that they Create new petals.

1.1 Petals on flowers

Probably most of us have never takes the time to examine very Carefully the numbers of - arrangment of petals on a flowers. If we were do so, we would find that the numbers of palels on a flowers that still has all of petals intact and has not lost any. for many flowers is a Fibonacci numbers.

⇒ 1. Petal : White Cally Lily.

⇒ 2 Petals : Euphorbia

⇒ 3 Petals : Trillium grandiflorum.

⇒ 5 Petals : Columbine, Periwinkle.

⇒ 8 petals : Cosmosbipinnatus, Berlamdiera.

⇒ 13. petals : Marigold, Black-eyed Susan, Cineraria.

⇒ 21 petals : Shasta daisy

⇒ 34 petals : Sunflower, field daisies.

⇒ 55 petals : michelmas daisies.

⇒ 89 petals : The asteraceae family.

1 Pelals

2 Pelals

3 Petals

5 Petals

8 Petals

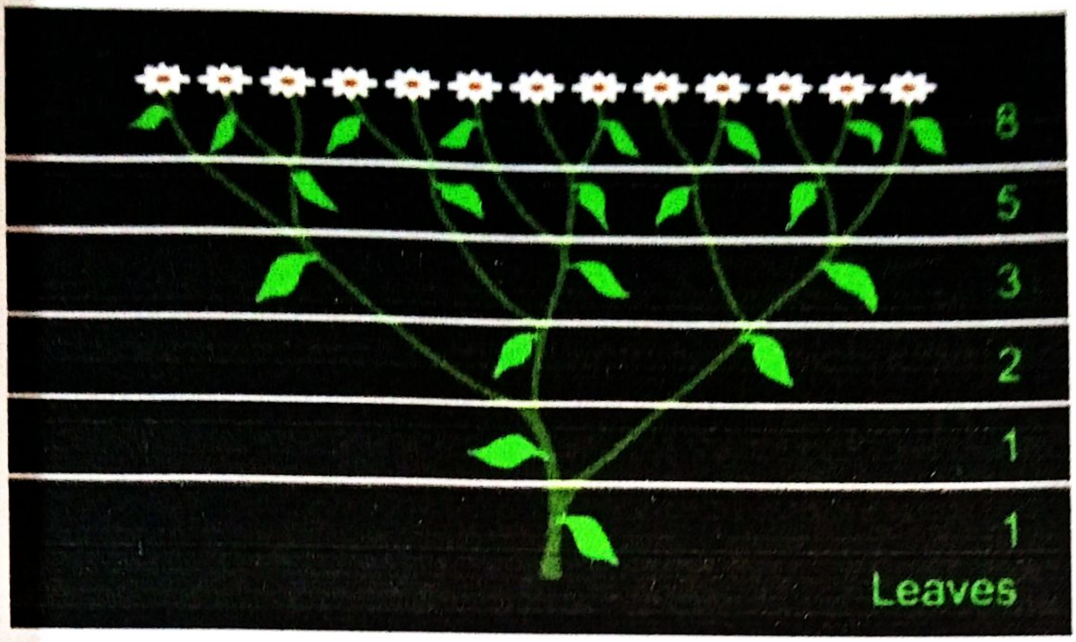
13 Petals

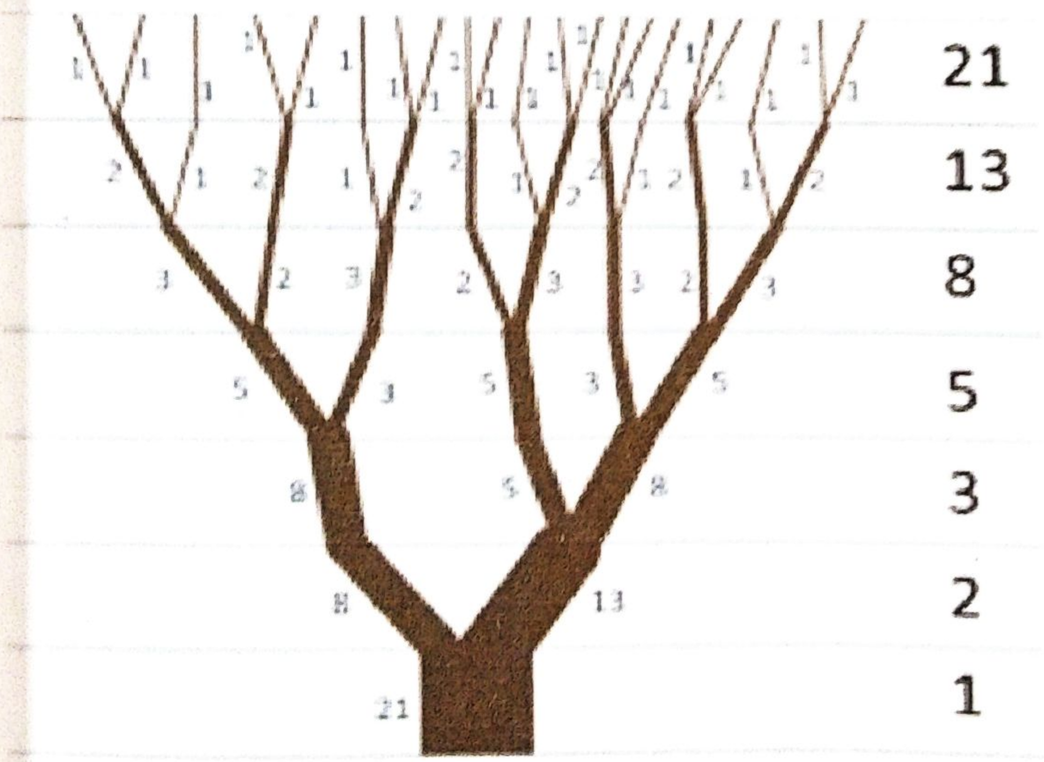
Plants show the Fibonacci numbers in the arrangements of their leaves, Three clockwise rotations, Passing five leaves two Counter-clokwise rotations. Sneezewort (Achillea ptarmica) also follows the Fibonacci numbers.

why do these arrangement occur?

In the case of leaf arrangement, or phyllotaxis, Some of the cases may be related to maximizing the space for each leaf, or the average amount of light falling on each one.

These following Pictures are very Common to us, we Can see the patterns of leaves just out of single step of our houses. All of these follow the Fibonacci Numbers.


