FRICTION

→ Contact force is having 2 components (i) Normal reaction → $\perp^{lr}$ to surface
(ii) friction → $//^{ld}$ to surface.

→ whenever an object moves or tends to move on a rough surface, the component of contact force parallel to surface, oppose the relative motion of the object w.r. to the surface in contact.

→ The frictional force oppose the tendency of contacting surfaces to slip one relative to the other.

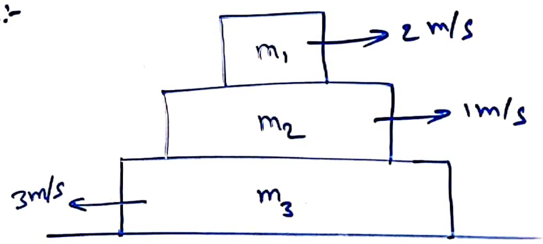
→ The force of friction comes into action only when there is a relative motion or there is a tendency of relative motion between the two contact surfaces.

→ Frictional force always oppose the relative motion or impending motion

→ Direction of frictional force is parallel to the surfaces in contact & opposite to the relative motion between two contact surfaces.

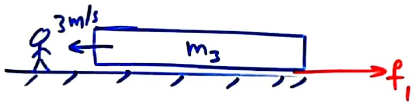
→ Friction arises due to roughness & irregularities on surfaces in contact, interlocks are developed which opposes the relative motion between surfaces.

Ex:-

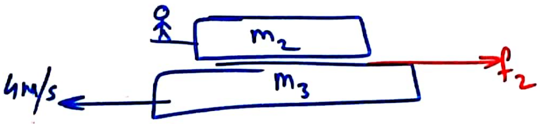


All contacts are rough. Find direction of frictional forces on each block.

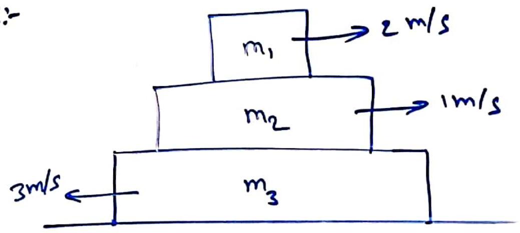
Sol:- To find direction of friction on m_3 due to ground assume a person sitting on ground & find direction of motion of block w.r.to him. Now the direction of friction is opposite to the direction of relative motion.



→ To find direction of friction on m_3 due to m_2, assume person sitting on m_2 & find direction of motion of m_3 w.r.to him. Now direction of friction is opposite to rel. motion.

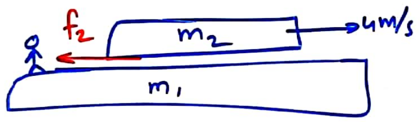


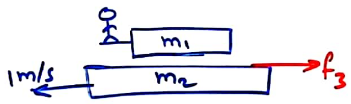
Ex:-



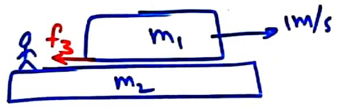
All contacts are rough. Find direction of frictional forces on each block.

Sol:- → direction of friction on m_2





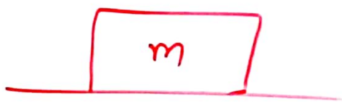
→ direction of friction on m_1



Note:- Friction always exist in nature as action- reaction pair.

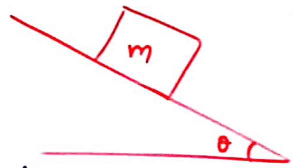
→ by polishing the surfaces moderately frictional force decreases, if surfaces are heavily polished, then friction will increase — known as cold welding.

Q) A block is resting on rough horizontal surface. Is there any frictional force acting on block?

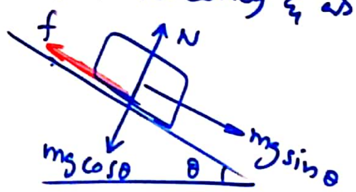


Sol:- No, because it is not having any motion tendency w.r.to surface on which it is placed.

Q) A block is resting on rough inclined surface. Is there any frictional force acting on block?



Sol:- Yes, due to $mg\sin\theta$ component it is having motion tendency & as the block is not moving, $f = mg\sin\theta$.



Types of friction :-

Friction
- Static friction
  - self-adjusting
  - Limiting
- sliding friction (or) kinetic friction
- rolling friction
- fluid friction

Static friction :-

→ It exists between the two surfaces when there is tendency of relative motion but no relative motion along the two contact surface.

→ Static friction arises when there is no relative slipping between two contact surfaces, even some external force trying to slide it.

→ It is a self-adjusting force.

→ Maximum value of static friction is known as limiting friction (f_L).

→ Static friction is independent on area of contact.

→ Static friction depends on nature of surfaces in contact.

→ Maximum value of static friction, $f_L \propto N$ → normal reaction.

$$\boxed{f_L = \mu_s N}$$

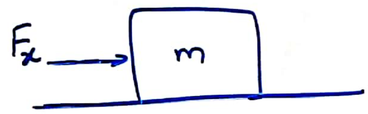
μ_s → Coefficient of static friction.

μ_s depends only on nature of the surfaces in contact,

μ_s is independent on 'N'.

→ Generally $\mu_s < 1$, but in some cases it may exceed 1.

→



If $F_x < f_L$, body doesn't slide, then $f_s = F_x$

$F_x = f_L$, body tends to slide, then $f_s = f_L$

→ Generally $f_s \leq \mu N$

Kinetic friction (f_k):—

The friction that acts when there is relative motion between two surfaces in contact is called sliding (or) Kinetic (or) dynamic friction.

→ Kinetic friction is not a self adjusting force & is slightly less than limiting friction.

→ Kinetic friction is independent on area of contact, and almost independent on velocity (velocity is not too large (or) not too small)

→ Kinetic friction depends on material surfaces in contact.

→ $f_k \propto N \Rightarrow \boxed{f_k = \mu_k N}$

N — Normal reaction

μ_k — coefficient of kinetic friction.

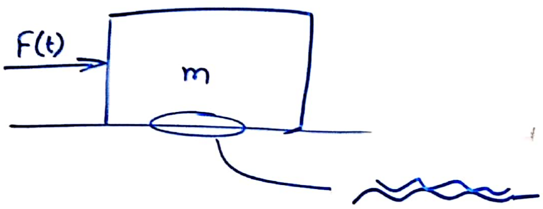
→ μ_k depends on nature of contact surfaces.

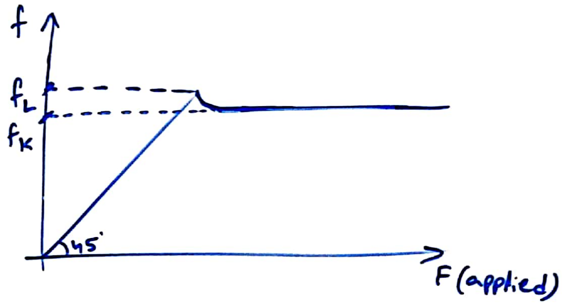
→ direction of f_k is opposite to the direction of relative motion.

→ by experimental observations, $f_k < f_L \Rightarrow \mu_k N < \mu_s N \Rightarrow \mu_k < \mu_s$

Note:- coefficient of friction doesn't have units & dimensions.

Ex:-





→ If we slowly increase the force, upto a certain critical value f_L box doesn't slide. If $F = f_L$, then body tends to slide.

→ If $F > f_L$, then box will slide, once sliding has begun, the surfaces do not have time to settle down on to each other completely. As a result less force is required to keep moving than to start the motion.

