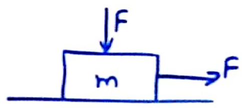
Laws of Motion

→ Kinematics deals with motion of an object with out considering cause of motion.

→ Dynamics deals with motion of an object by considering the cause of motion - Force.

→ Kinematic Variables :- $\bar{s}, \bar{u}, \bar{v}, \bar{a}, t$

→ Dynamic Variables :- $\bar{s}, \bar{u}, \bar{v}, \bar{a}, t, \bar{F}$ & m.



FORCE ($\bar{F}$) :-

A pull or push which changes or tends to change the state of rest (or) of uniform motion (or) direction of motion of any object is called force

→ It is a vector quantity.

$\boxed{1N = 10^5 \text{ dyne}}$

→ S.I unit – Newton ; C.G.S unit – dyne

$\bar{F}$ → magnitude, → direction, → Point of application of force (Point where F is applied)

The direction & Point of application of force both decide line of action of force.

→ Magnitude & direction of force decide effect on translation motion.

→ Magnitude & line of action of force decide effect on rotation.

Effects of Force :-

→ It produces or tries to produce motion in a body at rest

→ It stops or tries to stop a moving body.

→ It changes the direction of motion & shape of body

→ All forces in nature are classified into four fundamental forces.

1) Electro magnetic forces → due to charge on bodies
2) Gravitational forces → due to mass of objects

} long range forces.

3) Strong forces → between nucleons
4) weak forces → between nucleus & electrons

} Short range/Nuclear force (always attractive)

→ Among the basic forces <u>Gravitational force is weakest</u> force.

$$F_G < F_{weak} < F_{EM} < F_{strong}$$

$$F_{weak} \Downarrow 10^{25} F_G, \quad F_{EM} \Downarrow 10^{36} F_G, \quad F_{strong} \Downarrow 10^{38} F_G$$

<u>Classification of forces</u>:-

<u>Force</u>

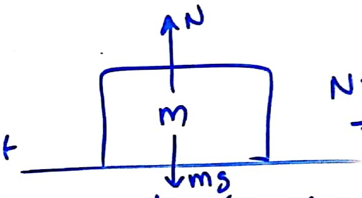
- System wise
  - External
  - Internal
- Source (origin)
  - Field (Non-contact) → (Gravitational, magnetic, electric)
  - Contact → (Tension, friction, Normal reaction, spring force)
- Reference frame
  - Real
  - Pseudo
- Work done
  - Conservative
  - Non-conservative.

$\underbrace{\text{(Tension, friction, Normal reaction, spring force)}}_{\text{Electromagnetic in nature.}}$

Free Body Diagram (FBD):- Pictorial representation of all forces acting on a body.

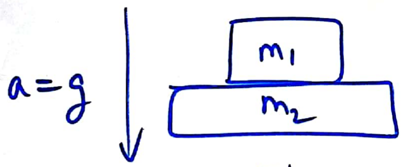
contact forces:- The force exerted by one surface over the surface of another body when they are physically in contact with each other.

(i) **Normal reaction** :- The component of contact force which is $\perp^{lr}$ to contact surface.



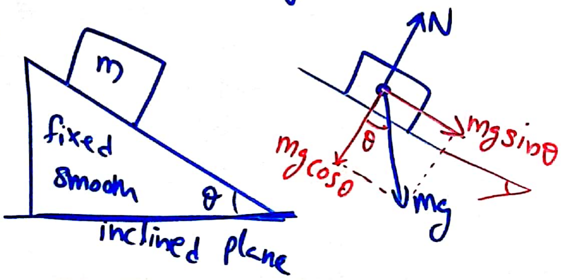
$\underline{N = mg}$

→ whenever two surfaces in contact & press with each other, then only Normal reaction arises.

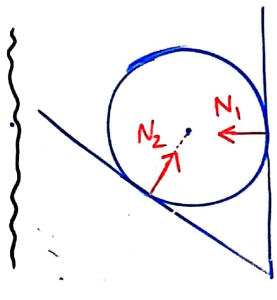


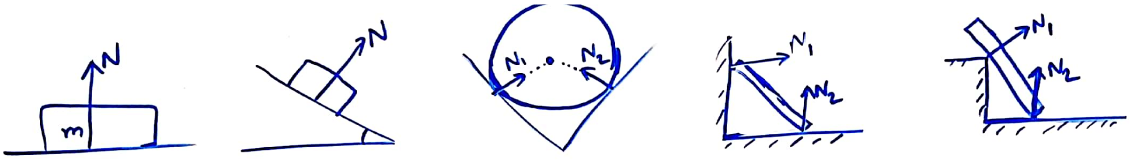
bet. m_1 & m_2 normal reaction $\underline{N = 0}$

→



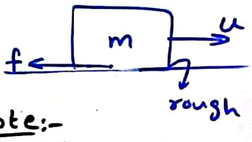
$N = mg\cos\theta$





Friction (f):-

The component of contact force tangential to the contact surface, which tries to oppose the relative motion of object.

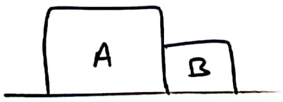


Note:-

→ Contact force, $R = \sqrt{N^2 + f^2}$

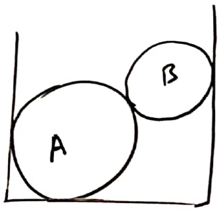
→ on smooth surface contact force is nothing but Normal reaction.

Q) Two blocks are kept on smooth surface as shown in fig. Draw normal force exerted by A on B.

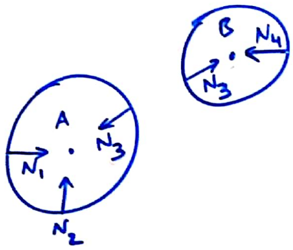


Sol:- Normal force exerted by A on B is zero, since normal reaction comes into role when one surface presses the other.

Q) Draw normal forces on both balls.



Sol:-



Tension (T):-

It is an internal molecular force which appear in a string when it is taut (or) stretch by some external force.

→ Tension has always pulling tendency.

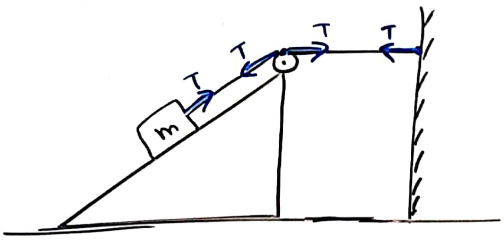
→ direction of tension is always act along the length of string.

→ direction of tension is always away from tied ends (or) points of contact of string.

→ In an ideal string, at any point on string tension remains constant. (mass less & friction less)

→ If string is not ideal, then tension varies at different points on string.

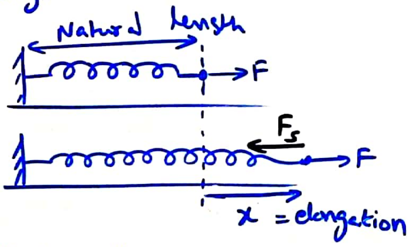
→ In these diagrams string is ideal.

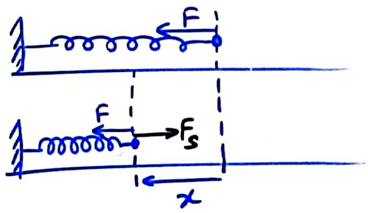


Spring force (F_s) :-

Whenever spring is compressed (or) extended, the elastic force developed in spring which helps the spring to restore its original position is known as spring force.

→ A spring has tendency to apply to restoring force opposite to the deformation of spring.





So, restoring force F_s is opposite to direction of deformation & proportional to deformation length

$\therefore \bar{F}_s \propto -\bar{x} \Rightarrow \boxed{\bar{F}_s = -K\bar{x}}$

$\Rightarrow \boxed{F_s = Kx}$, where x is elongation (or) compression in spring.

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→ If vector sum of all forces is zero, then those forces are said to be balanced forces, if not unbalanced forces.

Newton's laws of motion :-

Newton gave three laws useful to study the motion of bodies.

Newton's first law :-

According to this law, a body continues to be in its state of rest (or) uniform motion, unless it is acted up on by some unbalanced external force.

This means a body, on its own cannot change its state of rest (or) uniform motion (or) direction. This inability of a body to change its state by itself is called inertia of a body. This law explains about inertia, So this law is also known as law of inertia.

→ First law gives qualitative definition of force.

→ inertia $\propto$ mass of body

→ inertia always tries to oppose sudden change in state of body.

Types of inertia :-

a) **inertia of rest** :- Inability of a body to change its state of rest by itself.

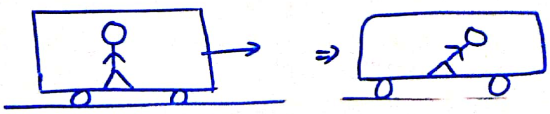
Ex:- (1) If a person standing in stationary bus & bus started moving suddenly then person feels backward jerk due to inertia of rest.

(2) The dust particles on a carpet fall off when it is beaten with a stick.

(3) only the carrom coin at the bottom of a pile is removed when a fast moving striker hits it.

b) **inertia of motion** :- Inability of a body to change its state of uniform motion by itself.

Ex:- (1) If a person standing in a moving bus & bus is stopped suddenly, then person feels forward jerk due to inertia of motion.

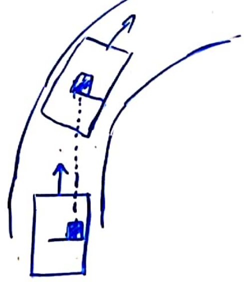


(2) An athelete runs a certain distance before taking long jump.

(3) A person trying to get down from running bus he falls forward.

(c) inertia of direction :- Inability of a body to change its direction of motion by itself.

Ex:- (1) If a person sitting in bus feel outward jerk, when bus takes turn suddenly.



(2) when a knife is sharpened by pressing it on a grinding stone, the sparks fly off tangentially to grinding stone.

→ Newton first law, explains about inertia & gives qualitative definition of force i.e, if $\bar{a}=0$ then $\bar{F}=0$, if $\bar{a}\neq 0$ then $\bar{F}\neq 0$

Linear momentum ($\bar{P}$) :-

The quantity of motion of a moving body.

→ It is also defined as product of mass & velocity of body.

$$\boxed{\bar{P} = m\bar{V}}$$

→ It is a vector quantity

S.I unit kg.m/s | Dimensional formula = $[MLT^{-1}]$

CGS unit g.cm/s | → direction of momentum is in the direction of velocity.

Newton's 2^{nd} law of motion :-

The rate of change of momentum of a body is directly proportional to the applied net external force and the change takes place in the direction of force.

$$\bar{F} \propto \frac{d\bar{P}}{dt} \Rightarrow \bar{F} = K\frac{d\bar{P}}{dt} \Rightarrow \bar{F} = K \cdot m\frac{d\bar{v}}{dt}$$

→ by experimental obsevations $K = 1$ so $\boxed{\bar{F} = m\frac{d\bar{v}}{dt}} \Rightarrow \boxed{\bar{F} = m\bar{a}}$

→ here $\bar{F}$ → net external force

m → mass of body

$\bar{a}$ → acceleration of body.

→
$$\left.\begin{aligned} F_x &= \frac{dP_x}{dt} = ma_x \\ F_y &= \frac{dP_y}{dt} = ma_y \\ F_z &= \frac{dP_z}{dt} = ma_z \end{aligned}\right\} \quad \bar{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

→ by Newton's second law we can able to find force acting on an object, So we can say second law gives quantitative definition of force.

→ $\bar{F} = m\bar{a}$, $\bar{F} = m\left(\frac{\bar{v}-\bar{u}}{t}\right)$ (or) $F = m\left(\frac{v^2-u^2}{2s}\right)$

→ Newton's 2nd law contains first law. (as $a=0$ if $F=0$).

→ $$\bar{F} = \frac{d\bar{P}}{dt} = \frac{d}{dt}(mv)$$

if m is constant, then $\bar{F} = m\frac{d\bar{v}}{dt}$

if v is constant & m is vary with time, then $\bar{F} = \bar{v}\frac{dm}{dt}$

if both m & v vary with time, then $F = m\frac{dv}{dt} + v\frac{dm}{dt}$

→ If external force acting on a system is zero, then momentum of the system remains constant → conservation of linear momentum.

$$F_{ext} = \frac{d\bar{P}}{dt}$$

if $F_{ext} = 0$, then $\frac{d\bar{P}}{dt} = 0 \Rightarrow \int_{P_i}^{P_f} d\bar{P} = 0 \Rightarrow \Delta\bar{P} = 0 \Rightarrow \bar{P}_f - \bar{P}_i = 0 \Rightarrow \boxed{\bar{P}_f = \bar{P}_i}$

Impulse & Impulsive force:-

A large amount of force acting on a body in short interval of time is known as impulsive force.

Impulse $= \int \bar{F}\,dt = \int d\bar{P} = \Delta\bar{P}$; Also impulse $= F \times t$; Impulse is a vector quantity

S.I unit of impulse = kg.m/s

→ Area under $\bar{F}$-t graph with time axis gives impulse.

Ex:- (i) A cricketer pull back his hands back while catching a fast moving ball to increase time of contact there by reduce impulsive force.

(ii) Shock absorbers are fixed to vehicles to reduce impulsive force

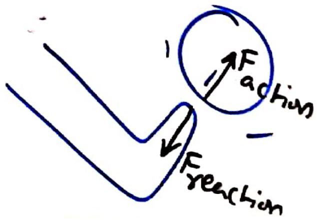
Newton's 3rd law of motion:-

→ For every action there is an equal and opposite reaction.

→ Force acting on body under our observation is action & force acting on body due to action is reaction.

→ Action & reaction forces are equal in magnitude & opposite in direction, and acts on different bodies

$$\overline{F}_{action} = -\overline{F}_{reaction}$$



Q) A force of 100 dyne acts on a mass of 5gm for 10 sec. Find velocity produced.

Sol:-
$$\bar{F} = m\bar{a} \Rightarrow \bar{F} = m\left(\frac{\bar{v}-\bar{u}}{t}\right) \Rightarrow 100 = \frac{5(\bar{v}-0)}{10} \Rightarrow \bar{v} = 200 \text{ cm/s.}$$

Q) If a bullet of mass 5gm moving with a velocity of 100 m/s penetrates a wooden block upto 6cm. Find the avg. force acting on bullet by wooden block.

Sol:-
$$\bar{F} = m\bar{a} \Rightarrow F = m\left(\frac{v^2-u^2}{2S}\right) = 5\times10^{-3}\left(\frac{0-100^2}{2\times6\times10^{-2}}\right) = -417 \text{ N}$$

$\therefore$ Retarding force on bullet is $\underline{\underline{417 \text{ N}}}$

Q) A body of mass 5 kg starts from the origin with an initial velocity of $\bar{u} = (30\hat{i}+40\hat{j})$ m/s. A constant force of $\bar{F} = (-\hat{i}-5\hat{j})$N acts on body. Find the time in which Y-component of velocity is zero.

Sol:-
$$F_y = \frac{dP_y}{dt} \Rightarrow F_y = m a_y \Rightarrow F_y = m\left(\frac{V_y-U_y}{t}\right) \Rightarrow -5 = 5\left(\frac{0-40}{t}\right) \Rightarrow t = \underline{\underline{40 \text{ sec}}}.$$

Q) A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 m/s. How long does the body take to stop.

Sol:- $\bar{F} = m\left(\frac{\bar{v}-\bar{u}}{t}\right) \Rightarrow -50 = 20\left(\frac{0-15}{t}\right) \Rightarrow t = 6$ sec.

Q) The velocity of a body of mass 2 kg as a function of 't' is given by $V = 2t\hat{i} + t^2\hat{j}$. Find the momentum & force acting on it, at time $t = 2$ sec.

Sol:- $\bar{P} = m\bar{V} \Rightarrow \bar{P} = 2(2t\hat{i} + t^2\hat{j}) = 4t\hat{i} + 2t^2\hat{j}$

$\bar{F} = \frac{d\bar{P}}{dt} \Rightarrow \bar{F} = \frac{d}{dt}(4t\hat{i} + 2t^2\hat{j}) = 4\hat{i} + 4t\hat{j}$

at $t = 2$ sec, $\bar{P} = 4(2)\hat{i} + 2(2)^2\hat{j} = 8\hat{i} + 8\hat{j}$

$\bar{F} = 4\hat{i} + 4(2)\hat{j} = 4\hat{i} + 8\hat{j}$

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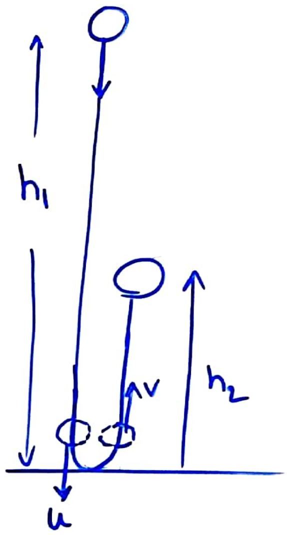
Q) A ball of mass 'm' dropped from a height 'h_1' hits the floor and rebounds to a height 'h_2'. If ball is in contact with the ground for 't' seconds. Find force exerted by ground on ball.