8
5
3
2
1
1
Leaves

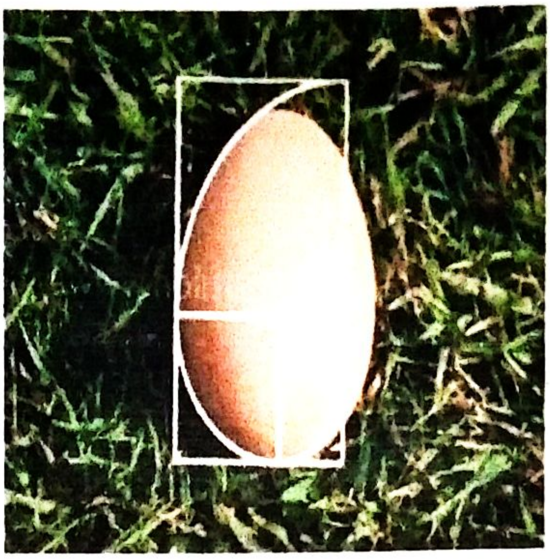
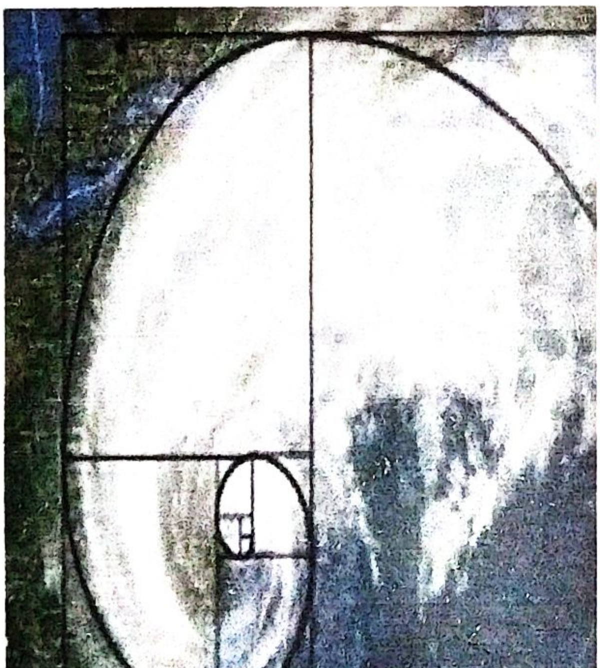
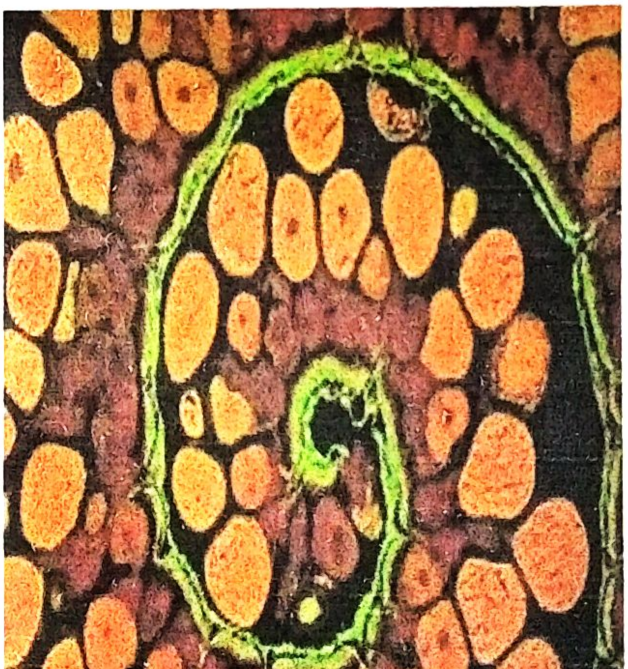

21
13
8
5
3
2
1

1.2 Fibonacci Spiral.

The Fibonacci numbers are found in the arrangement of seeds on flower heads.

There are 55 spirals spiraling outwards and 34 spirals spiraling inwards in most daisy or sunflower blossoms.

Pinecones clearly show the Fibonacci spirals. Fibonacci spirals can be found in Cauliflower. The Fibonacci numbers can also be found in Pineapples and Bananas. Bananas have 3 or 5 flat sides and Pineapple scales have Fibonacci spirals in set of 8, 13 and 21 Inside the fruit of many plants we can observe the presence of Fibonacci order.

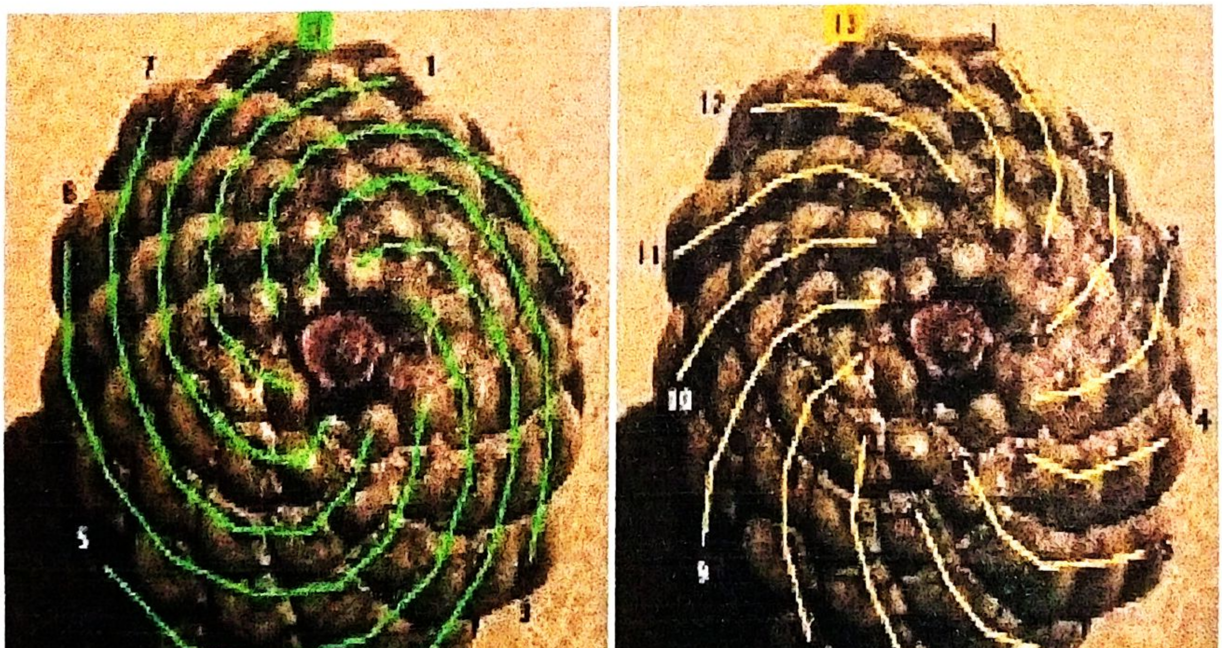





Fibonacci spiral are also found in various fields associated in nature.