Rolling friction (f_R):-

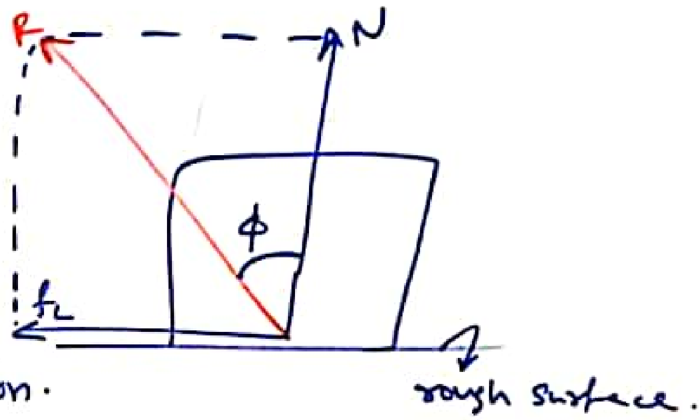
→ when a body (say a wheel or sphere) rolls on a surface, the resistance offered by surface is called rolling friction.

→ In rolling motion, contact point doesn't slip on surface.

→ Rolling friction ∝ deformation in either of contact surface, Rolling friction ∝ Area of contact.

→ It depends on area of contact. $f_R = \mu_R N$ & $\mu_S > \mu_K > \mu_R$

Angle of friction :- (ϕ)



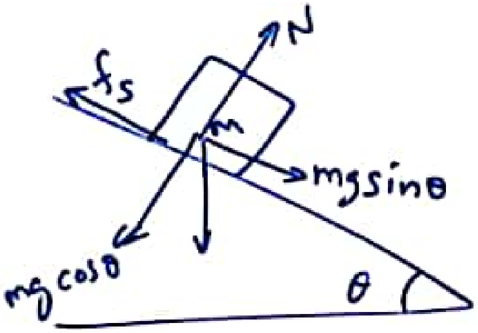
→ Angle between resultant of N & f_L and Normal reaction N is known as angle of friction.

$$\tan\phi = \frac{f_L}{N} \Rightarrow \tan\phi = \frac{\mu_s N}{N}$$

$$\Rightarrow \phi = \tan^{-1}(\mu_s)$$

Angle of repose :-

If a body placed on an inclined plane, and if its angle of inclination gradually increases, then at same angle of inclination the body will ready or tends to slide.



→ if $\theta < \alpha$, block doesn't slide

if $\theta = \alpha$, block is ready to slide

if $\theta > \alpha$, block will slide.

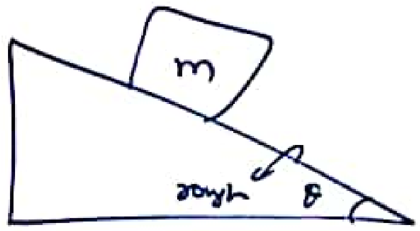
→ then α → angle of repose.

$$mg\sin\alpha = f_L$$

$$mg\sin\alpha = \mu_s N \Rightarrow mg\sin\alpha = \mu_s\, mg\cos\alpha$$

$$\tan\alpha = \mu_s \Rightarrow \boxed{\alpha = \tan^{-1}(\mu_s)}$$

→ Numerically angle of friction & angle of repose are same.

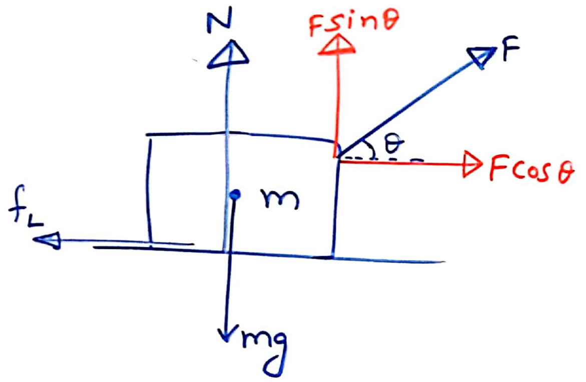


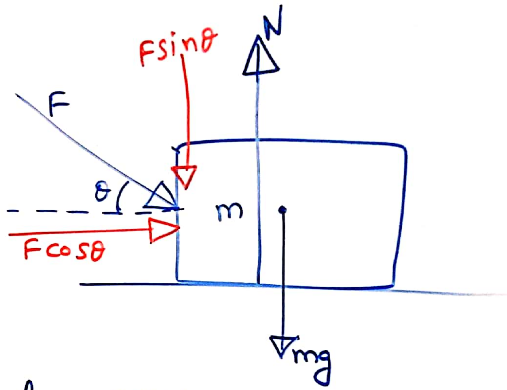
→ $\tan\theta < \mu_s$ ⟶ block doesn't slide

→ $\tan\theta = \mu_s$ ⟶ block is ready to slide

→ $\tan\theta > \mu_s$ ⟶ block will slide down on inclined plane.

→ Pulling is easier than pushing on a rough surface :-





from F.B.D of 'm'

$$N + F\sin\theta = mg$$

$$N = mg - F\sin\theta$$

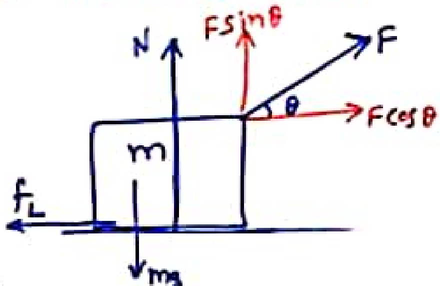
So, because of pulling force, N decreases, there by $f_L = \mu_s N$ is also decreases.

→ to slide the block, $F\cos\theta > f_L$

from F.B.D. of m, $N = mg + F\sin\theta$, so because of pushing force, N increases, there by, $f_L = \mu_s N$ increases.

→ Hence it is easier to slide the object by pulling instead of pushing.

Q) A body of mass 'm' rests on a horizontal rough surface which has a coefficient of static friction 'μ'. It is desired to make the body move by applying the minimum possible force F. Find magnitude & direction of force F, has to be applied.



Sol:- min. force required to slide the object is

$$F\cos\theta = f_L$$

$$F\cos\theta = \mu N$$

$$F\cos\theta = \mu(mg - F\sin\theta)$$

$$F\cos\theta + \mu F\sin\theta = \mu mg$$

$$F = \frac{\mu mg}{\cos\theta + \mu\sin\theta} \quad \text{—— ①}$$

→ here F is minimum, when

$\cos\theta + \mu\sin\theta$ = maximum.

by maxima condition,

$$\frac{d}{d\theta}(\cos\theta + \mu\sin\theta) = 0$$

$$\Rightarrow -\sin\theta + \mu\cos\theta = 0$$

$$\Rightarrow \mu = \tan\theta \Rightarrow \boxed{\theta = \tan^{-1}(\mu)} \quad \text{—— ②}$$

by Sub. ② in ①

$$F_{min} = \frac{\mu mg}{\frac{1}{\sqrt{\mu^2+1}} + \mu \cdot \frac{\mu}{\sqrt{\mu^2+1}}}$$

$$\boxed{F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}}$$

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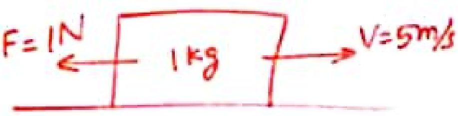
Q) Find the direction of f_k

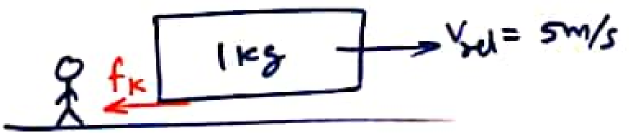
(a) on block, exerted by ground.

