

- ii) Learn about the relation between basic feasible solutions and extreme points.
- iii) Understand the theory of the simplex method used to solve linear programming problems.
- iv) Learn about two-phase and big-M methods to deal with problems involving artificial variables.
- v) Learn about the relationships between the primal and dual problems.
- vi) Solve transportation and assignment problems.
- vii) Apply linear programming method to solve two-person zero-sum game problems.

**Unit 1: The Linear Programming Problem:** Standard, Canonical and matrix forms, Graphical solution. Hyperplanes, Extreme points, Convex and polyhedral sets. Basic solutions; Basic Feasible Solutions; Reduction of any feasible solution to a basic feasible solution; Correspondence between basic feasible solutions and extreme points.

[1] Chapter 1 (Section 1.1, 1.4 and 1.6)

[2] Chapter 2 (Sections 2.16, 2.19 and 2.20), Chapter 3 (Sections 3.2, 3.4 and 3.10)

**Unit 2: Simplex Method:** Optimal solution, Termination criteria for optimal solution of the Linear Programming Problem, Unique and alternate optimal solutions, Unboundedness; Simplex Algorithm and its Tableau Format; Artificial variables, Two-phase method, Big-M method.

[1] Chapter 3 (Sections 3.3, 3.6, 3.7 and 3.8)

**Unit 3: Motivation and Formulation of Dual problem;** Primal-Dual relationships; Fundamental Theorem of Duality; Complimentary Slackness.

[1] Chapter 4 (Sections 4.1 to 4.3)

[1] Chapter 6 (Section 6.1, and 6.2, up to Example 6.4)

#### **Unit 4: Applications**

**Transportation Problem:** Definition and formulation; Methods of finding initial basic feasible solutions; North West corner rule. Least cost method; Vogel's Approximation method; Algorithm for solving Transportation Problem.

**Assignment Problem:** Mathematical formulation and Hungarian method of solving.

**Game Theory:** Basic concept, Formulation and solution of two-person zero-sum games, Games with mixed strategies, Linear Programming method of solving a game.

[3] Chapter 5 (Sections 5.1, 5.3, and 5.4)

[2] Chapter 11 (Sections 11.12, and 11.13)

#### **Text Books:**

1. Bazaraa, Mokhtar S., Jarvis, John J. and Sherali, Hanif D. (2010). *Linear Programming and Network Flows* (4th ed.). John Wiley and Sons.
2. Hadley, G. (1997). *Linear Programming*. Narosa Publishing House. New Delhi.
3. Taha, Hamdy A. (2010). *Operations Research: An Introduction* (9th ed.). Pearson.

#### **Reference Books:**

1. Hillier, Frederick S. & Lieberman, Gerald J. (2015). *Introduction to Operations Research* (10th ed.). McGraw-Hill Education (India) Pvt. Ltd.
2. Thie, Paul R., & Keough, G. E. (2014). *An Introduction to Linear Programming and Game Theory*. (3rd ed.). Wiley India Pvt. Ltd.

### **SEMESTER-IV**

#### **MAT-HC-4016: Multivariate Calculus**

Total Marks: 100 (Theory 80, Internal assessment 20)

Per week: 5 lectures 1 Tutorial, Credits 6, Each unit carry equal credit

(Use of Scientific calculator is allowed)

**Course Objectives:** To understand the extension of the studies of single variable differential and integral calculus to functions of two or more independent variables. Also, the emphasis will be on the use of Computer Algebra Systems by which these concepts may be analyzed and visualized to have a better understanding. This course will facilitate to become aware of applications of multivariable calculus tools in physics, economics, optimization, and understanding the architecture of curves and surfaces in plane and space etc.

**Course Learning Outcomes:** This course will enable the students to:

- i) Learn the conceptual variations when advancing in calculus from one variable to multivariable discussion.
- ii) Understand the maximization and minimization of multivariable functions subject to the given constraints
- iii) Learn about inter-relationship amongst the line integral, double and triple integral formulations.
- iv) Familiarize with Green's, Stokes' and Gauss divergence theorems

**UNIT 1:** Functions of several variables, Level curves and surfaces, Limits and continuity, Partial differentiation, Higher order partial derivative, Tangent planes, Total differential and differentiability, Chain rule, Directional derivatives, The gradient, Maximal and normal property of the gradient, Tangent planes and normal lines.

[1] Chapter 11 (Sections 11.1 and 11.2, 11.3 and 11.4, 11.5, 11.6)

**UNIT 2:** Extrema of functions of two variables, Method of Lagrange multipliers, Constrained optimization problems; Definition of vector field, Divergence and curl.

[1] Chapter 11 [Section 11.7 (up to page 605)], Section 11.8 (pages 610-614)], Chapter 13 (Section 13.1)

**UNIT 3:** Double integration over rectangular and nonrectangular regions, Double integrals in polar coordinates, Triple integral over a parallelepiped and solid regions, Volume by triple integrals, triple integration in cylindrical and spherical coordinates, Change of variables in double and triple integrals.