It is seen spiral, sea shells, waves, Combination of colours, roses etc in so many things created in nature. But very few of us have time to study this phenomenon.

Nature isn't trying to use the Fibonacci numbers: They are appearing as a by-product of a deeper physical process. That is why the spirals are imperfact, The plant is responding to physical constrains, not to a mathematical rule.

The basic idea is that the phenomenon, the position of each new growth is about 222.5 degrees away from the previous one, because it provides, on average, the maximum space for all the shoots. This angle is called the Golden angle and it devides the complete 360° circle in the 0.618033...


7
1
6
2
5
3
4
13
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13
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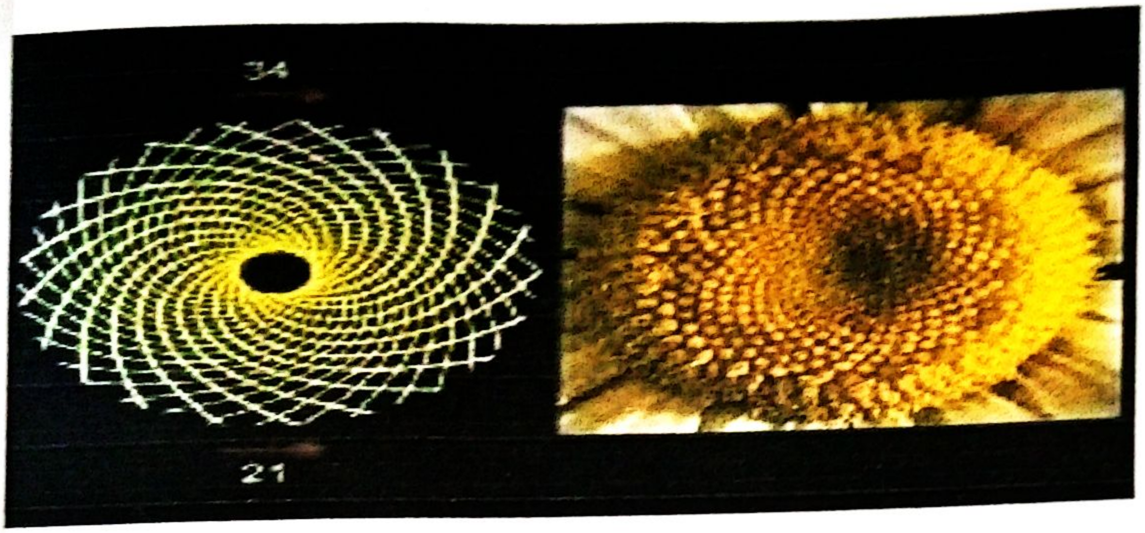
21
34


34
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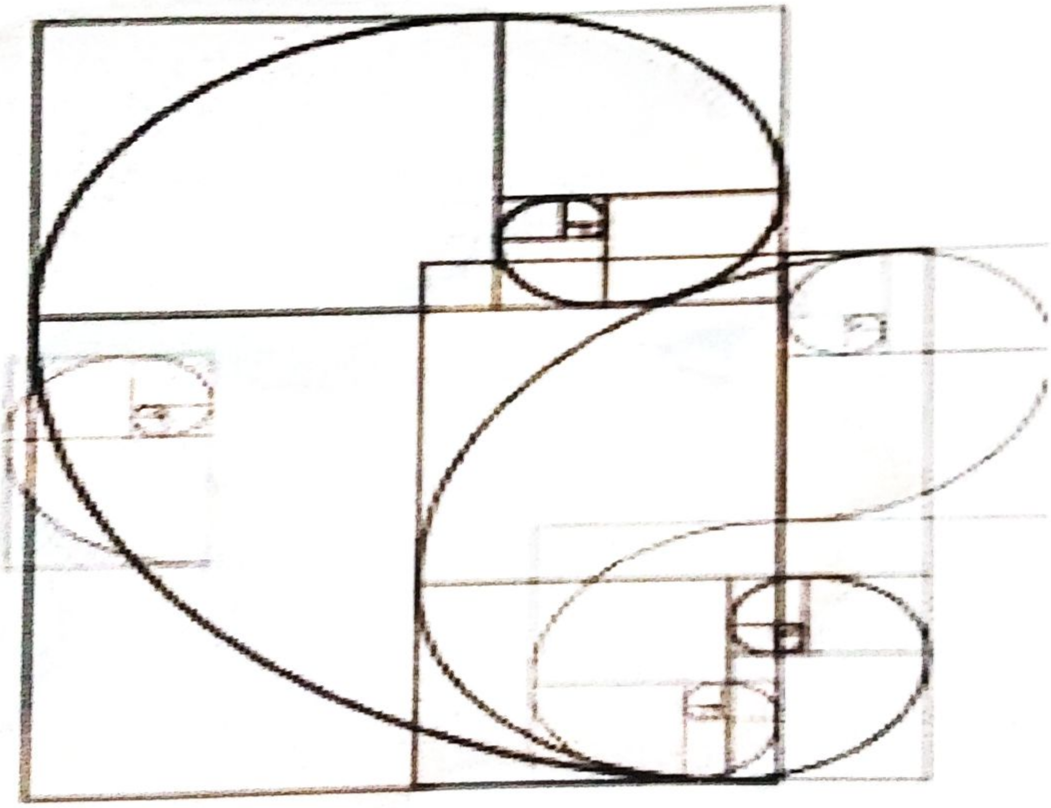
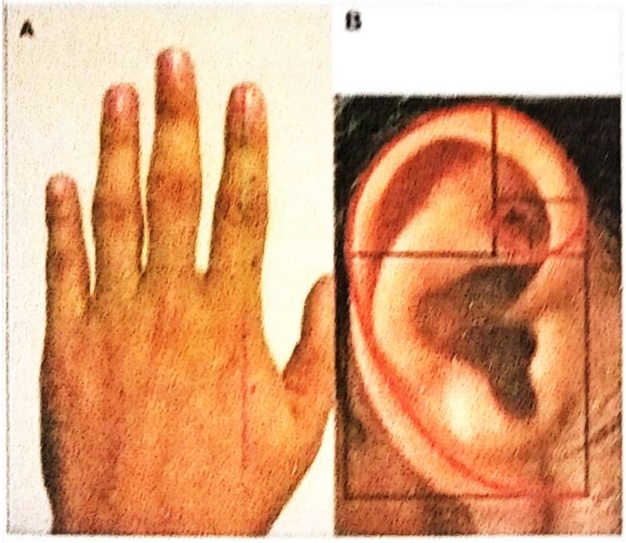