(b) on the ground, exerted by the block.



Sol:-

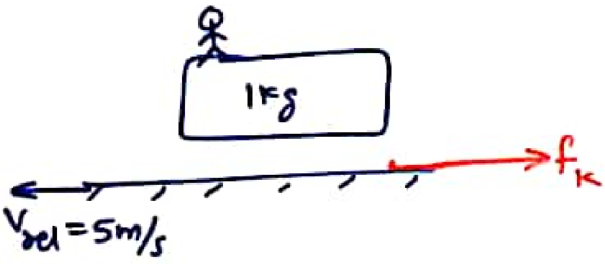
(a)



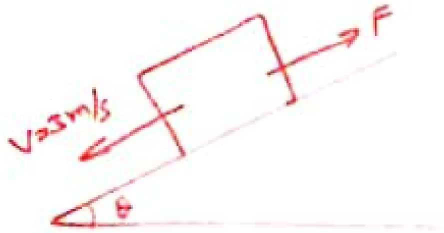
Ans:- (a) left

(b) Right.

(b)

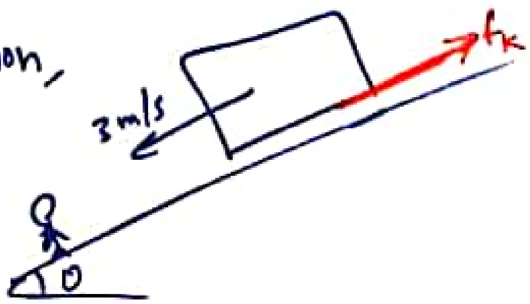


Q) In the following fig, find direction & nature of friction on block.



Sol :- → As the block is sliding w.r.t inclined plane, friction is kinetic in nature.

→ direction of f_k is opposite to the direction of relative motion,



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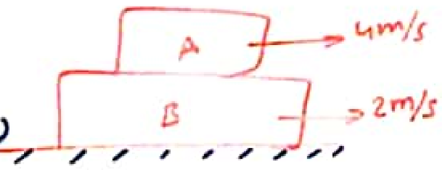
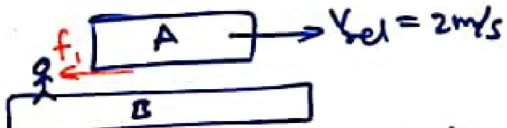
Q) All surfaces are rough. Find direction of frictional force on each block & ground at this instant.



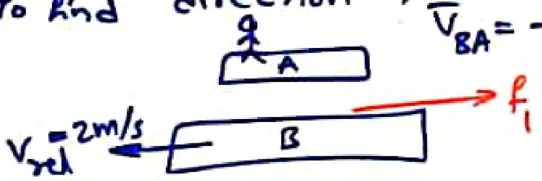
$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B = (+4)-(+2) = +2\,m/s.$

Sol :-



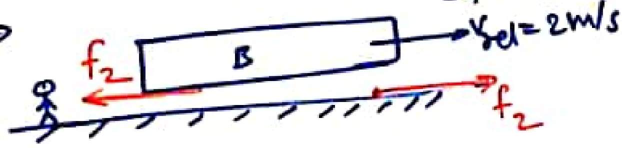
→ To find direction of friction on top surface of 'B'

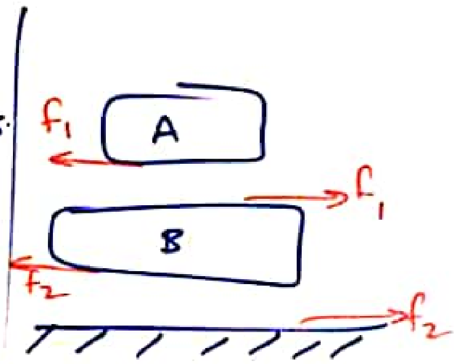
$\vec{V}_{BA} = -\vec{V}_{AB} = -2\,m/s$



$\vec{V}_{BG} = \vec{V}_B - \vec{V}_G = (+2) - 0 = +2\,m/s.$

→



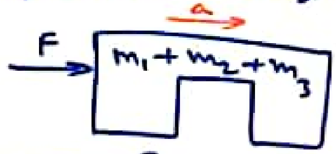


→ At contact, friction exists as action-reaction pair

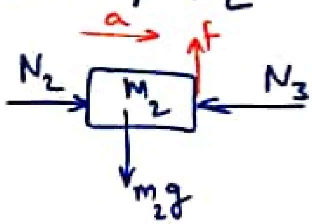
Q) only between m_2 & m_3 surfaces are rough. All other contact surfaces are smooth. coefficient of friction between m_2 & m_3 is μ. Find the minimum value of 'F', so that block m_2 doesn't slip relative to m_1 & m_3.

Sol:- → As there is no slipping between three blocks all blocks will move together as a single block.

$$\therefore \quad a = \frac{F}{m_1+m_2+m_3} \quad \text{——— (1)}$$



→ F.B.D of m_2

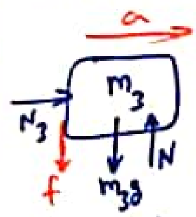


As block m_2 doesn't slipping down,

$$m_2 g \le f_L$$

$$\Rightarrow m_2 g \le \mu N_3 \quad \text{——— (2)}$$

→ F.B.D of m_3



$$N_3 = m_3 a \quad \text{——— (3)}$$

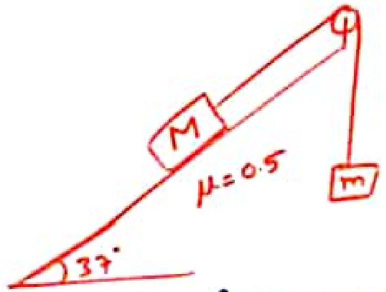
by sub. (3) & (1) in (2)

$$m_2 g \le \mu m_3 \frac{F}{m_1+m_2+m_3}$$

$$F \ge \frac{m_2 g}{\mu m_3}(m_1+m_2+m_3)$$

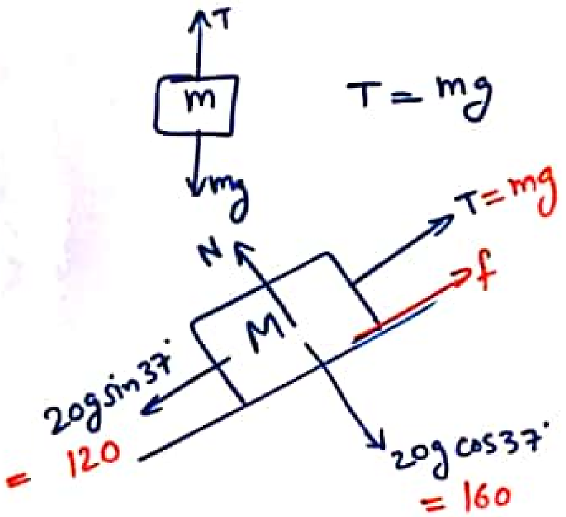
$$\therefore \boxed{F_{min} = \frac{m_2 g\,(m_1+m_2+m_3)}{\mu m_3}}$$

Q) In the arrangement shown, if $M = 20\,kg$, then determine the minimum & maximum values of block m to keep the heavy block stationary.