Sol:- Let us assume velocity of ball just before impact & after impact as u & V.



$$\bar{u} = -\sqrt{2gh_1}\ \hat{j} \quad \& \quad \bar{V} = \sqrt{2gh_2}\ \hat{j}$$

∴ Force exerted by ground on ball is

$$\bar{F} = m\bar{a} \Rightarrow \bar{F} = m\left(\frac{\bar{V}-\bar{u}}{t}\right)$$

$$\bar{F} = \left[\frac{m\left(\sqrt{2gh_2} + \sqrt{2gh_1}\right)}{t}\right]\hat{j}$$

∴ direction of force exerted by ground on ball is upward.

Q) A machine gun fires a bullet of mass 'm' with velocity 'V'. The man holding it can exert a maximum force F on the gun. How many bullets can he fire per second at the most?

Sol:- Force exerted by gun after firing bullet is

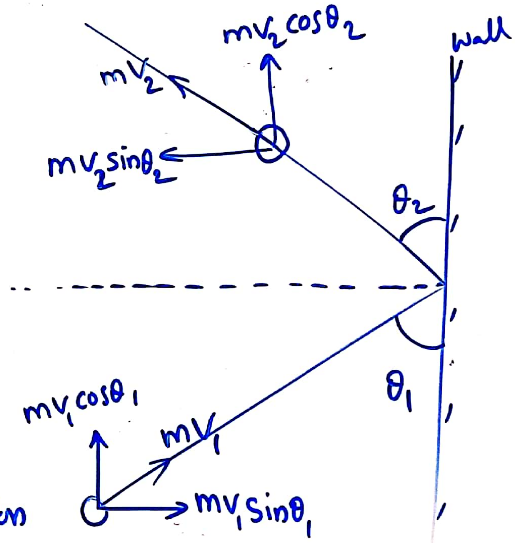
$$\bar{F} = \frac{\Delta \bar{P}}{\Delta t}$$

$$\Rightarrow \quad F = \frac{N \cdot mV}{t}$$

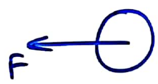
$$F = n\,mV \quad (n \rightarrow \text{no. of bullets fired/second})$$

$$\therefore \quad n = \frac{F}{mV}$$

Q) A ball of mass 'm' moving on floor strikes a smooth wall with a velocity 'V_1' at angle 'θ_1' with wall & rebounds with speed 'V_2' making an angle 'θ_2' with wall. Find the change in momentum of ball.



Sol:- → At time of collision, forces acting on ball due to wall is



→ by N-II law, we can say change in momentum takes place in the direction of Force

→ As there is no force along the direction of wall, no change in momentum along the direction of wall

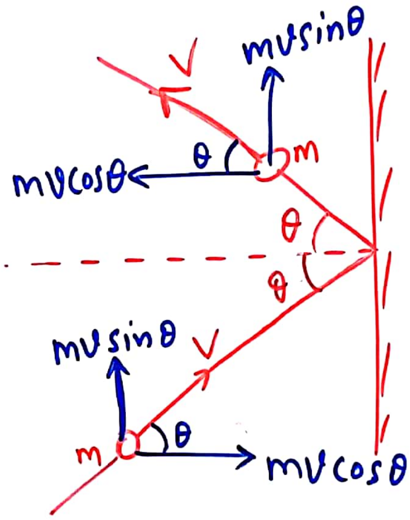
$$\therefore \quad mV_2 \cos\theta_2 = mV_1 \cos\theta_1$$

→ $\bar{P}_i = mv_1 \sin\theta_1 \hat{i} + mv_1 \cos\theta_1 \hat{j}$

$\bar{P}_f = -mv_2 \sin\theta_2 \hat{i} + mv_2 \cos\theta_2 \hat{j}$

$$\therefore \Delta\bar{P} = \bar{P}_f - \bar{P}_i = -(mv_2 \sin\theta_2 + mv_1 \sin\theta_1)\hat{i}$$

Q) Find change in momentum of ball of mass m



Sol:-

initial momentum, $\bar{P}_i = mv\cos\theta\,\hat{i} + mv\sin\theta\,\hat{j}$

final momentum, $\bar{P}_f = -mv\cos\theta\,\hat{i} + mv\sin\theta\,\hat{j}$

$\therefore$ Change in momentum is $\Delta\bar{P} = \bar{P}_f - \bar{P}_i$

$$\Delta\bar{P} = -2mv\cos\theta\,\hat{i}$$

Equilibrium :-

→ If net force & net torque acting on a body are zero, then the body is said to be in equilibrium.

→ If net external force acting on a body is zero, then the body is said to be in translational equilibrium.

In translational equilibrium body may be in rest ($v=0$) or in uniform motion ($v = \text{const}$).

→ If net torque acting body is zero, then it is said to be in rotational equilibrium.

→ If body is in translational equilibrium, & we need to find forces acting on it, then first draw F.B.D & use Lami's theorem (or) resolution of vectors concept.

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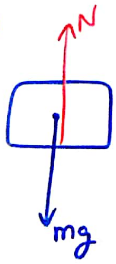
Q) Find normal reaction force exerted by ground on block.

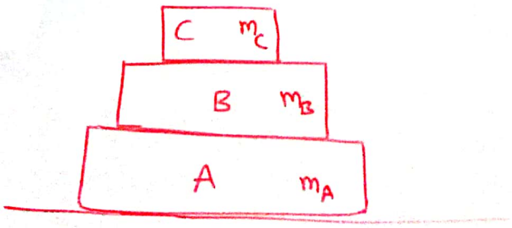
Sol:- F.B.D of m

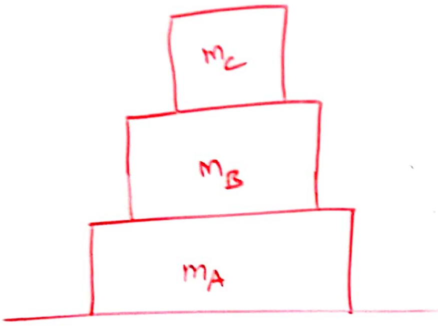


As the body is in equilibrium,
upward forces balances with downward forces
& left ward forces balances with right ward forces.

$\therefore$ $N = mg$.

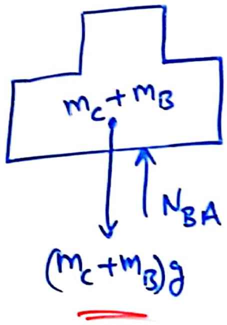
Q) Find normal reaction between blocks A & B and B & C





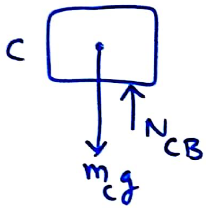
Alternate method for (ii)

(ii)



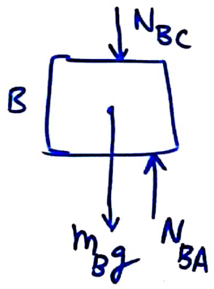
$N_{BA} = (m_C + m_B)g$.

(i) F.B.D of 'C'



$N_{CB} = m_C g$ $N_{CB} \rightarrow$ force exerted by B on C

(ii) F.B.D of 'B'



$m_B g + N_{BC} = N_{BA}$ (Acc. to N-III law $N_{BC} = N_{CB}$)

$m_B g + m_C g = N_{BA}$ $N_{BA} \rightarrow$ force exerted by A on B

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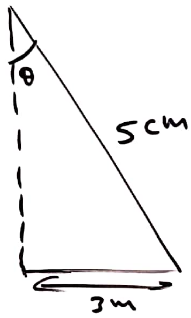
Q) Find tension in string. sphere having mass 1 kg.

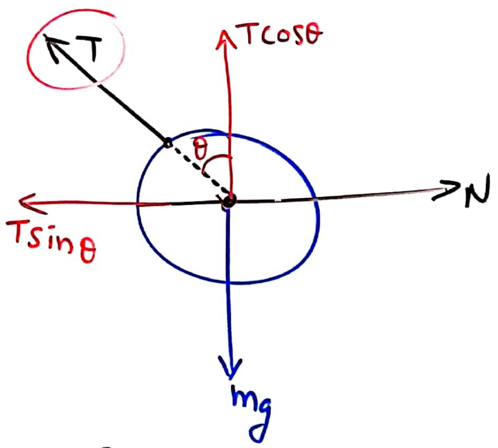


$\sin\theta = \frac{3}{5}$

$\theta = 37^\circ$

String makes an angle 37° with vertical.

Sol :-



F.B.D of sphere.

As sphere is in equilibrium

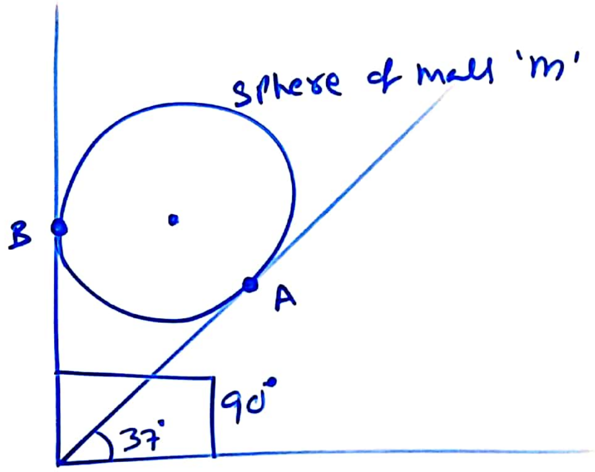
$T\cos\theta = mg$ —— (1)

$T\sin\theta = N$ —— (2)

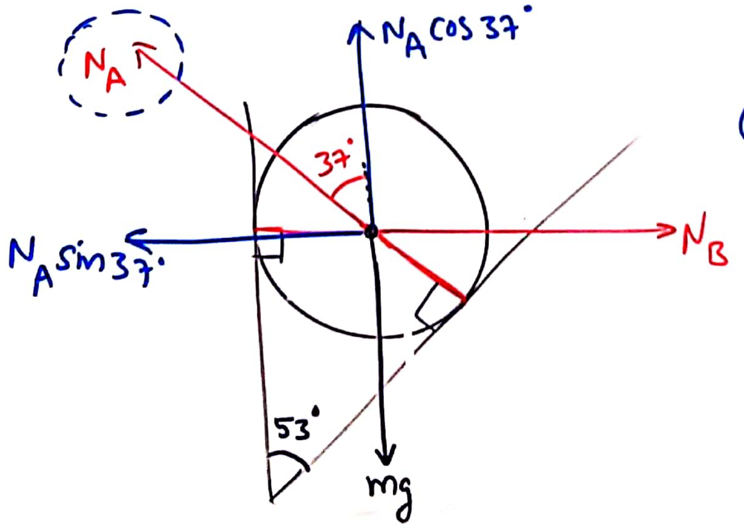
from (1), $T = \frac{mg}{\cos\theta} = \frac{1\times 10}{4/5}$

$T = 12.5$ Newton.

Q) Find Normal reaction forces exerted by contact surfaces on sphere.



Sol:-



As sphere is in equilibrium,

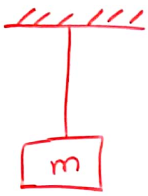
(1) — $N_A \cos 37° = mg \Rightarrow N_A = \frac{5mg}{4}$

(2) — $N_A \sin 37° = N_B$

$\therefore N_B = \frac{5mg}{4} \times \frac{3}{5}$

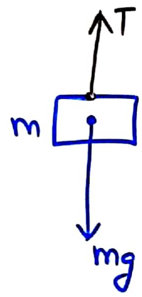
$= \frac{3mg}{4}$.

Q) Find Tension in ideal string



Sol:-

F.B.D of m

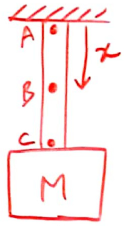


As the block is in equilibrium,

$$T = mg$$

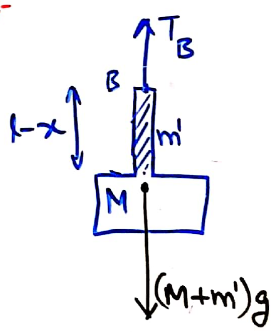
Note:- In an ideal string at any point Tension remains constant.

Q) Find tension in non-ideal string at points A, B, C as shown in fig. [mass of string - m, length of string - l]



→ mass is distributed uniformly through out the string.

Sol:-



$l \longrightarrow m$

$l-x \longrightarrow m'$

$$m' = \frac{m}{l}(l-x)$$

$$T_B = (M+m')g.$$

$$\boxed{T_B = \left[M + \frac{m}{l}(l-x)\right]g}$$

At A, $x=0$

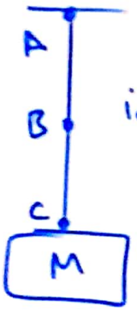
$$\therefore T_A = \left[M + \frac{m}{l}(l-0)\right]g = (M+m)g.$$

At C, $x=l$

$$\therefore T_C = \left[M + \frac{m}{l}(l-l)\right]g = Mg$$

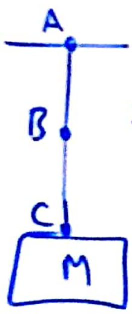
Note :-

Tension in a string at a Point depends on amount of mass hanging below that Point.



ideal string

$$T_A = T_B = T_C = Mg.$$

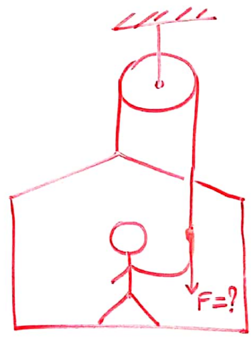


string is having uniform mass distribution

$$T_A \neq T_B \neq T_C$$

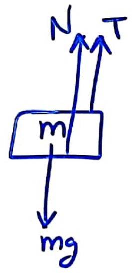
$$T_A > T_B > T_C$$

Q) A man of mass 'm' stands on a frame of mass 'M'. He pulls on a light rope which passes over pulley. The other end of rope is attached to the frame. For the system to be in equilibrium, what force must the man exert on rope?



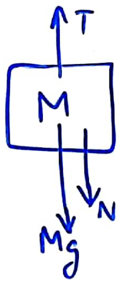
Sol:-

F.B.D of man,



$$N + T = mg \quad \text{—} \ (1)$$

F.B.D of box

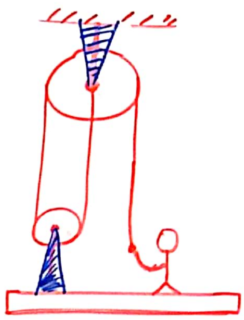


$$T = N + Mg \quad \text{—} \ (2)$$

(1) + (2) $\Rightarrow 2T = (M+m)g$

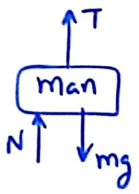
$$\boxed{T = \frac{(M+m)g}{2}}$$

Q) what force must man exert on rope to keep platform in equilibrium.



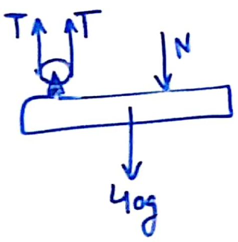
[mass of platform = 40 kg
mass of man = 50 kg
pulley's are ideal]

Sol:- F.B.D of man



$T + N = 50g$ — ①

F.B.D of platform



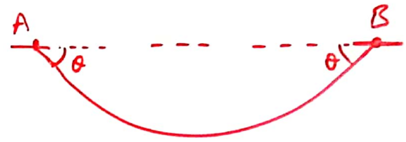
$2T = N + 40g$ — ②

① + ②

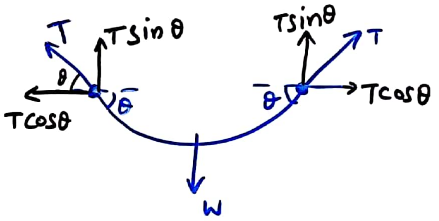
$3T = 90g \Rightarrow T = 30g$

$\therefore T = 300$ Newton

Q) A flexible chain of weight W hangs between two fixed points A & B at same level. The inclination of chain with the horizontal at two supports is 'θ'. What is tension in chain at the end point.



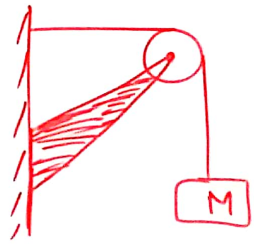
Sol:-



$\therefore \; 2T\sin\theta = W$

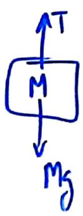
$$T = \frac{W}{2\sin\theta} = \frac{W}{2}\operatorname{cosec}\theta$$

Q) A string of negligible mass going over a clamped pulley of mass 'm' supports a block of mass M as in fig. Find force acting on pulley by the clamp.



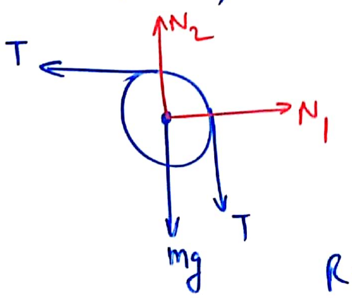
Sol:-

F.B.D of block



$T = Mg$

F.B.D of Pulley



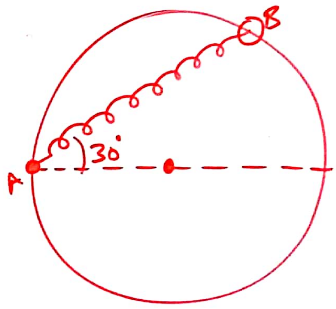
$N_1 = T$

$N_2 = T + mg$

$\therefore$ force acting on pulley by clamp is

$$R = \sqrt{N_1^2 + N_2^2} = \sqrt{T^2 + (T+mg)^2} = \sqrt{(Mg)^2 + [(M+m)g]^2}$$

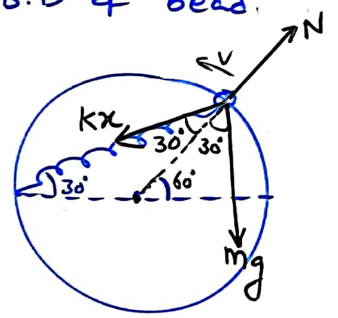
Q) A bead of mass 'm' is attached to one end of a spring of natural length 'R' and spring constant $K = \frac{(\sqrt{3}+1)mg}{R}$. The other end of spring is fixed at point A on a smooth vertical ring of radius R as shown. What is the normal reaction at 'B' just after the bead is free to move.