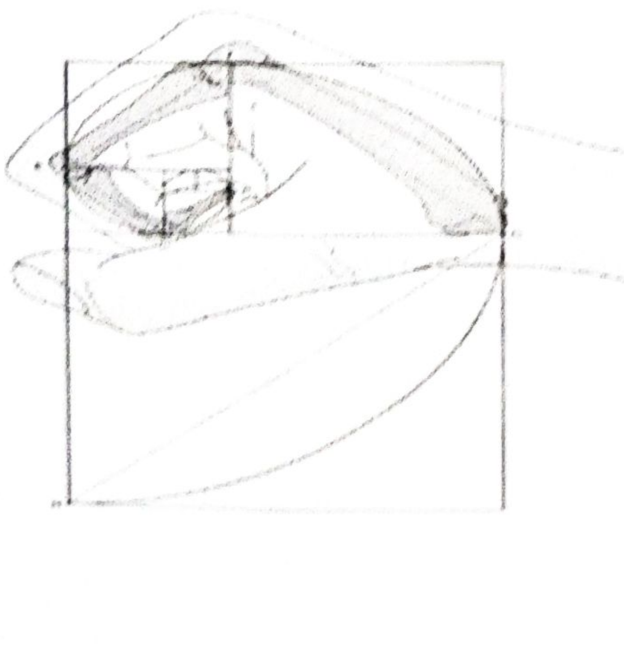
1.3 Organs of Human Body

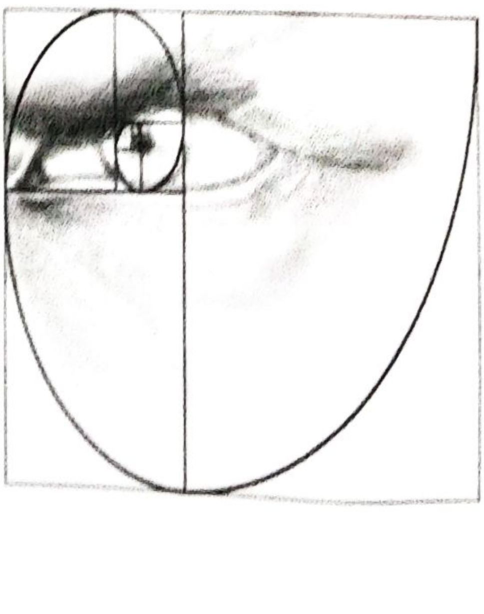
Human exhibit Fibonacci characteristics. Every human has two hands, each one of these has five fingers and each finger has three parts which are seperated by two Knuckles. All of these numbers fit into the sequence, Morever the lengths of bones in a hand are in Fibonacci numbers.

The Cochlea of the inner ear forms a Golden Spiral.


Golden spiral
on Joconda?


A
B

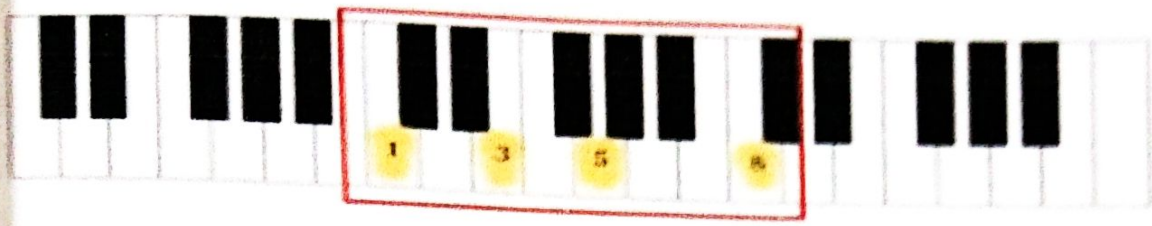




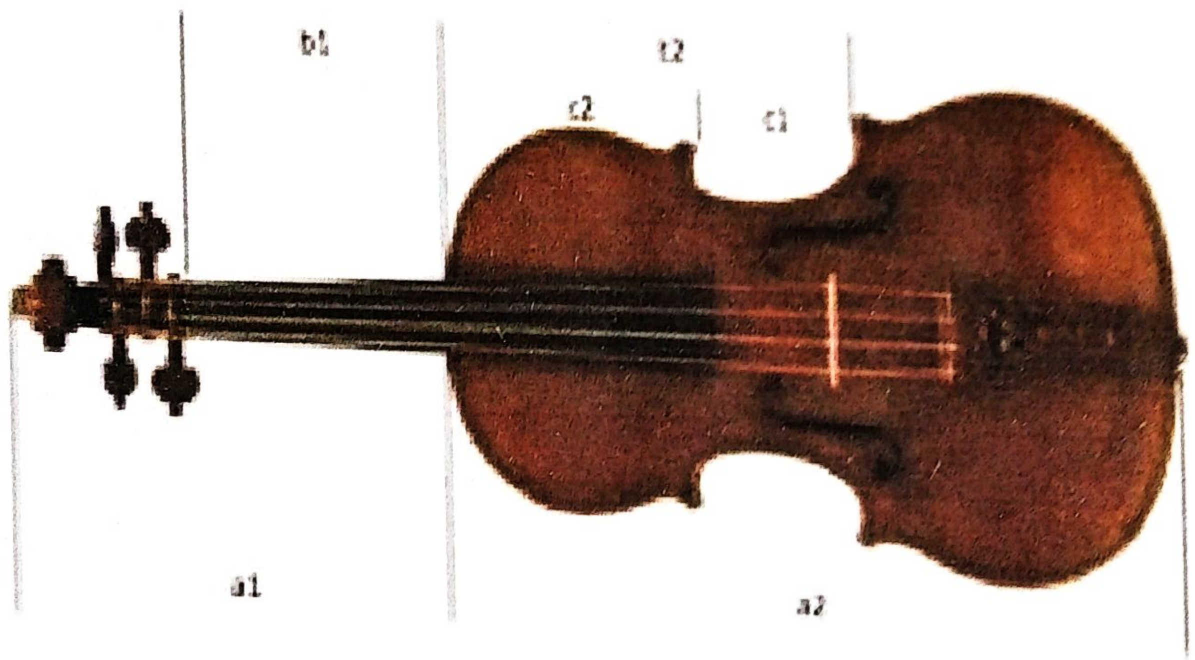
1.4 Fibonacci in Music.

The Fibonacci sequence of numbers and the golden ratio are manifested in music widely.
The numbers are present in the octave, the fondational unit of melody and harmony.
Stradivarius used the golden ratio to make the greatest string instruments ever created.