Sol:-

→ To find min. value of 'm'



$T = mg$

As block is not sliding

$120 - mg \le f_L$

$120 - mg \le \mu N$

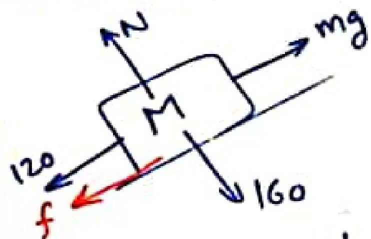
$120 - mg \le \frac{1}{2} \times 160$

$mg \ge 120 - 80$

$m \ge 4\,kg.$

$\boxed{m_{min} = 4\,kg}$

→ To find max. value of 'm'



As block is not sliding,

$mg - 120 \le f_L$

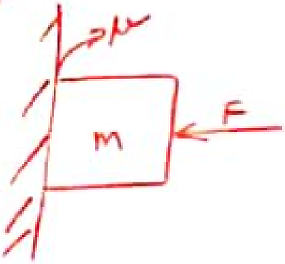
$mg - 120 \le \mu N$

$mg - 120 \le 80$

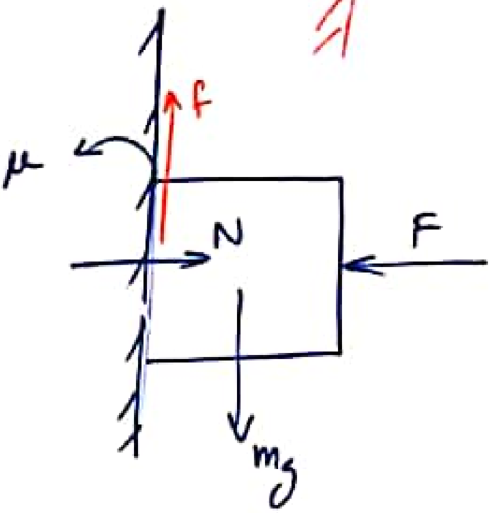
$mg \le 200 \Rightarrow m \le 20\,kg$

$\boxed{m_{max} = 20\,kg}$

Q) what should be the min. value of 'F' so that block is stationary?



Sol:-



$N = F$

As block is stationary
driving force $\leq f_L$

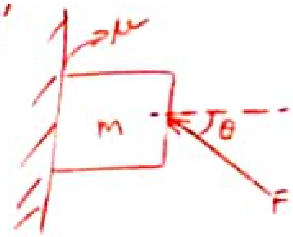
$mg \leq \mu N$

$mg \leq \mu F$

$F \geq \frac{mg}{\mu}$.

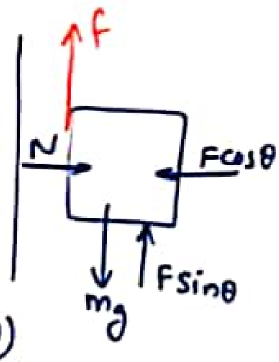
$\boxed{F_{min} = \frac{mg}{\mu}}$.

Q) what should be max & min values of 'F' so that block is stationary?



Sol:- if $mg > F\sin\theta$

As block is stationary



driving force $\leq f_L$

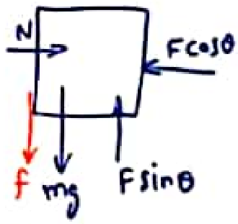
$\Rightarrow \quad mg - F\sin\theta \leq \mu(F\cos\theta)$

$$\Rightarrow \quad F \geq \frac{mg}{\sin\theta + \mu\cos\theta}$$

$$F_{min} = \frac{mg}{\sin\theta + \mu\cos\theta}$$

if $mg < F\sin\theta$

As block is stationary



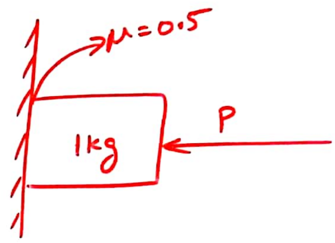
driving force $\leq f_L$

$\Rightarrow \quad F\sin\theta - mg \leq \mu F\cos\theta$

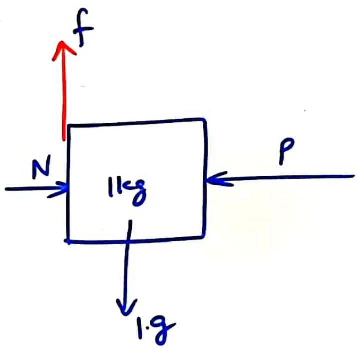
$$\Rightarrow \quad F \leq \frac{mg}{\sin\theta - \mu\cos\theta}$$

$$\Rightarrow \quad F_{max} = \frac{mg}{\sin\theta - \mu\cos\theta}$$

Q) Minimum force required to keep a block of mass 1 kg at rest against a rough vertical wall is 'P'. If a force $\frac{P}{2}$ is applied then find acceleration of block.



Sol:-



As the block is stationary,

driving force $\leq f_L$

$$1 \cdot g \leq \mu N$$

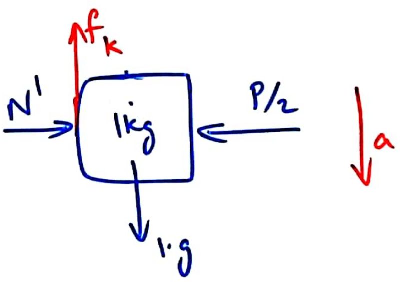
$$g \leq \mu P$$

$$P \geq \frac{g}{\mu}$$

As the applied force is minimum,

$$\therefore \quad P = \frac{g}{\mu}$$

→ Now applied force reduced to half, block slides down.

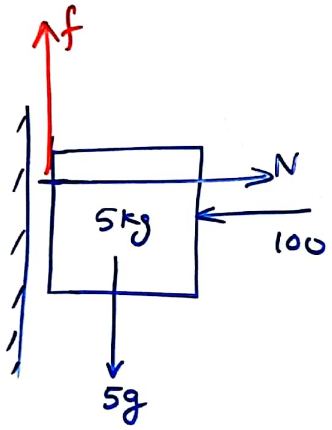


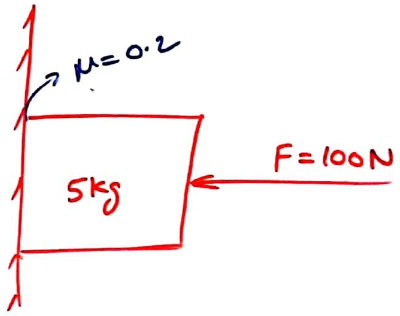
$$a = \frac{1 \cdot g - f_K}{m} = \frac{g - \mu N'}{m} = \frac{g - \mu \cdot P/2}{m}$$

$$a = \frac{g - g/2}{m} = \frac{g}{2m} = \frac{10}{2 \times 1} = 5 \, m/s^2$$

Q) Determine the magnitude of frictional force on block & also find the acceleration of block. ($\mu = 0.2$)

Sol:-





driving force $= 5g = 50$

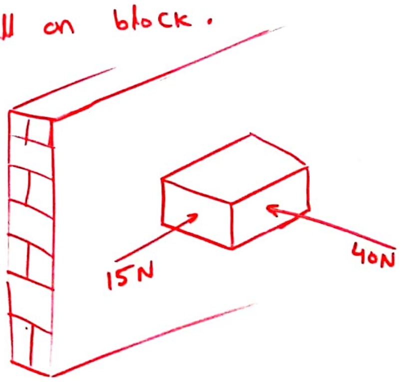
$f_L = \mu N = 0.2 \times 100 = 20.$

As driving force $> f_L$, block will slide down.

$\therefore \; f_k = \mu N = 20$

$$a = \frac{50 - f_k}{m} = \frac{30}{5} = 6 \, m/s^2.$$

Q) A block of mass 2 kg is pushed normally against a rough vertical wall with a force of 40 N. & $\mu_s = 0.5$. Another horizontal force of 15 N is applied on the block in a direction parallel to the wall. Will the block move? If yes, in which direction & with what acceleration? If no, find the frictional force exerted by wall on block.