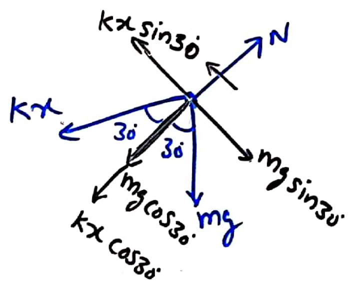
Sol:- From fig, elongation in spring is

$$x = AB - R = 2R\cos 30^\circ - R = (\sqrt{3}-1)R$$

F.B.D of bead.



by resolving mg & kx into components, such that one is along N, other is $\perp^{r}$ to N.

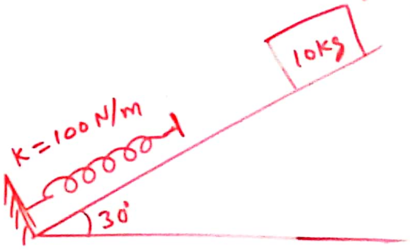


$$N = mg\cos 30^\circ + kx\cos 30^\circ$$

$$= mg\frac{\sqrt{3}}{2} + \frac{(\sqrt{3}+1)mg}{R} \times (\sqrt{3}-1)R \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{2}mg$$

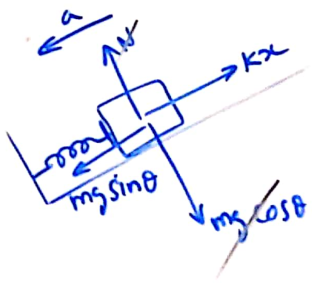
Q) A 10 kg block is released from rest at the top of an smooth incline & brought to rest momentarily after compressing the spring by 2 metre. What is the speed of mass just before it reaches the spring?



Sol:- Net force acting on block at an instant when spring compressed by 'x' is

$$F_{net} = ma$$

$$\Rightarrow mg\sin\theta - Kx = m \cdot v \cdot \frac{dv}{dx}$$



$$\Rightarrow \int_{v}^{0} v \cdot dv = \int_{0}^{2} \left(g\sin\theta - \frac{K}{m}x\right) dx$$

$$\Rightarrow -\frac{v^2}{2} = \left[5x - \frac{10x^2}{2}\right]_0^2$$

$$-\frac{v^2}{2} = -10 \Rightarrow v = \sqrt{20} \text{ m/s}$$

Q) A man of mass 60 kg is holding block of mass 10 kg in equilibrium in two different ways as shown in fig(a) & fig(b). Find the normal force exerted by the floor on man in both the cases.

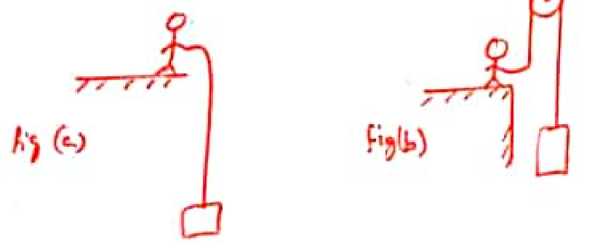
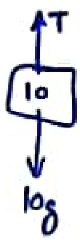


fig (a) fig(b)

Sol:-

fig (a)

F.B.D of block.



$T = 10g$ —(1)

F.B.D of man

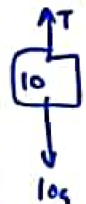
$N_1 = T + 60g$ —(2)

from (1) & (2)

$N_1 = 70g = 700$ newton.

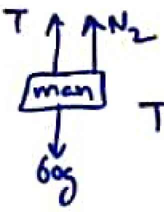
fig (b)

F.B.D of block



$T = 10g$ —(1)

F.B.D of man



$T + N_2 = 60g$ —(2)

by (1) & (2)

$10g + N_2 = 60g$

$N_2 = 500$ newton.

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Accelerating objects :-

To solve problems involving objects that are in accelerated motion :-

1) Isolate the objects of a system & draw FBD. In FBD, show the direction of acceleration with an arrow.

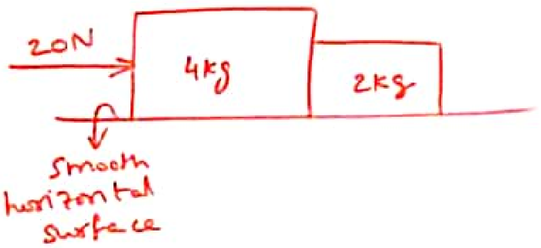
2) With reference to the direction of motion of objects, select suitable coordinate axes & resolve forces in F.B.D along the chosen axes.

3) Apply the eqn of Newton's 2nd law, $\Sigma F_x = ma_x$; $\Sigma F_y = ma_y$, for all objects

4) Above step will give eqns with several unknown quantities. By solving these eqns we get unknown quantities.

5) In case of composite problem of a number of bodies connected by strings & pulleys, find the relationship of accelerations of different bodies by constraint relationship (discussed later).

Q)

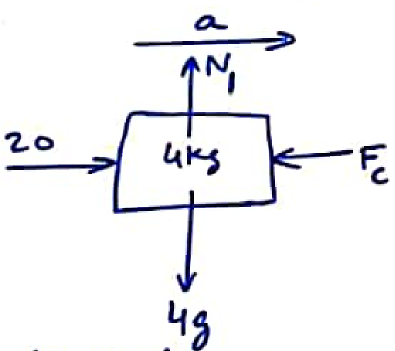


a) Find acceleration of each block.

b) Find normal reaction between 2 blocks (or) contact force (F_c)

Sol :-

F.B.D of 4 kg block

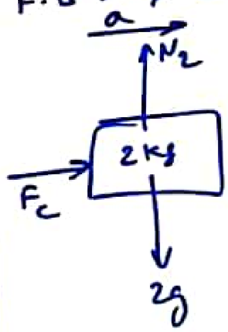


by N - II law, $F_{net} = ma$

$$20 - F_c = 4a \quad \text{—— (1)}$$

$$N_1 = 4g$$

F.B.D of 2 kg block



by N - II law

$$F_c = 2a \quad \text{—— (2)}$$

$$N_2 = 2g$$

by solving (1) & (2)

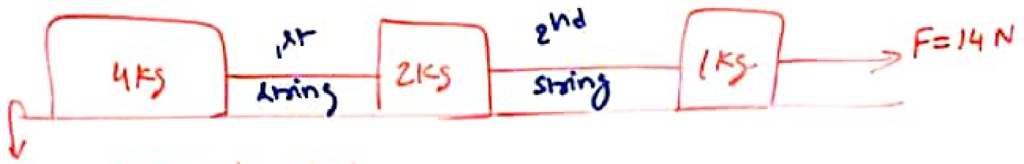
(1) + (2)

$$\Rightarrow 20 = 6a \Rightarrow a = \frac{10}{3}\ m/s^2$$

sub 'a' in eq (2)

$$F_c = 2\left(\frac{10}{3}\right) = \frac{20}{3} \text{ newton}$$

Q) strings are ideal.

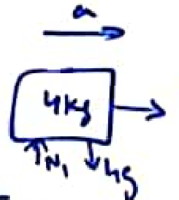


(i) Find acceleration of each block

(ii) Find tension in each string.

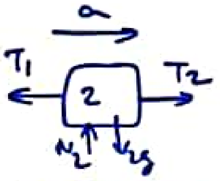
Sol:-

F.B.D of 4kg



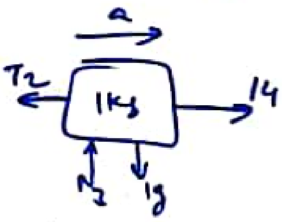
$T_1 = 4a$ — ①

F.B.D of 2kg



$T_2 - T_1 = 2a$ — ②

F.B.D of 1kg



$14 - T_2 = 1a$ — ③

by solving ①, ② & ③ we get T_1, T_2 & a

$a = 2m/s^2$, $T_1 = 8N$, $T_2 = 12N$.

Short cut:-

Consider all blocks as a system.

$\therefore a = \dfrac{F_{net\ ext}}{m_{total}}$

$a = \dfrac{14}{4+2+1} = 2m/s^2$.

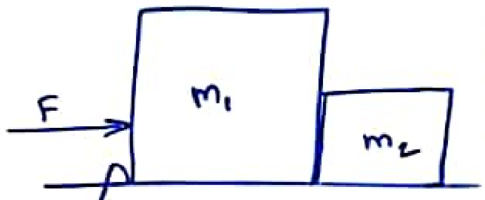
from F.B.D of 4kg

$T_1 = 4a = 8\ N$

from F.B.D of 1kg

$14 - T_2 = 2 \Rightarrow T_2 = 12N$

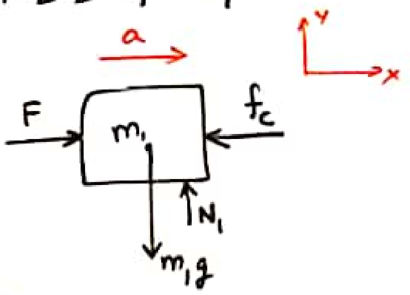
Q)



Smooth horizontal surface.

Find acceleration of each block & contact force between two blocks.

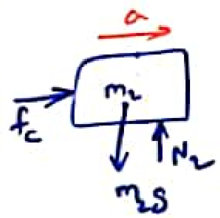
Sol:- F.B.D of 'm_1'



→ Along y-direction, block is not moving
$N_1 = m_1 g$

→ by N-II law along x-direction
$F - f_c = m_1 a$ —— ①

F.B.D of 'm_2'



by applying N-IInd law along x-direction

$f_c = m_2 a$ —— ②

by solving ① & ②

① + ②

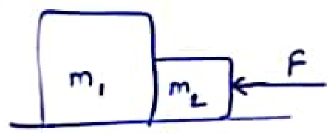
$\Rightarrow F = m_1 a + m_2 a$

$$\boxed{a = \frac{F}{m_1+m_2}}$$

by sub 'a' in ②

$$\boxed{f_c = \frac{m_2 F}{m_1+m_2}}$$

Q)

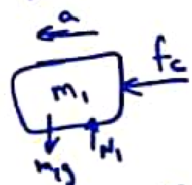


Find contact force bet blocks.

Sol:- $a = \frac{F_{net\ external}}{m_{total}}$

$= \frac{F}{m_1+m_2}$ (←)

F.B.D of m_1



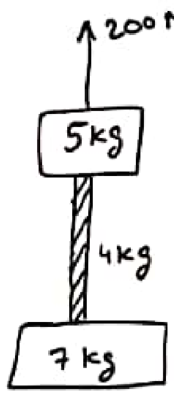
$f_c = m_1 a = m_1 \frac{F}{m_1+m_2}$ //

Q)

a) Find acceleration of the system.

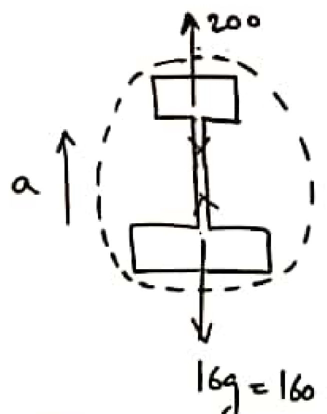
b) what is the tension at top of heavy rope ?

c) what is the tension at mid point of the rope. (take $g = 10 m/s^2$)



Sol :- a) consider 3 objects together as a system.

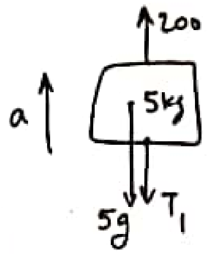
$$\therefore a = \frac{F_{net\ external}}{m_{total}}$$



$$a = \frac{200 - 160}{5+4+7}$$

$$= \frac{40}{16} = 2.5 m/s^2$$

(b) F.B.D of 5 kg

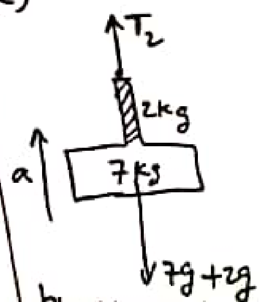


by N-II law

$$200 - (50 + T_1) = 5a$$

$$T_1 = 150 - 5(2.5) = 137.5 N$$

(c)



by N-II law

$$T_2 - (9g) = 9a$$

$$T_2 = 112.5 N$$

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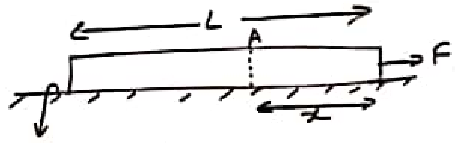
Q) Find the tension in a rope at section A, at a distance 'x' from right end.

→ mass is distributed uniformly throught the rope.

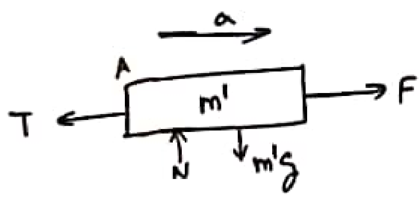


Smooth Surface.

Sol:-

$$a = \frac{F_{net\ external}}{m_{tot}} = \frac{F}{m}$$

F.B.D of 'Part x'



by N-II law, $F - T = m'a$

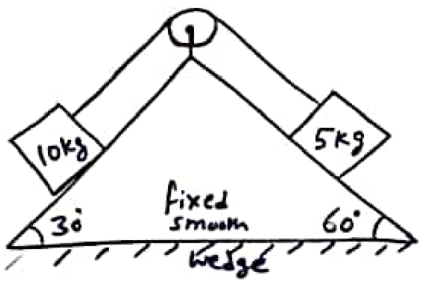
$$\Rightarrow F - T = \left(\frac{m}{l} x\right) a$$

$$F - T = \left(\frac{m}{l} \cdot x\right) \cdot \frac{F}{m}$$

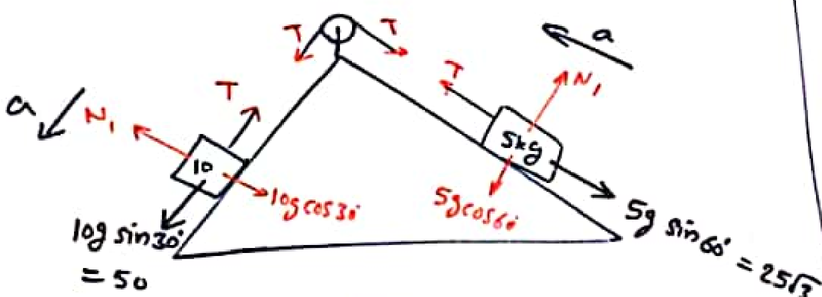
$$F - T = \frac{Fx}{l}$$

$$\Rightarrow T = F - \frac{Fx}{l} = F\left(1 - \frac{x}{l}\right)$$

Q) Find acceleration of each block & tension in string.



Sol:-



from F.B.D of 10 kg block.

$10g\sin 30° - T = 10a$

by sub. 'a', we get T

→ Assume 2 blocks together as system.

As $10\,g\sin 30° > 5g\sin 60°$

$$a = \frac{F_{\text{net pulling}}}{m_{\text{total}}} = \frac{10g\sin 30° - 5g\sin 60°}{10+5} = \frac{50-25\sqrt{3}}{15}\ m/s^2.$$

Frame of reference :-

It is a conveniently choosen co-ordinate system with clock, with reference to this position & motion of an object can be observed in space.

→ Frame of reference is of two types.

(i) inertial frame of reference :- Newton laws of motion are valid

(ii) non-inertial frame of reference :- Newton laws of motion are invalid.

(i) Inertial frame of reference :-

→ A frame of reference which is at rest (or) moving with constant velocity is called inertial frame of reference.

→ Inertial frame of reference is also known as zero accelerated frame of reference (or) Newtonian (or) Galilean frame of reference.

→ In inertial frame of reference Newton laws of motion holds good.

→ Ideally no inertial frame exist in universe, but while solving problems we will assume frame of reference fixed to earth as inertial frame of reference.

→ Actually earth is not an inertial frame because of its rotation.

→ Only real forces present in FBD of an object in inertial frame of reference.

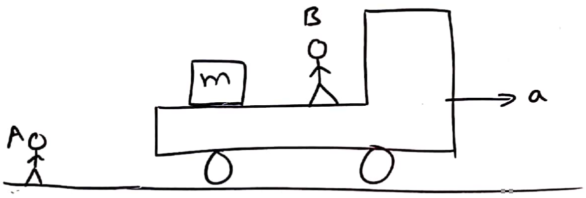
Non- inertial frame of reference :-

→ Accelerated frame of reference is called Non - inertial frame of reference.

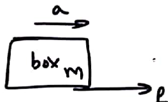
→ In non - inertial frame body appears to be accelerated even if there is no force acting on body.

→ So, in non - inertial frame of reference Newton laws of are invalid, but by considering an imaginary force known as "Pseudo force" we can apply Newton laws of motion.