Howat's (1983) research on Debussy's works shows that the composere used the golden ratio and Fibonacci numbers to structure his music. The Fibonacci Composition reveals the inherent aesthetic appeal of this mathematics phonomenon. Fibonacci numbers harmonize natuarally and exponential growth which the Fibonacci sequence typically defines in nature is made present in music by using Fibonacci notes.



$$\frac{a1 + a2}{a2} = \frac{a2}{a1} = \frac{b2}{b1} = \frac{b2}{c2} = \frac{c2}{c1} = \phi$$

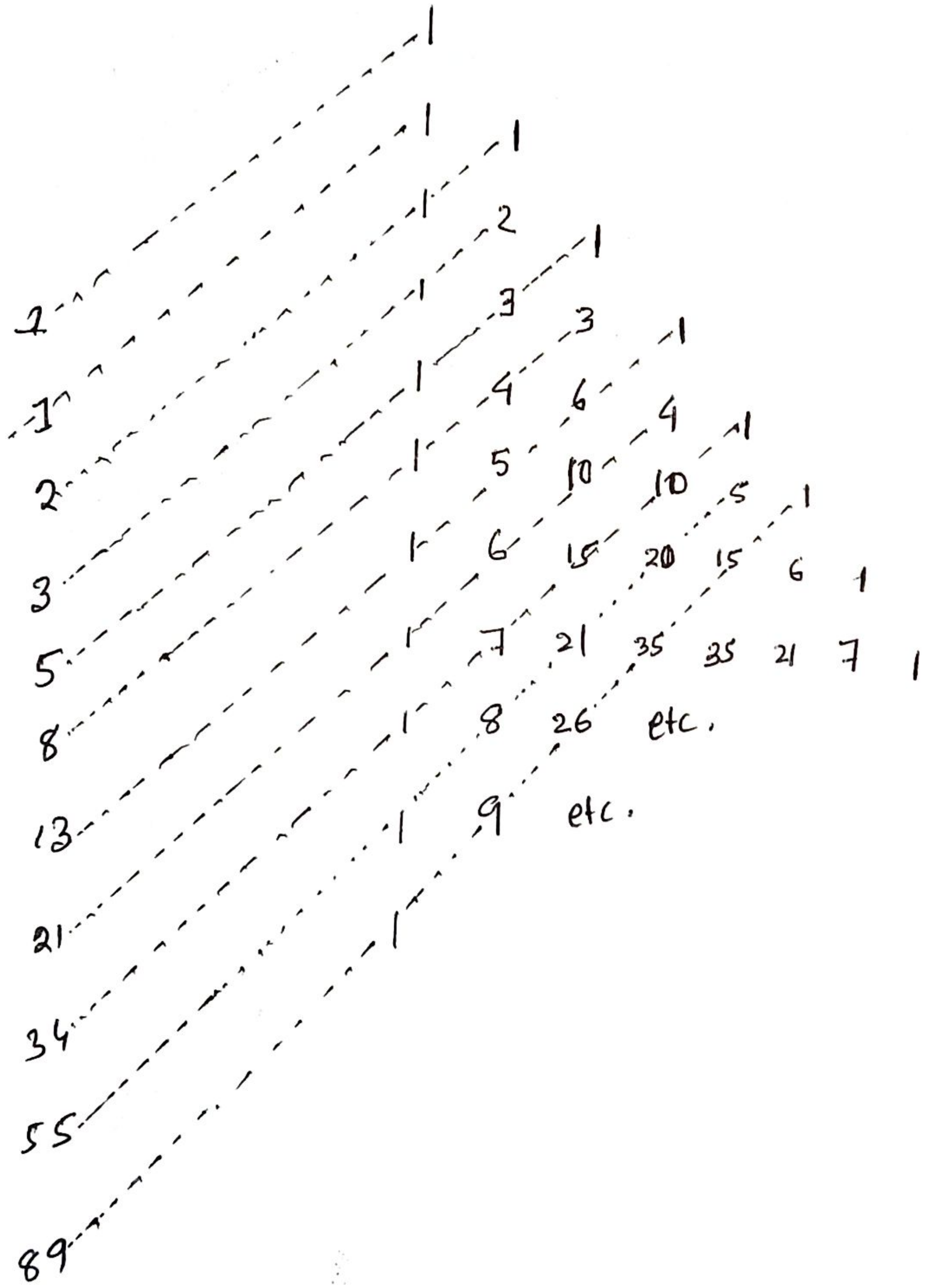


1.5 Fibonacci Numbers in Pascal's Triangle.

The Fibonacci Numbers are also applied in pascal's Triangle. Entry is sum of the two numbers either side of it, but in the row above. Diagonal sums in Pascal's Triangle are the Fibonacci numbers. Fibonacci numbers can also be found using a formula.

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

Diagonal Sums



1.6 The Golden Section.

Represented by the Greek Letter Phi

$(\phi) = 1.6180339887$

How did 1.6180339887.... Come from?

Let's look at the ratio of each numbers in The Fibonacci sequence to the one before it.