Sol:-

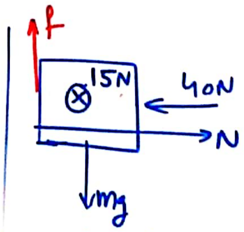
→ driving force = Resultant of weight & horizontal force applied

$$= \sqrt{(2g)^2 + (15)^2} = 25$$

→ If driving force $\leq f_L$, then block doesn't slip

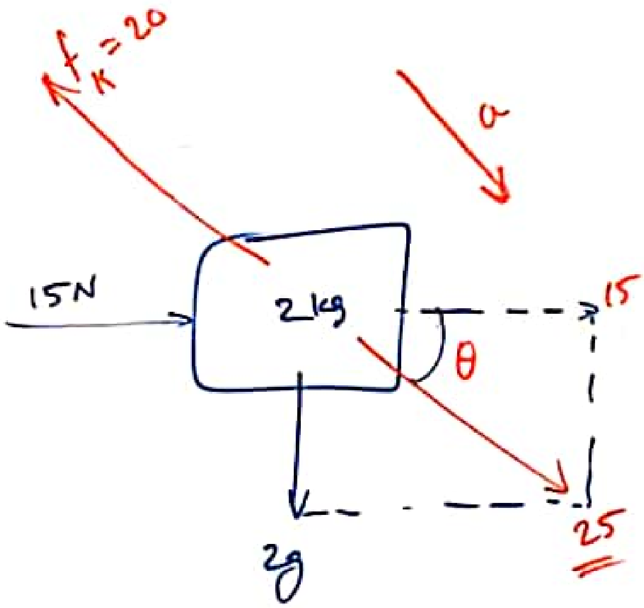
$$f_L = \mu_s N$$

$$= 0.5 \times 40 = 20$$



∴ As driving force $> f_L$, block will slip

$$\therefore\ a = \frac{\text{driving force} - f_k}{m} = \frac{25-20}{2} = 2.5\ m/s^2$$



Front view

$$a = \frac{25-20}{2}$$

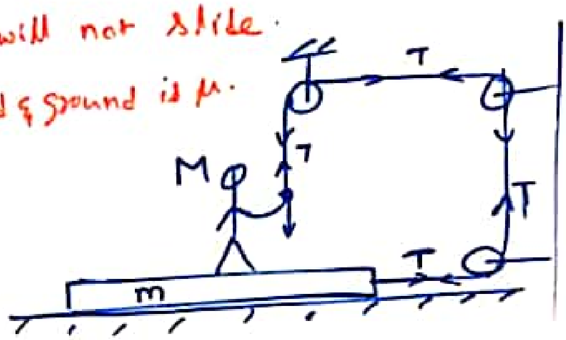
$$= 2.5\ m/s^2$$

$$\tan\theta = \frac{20}{15}$$

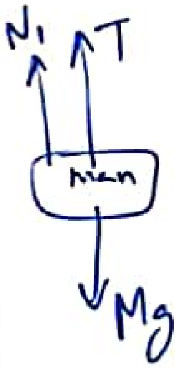
$$\tan\theta = \frac{4}{3}$$

$\theta = 53^\circ$ with horizontal

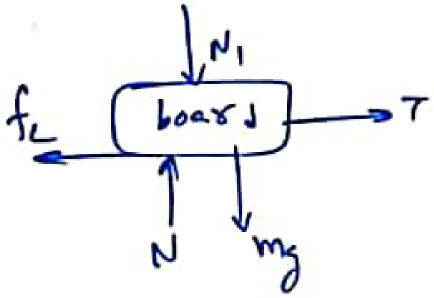
Q) Find the max. force applied by man on rope, so that board will not slide. friction coeff. bet. board & ground is μ.



Sol:-



$N_1 + T = Mg$ — ①



$N_1 + mg = N$ — ②

$T = f_L$ — ③

⇒ ③

$T = \mu_s N$

$T = \mu (N_1 + mg)$ from ②

$T = \mu (Mg - T + mg)$ from ①

$$\boxed{T = \frac{\mu g (M+m)}{1+\mu}}$$

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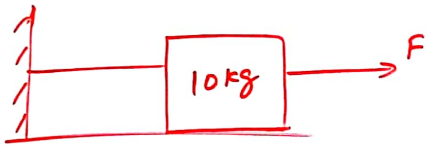
Q) calculate the tension in string connecting to the wall & block if

(i) $F = 20N$

(ii) $F = 60\ N$



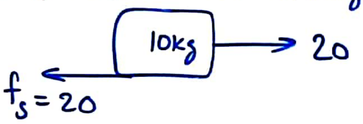
$\mu_s = 0.3,\ \mu_k = 0.25$

Sol:- here F is driving force.

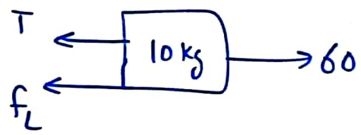
→ if driving force $> f_L$, then block will slip & then tension will develop in string

$$f_L = \mu_s N = 0.3 \times 10 \times g = 30$$

(i) here driving force $< f_L$, so block doesn't slip, friction is static in nature & tension in string is zero.



(ii) here driving force $> f_L$, then tension develops in string



$T + f_L = 60 \Rightarrow T = 60 - 30 = 30N$

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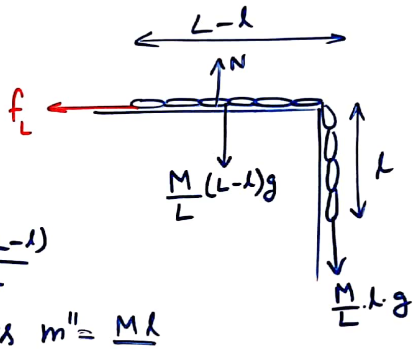
Q) A uniform chain of length L, hangs partially from a rough horizontal table. The chain is kept in equilibrium by friction. If the maximum length that can hang without slipping is 'l'. Find coefficient friction between table & chain.



Sol:- Let mass of chain of length L is 'm'

$L \longrightarrow m$

$L-l \longrightarrow m'$ $\quad m' = \frac{m(L-l)}{L}$

||ly mass of chain of length l is $m'' = \frac{Ml}{L}$

→ As max. length of chain is hanging, driving force $= f_L$, so that chain is in equilibrium.

$$\therefore \frac{M}{L}.l.g = \mu_s N$$

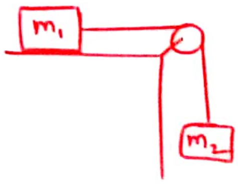
$$\frac{M}{L}.l.g = \mu \frac{M}{L}(L-l)g$$

$$\Rightarrow l = \mu(L-l) \Rightarrow \boxed{\mu = \frac{l}{L-l}} = \frac{\text{length of the hanging part}}{\text{remaining length on table}}$$

Note :- A uniform rope is kept on horizontal rough table such that $\left(\frac{1}{n}\right)^{th}$ part of its length is hanging over the edge of table. If the rope is in limiting equilibrium, then

$$\mu = \frac{\text{length of hanging part}}{\text{remaining length on table}} \Rightarrow \mu = \frac{L/n}{L - L/n} \Rightarrow \boxed{\mu = \frac{1}{n-1}}$$

Q) In the fig. shown, if m_1 is in limiting equilibrium, then find coeff. of friction between block m_1 & table.