Ex:-



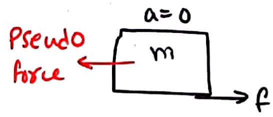
→ w.r.to person A, box is moving with acceleration, & on which frictional force acts towards right.



$\boxed{f=ma}$

Person A → inertial frame

→ w.r.to person B, box is at rest.



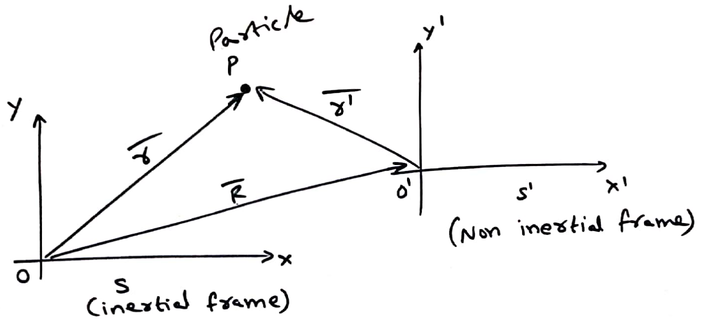
Person B → non - inertial frame

Pseudo force = f = ma.

→ So while drawing F.B.D of an object in non - inertial frame draw Pseudo force also with real forces acting on body.

→ direction of Pseudo force is opposite to direction of acceleration of frame & Magnitude is product of mass of body & acceleration of frame.

Newton's second law of motion in Non-inertial frame:-



$\bar{r} \rightarrow$ Position vector of particle w.r.to inertial frame.

$\bar{r}' \rightarrow$ P.V. of particle w.r.to non-inertial ".

$\bar{R} \rightarrow$ P.V of origin of S' w.r.to S.

$\bar{a}' \rightarrow$ acceleration of particle w.r.to S'

$\bar{a} \rightarrow$ " " " w.r.to S

$\bar{A} \rightarrow$ acceleration of S' w.r.to S.

$\rightarrow$ from fig, $\bar{r} = \bar{R} + \bar{r}'$

$\Rightarrow \bar{r}' = \bar{r} - \bar{R}$

by differentiating above eqn w.r.to time by twice,

$$\frac{d^2 \bar{r}'}{dt^2} = \frac{d^2}{dt^2}\bar{r} - \frac{d^2}{dt^2}\bar{R}$$

$$\Rightarrow \bar{a}' = \bar{a} - \bar{A}$$

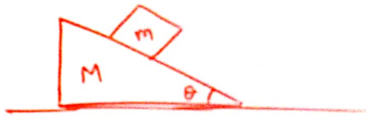
$$m\bar{a}' = m\bar{a} - m\bar{A}$$

$$m\bar{a}' = \bar{F}_{real} + (-m\bar{A})$$

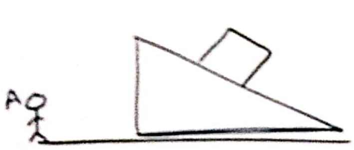
$$\Rightarrow \boxed{\bar{F}_{real} + \bar{F}_{Pseudo} = m\bar{a}'}$$

Scanned with CamScanner

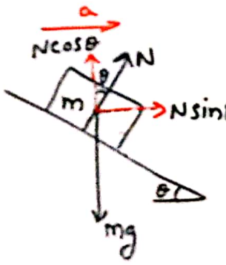
Q) A block of mass 'm' is placed on smooth wedge of mass M. With what acceleration wedge should move, so that during motion block remains at rest w.r.to wedge?



Sol:- w.r.to inertial frame, i.e w.r.to person A



→ F.B.D of block 'm' w.r.to person A



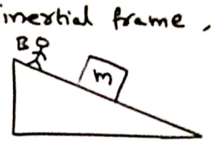
$N\sin\theta = ma$ —— ①

$N\cos\theta = mg$ —— ②

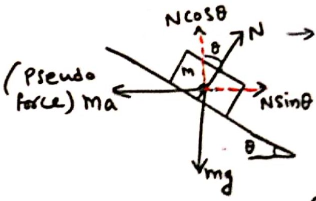
$\frac{①}{②} \Rightarrow a = g\tan\theta$

Method-II

w.r.to non-inertial frame, i.e w.r.to wedge (or) person B.



→ F.B.D of block 'm' w.r.to wedge (or) person B.



→ As block 'm' is rest w.r.to B, balance all forces.

$N\sin\theta = ma$ —— ①

$N\cos\theta = mg$ —— ②

$\frac{①}{②} \Rightarrow a = g\tan\theta$.

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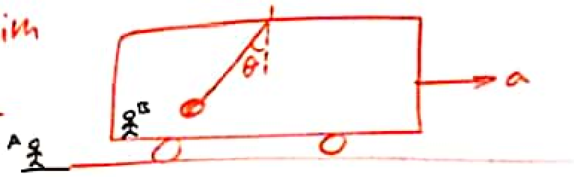
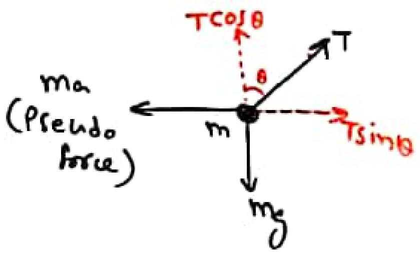
Q) A pendulum of mass m is hanging from ceiling of a car moving with acceleration a. Find angle made by string with vertical.



Sol:-

by non-inertial frame

F.B.D of Pendulum w.r.to car



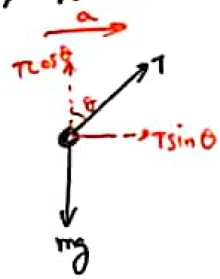
$$T\sin\theta = ma \quad \text{—} (1)$$

$$T\cos\theta = mg \quad \text{—} (2)$$

$$\frac{(1)}{(2)} \Rightarrow \tan\theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

by inertial frame:-

F.B.D of Pendulum w.r.to ground



by N-II law,

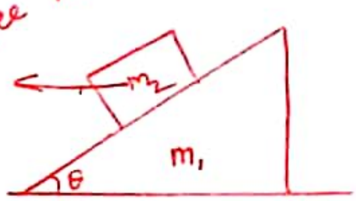
$$T\sin\theta = ma \quad \text{—} (1)$$

$$T\cos\theta = mg \quad \text{—} (2)$$

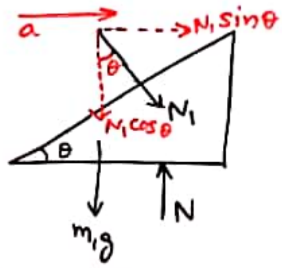
$$\frac{(1)}{(2)} \Rightarrow \tan\theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

Q) All contact surfaces are smooth. Find the acceleration of wedge & acceleration of block w.r.to wedge

→ wedge is free to move



Sol:- F.B.D of wedge m_1



by N-II law,

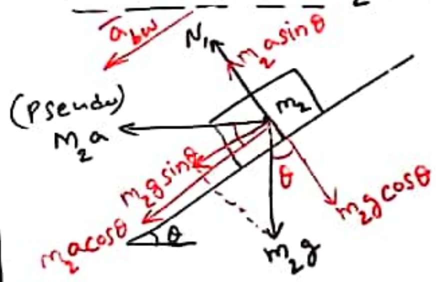
$$N_1 \sin\theta = m_1 a \quad \text{--- (1)}$$

$$N = m_1 g + N_1 \cos\theta \quad \text{--- (2)}$$

$\bar{a}$ → acceleration of wedge w.r.to ground.

$\bar{a}_{bw}$ → acceleration of block w.r.to wedge.

F.B.D of block m_2 w.r.to wedge m_1



by N-II law,

$$m_2 g \sin\theta + m_2 a \cos\theta = m_2 a_{bw} \quad \text{--- (3)}$$

$$N_1 + m_2 a \sin\theta = m_2 g \cos\theta \quad \text{--- (4)}$$

by solving 4 eqn's, we get 4 unknowns, a, a_{bw}, N_1, N

by sub (4) in (1)

$$(m_2 g \cos\theta - m_2 a \sin\theta)\sin\theta = m_1 a_w$$

$$\boxed{a = \frac{m_2 \sin\theta (g\cos\theta - a\sin\theta)}{m_1}}$$

Sub. 'a' in (3), then we get

$a_{bw} =$ ___

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Multiply eq ④ by $\sin\theta$ & sub. eq ① in it

$$N_1 \sin\theta + m_2 a \sin^2\theta = m_2 g \cos\theta \sin\theta$$

$$\Rightarrow m_1 a + m_2 a \sin^2\theta = m_2 g \cos\theta \sin\theta$$

$$\Rightarrow a = \frac{m_2 g \sin\theta \cos\theta}{m_1 + m_2 \sin^2\theta}$$

$$\Rightarrow \boxed{a = \frac{g \sin\theta \cos\theta}{\left(\frac{m_1}{m_2}\right) + \sin^2\theta}}$$

Sub. 'a' in ③,

$$m_2 g \sin\theta + m_2 \frac{g \sin\theta \cos\theta}{\frac{m_1}{m_2} + \sin^2\theta} \cos\theta = m_2 a_{bw}$$

$$\Rightarrow a_{bw} = g \sin\theta + \frac{g \sin\theta \cos^2\theta}{\frac{m_1}{m_2} + \sin^2\theta} = \frac{g \sin\theta \left[\left(\frac{m_1}{m_2} + \sin^2\theta\right) + \cos^2\theta\right]}{\frac{m_1}{m_2}\left(1 + \frac{m_2}{m_1} \sin^2\theta\right)}$$

$$= \frac{g \sin\theta \left[\frac{m_1}{m_2} + 1\right]}{\frac{m_1}{m_2}\left[1 + \frac{m_2}{m_1} \sin^2\theta\right]} \Rightarrow \boxed{a_{bw} = \frac{g \sin\theta \left(1 + \frac{m_2}{m_1}\right)}{1 + \frac{m_2}{m_1} \sin^2\theta}}$$

$$\bar{a}_w = a\hat{i}$$

$$\bar{a}_{bw} = -a_{bw} \cos\theta\, \hat{i} - a_{bw} \sin\theta\, \hat{j}.$$

If we need to find acceleration of block w.r.to ground, then

$$\bar{a}_{bw} = \bar{a}_b - \bar{a}_w$$

$\bar{a}_b \rightarrow$ acceleration of block w.r.to ground.

$$\Rightarrow \boxed{\bar{a}_b = \bar{a}_{bw} + \bar{a}_w}$$

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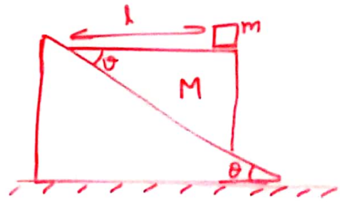
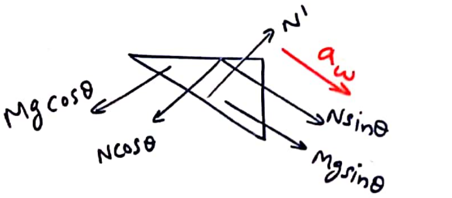
Q) A wedge of mass M lies on smooth fixed wedge of inclination 'θ'. A particle of mass m lies on wedge of mass M as shown. All surfaces are frictionless. & the distance of block from fixed wedge is 'l'. Find



(i) normal reaction between m & M

(ii) the time after which the particle will hit the fixed wedge.

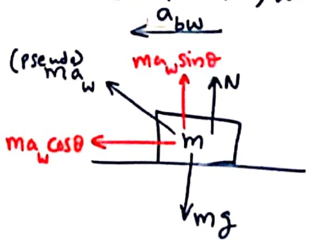
Sol:- F.B.D of wedge M, w.r.to ground



$$N\sin\theta + Mg\sin\theta = Ma_w \quad \text{——} ①$$

w.r.to wedge M, block m moves horizontally left

→ F.B.D of block 'm', w.r.to wedge M



$$N + ma_w\sin\theta = mg \quad \text{——} ②$$

$$ma_w\cos\theta = ma_{bw} \quad \text{——} ③$$

→ by solving eqns ①, ②, ③, we get

$$a_w = \frac{(M+m)g\sin\theta}{M+m\sin^2\theta}, \quad N = \frac{Mmg\cos^2\theta}{M+m\sin^2\theta}, \quad a_{bw} = \frac{(M+m)g\sin\theta\cos\theta}{M+m\sin^2\theta}$$

∴ time taken by particle to hit the fixed wedge is

$$S = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2l}{a_{bw}}} \Rightarrow \boxed{t = \sqrt{\frac{2l(M+m\sin^2\theta)}{(M+m)g\sin\theta\cos\theta}}}$$

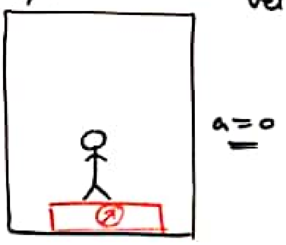
Apparent weight in lift :-

→ Generally weighing machine measures Normal reaction.

→ Consider a weighing machine kept in lift & a person of mass 'm' stands on it.

Case (i) :-

→ if Lift is at rest (or) moving up/down with constant velocity



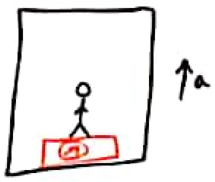
F.B.D of Person

As $a = 0$, $\boxed{N = mg}$

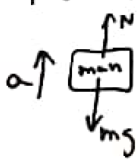
→ Apparent weight = actual weight.

Case (ii)

→ if lift is accelerating upward (or) decelerating downward



F.B.D of person



$N - mg = ma$

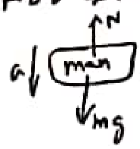
$\boxed{N = m(g+a)}$

→ Apparent wt > actual wt

Case (iii)

→ if lift is accelerating down (or) decelerating upward

F.B.D of Person



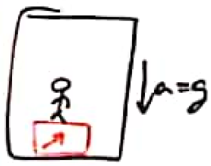
$mg - N = ma$

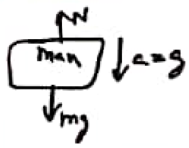
$\boxed{N = m(g-a)}$

→ Apparent wt < actual wt

Case (iv):-

→ if lift is falling freely





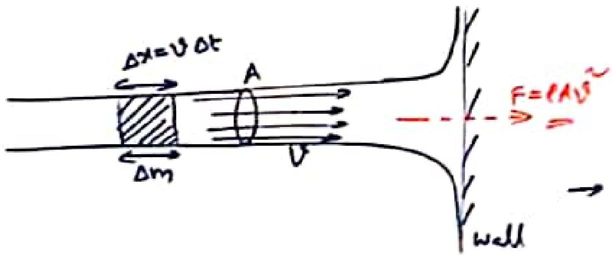
$mg - N = ma$

$N = 0$

Note :- Spring balance measures Tension.

If a spring balance is connected to ceiling of lift & a block of mass 'm' is suspended to it, then its app. weight is similar to previous case.

Force exerted by liquid jet on wall :-



Consider a jet of liquid of density 'ρ' & pipe of cross sectional area A, liquid eject out from pipe with speed v.

→ Now this liquid jet hit a wall & liquid splashes along the wall.

→ Force exerted by wall on liquid jet

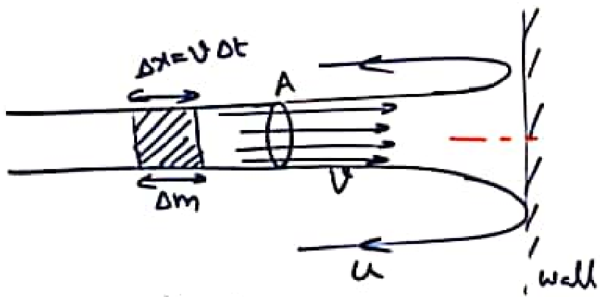
$$F_x = \frac{\Delta P_x}{\Delta t} = \frac{0 - \Delta m \cdot v}{\Delta t}$$

$$= -\frac{(A\Delta x\,\rho)v}{\Delta t} = \frac{-(Av\Delta t\,\rho)v}{\Delta t}$$

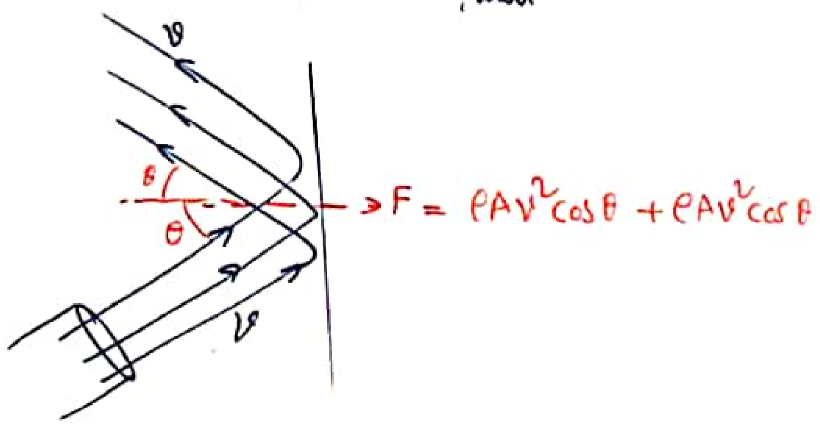
$$= -\rho A v^2$$

∴ In this case, force exerted by liquid on wall is $\boxed{F = \rho A v^2}$

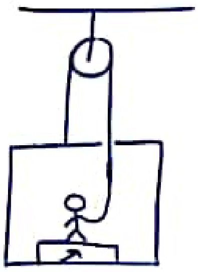
→ If liquid jet rebound with speed u, after hitting the wall, then force exerted by liquid jet on wall is $F = \rho A v^2 + \rho A u^2$



→



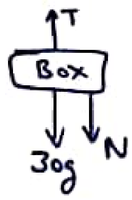
Q) Mass of man = 60 kg
Mass of box = 30 kg | Pulley, string are ideal.