$\frac{1}{1} = 1$

$\frac{2}{1} = 2$

$\frac{3}{2} = 1.5$

$\frac{5}{3} = 1.666$

$\frac{8}{5} = 1.6$

$\frac{13}{8} = 1.625$

$\frac{21}{13} = 1.61538$

$$\frac{34}{21} = 1.61905$$

$$\frac{55}{34} = 1.61764$$

$$\frac{89}{55} = 1.61861$$

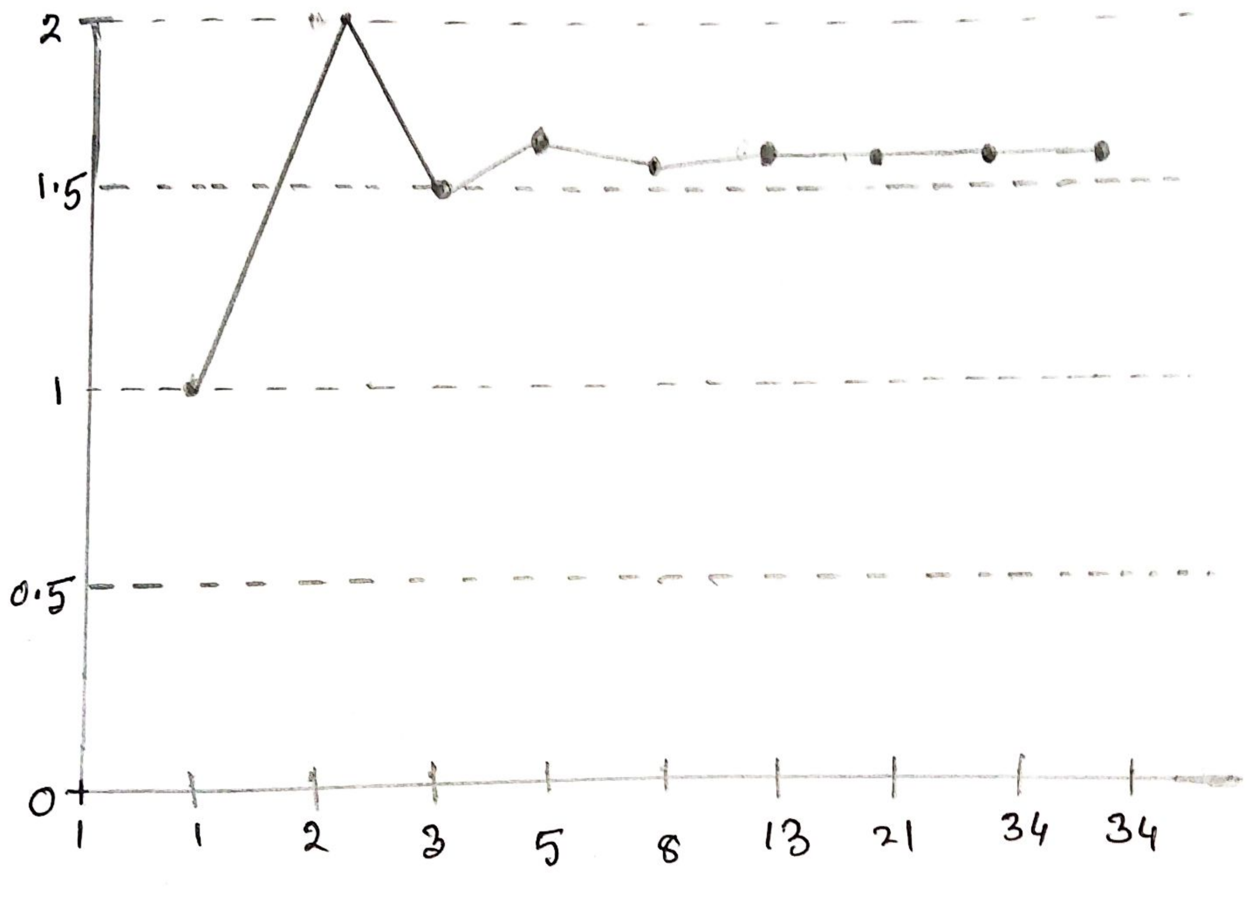
$$\frac{144}{89} = 1.61798$$

$$\frac{233}{144} = 1.61806$$

If we keep going, we get an interesting number which mathematicians call " Phi" (Golden Ratio or Gollen Ratio). It is denoted by ϕ and the value of $\phi = 1.6180339887$

$$\lim_{n \to \infty} \frac{F_{n+1}}{F_n} = 1.618$$

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \ldots$$



Fibonacci Numbers

If we take two successive terms of the series, a, b and $a+b$, then,

$$\frac{b}{a} \cong \frac{a+b}{b}$$

$$\cong \frac{a}{b} + 1$$

we define the golden section, ϕ (phi) to be the limit of b/a, so,

$$\phi = \frac{1}{\phi} + 1$$

$$\phi^2 = \phi + 1$$

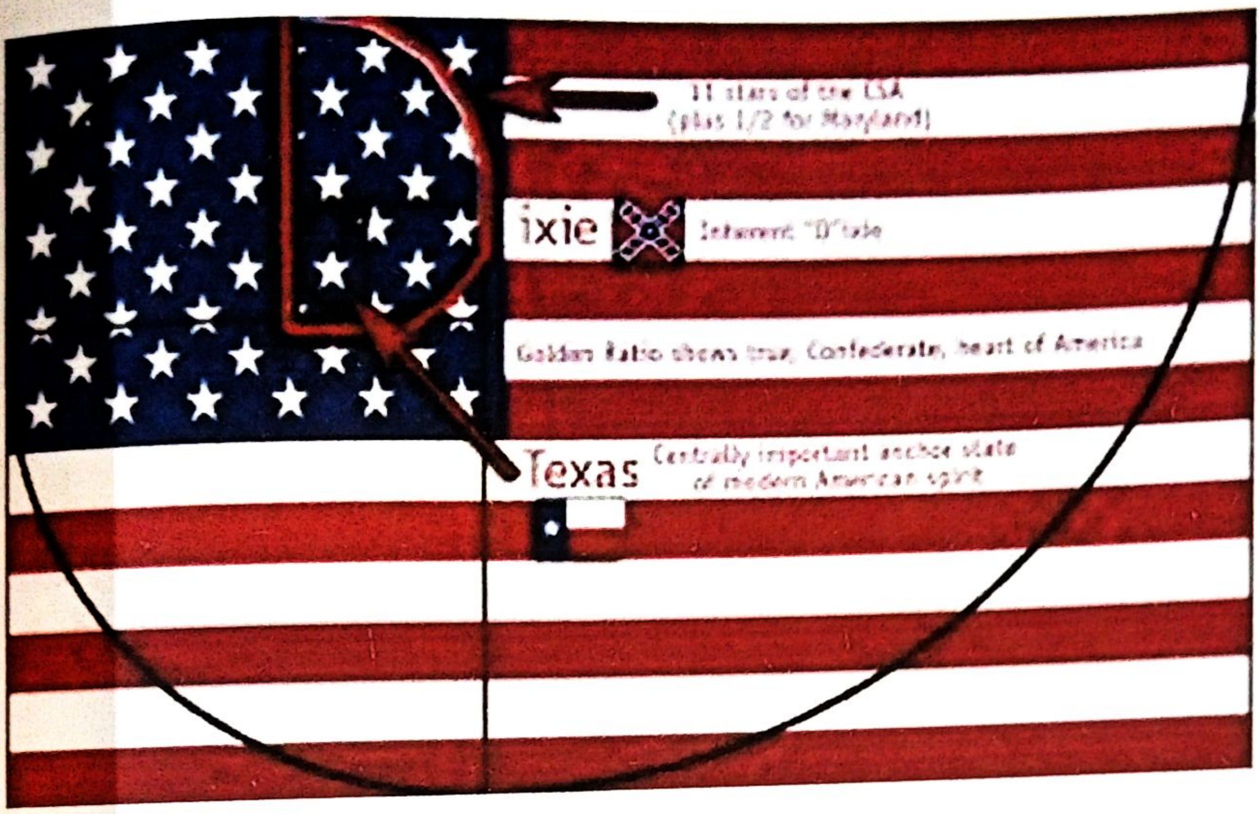
$$\Rightarrow \phi^2 - \phi - 1 = 0$$

$$\therefore \phi = \frac{1+\sqrt{5}}{2} \simeq 1.618.$$

1.7 Applications of Golden ratio.

Leonardo da Vinci showed that in a 'perfect man' there were lots of measurements that followed the Golden Ratio. The Golden (Divine) Ratio has been talked about for thousands of years.