[1] Chapter 12 (Sections 12.1-12.4)

**UNIT 4:** Line integrals, Applications of line integrals: Mass and Work, Fundamental theorem for line integrals, Conservative vector fields, Green's theorem, Area as a line integral; Surface integrals, Stokes' theorem, The Gauss divergence theorem.

[1] Chapter 12 (Sections 12.5 and 12.6) Chapter 13 (Section 13.2, 13.3), [Sections 13.4 (pages 712 to 716), 13.5 (pages 723 to 726)]

**Textbook:**

[1] Strauss, Monty J., Bradley, Gerald L., & Smith, Karl J. (2007). *Calculus* (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. (Pearson Education). Delhi. Indian Reprint 2011

**Reference Books:**

1. Marsden, J. E., Tromba, A., & Weinstein, A. (2004). *Basic Multivariable Calculus*. Springer (SIE). First Indian Reprint.
2. G.B. Thomas and R.L. Finney, *Calculus*, 9th Ed., Pearson Education, Delhi, 2005.
3. M. J. Strauss, G. L. Bradley and K. J. Smith, *Calculus* (3 Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2007.
4. James Stewart, *Multivariable Calculus, Concepts and Contexts*, 2nd Ed., Brooks /Cole, Thomson Learning, USA, 2001.

**MAT-HC-4026: Numerical Methods (including practical)**

Total marks: 100: (Theory: 60, Practical 20, Internal Assessment: 20)

Per week: 4 Lectures, 2 Practical, Credits 6(4+2), *Each unit carry equal credit*

**Course Objectives:** To comprehend various computational techniques to find approximate value for possible root(s) of non-algebraic equations and to find the approximate solutions of system of linear equations and



ordinary differential equations. Also, use of Computer Algebra System (CAS) by which the numerical problems can be solved both numerically and analytically, and to enhance the problem solving skills.

**Course Learning Outcomes:** The course will enable the students to:

- i) Learn some numerical methods to find the zeroes of nonlinear functions of a single variable and solution of a system of linear equations, up to a certain given level of precision.
- ii) Know about methods to solve system of linear equations, such as False position method, Fixed point iteration method, Newton's method, Secant method and LU decomposition.
- iii) Interpolation techniques to compute the values for a tabulated function at points not in the table.
- iv) Applications of numerical differentiation and integration to convert differential equations into difference equations for numerical solutions.

**UNIT 1:** Algorithms, Convergence, Bisection method, False position method, Fixed point iteration method, Newton's method, Secant method, LU decomposition.

[1] Chapter 1 (Sections 1.1-1.2), Chapter 2 (Sections 2.1-2.5), Chapter 3 (Section 3.5, 3.8).

**UNIT 2:** Lagrange and Newton interpolation: linear and higher order, finite difference operators.

[1] Chapter 5 (Sections 5.1, 5.3) [2] Chapter 4 (Section 4.3).

**UNIT 3:** Numerical differentiation: forward difference, backward difference and central difference. Integration: trapezoidal rule, Simpson's rule, Euler's method.

[1]: Chapter 6 (Sections 6.2, 6.4), Chapter 7 (Section 7.2)

**Note:** Emphasis is to be laid on the algorithms of the above numerical methods.

**Practical / Lab work to be performed on a computer:**

Use of computer aided software (CAS), for example *Matlab / Mathematica / Maple / Maxima* etc., for developing the following Numerical programs:

- (i) Calculate the sum  $1/1 + 1/2 + 1/3 + 1/4 + \dots + 1/N$ .
- (ii) To find the absolute value of an integer.
- (iii) Enter 100 integers into an array and sort them in an ascending order.
- (iv) Any two of the following
  - (a) Bisection Method
  - (b) Newton Raphson Method
  - (c) Secant Method
  - (d) Regula Falsi Method
  - (v) LU decomposition Method
  - (vi) Gauss-Jacobi Method
  - (vii) SOR Method or Gauss-Siedel Method
  - (viii) Lagrange Interpolation or Newton Interpolation
  - (ix) Simpson's rule.

**Note:** For any of the CAS *Matlab / Mathematica / Maple / Maxima* etc., Data types-simple data types, floating data types, character data types, arithmetic operators and operator precedence, variables and constant declarations, expressions, input/output, relational operators, logical operators and logical expressions, control statements and loop statements, Arrays should be introduced to the students.

**Text Books:**

1. B. Bradie, *A Friendly Introduction to Numerical Analysis*, Pearson Education, India, 2007.
2. M. K. Jain, S. R. K. Iyengar and R. K. Jain, *Numerical Methods for Scientific and Engineering Computation*, New age International Publisher, India, 5th edition, 2007.

**Reference Book:**

1. C. F. Gerald and P. O. Wheatley, *Applied Numerical Analysis*, Pearson Education, India, 7th edition, 2008

### **MAT-HC-4036: Ring Theory**

Total Marks: 100: (Theory 80 Internal Assessment: 20)

Per week: 5 Lectures, 1 Tutorial, Credits 6, *Each unit carry equal credit*

**Course Objectives:** The objective of this course is to introduce the fundamental theory of rings and their corresponding homomorphisms. Also introduces the basic concepts of ring of polynomials and irreducibility tests for polynomials over ring of integers.