(i) If the man manages to keep the box at rest, then what is the weight shown by weighing machine?
(ii) What force should he exert on the rope to get his correct weight on the weighing machine?

Sol:-

(i) F.B.D of box



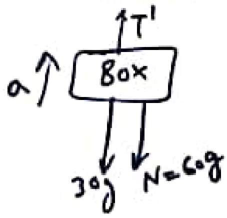
$T = N + 30g$ — ①

F.B.D of man

$T + N = 60g$ — ②

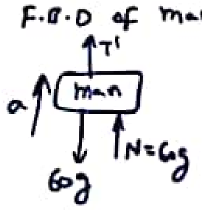
② − ① ⇒ $2N = 30g \Rightarrow N = 15g = 15\ kg.wt$

(ii) F.B.D of box



$T' - 90g = 30a$ — ①

F.B.D of man



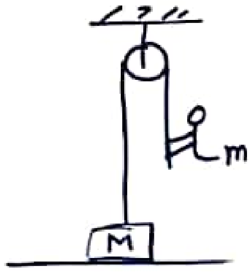
$T' + 60g - 60g = 60a$

$T' = 60a$ — ②

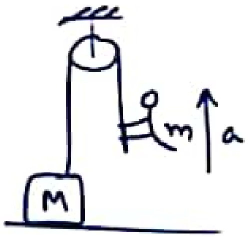
sub ② in ①

$T' - 90g = \frac{T'}{2} \Rightarrow \frac{T'}{2} = 90g \Rightarrow T' = 180g$
$= 1800\ N$

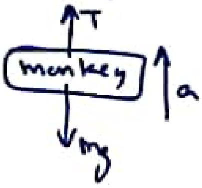
Q) With what acceleration monkey should climb up along the rope of negligable mass, so as to lift the block from the floor. $(m<M)$



Sol:-



F.B.D of monkey



$T - mg = ma$ —— ①

F.B.D of block, M

When it looses contact, $N=0$

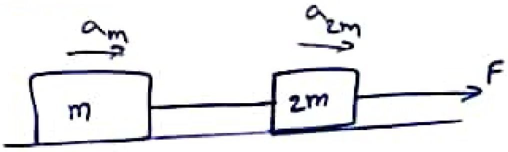
$T = Mg$ —— ②

by sub. ② in ①, $Mg - mg = ma \Rightarrow a = \left(\frac{M}{m} - 1\right)g$

$\therefore a \geq \left(\frac{M}{m} - 1\right)g$ //

Connecting bodies :-

1)



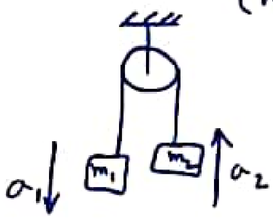
$$a_{2m} = a_m = a$$

$$a = \frac{F}{m+2m} = \frac{F}{3m}.$$

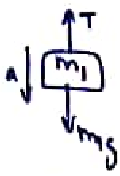
$$T = ma = \frac{mF}{3m} = \frac{F}{3}.$$

2) Atwood's machine

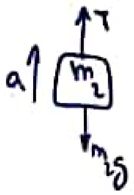
$(m_1 > m_2)$



$$a_1 = a_2 = a$$



$$m_1g - T = m_1a \quad \text{— ①}$$



$$T - m_2g = m_2a \quad \text{— ②}$$

① + ② ⇒

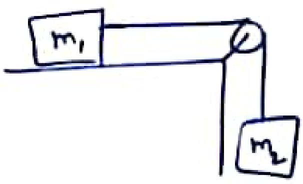
$$a = \frac{(m_1 - m_2)g}{m_1 + m_2}$$

$$a = \frac{F_{net\ pulling}}{m_{total}}$$

by sub a in ①

$$T = \frac{2m_1m_2}{m_1+m_2}g.$$

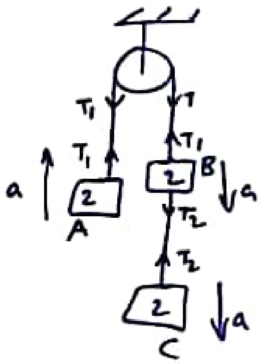
3)



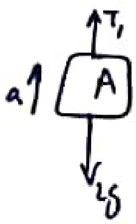
$$a = \frac{F_{net\ pulling}}{m_{total}} = \frac{m_2 g}{m_1+m_2}$$

$$T = m_1 a \Rightarrow T = \frac{m_1 m_2 g}{m_1+m_2}$$

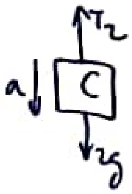
4)



$$a = \frac{F_{net\ pulling}}{m_{total}} = \frac{(2g+2g)-2g}{2+2+2} = g/3$$

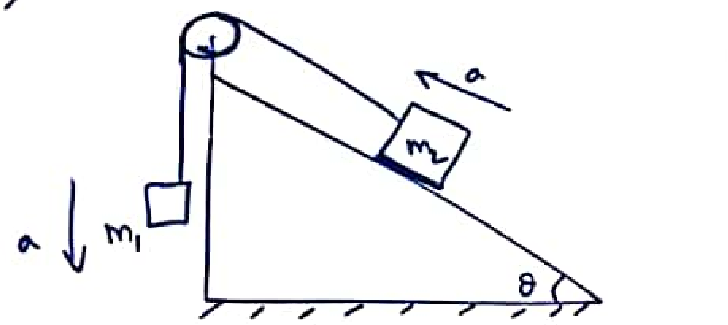


$$T_1 - 2g = 2a \Rightarrow T_1 = 2g + \frac{2g}{3} = \frac{8g}{3}.$$



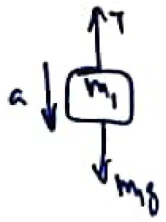
$$2g - T_2 = 2a \Rightarrow T_2 = 2g - 2a = 2g - \frac{2g}{3} = \frac{4g}{3}.$$

5)



$$a = \frac{F_{net\ pulling}}{M_{total}} = \frac{m_1 g - m_2 g\sin\theta}{m_1 + m_2}$$

F.B.D of m_1

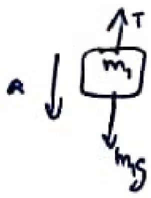


$m_1 g - T = m_1 a.$

$$T = m_1 g - m_1\left(\frac{m_1 g - m_2 g\sin\theta}{m_1 + m_2}\right).$$

(or)

F.B.D of m_1



$m_1 g - T = m_1 a$ —— ①

F.B.D of m_2

$T - m_2 g\sin\theta = m_2 a$ —— ②

① + ② $\Rightarrow$ $m_1 g - m_2 g\sin\theta = m_1 a + m_2 a$

$$a = \frac{m_1 g - m_2 g\sin\theta}{m_1 + m_2}.$$

by sub. 'a' in ①, we get T.

Constrained motion :-

→ Motion of a body restricted under some physical conditions is called constrained motion.

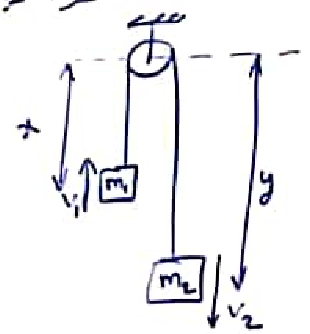
→ The eq^ns showing the relation of the motions of a system of bodies, in which motion of one body is constrained by the motion of other bodies are called constraint relations. The eq^n which gives relation between displacements (or) velocities (or) accelerations of objects in a system is called constraint eq^n.

→ Applying Newton's laws alone is not sufficient in some cases where the no. of eq^ns are less than the no. of unknowns. At this time we need to apply constraint eq^n.

String constraints :-

Along the length of string component velocities of objects must be same, as string always is tight & inextensible.

Ex:- Simple constraint



$$V_1 = -\frac{dx}{dt}$$

$$V_2 = +\frac{dy}{dt}$$

String length = L

$$x + y = L$$

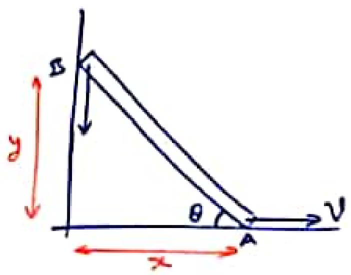
diff. w.r. to time on b.s

$$\frac{dx}{dt} + \frac{dy}{dt} = 0$$

$\Rightarrow -V_1 + V_2 \Rightarrow V_1 = V_2$

$\therefore a_1 = a_2$

Ex:- At the instant shown in fig
Find the velocity of end 'B'.



Sol:- $x^2 + y^2 = L^2$

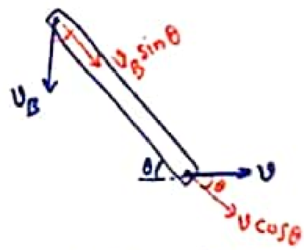
diff. w.r.to time

$$2x \cdot \frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2x \cdot V + 2y \cdot (-V_B) = 0$$

$$\Rightarrow V_B = V \cdot \frac{x}{y} = V \cdot \cot\theta$$

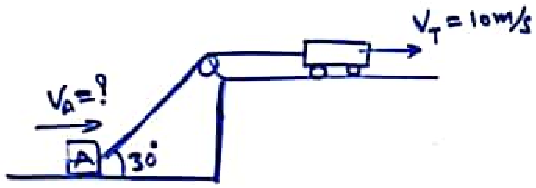
(or) As the rod is rigid, along the its length component velocities of A & B must be same.



$$V_B \sin\theta = V\cos\theta$$

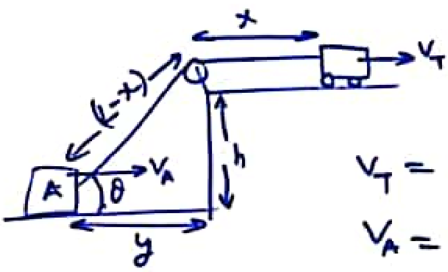
$$\boxed{V_B = V\cot\theta}$$

Q)



Find velocity of block 'A', at this instant.

Sol:-



$$V_T = +\frac{dx}{dt}$$

$$V_A = -\frac{dy}{dt}$$

→ As string is extensible, its length 'L' remains constant.

→ from fig, $y^2 + h^2 = (L-x)^2$

by differentiating w.r.to time

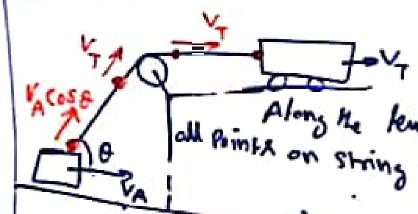
$$2y \cdot \frac{dy}{dt} + 2h\frac{dh}{dt} = 2(L-x)\left[\frac{dL}{dt} - \frac{dx}{dt}\right]$$

$$y \cdot (-V_A) + 0 = (L-x)\left[0 - V_T\right]$$

$$V_A = \frac{L-x}{y} \cdot V_T$$

$$V_A = \sec\theta \cdot V_T \Rightarrow V_A = \frac{2}{\sqrt{3}} \times 10 = \frac{20}{\sqrt{3}}\ m/s.$$

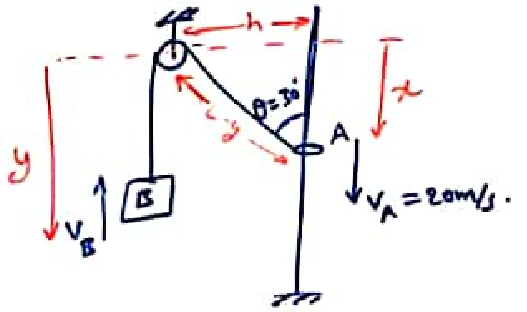
Method. II :-



Along the length of string, velocities of all points on string are same. $V_T = V_A\cos\theta$

$$\Rightarrow V_A = V_T\sec\theta = 10 \times \frac{2}{\sqrt{3}} = \frac{20}{\sqrt{3}}\ m/s$$

Q) Find V_B at this instant.



Sol:- from fig, $V_A = +\frac{dx}{dt}$, $V_B = -\frac{dy}{dt}$

$$h^2 + x^2 = (L-y)^2$$

diff w.r.t time on b.s

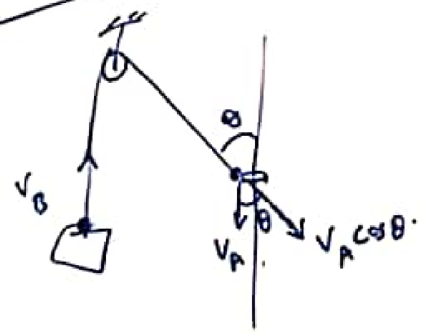
$$\Rightarrow \quad 0 + 2x \cdot \frac{dx}{dt} = 2(L-y) \cdot \left[0 - \frac{dy}{dt}\right]$$

$$\Rightarrow \quad x \cdot V_A = (L-y) \cdot V_B .$$

$$\Rightarrow \quad V_B = V_A \cdot \frac{x}{L-y} = V_A \cos\theta = 20 \times \frac{\sqrt{3}}{2}$$

$$= 10\sqrt{3} \text{ m/s}$$

Method- II



$$V_B = V_A \cos\theta$$

$$= 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ m/s}$$

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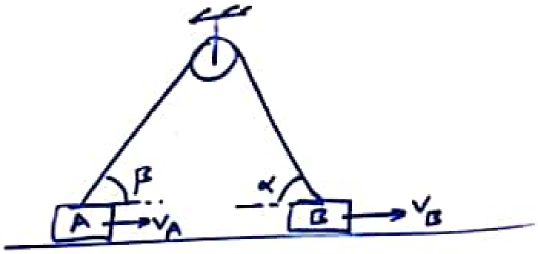
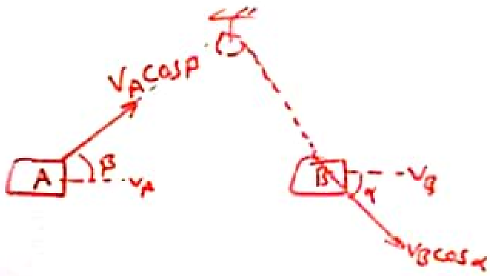
Q)



Find relation between V_A & V_B

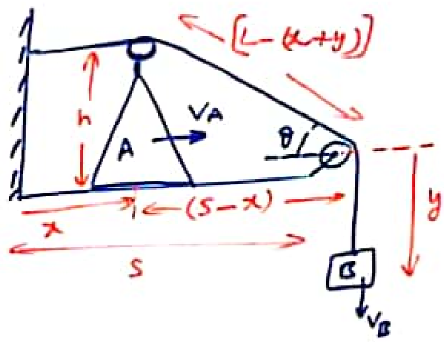
Sol :-



Along the length of string component velocities of A & B must be same.

$$\therefore \quad V_A \cos\beta = V_B \cos\alpha$$

Q)



Find relation between V_A & V_B, at this instant.

Sol :- $V_A = +\frac{dx}{dt}$, $V_B = +\frac{dy}{dt}$

from h_s, $(s-x)^2 + h^2 = [L-(x+y)]^2$

diff. w.r.to time

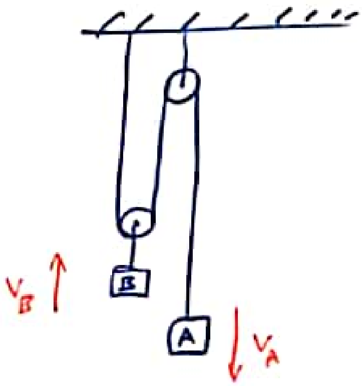
$$2(s-x)\cdot\left(0-\frac{dx}{dt}\right) + 0 = 2[L-(x+y)]\left(0-\frac{dx}{dt}-\frac{dy}{dt}\right)$$

$$(s-x)\frac{dx}{dt} = [L-(x+y)]\left(\frac{dx}{dt}+\frac{dy}{dt}\right)$$

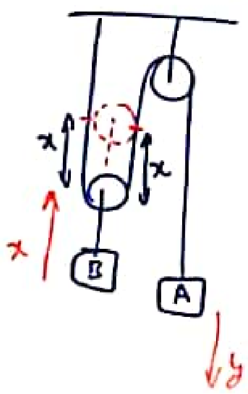
$$\Rightarrow (s-x)\cdot V_A = [L-(x+y)]\,(V_A+V_B)$$

$$\Rightarrow V_A = \sec\theta\,(V_A+V_B)$$

Q)



Sol :- displacement method:



$y = 2x$

diff. w.r.t time.

$$\frac{dy}{dt} = 2\frac{dx}{dt}$$

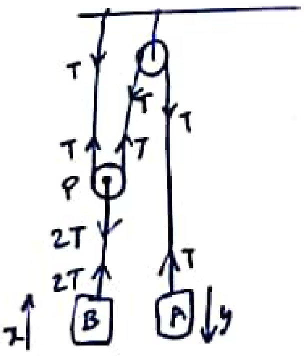
$$\Rightarrow \boxed{V_A = 2V_B}$$

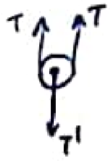
again by differentiating with time, $a_A = 2a_B$ //

Tension (or) Virtual work method:-

→ work done by internal force in a system = 0

→ work done by tension = 0.