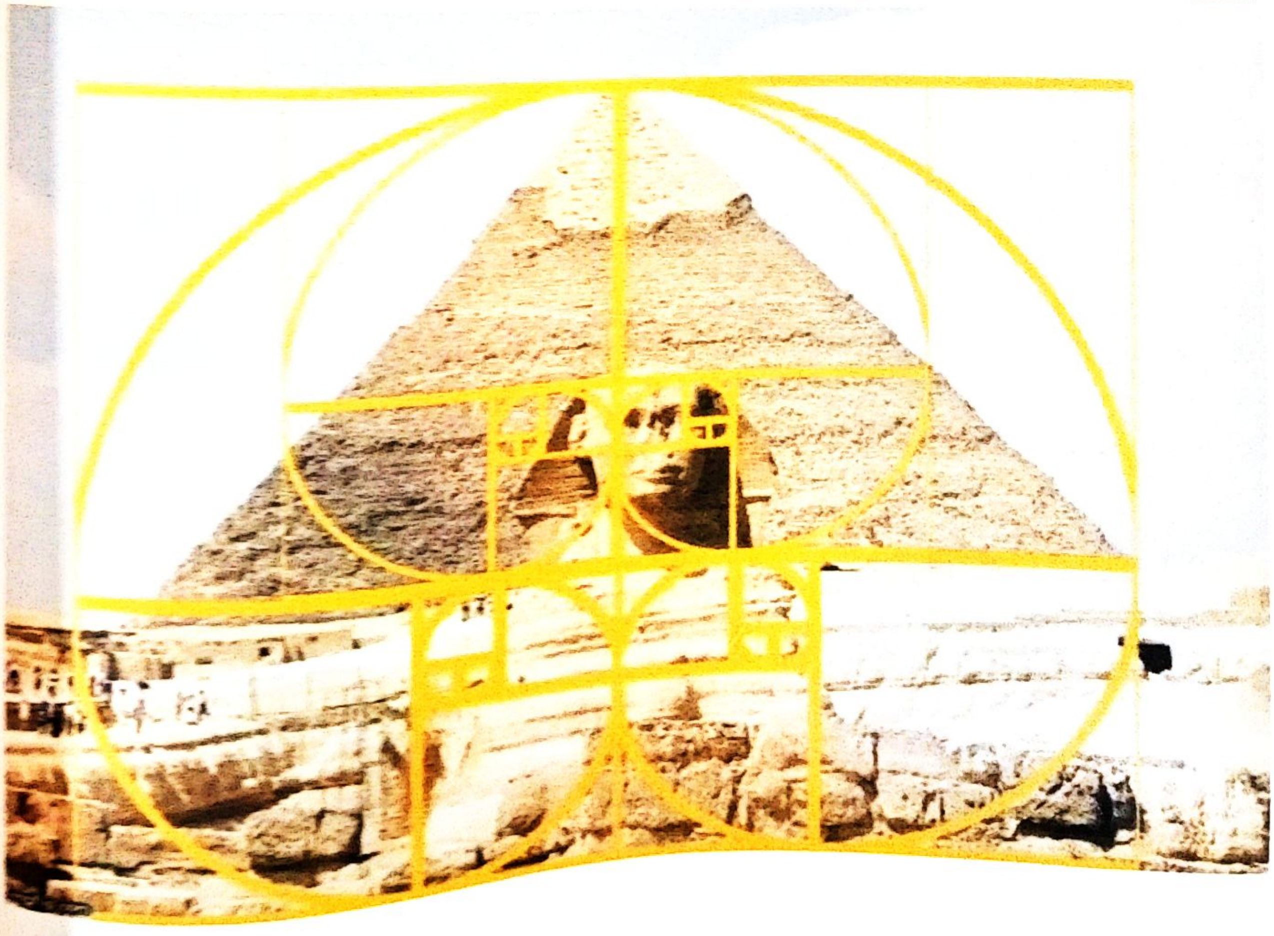
The Golden ratio is widely used in Geometry. It is the ratio of the side of a regular pentagon to its diagonal. The diagonal cut each other with the golden ratio. Pentagram describes a star which forms parts of many flags. This Five-point Systemmetry with Golden proportions is found in Starfish which have five arms.


11 stars of the CSA
(plus 1/2 for Maryland)
ixie
Golden Ratio shows true, Confederate, heart of America
Texas
Centrally important anchor state
of modern American spirit

The Golden Ratio is also frequently seen in natural architecture also.

It can be found in the great pyramid in Egypt. Perimeter of the pyramid, divided by twise its vertical height is the value of ϕ.

Golden section appears in many of the proportions of the parthenon in Greece. Front elevation is built on the golden section (0.618 times as wide as it is tall).



Some example

Example 1. Find the Fibonacci numbers when $n=5$, using recursive relation.

Sol^n. The formulla to calculate the Fibonacci sequence is

$$F_n = F_{n-1} + F_{n-2}$$

Take $F_0 = 0$ and $F_1 = 1$

using the formulla, we get

$$F_2 = F_1 + F_0 = 0+1 = 1$$

$$F_3 = F_2 + F_1 = 1+1 = 2$$

$$F_4 = F_3 + F_2 = 2+1 = 3$$

$$F_5 = F_4 + F_3 = 3+2 = 5$$

Therefore, the Fibonacci number is 5. when $n=5$

Example 2.

Find the Fibonacci number using the Golden ratio when $n=6$.

Solution:

The formulla to calculate the Fibonacci number using the golden ratio is

$$x_n = [\phi^n - (1-\phi)^n] / \sqrt{5}$$

we know that ϕ is approximatly to 1.618.

$n=6$

Now, Substitute the values in the formula,

We have get $x_n = [\phi^n - (1-\phi)^n] / \sqrt{5}$

$$x_6 = [(1.618)^6 - (1-1.618)^6] / \sqrt{5}$$

$$x_6 = [17.942 - (0.618)^6] / 2.236$$

$$x_6 = [17.942 - 0.056] / 2.236$$

$$x_6 = 17.886 / 2.236$$

$$x_6 = 7.999$$

$$x_6 = 8 \text{ (Rounded value)}$$

$\therefore$ The Fibonacci number in the sequence is 8 when $n=6$.

Example 3.

Starting with 0 and 1, Write the first 5 Fibonacci numbers.