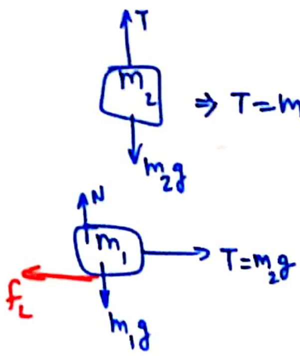


Sol :- if an object is in limiting equilibrium,
driving force = limiting friction

$\Rightarrow$ $m_2 g = \mu_s N$

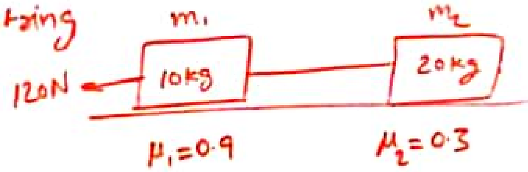
$m_2 g = \mu m_1 g$

$\Rightarrow \mu = \frac{m_2}{m_1}$



$\Rightarrow T = m_2 g.$

Q) In this situation, find the tension in string

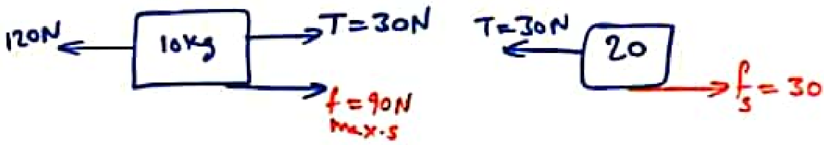


Sol:- $f_{L_1} = \mu_{s_1} N_1 = 0.9 \times 10 \times 10 = 90\,N$

$f_{L_2} = \mu_{s_2} N_2 = 0.3 \times 20 \times 10 = 60\,N$

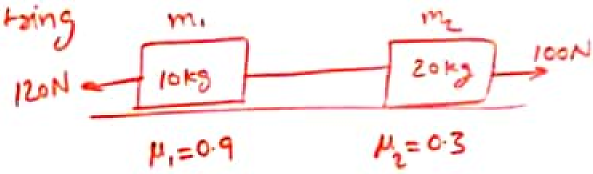
→ if driving force on system (i.e 120 N) $> f_{L_1} + f_{L_2}$, then only both blocks will move.

→ As driving force $< f_{L_1} + f_{L_2}$, no block will slip on ground & friction is static.



∴ Tension in string = 30 N

Q) In this situation, find the tension in string



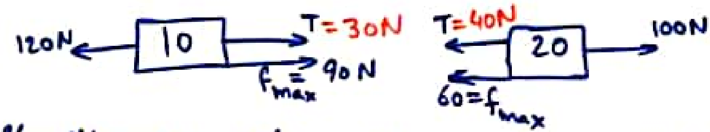
Sol:- $f_{L_1} = \mu_{s_1} N_1 = 0.9 \times 10 \times 10 = 90\,N$

$f_{L_2} = \mu_{s_2} N_2 = 0.3 \times 20 \times 10 = 60\,N$

→ if driving force on system (i.e $120 - 100 = 20N$) $> f_{L_1} + f_{L_2}$, then only both blocks will move.

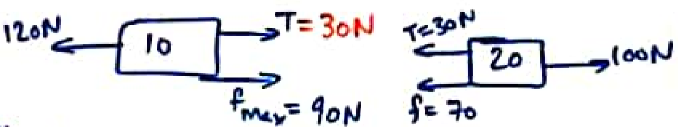
→ As driving force $< f_{L_1} + f_{L_2}$, no block will slip on ground & friction is static.

Assumption-1 :- on both blocks, static friction is maximum.



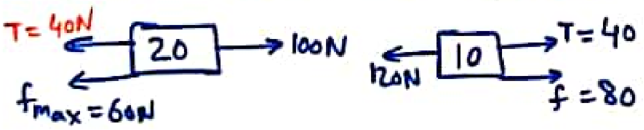
As we got, different T values from F.B.D, this assumption is wrong. ✗

Assumption-2 :- only on block m_1, friction is limiting



As we got, on 20kg block $f_{static} > f_L$, this assumption is wrong ✗

Assumption-3 :- only on block m_2, friction is limiting.



As we got, on 10kg block, $f < f_L$, this assumption is correct. ✓

∴ Tension in string = 40N

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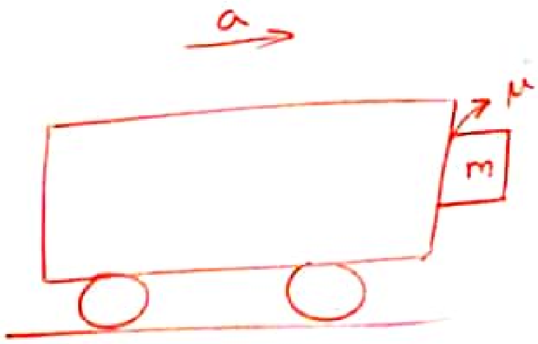
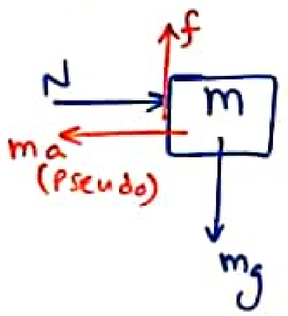
Q) What will be the minimum acceleration of bus, so that block 'm' doesn't slip down?



Sol:- According to the frame of bus block is stationary.

→ F.B.D of 'm' w.r.to bus



$N = ma$ —— ①

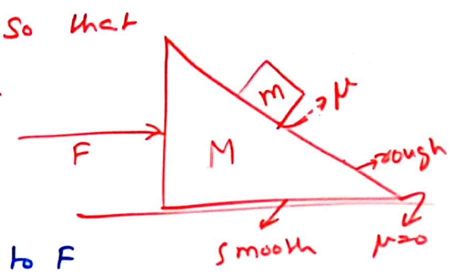
$mg \le f_L$ —— ②

② ⇒ $mg \le \mu_s N$

$mg \le \mu\, ma$

$a \ge \dfrac{g}{\mu}$ ⇒ $\boxed{a_{min} = \dfrac{g}{\mu}}$

Q) Find the min & max. value of 'F' so that block is stationary w.r.t wedge.

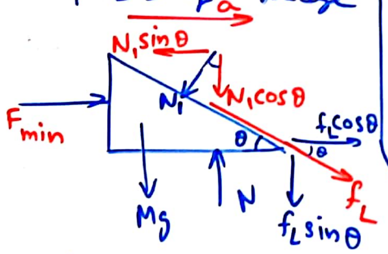


Sol:- Let wedge is accelerating with 'a' due to F

Acc. to wedge block is stationary. So, $F=(M+m)a$.

→ To find F_{min},

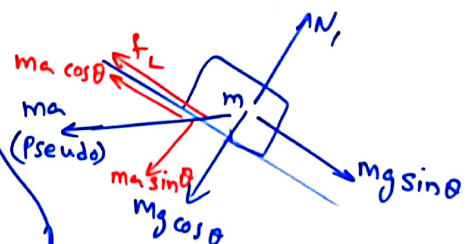
F.B.D of a wedge



$$F_{min} + f_L \cos\theta - N_1 \sin\theta = Ma$$

$$F_{min} + \mu N_1 \cos\theta - N_1 \sin\theta = Ma \quad (1)$$

F.B.D of block w.r.to wedge



$$N_1 = mg\cos\theta + ma\sin\theta \quad (2)$$

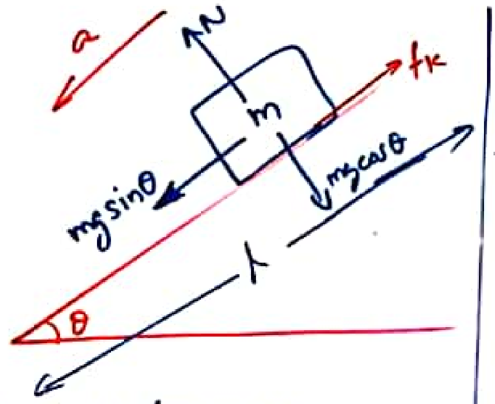
$$f_L + ma\cos\theta = mg\sin\theta$$

$$\mu N_1 + ma\cos\theta = mg\sin\theta \quad (3)$$

by solving (1), (2), (3)

we get $F_{min} =$ ______

Motion of a block on sliding down on rough inclined plane:-



$f_L = \mu_s N = \mu_s (mg\cos\theta)$.