As pulley is massless, $F_{net} = 0$

$\Rightarrow T' = 2T$ //

$W_T = 0$

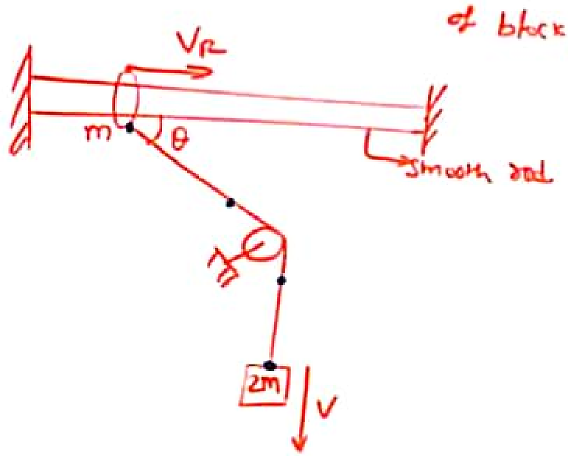
$\Rightarrow (2T\cdot x) + (-T\cdot y) = 0$

$\Rightarrow 2x = y$ ⇒ by differentiating with time

$2V_B = V_A$

→ again by differentiating, $2a_B = a_A$

Q) Find relation between velocities of block & ring



$$\frac{d}{dt}(\sec\theta) = \frac{d}{d\theta}(\sec\theta)\cdot\frac{d\theta}{dt}$$

$$= (\sec\theta\cdot\tan\theta)\frac{d\theta}{dt}$$

Sol:- Along the length of string component velocity of ring is $V_R\cos\theta$, which is equal to component of velocity of block along string

$\therefore\ V = V_R\cos\theta$

$\boxed{V_R = V\sec\theta}$; $a_R \neq a\sec\theta$

As time, θ is $\uparrow$

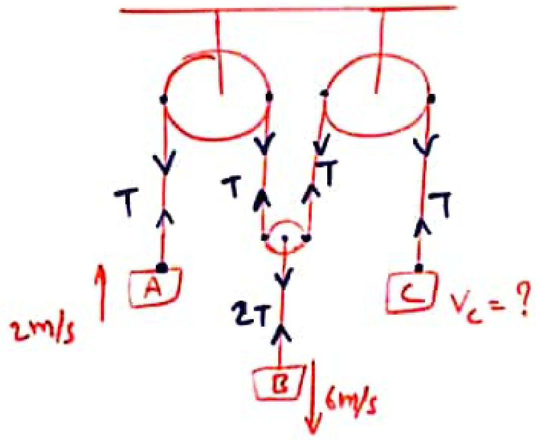
$$\frac{d}{dt}V_R = \frac{d}{dt}(V\sec\theta)$$

$$\Rightarrow \frac{d}{dt}V_R = V\frac{d}{dt}\sec\theta + \sec\theta\cdot\frac{dV}{dt}$$

$$\Rightarrow a_R = V(\sec\theta\cdot\tan\theta)\frac{d\theta}{dt} + \sec\theta\cdot a$$

$\therefore\ a_R \neq a\sec\theta$

Q) Find velocity of block 'C'.



Sol :- $\Sigma T \cdot v = 0$.

$\Rightarrow (T)(2) - (2T)(6) + T(V_c) = 0$

$\Rightarrow V_c = +10$

$\Rightarrow V_c = 10\,m/s \uparrow$

(or).

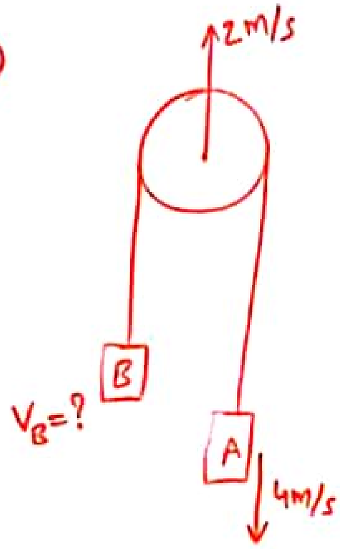
Method - II :-

$-2+0+0+6+6+0+0+\bar{V}_c = 0$

$\bar{V}_c = -10$.

$V_c = 10\,m/s \uparrow$

Q)



Find velocity of block 'B'.

Sol:-

w.r.to pulley, if one block moves up then other block moves down with same speed.

$$\Rightarrow \bar{V}_{AP} = -\bar{V}_{BP}$$

$$\Rightarrow \bar{V}_A - \bar{V}_P = -(\bar{V}_B - \bar{V}_P)$$

$$2\bar{V}_P = \bar{V}_A + \bar{V}_B$$

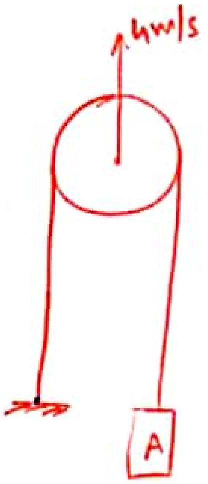
$$\boxed{\bar{V}_P = \frac{\bar{V}_A + \bar{V}_B}{2}}$$

$$+2 = \frac{-4 + \bar{V}_B}{2}$$

$$\bar{V}_B = +8 \text{ m/s}$$

$$\Rightarrow V_B = 8\text{ m/s} (\uparrow)$$

Q) Find velocity of block 'A'.



Sol :-

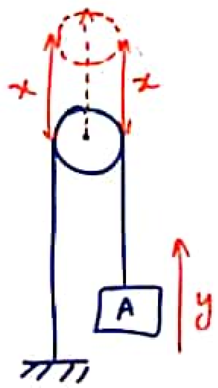
$$\bar{V}_p = \frac{\bar{V}_A + \bar{V}_B}{2}$$

$$\Rightarrow +4 = \frac{\bar{V}_A + 0}{2}$$

$$\Rightarrow \bar{V}_A = +8 \text{ m/s}$$

$$= 8 \text{ m/s} (\uparrow)$$

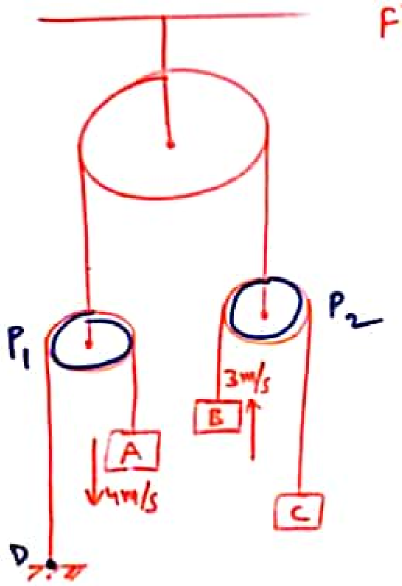
(or)



$$y = 2x.$$

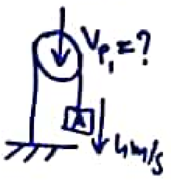
$$\Rightarrow \frac{dy}{dt} = 2\frac{dx}{dt} \Rightarrow \bar{V}_A = 2\bar{V}_p \Rightarrow V_A = 2(4) = 8 \text{ m/s}$$

Q) Find velocity of block 'C'.



Sol:- $P_1, P_2 \longrightarrow$ movable pulleys.

→ Velocities of pulleys P_1 & P_2 are equal in magnitude & opposite in direction.



$$\bar{V}_{P_1} = \frac{\bar{V}_A + \bar{V}_D}{2}$$

$$V_{P_1} = \frac{-4+0}{2}$$

$V_{P_1} = 2\,m/s\,(\downarrow)$.

$\therefore V_{P_2} = 2\,m/s\,(\uparrow)$

$$\bar{V}_{P_2} = \frac{\bar{V}_B + \bar{V}_C}{2}$$

$$\Rightarrow +2 = \frac{+3 + \bar{V}_C}{2}$$

$\Rightarrow \bar{V}_C = +1\,m/s$

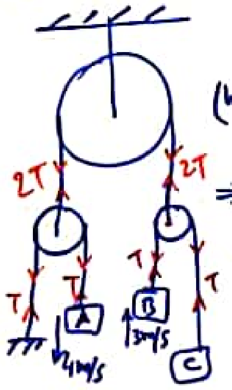
$V_C = 1\,m/s\,(\uparrow)$.

Alternate method:-

Virtual work (or) Tension method:-

→ Work done by internal force on a system is zero.

$\Sigma T \cdot x$ (or) $\Sigma T \cdot v$ (or) $\Sigma T \cdot a = 0$



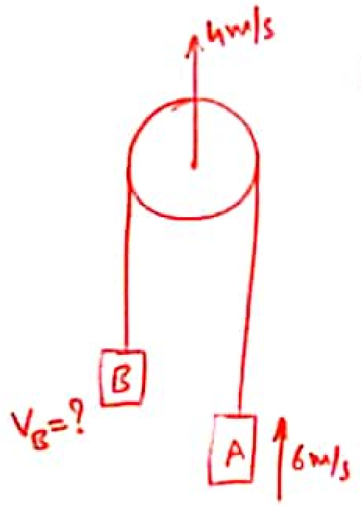
$(W_T)_A + (W_T)_B + (W_T)_C = 0$

$\Rightarrow -T4 + T \cdot 3 + TV_C = 0$

$\Rightarrow V_C = +1\,m/s$

$\therefore V_C = 1\,m/s\,(\uparrow)$

Q)



Find velocity of block 'B' w.r.to ground & velocity of block 'B' w.r.to pulley.

Sol:- w.r.to pulley, if one block moves up then other block moves down with same speed.

$$\Rightarrow \bar{V}_{AP} = -\bar{V}_{BP}$$

$$\Rightarrow \bar{V}_A - \bar{V}_P = -(\bar{V}_B - \bar{V}_P)$$

$$2\bar{V}_P = \bar{V}_A + \bar{V}_B$$

$$\boxed{\bar{V}_P = \frac{\bar{V}_A + \bar{V}_B}{2}}$$

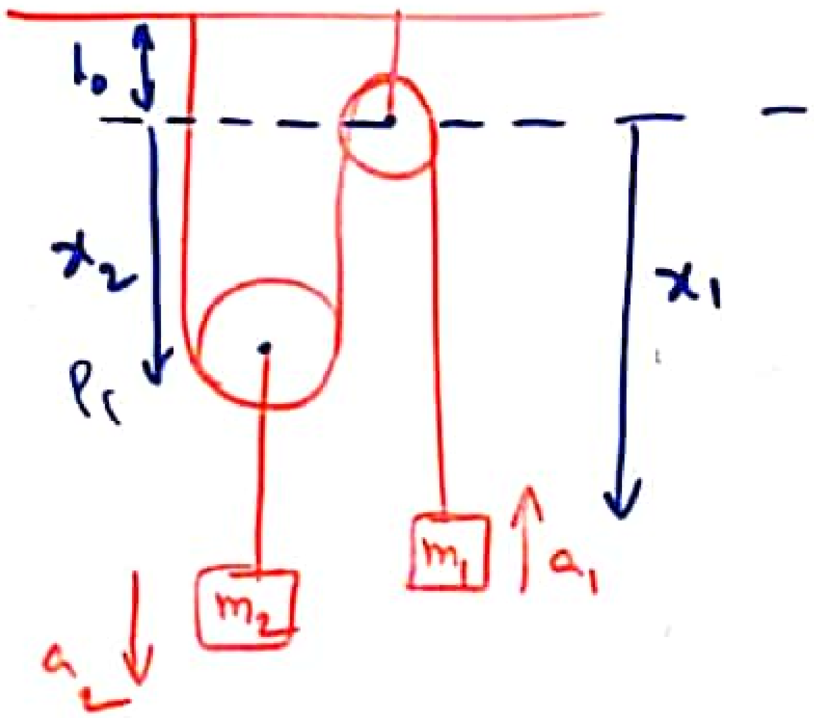
$$+4 = \frac{+6 + \bar{V}_B}{2}$$

$$\bar{V}_B = +2 \Rightarrow \bar{V}_B = 2\,m/s \uparrow \quad — (1)$$

$$\bar{V}_{BP} = \bar{V}_B - \bar{V}_P = (+2) - (+4)$$

$$= -2\,m/s = 2\,m/s \downarrow.$$

Q) Find constraint eqn.



String length $= L$

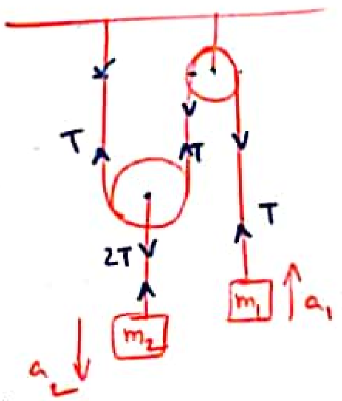
$$l_0 + 2x_2 + x_1 = L$$

$$\frac{d}{dt} l_0 + \frac{d}{dt} 2x_2 + \frac{d}{dt} x_1 = \frac{d}{dt} L$$

$$\Rightarrow 0 + 2(+v_2) + (-v_1) = 0$$

$$\Rightarrow \boxed{2v_2 = v_1} \Rightarrow 2\frac{dv_2}{dt} = \frac{dv_1}{dt} \Rightarrow \boxed{2a_2 = a_1}$$

Q) Find constraint eqⁿ.



Virtual work method:-

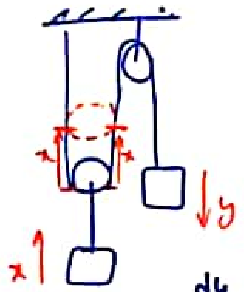
$$\Sigma T \cdot a = 0$$

$$\Rightarrow \; -2T a_2 + T a_1 = 0$$

$$\Rightarrow \; \boxed{a_1 = 2a_2}$$

displacement method:-

→ if block m_2 moves up by x, then P_1 will also move up by x & $2x$ amount of string will slack, so block m_1 falls down by $2x$ to keep the string taut.



$$y = 2x.$$

$$\frac{dy}{dt} = 2\frac{dx}{dt}.$$

$$\Rightarrow \; V_1 = 2V_2$$

$$\frac{dV_1}{dt} = 2\frac{dV_2}{dt} \Rightarrow \boxed{a_1 = 2a_2}$$

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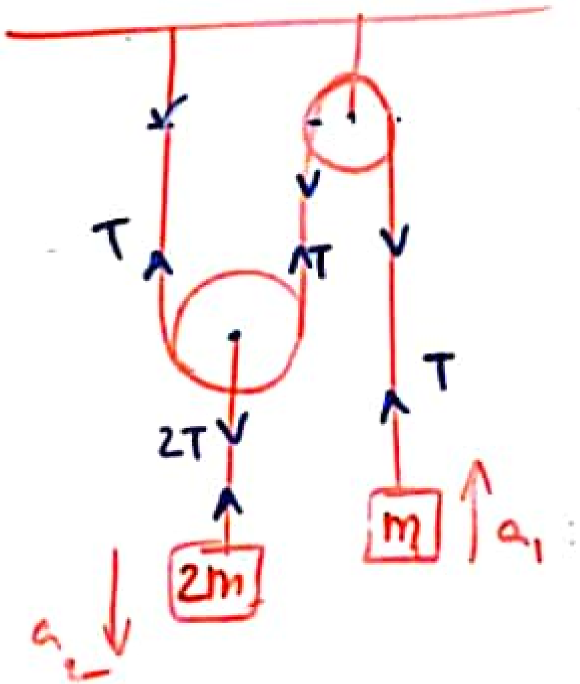
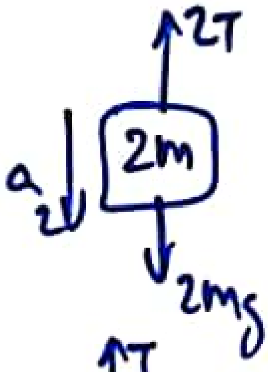
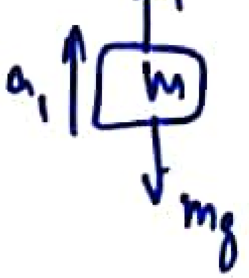
Q) Find acceleration of each block.





$$2mg - 2T = 2ma_2 \quad \text{---} \ (1)$$

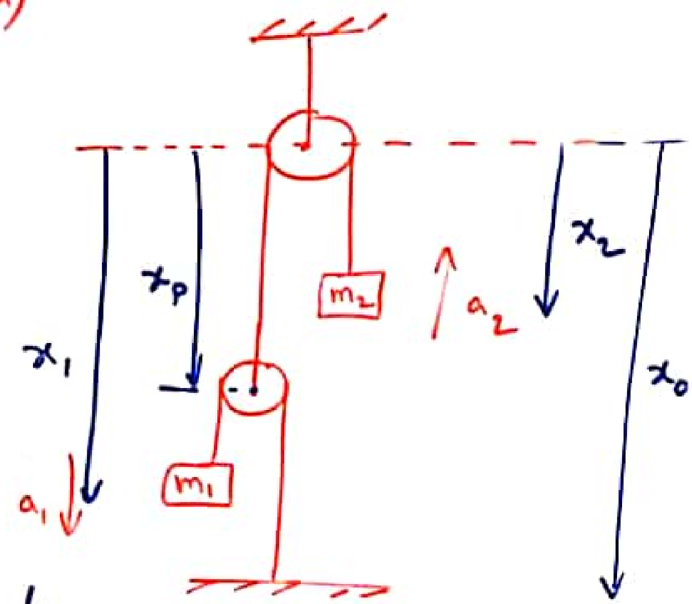


$$T - mg = ma_1 \quad \text{---} \ (2)$$

$$a_1 = 2a_2 \quad \text{---} \ (3)$$

by soln (1), (2), (3) we get a_1 & a_2

a)