Solution :- The Formula for the Fibonacci Sequence is $F_n = F_{n-1} + F_{n-2}$

The First and second terms are 0 and 1 respectively.

$F_1 = 0$, and $F_2 = 1$

$F_3 = F_1 + F_2 = 0 + 1 = 1$ is the 3rd term.

$F_4 = F_3 + F_2 = 1 + 1 = 2$ is the 4th term

$F_5 = F_4 + F_3 = 2 + 1 = 3$ is the 5th term.

∴ The Fibonacci Sequence's First Five terms are 0, 1, 1, 2, 3.

Example 4.

Find the Fibonacci series next term : 0, 1, 1, 2, 3, 5 ....

Solution: The preceding two Fibonacci terms are the next term in the series.

i.e, $F_1 = 0$, $F_2 = 1$,

$F_3 = 1$, $F_4 = 2$, $F_5 = 3$,

$F_6 = 5$, $F_7 = F_6 + F_5 = 8$

$F_8 = F_7 + F_6$

$= 8 + 5 = 13$

$F_9 = F_8 + F_7 = 13 + 8 = 21$

$\therefore$ The Fibonacci series next terms:

0, 1, 1, 2, 3, 5, 8, 13, 21, ...

CHAPTER 2.

FIBONACCI IN CODING

Recently Fibonacci sequence and golden ratio are of great interest to the researchers in many fields of Science incliding high energy physics, quantum mechanics, Cryptography and Coding. Raghu and Ravishankarr developed a paper of application classical encryption techniques for securing data. (Raphel and Sundaram) showed that Communication may be secured be use of Fibonacci numbers. Similar application of Fibonacci in Cryptography is described here by a simple illustration. Suppose that original message" CODE" to be Encrypted.

It is sent through an unsecured channel. Security key is chosen based on the Fibonacci number. Any one character may be chosen as a first security key to generate cipher text and then Fibonacci sequence can be used. Agarwal at (2015) used Fibonacci sequence for encryption data.

2.1 Method of Encryption.

(a) For instance, let the first security key chosen be 'k'.

Plain Text C O D E

Characters: k, l, m, o, p, q, r, s, t, u, v

. y, x, y, z, a, b, c, d, e, f g h j k l...

Fibonacci : 1 2 3 5 ....

Chipher Text : k l m o

Cipher Text is converted into unicode symbols and saved in a text file. The text file is transmitted over the transmission medium. It is the first level of security

(b) Chipher text to Unicode.

In the second level of secuirity, the ASCII code of each character obtained from the cipher text plus the ASCII code of its previous character, and text character is added to the ASCII code of the equivalant character in the original message. Here, ASCII codes of four characters are used as a security key to further ~~ee~~ encode the characters available in the chiper text to unicode symbols.

For instance

ASCII Code of previous character + ASCII Code of Equivalant character + ASCII Code of Next character + ASCII code of chiper code character

⇒ SUM

K I M O

$109(m)+111(o)+112(p)+69(E)=401$

$108(l)+109(m)+110(n)+66(D)=405$

$107(K)+108(l)+109(m)+79(O)=403$

$106(j)+107(K)+108(l)+67(c)=388$

By looking at the symbols in a text file no unknown person can identify what it is and the message cannot be retrieved unless the re-trivial procedure is known.

2.2 Decription method.

The Description process follows a reverse process of Encryption. Recipent extracted each symbol from the recived text file and mapped to find its hexadecimal value. obtain value is Converted into a decimal value to find out the plain text using the Key. without Knowledge of the Key an unknown person cannot understant the existance of any secret message.

CHAPTER 3.

The Mystery of Phi (1.618) and phi (0.618)

why is the Golden Ratio (1.61803 ...) of such interest to traders?

Known by various names since the ancient Egyptians and Pythagoreans 570-490 BC. the first definition comes from Euclied (325-265 BC), vie Divine Proportion, or Divina proporthane, by Luca Pacioli (1445-1517) was the earliest known treatise devoted to the subject and was illustrated by Leonardo da vinci, who coined the name section aurea or the "Golden section".

By any of the historical names, it is of little interest to investors but investors rarely face the day-to-day battlefield of rapid global market action.

antiquity of a nautilus seashell, or the spiral that forms at the start of life as a ferm begins to unfurl from the ground on a spring day. Plato (circa 427-347 BC) in the Republic asks the reader to 'take a line and divide it unevenly". Under a Pythagorean oath of silence not to reveal the secrets of the mysteries, plato posed questions in hopes in hopes of provoking an insightful response. So why does he use a line, rather than numbers? Why does he ask you to divide it unevenly?

To answer plato and how this may connect to the needs of all traders today. We first must understand ratio and proportion. All living things from large to small, including markets, must abide by the same divine blueprint. As a result, the more traders understand how a ratio called phi (1.618)

Traders on the frontlines, however, live and die on their ability to measure risk and Control their Capital drawdown, To enter the tough markets of today requires the foresight to call what price a market will move toward. of equal importance is our need to know where a market will move foward. of equal important is our need to know where a market should not be trading so we can execute a timely exit plan.

So, how does an eloquent ratio that has Captured people's interest from the time of antiquity help and influence the decision a trader must face today? The Golden Ratio is a universal law that explains how everything with a growth and decay cycle evolves. Be it spiral of the soler system, the spiral of hurricane, the Growth pattern of a influences our lives, the clearer we can look into the face of a grander plan and witness a force acting upon our markets that demands our respect, comprehension, and humility. As a trader's knowledge deepends,, he can see in advance a mathematical grid that determines future market movement. But few people truly understant the forces at play and never develop beyond the basic theory that is insufficient to adjust to the nutural expansion and contraction cycles found within all markets in both price and time.

Conclusion.

The Fibonacci numbers are Nature's numbering system. They appear everywhere in nature, from the leaf arrangement in plants, to the pattern of the florets of a flower, the bracts of a pinecone, or the scales of a pineaple.

The Fibonacci numbers are therefore applicable to the growth of every living thing, including a single cell, a grain of wheat, a hive of bees, and even all of mankind, Nature follows the fibonacci numbers astonishingly. But very little we observe the beauty of nature.

If we study the pattern of various natural things minutely we observe that many of the natural things around us follow the Fibonacci numbers in real life which is creats strange among us.

The study of nature is very important for the learners. It increases the inquisitiveness among the learners. The topic is chosen so that learners could be interested towards the study of nature around them. Security in Communication System is an interesting topic at present as India is going towards digitalization everywhere.

A little bit of Concept for Securing data is also provided in this model. Let us finish by the words of Leonardo da Vinci "Learn how to see, Realize that everything Connects to everything else".

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