→ If $mg\sin\theta > f_L$, block will slide & friction is kinetic in nature.

$$a = \frac{mg\sin\theta - f_K}{m}$$

$$\boxed{a = g\sin\theta - \mu_K g\cos\theta}$$

→ for smooth incline, $\mu = 0$

$s = ut + \frac{1}{2}at^2$

→ assume initial velocity of block as zero.

$$l = 0 + \frac{1}{2}(g\sin\theta - \mu_K g\cos\theta)t^2$$

$$\boxed{t = \sqrt{\frac{2l}{g(\sin\theta - \mu_K\cos\theta)}}}$$

l → length of incline

t → time taken by block to reach bottom of incline.

→ velocity with which block reaches to bottom is $v^2 - u^2 = 2as$

$$\boxed{v = \sqrt{2g(\sin\theta - \mu_K\cos\theta)\cdot l}}$$

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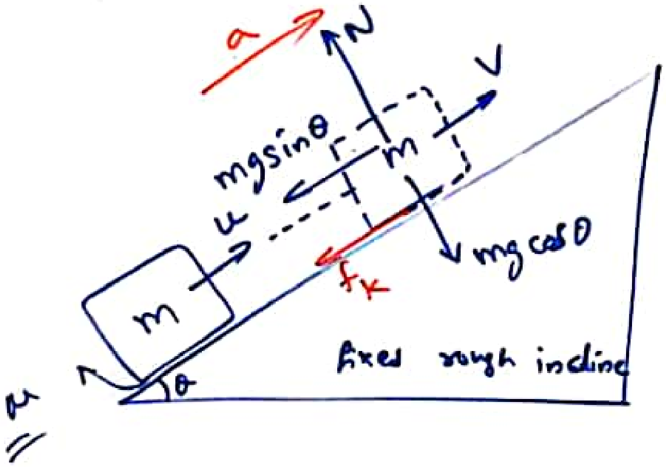
Motion of a block sliding up on fixed rough incline :-



When block is sliding up the rough incline, it moves with constant deceleration.

$$a = \frac{-(mg\sin\theta + f_k)}{m}$$

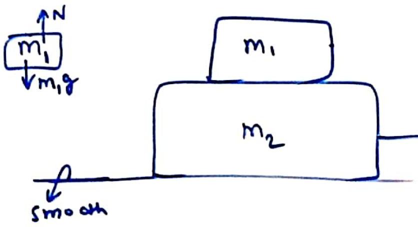
$$a = -(g\sin\theta + \mu_k g\cos\theta)$$

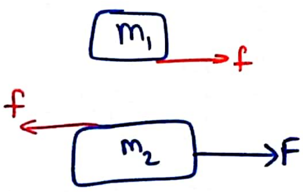
$$\boxed{a = -g(\sin\theta + \mu_k\cos\theta)}$$

Block - on - block concept :-

Case (i):-

→ Consider two blocks m_1 & m_2 kept one above other as shown & between the blocks friction coefficient μ. Ground is smooth.



- maximum static friction between blocks is
  $$f_L = \mu N = \mu m_1 g$$
- If we apply force on bottom block, upper block will also move due to friction.
- 

- maximum acceleration of upper block is
  $$a_{1_{max}} = \frac{f_{max}}{m_1} = \frac{\mu m_1 g}{m_1} = \mu g$$
- As ground is smooth, whatever may be the value of F (>0), both blocks will move together (or) seperately based on value of driving force F.
- Maximum value of F for which both blocks move together is
  $$F_{max} = (m_1+m_2)\, a_{1_{max}}$$
  $$\boxed{F_{max} = (m_1+m_2)\mu g}$$
- As $F \le (m_1+m_2)\mu g$, both blocks will move together & friction between blocks is static in nature.
- As $F > (m_1+m_2)\mu g$, both blocks will move seperately & friction between blocks is kinetic in nature.

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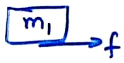
- If both blocks are moving together,

$$a_1 = a_2 = \frac{F}{m_1 + m_2}.$$

& friction between the blocks is

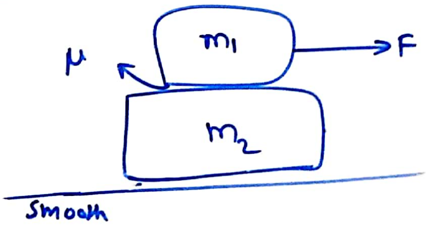
$$f = m_1 a_1 = \frac{m_1 F}{m_1+m_2}$$



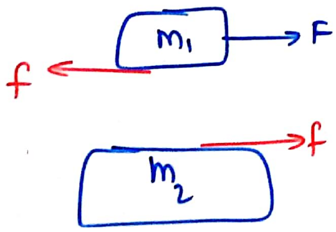
- If both blocks are moving seperately,

$$a_1 = \frac{f_k}{m_1} = \mu_k g \text{ \& } a_2 = \frac{F - f_k}{m_2} = \frac{F - \mu_k m_1 g}{m_2}.$$

Case (ii) :-



- Lower block moves due to friction acts on it.



- Maximum acceleration of lower block is

$$a_{2_{max}} = \frac{f_{max}}{m_2} = \frac{\mu m_1 g}{m_2}$$

- As ground is smooth, whatever may be the value of F (>0), both blocks will move together (or) seperately based on value of driving force F.

- Maximum value of F for which both blocks move together is

$$F_{max} = (m_1+m_2)\, a_{2_{max}}$$

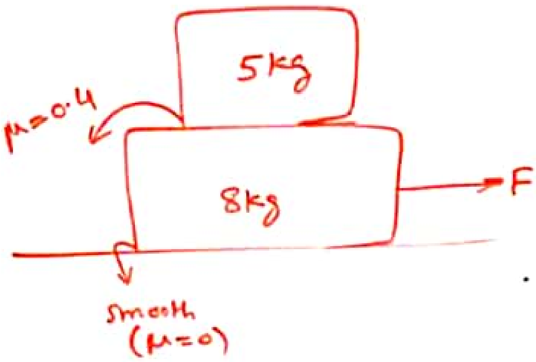
$$\boxed{F_{max} = (m_1+m_2)\frac{\mu m_1 g}{m_2}}$$

- If both blocks move together,

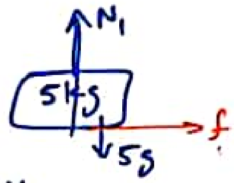
$a_1 = a_2 = \frac{F}{m_1+m_2}$ & friction between blocks is

$$f = m_2 a_2 = \frac{m_2 F}{m_1+m_2}.$$

Q)



Find max. value of F, so that both blocks move together.