Length method:-

$$x_2 + x_p = l_2 \quad \text{—— ①}$$

$$(x_1 - x_p) + (x_0 - x_p) = l_1 \quad \text{—— ②}$$

$$\Rightarrow x_1 - 2x_p + x_0 = l_1$$

$$\Rightarrow x_1 - 2(l_2 - x_2) + x_0 = l_1 \text{ (from ①)}$$

$$x_1 - 2l_2 + 2x_2 + x_0 = l_1$$

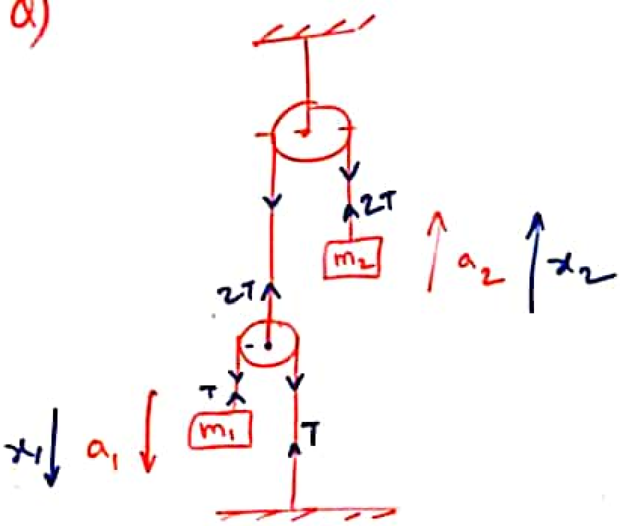
diff. w.r.t. time

$$V_1 - 0 + 2(-V_2) + 0 = 0$$

$$V_1 = 2V_2$$

$$\Rightarrow \boxed{a_1 = 2a_2}$$

a)



virtual work method:-

$$\Sigma T \cdot x = 0$$

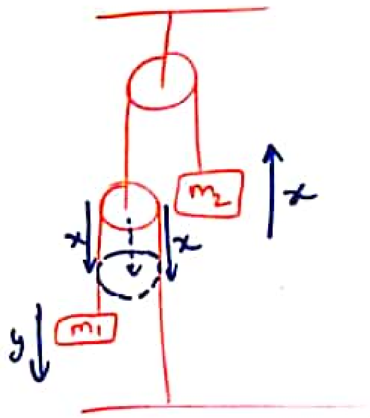
$$-T x_1 + 2T \cdot x_2 = 0$$

$$x_1 = 2x_2$$

by differentiating twice w.r.t time

$$a_1 = 2a_2$$

displacement method:-

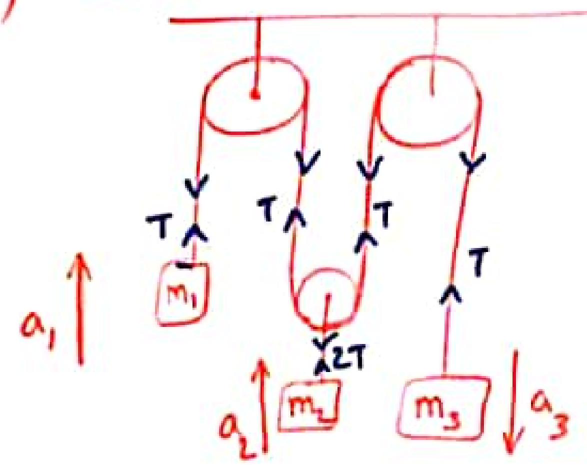


$$y = 2x$$

by differing twice w.r.t time

$$a_1 = 2a_2$$

Q) write constraint eqn.



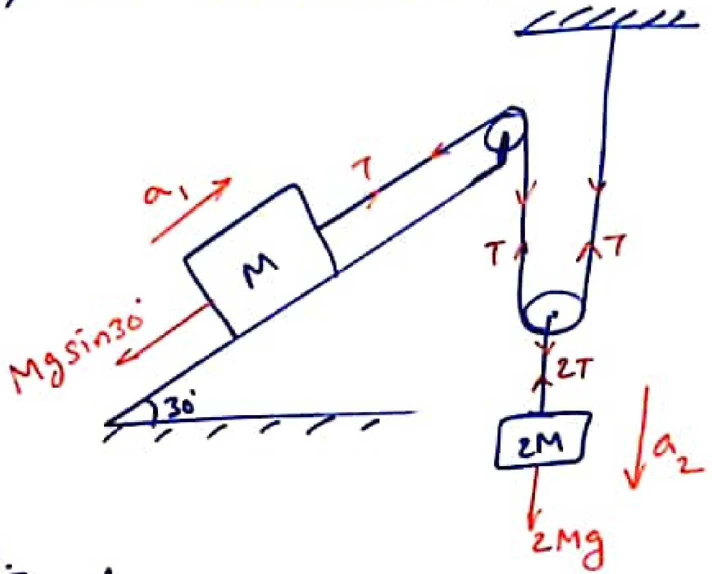
Sol:- **Virtual work method:-**

$$\Sigma T a = 0$$

$$\Rightarrow T a_1 + 2T a_2 - T a_3 = 0$$

$$\Rightarrow a_3 = a_1 + 2a_2$$

Q) Find acceleration of 'M'.



Sol:- from F.B.D of 'M'

$$T - Mg\sin 30° = Ma_1 \quad \text{--- (1)}$$

from F.B.D of 2M,

$$2Mg - 2T = 2Ma_2 \quad \text{--- (2)}$$

→ by virtual work method

$$\Sigma T \cdot a = 0 \Rightarrow Ta_1 - 2Ta_2 = 0$$

$$a_1 = 2a_2 \quad \text{--- (3)}$$

by solving above 3 eqn's, we get T, a_1, a_2

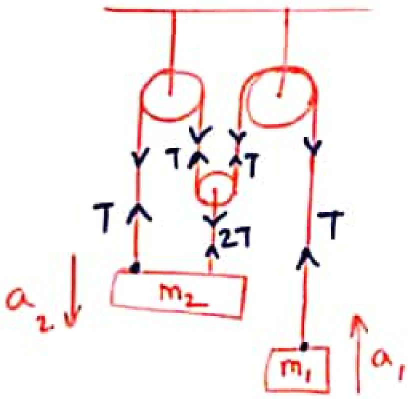
$$(1) \Rightarrow T - \frac{Mg}{2} = Ma_1$$

$$(2) \Rightarrow 2Mg - 2T = Ma_1$$

$$2\times(1) + (2) \Rightarrow Mg = 3Ma_1$$

$$\Rightarrow a_1 = g/3.$$

Q) Find constraint eqn.



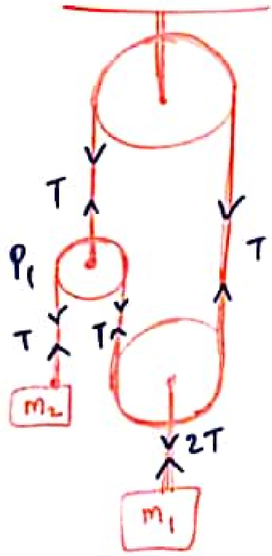
Sol:- by virtual work method,

$$\Sigma T a = 0$$

$$\Rightarrow -3Ta_2 + Ta_1 = 0$$

$$\Rightarrow \boxed{a_1 = 3a_2}$$

Q) Find accelerations of m_1 & m_2 $(m_1 > m_2)$



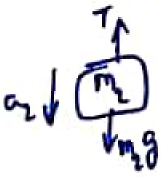
Ans:- g downward.

Sol:-

From F.B.D of movable pulley 'P_1'

$F_{net} = m a$, $m \to 0$

$2T - T = 0 \Rightarrow T = 0$

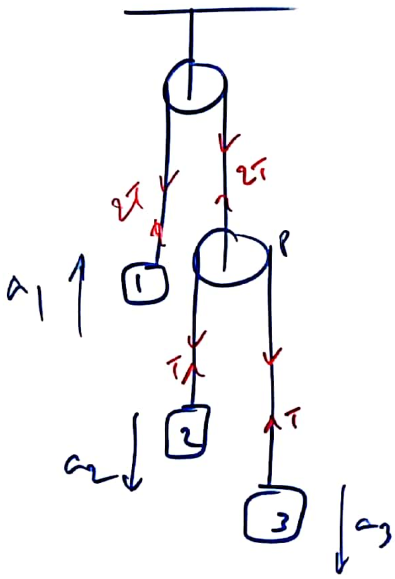


$\Rightarrow m_2 g - T = m_2 a_2 \Rightarrow a_2 = g$

$m_1 g - 2T = m_1 a_1$

$a_1 = g$

Q) Find a_1, a_2, a_3



from F.B.D's,

$$2T - g = 1.a_1 \quad \text{--- (1)}$$

$$2g - T = 2a_2 \quad \text{--- (2)}$$

$$3g - T = 3a_3 \quad \text{--- (3)}$$

$$\bar{a}_{2P} = -\bar{a}_{3P}$$

$$\bar{a}_2 - \bar{a}_P = -(\bar{a}_3 - \bar{a}_P)$$

$$2\bar{a}_P = \bar{a}_2 + \bar{a}_3$$

$$\Rightarrow 2a_1 = a_2 + a_3 \quad \text{--- (4)}$$

(2) × 3 + (3) × 2

$$\Rightarrow 12g - 5T = 12a_1$$

$$2T - g = a_1 \quad \times 2.5$$

$$9.5g = 14.5a_1$$

$$\boxed{a_1 = \frac{19g}{29}}$$

from (1) $2T = a_1 + g$

$$T = \frac{19g}{58} + \frac{g}{2}$$

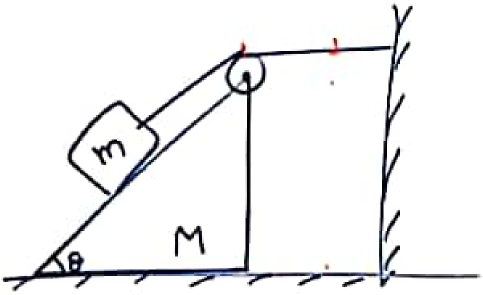
$$T = \frac{48g}{58} = \frac{24g}{29}$$

Sub. T in (2)

$$a_2 = g - \frac{T}{2} = g - \frac{24g}{58}$$

$$a_2 = \frac{34g}{58} \Rightarrow \boxed{a_2 = \frac{17g}{29}}$$

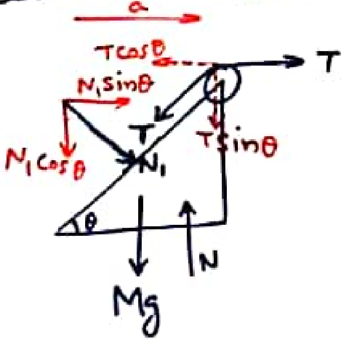
Q)



All contacts are smooth. wedge is free to move. M & m are always in contact.
Find acceleration of M.

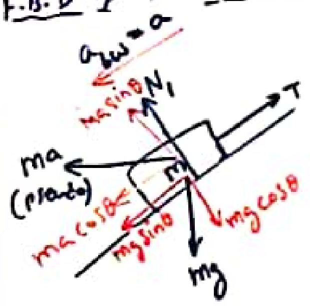
Sol:-

F.B.D of wedge.



$$(T + N_1 \sin\theta) - T\cos\theta = Ma \quad \text{—(1)}$$

F.B.D of block 'm' w.r.t wedge:-



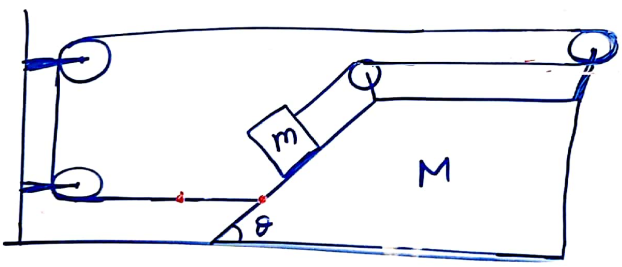
$$ma\cos\theta + mg\sin\theta - T = ma \quad \text{—(2)}$$

$$N_1 + ma\sin\theta = mg\cos\theta \quad \text{—(3)}$$

by solving (1), (2), (3), we get T, N_1 & a

$$\therefore a = \frac{mg\sin\theta}{M + 2m(1-\cos\theta)}$$

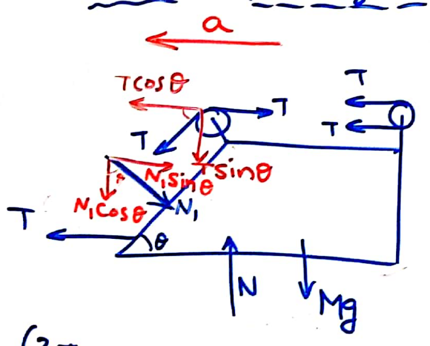
Q)



All contacts are smooth.
Find acceleration of wedge 'M' & tension in string.

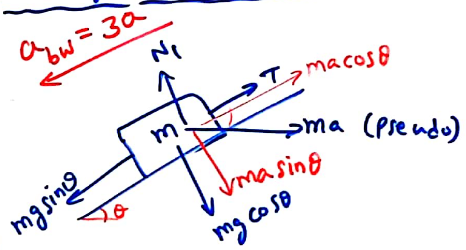
Ans:- $\boxed{a = \dfrac{2mg\sin\theta}{M + m(7 + 5\cos\theta)}}$

Sol:- F.B.D of wedge 'M'.



$$(3T + T\cos\theta) - (T + N_1\sin\theta) = Ma \quad \text{—— (1)}$$

F.B.D of block 'm' w.r.to wedge :-



$$mg\sin\theta - (T + ma\cos\theta) = m(3a) \quad \text{—— (2)}$$

$$N_1 = ma\sin\theta + mg\cos\theta \quad \text{—— (3)}$$

by solving (1), (2), (3), we get N_1, T, a

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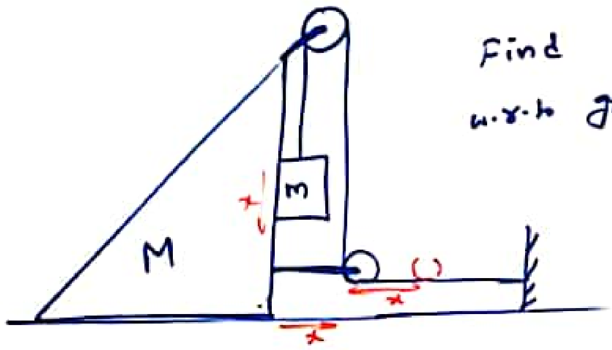
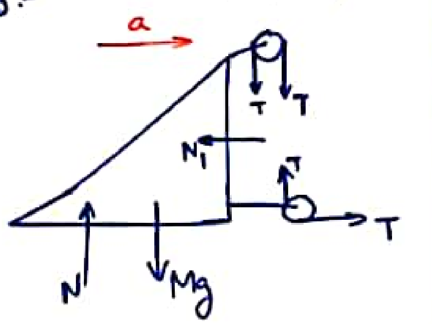
Q) Find acceleration of block 'm' w.r.t ground.



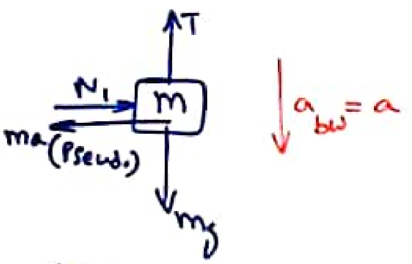
Sol:-

F.B.D of wedge 'M'



$$T - N_1 = Ma \quad \text{—} (1)$$

F.B.D of block 'm', w.r.t wedge,



$$mg - T = ma \quad \text{—} (2)$$

$$N_1 = ma \quad \text{—} (3)$$

Sub (3) in (1) $T - ma = Ma$

(2) $mg - T = ma$

$$mg = (M+2m)a$$

$$\Rightarrow a = \frac{mg}{M+2m}$$

$$\therefore \bar{a}_b = \bar{a}_{bw} + \bar{a}_w = -a\hat{j} + a\hat{i}$$

$$a_b = a\sqrt{2} = \sqrt{2}\,mg/(M+2m)$$

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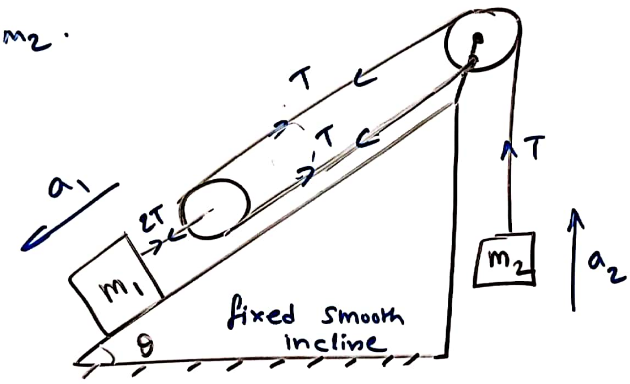
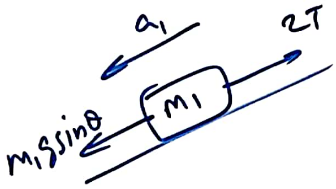
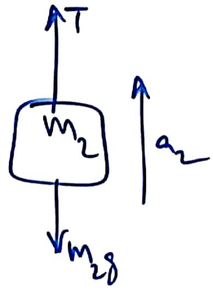
Q) Find accelerations of m_1 & m_2.