**Courses Learning Outcomes:** On completion of this course, the student will be able to:

- i) Appreciate the significance of unique factorization in rings and integral domains.
- ii) Learn about the fundamental concept of rings, integral domains and fields.
- iii) Know about ring homomorphism and isomorphism theorems of rings.
- iv) Learn about the polynomial rings over commutative rings, integral domains, Euclidean domains, and UFD

**UNIT 1:** Definition and examples of rings, properties of rings, subrings, integral domains and fields, characteristic of a ring. Ideals, ideal generated by a subset of a ring, factor rings, operations on ideals, prime and maximal ideals. Ring homomorphisms, properties of ring homomorphisms, Isomorphism theorems I, II and III, field of quotients.

[1]: Chapter 12, Chapter 13, Chapter 14, Chapter 15.

**UNIT 2:** Polynomial rings over commutative rings, division algorithm and consequences, principal ideal domains, factorization of polynomials, reducibility tests, irreducibility tests, Eisenstein criterion, unique factorization in  $\mathbb{Z}[x]$ . Divisibility in integral domains, irreducibles, primes, unique factorization domains, Euclidean domains.

[1]: Chapter 16, Chapter 17, Chapter 18.

#### **Text Books:**

1. Joseph A. Gallian, Contemporary Abstract Algebra, 4th Ed., Narosa Publishing House, New Delhi, 1999.

#### **Reference Books:**

1. John B. Fraleigh (2002), A First Course in Abstract Algebra, 7th Ed., Pearson.
2. M. Artin (2011), Abstract Algebra, 2nd Ed., Pearson.
3. D.A.R. Wallace (1998), Groups, Rings and Fields, Springer Verlag London Ltd.
4. G. Santhanam (2017), Algebra, Narosa Publishing House.

### **SKILL ENHANCEMENT COURSE**

#### **SEC-2**

#### **MAT-SE-4014: R Programming**

Total marks: 100 (Theory 60, Internal assessment 20, Practical 20)

Per week: 2 Lectures 2 Practical, Credits 4(2+2)

**Course Objectives:** The purpose of this course is to help using **R**, a powerful free software program for doing statistical computing and graphics. It can be used for exploring and plotting data, as well as performing statistical tests.

**Course Learning Outcomes:** This course will enable the students to:

- i) Become familiar with **R** syntax and to use **R** as a calculator.
- ii) Understand the concepts of objects, vectors and data types.
- iii) Know about summary commands and summary table in **R**.
- iv) Visualize distribution of data in **R** and learn about normality test.
- v) Plot various graphs and charts using **R**.



### **Unit 1: Getting Started with R - The Statistical Programming Language**

Introducing **R**, using **R** as a calculator; Explore data and relationships in **R**; Reading and getting data into **R**; combine and scan commands, viewing named objects and removing objects from **R**, Types and structures of data items with their properties, Working with history commands, Saving work in **R**; Manipulating vectors, Data frames, Matrices and lists; Viewing objects within objects, Constructing data objects and their conversions.

[1] Chapter 14 (Sections 14.1 to 14.4), [2] Chapter 2, Chapter 3

### **Unit 2: Descriptive Statistics and Tabulation**

Summary commands: Summary statistics for vectors, Data frames, Matrices and lists; Summary tables.

[2] Chapter 4

### **Unit 3: Distribution of Data**

Stem and leaf plot, Histograms, Density function and its plotting, The Shapiro-Wilk test for normality, The Kolmogorov-Smirnov test.

[2] Chapter 5

### **Unit 4: Graphical Analysis with R**

Plotting in **R**: Box-whisker plots, Scatter plots, Pairs plots, Line charts, Pie charts, Cleveland dot charts, Bar charts; Copy and save graphics to other applications.

[1] Chapter 14 (Section 14.7)[2] Chapter 7

Practical to be done in the Computer Lab using Statistical Software R:

[1] Chapter 14 (Exercises 1 to 3)

[2] Relevant exercises of Chapters 2 to 5, and 7

**Note:** The practical may be done on the database to be downloaded from <https://data.gov.in/>

### **Textbooks:**

1. Bindner, Donald & Erickson, Martin. (2011). *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*. CRC Press, Taylor & Francis Group, LLC.
2. Gardener, M. (2012). *Beginning R: The Statistical Programming Language*, Wiley Publications.

### **MAT-SE-4024: LaTeX and HTML (practical)**

Total marks: 100 (Theory 60, Internal assessment 20, Practical 20)

Per week: 2 Lectures 2 Practical, Credits 4(2+2)

**Course Objectives:** The purpose of this course is to acquaint students with the latest typesetting skills, which shall enable them to prepare high quality typesetting, beamer presentation and webpages

**Course Learning Outcomes:** After studying this course the student will be able to:

- i) Create and typeset a LaTeX document.
- ii) Typeset a mathematical document using LaTeX.
- iii) Learn about pictures and graphics in LaTeX.
- iv) Create beamer presentations.
- v) Create web page using HTML.

**Unit 1:** Elements of LaTeX; Hands-on-training of LaTeX; graphics in LaTeX; PSTricks; Beamer presentation

[1] Chapters 9,10, 11.