Sol :-

→ between the blocks, $f_L = \mu N_1$

$$= 0.4 \times 5g = 20\,N$$

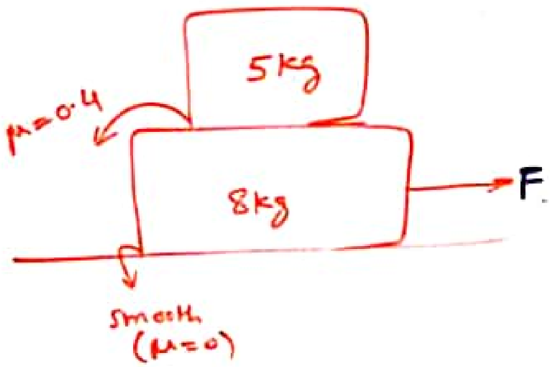
→ Max. acceleration of upper block is

$$a_{5_{max}} = \frac{f_L}{m_{upper}} = \frac{20}{5} = 4\ m/s^2.$$

→ Max. value of F, so that both blocks move together is

$$F_{max} = (5+8)\,4 = 52\,N$$

Q)



→ Find friction between the blocks & acceleration of each block.

(i) if $F = 39$

(ii) if $F = 60N$

Sol :- (i) assume both blocks move together,

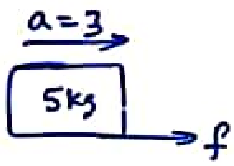
then $a = \frac{F}{m_1+m_2} = \frac{39}{13} = 3\ m/s^2$.

bet blocks

$f_L = \mu N_1$

$= 0.4 \times 5g$

$= 20\ N$

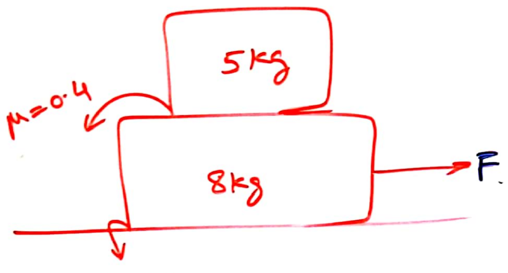


$f = 5(a)$

$= 5(3) = 15N$

As $f < f_L$, our assumption is correct.

Q)



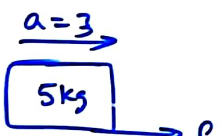
→ Find friction between the blocks & acceleration of each block.

(i) if $F = 39$

(ii) if $F = 65$ N

Sol :- (i) assume both blocks move together,

then $a = \dfrac{F}{m_1 + m_2} = \dfrac{65}{13} = 5 \ m/s^2$.



$f = 5(a)$

$= 5(5) = 25$ N

bet blocks

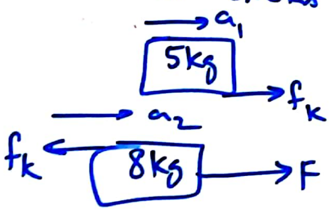
$f_L = \mu N_1$

$= 0.4 \times 5g$

$= 20$ N

As $f > f_L$, our assumption is wrong

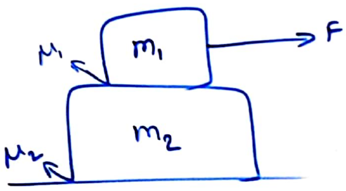
So, both blocks move seperately. & friction is kinetic, $f_k = \mu N = 20$N



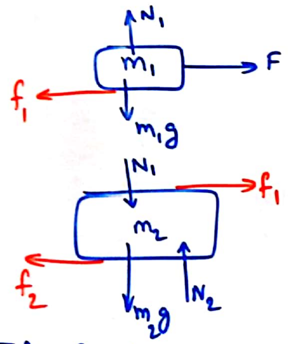
$a_1 = \dfrac{f_k}{5} = \dfrac{\mu N_1}{5} = \dfrac{20}{5} = 4 \ m/s^2$.

$a_2 = \dfrac{F - f_k}{8} = \dfrac{65 - 20}{8} = \dfrac{45}{8} \ m/s^2$

Case (iii) :-



→ here lower block moves (or) tends to move due to friction between m_1 & m_2.



$$f_{1max} = \mu_1 N_1 = \mu_1 m_1 g$$

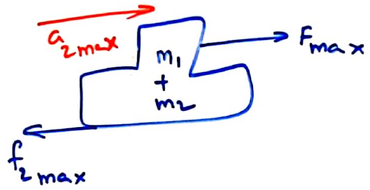
$$f_{2max} = \mu_2 N_2 = \mu_2 (m_1 + m_2) g$$

→ If $f_1 > f_{2max}$, then only m_2 will move.

→ Maximum acceleration of lower block is

$$a_{2max} = \frac{f_{1max} - f_{2max}}{m_2} = \frac{\mu_1 m_1 g - \mu_2 (m_1 + m_2) g}{m_2}$$

The maximum value of 'F', so that both blocks move together is



$$F_{max} - f_{2max} = (m_1 + m_2)\, a_{2max}$$

$$F_{max} = (m_1 + m_2)\left(\frac{\mu_1 m_1 g - \mu_2 (m_1 + m_2) g}{m_2}\right) + \mu_2 (m_1 + m_2) g.$$

→ If $F < F_{max}$, both blocks move together, then friction between blocks is static in nature.

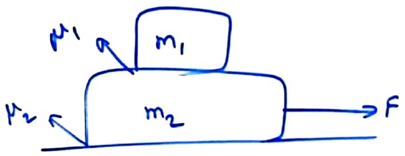
& $a = \dfrac{F - f_{2max}}{m_1 + m_2}$ & $f_1 < f_{1max}$

→ If $F > F_{max}$, both blocks move seperately, then friction between blocks is kinetic in nature.

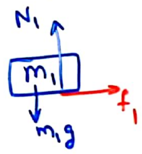
→ If $F < f_{1max}$ & $F < f_{2max}$ then no block will slip

→ If $F < f_{1max}$ & $F > f_{2max}$, then both blocks move together.

Case (iv) :-

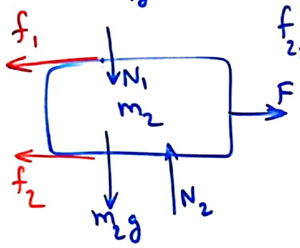


→ here if lower block moves, then only only upper block will move.



$$f_{1max} = \mu_1 N_1 = \mu_1 m_1 g.$$

$$f_{2max} = \mu_2 N_2 = \mu_2 (m_1+m_2) g.$$



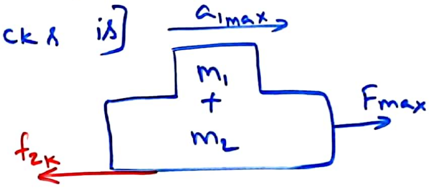
→ If $F > f_{2max}$, then only both blocks move together (or) seperately based on the value of F. here $f_1 \neq 0$

→ If $F \leq f_{2max}$, then no block will move. & $f_1 = 0$

→ maximum acceleration of upper block is

$$a_{1max} = \frac{f_{1max}}{m_1} = \mu_1 g$$

→ The maximum value of F, so that both blocks move together (or) [the minimum value of F, so that slipping takes place between the blocks is]



$$F_{max} - f_{2k} = (m_1+m_2) a_{1max}$$

$$F_{max} = (m_1+m_2)\mu_1 g + \mu_2 (m_1+m_2) g$$

$$\boxed{F_{max} = (\mu_1+\mu_2)(m_1+m_2) g}$$

→ If $f_{2max} < F \leq F_{max}$, then both blocks move together, then friction bet. blocks is static & friction bet. ground & bottom block is kinetic

$$a = \frac{F - f_{2k}}{m_1+m_2}, \quad f_1 = m_1 a, \quad f_2 = \mu_2 (m_1+m_2) g.$$

→ If $F > F_{max}$, then both block move seperately, then friction at each contact is kinetic

$$a_1 = \frac{f_{1k}}{m_1} = \mu_1 g, \quad a_2 = \frac{F - (f_{1k} + f_{2k})}{m_2} \quad \& \quad f_1 = f_{1k}, \; f_2 = f_{2k}$$