Sol:-





$$m_1 g \sin\theta - 2T = m_1 a_1 \quad \text{—} (1)$$

$$T - m_2 g = m_2 a_2 \quad \text{—} (2)$$

by virtual work method,

$$\Sigma T a = 0 \Rightarrow T a_2 - 2T a_1 = 0 \Rightarrow a_2 = 2a_1 \quad \text{—} (3)$$

(1) + (2) × 2

$$\Rightarrow m_1 g \sin\theta - 2m_2 g = m_1 a_1 + 2m_2 a_2$$

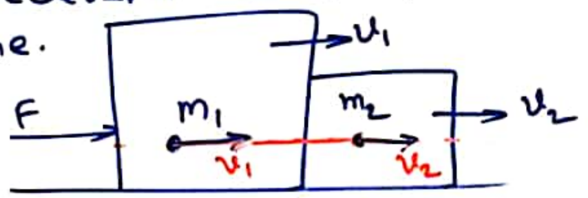
$$\Rightarrow m_1 g \sin\theta - 2m_2 g = m_1 a_1 + 4m_2 a_1$$

$$\Rightarrow m_1 g \sin\theta - 2m_2 g = (m_1 + 4m_2) a_1$$

$$\therefore a_1 = \frac{m_1 g \sin\theta - 2m_2 g}{m_1 + 4m_2}$$

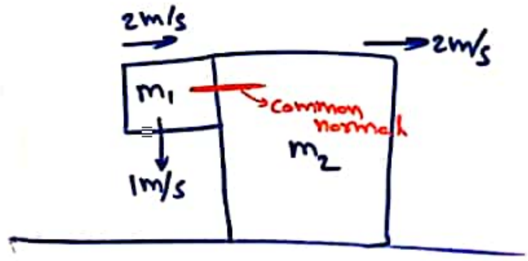
Normal constraints :-

→ With out loosing contact between two objects, along the common normal displacement (or) velocity (or) acceleration components of two objects must be same.

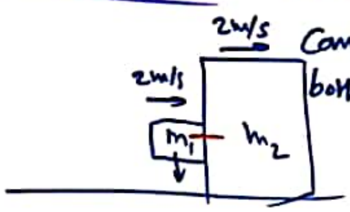


$v_1 = v_2$

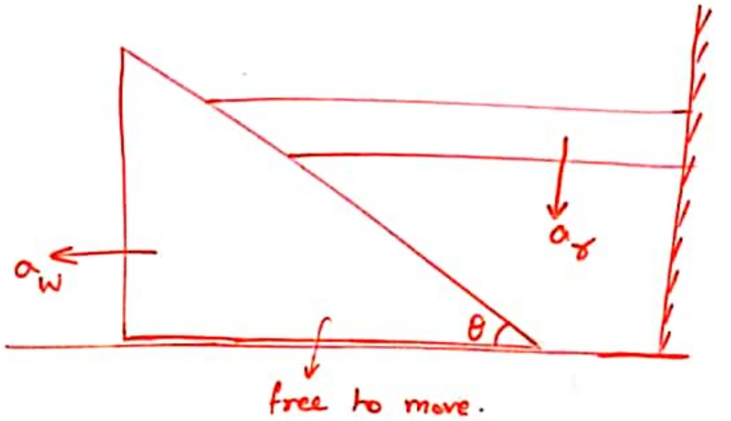
Ex:-



In this case bodies will not loose contact, since along common normal velocities of both objects are same.



Q)

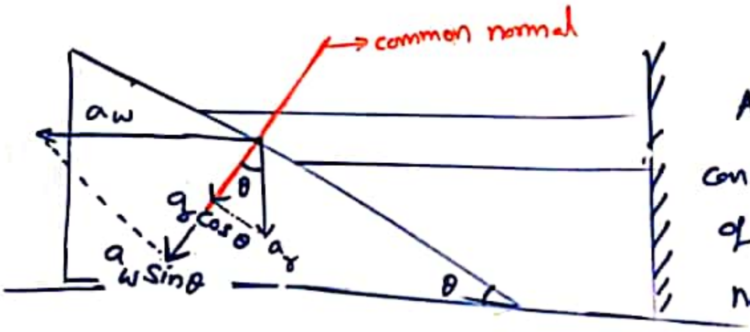


Find the relation between their accelerations.

$a_r \rightarrow$ acceleration of rod

$a_w \rightarrow$ acceleration of wedge.

Sol:-

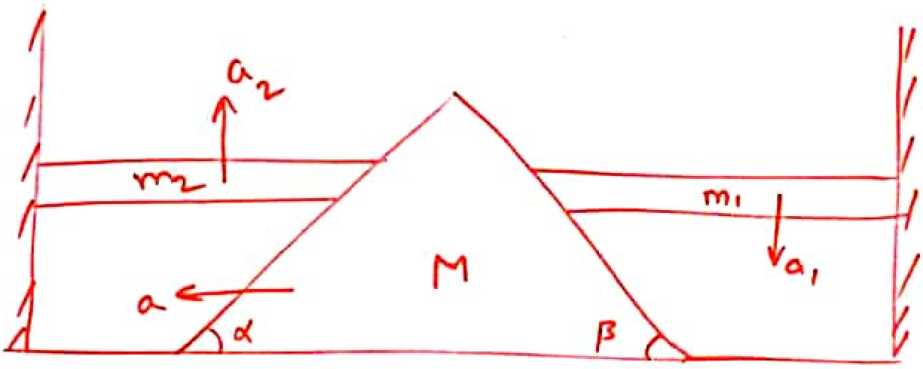


As the rod is not loosing contact with wedge, components of a_r & a_w along common normal must be same.

$$\therefore a_r\cos\theta = a_w\sin\theta$$

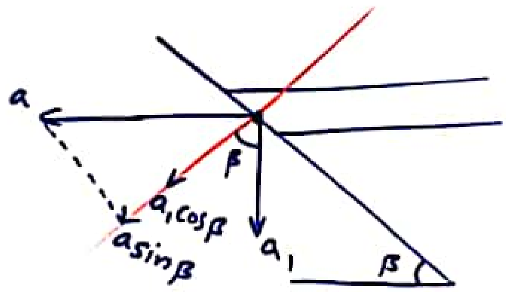
$$\Rightarrow \boxed{a_r = a_w\tan\theta}$$

Q)



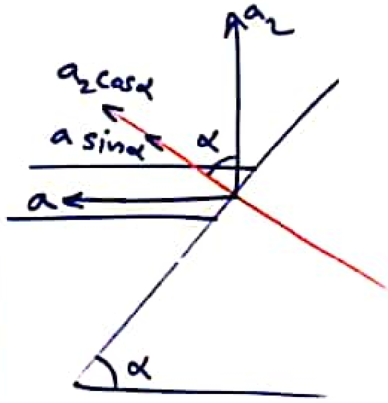
Find relation between a_1 & a and a & a_2.

Sol :-



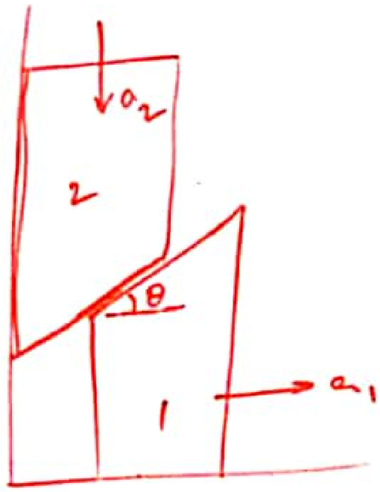
$$a_1 \cos\beta = a \sin\beta$$

$$a = a_1 \cot\beta \quad \text{—— (1)}$$



$$a_2 \cos\alpha = a \sin\alpha \Rightarrow a = a_2 \cot\alpha.$$

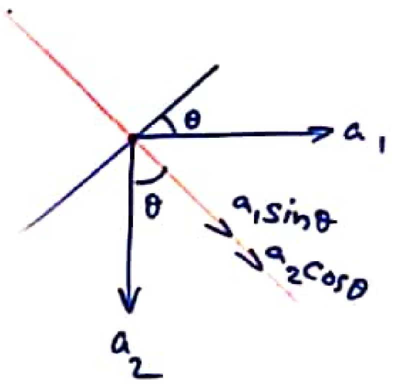
Q)



Find relation between

a_1 & a_2

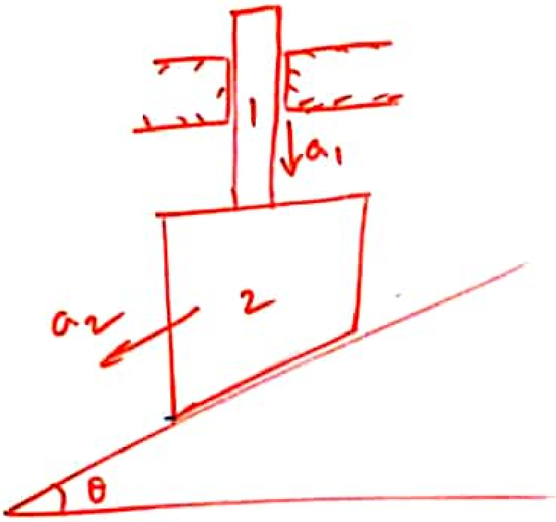
Sol :-



$$a_1 \sin\theta = a_2 \cos\theta.$$

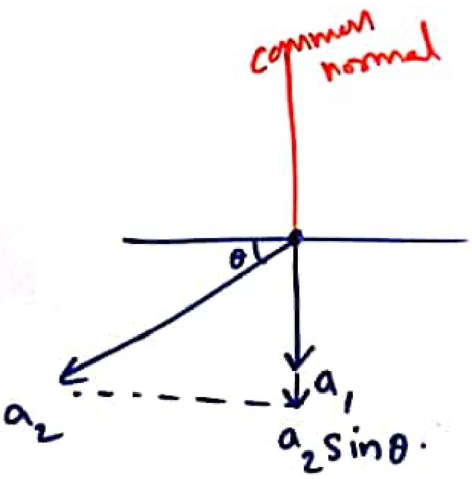
$$\underline{\underline{a_2 = a_1 \tan\theta}}$$

Q)



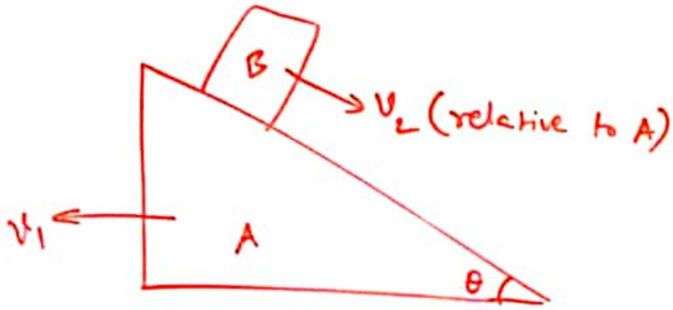
Find relation between a_1 & a_2

Sol :-



$$a_1 = a_2 \sin\theta$$

Q)



Find velocity of 'B' w.r. to ground.

Sol:-

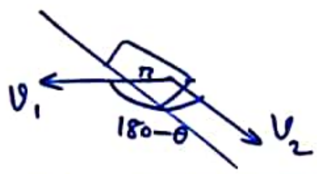
$$\overline{V}_{BW} = \overline{V}_B - \overline{V}_W$$

$$\overline{V}_B = \overline{V}_{BW} + \overline{V}_W$$

$$= \left(v_2\cos\theta\,\hat{i} - v_2\sin\theta\,\hat{j}\right) + \left(-v_1\,\hat{i}\right)$$

$$= \left(v_2\cos\theta - v_1\right)\hat{i} - v_2\sin\theta\,\hat{j}$$

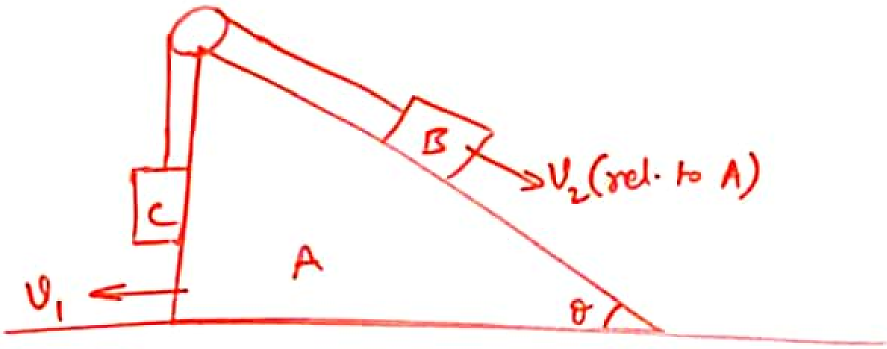
$$v_B = \sqrt{\left(v_2\cos\theta - v_1\right)^2 + \left(v_2\sin\theta\right)^2}$$



$$v_B = \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos(180-\theta)}$$

$$= \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$$

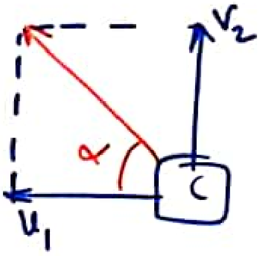
Q)



Find velocity of 'C' w.r. to ground.

Sol:-

$$\bar{V}_{CW} = V_2 \hat{j}$$



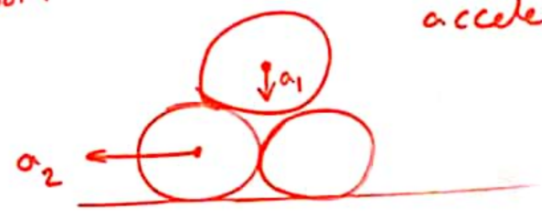
$$\bar{V}_C = \bar{V}_{CW} + \bar{V}_W$$

$$= V_2 \hat{j} - V_1 \hat{i}$$

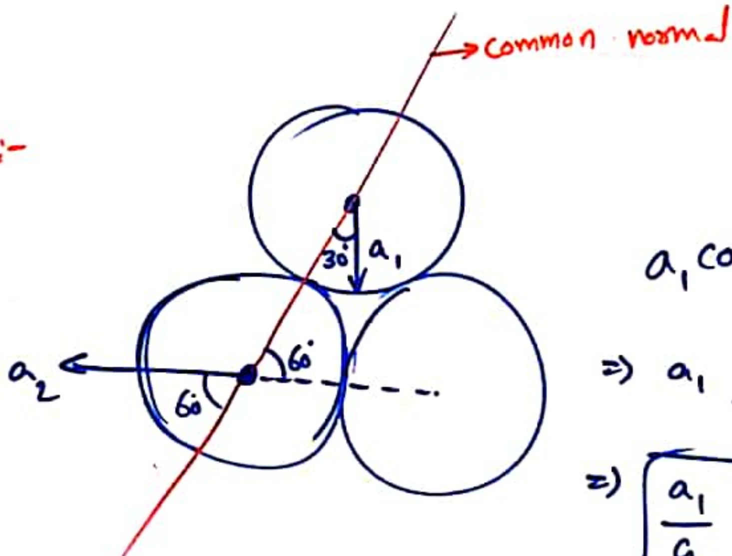
$$V_C = \sqrt{V_2^2 + V_1^2}$$

$$\tan\alpha = \frac{V_2}{V_1}$$

Q) Three identical cylinders are kept on horizontal surface. Find relation between accelerations a_1 & a_2.



Sol:-

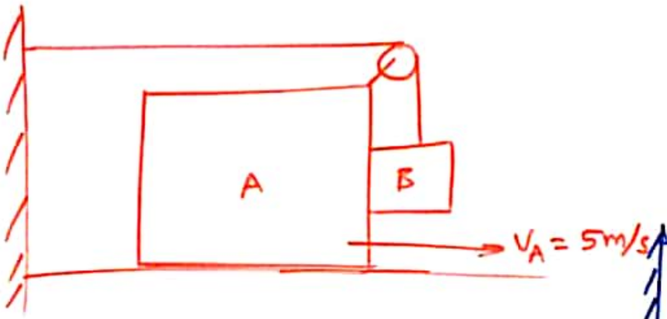


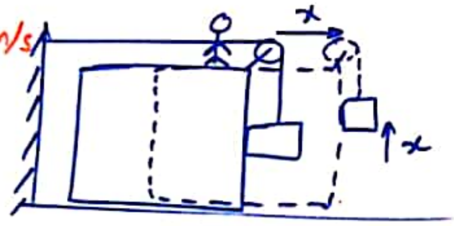
$$a_1 \cos 30^\circ = a_2 \cos 60^\circ$$

$$\Rightarrow a_1 \frac{\sqrt{3}}{2} = a_2 \frac{1}{2}$$

$$\Rightarrow \boxed{\frac{a_1}{a_2} = \frac{1}{\sqrt{3}}}$$

Q) Find velocity of 'B' w.r.to ground.





Sol:- → w.r.to A,

Velocity of B is upward = 5 m/s.

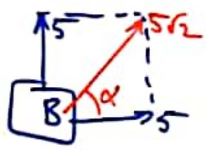
$$\bar{V}_{BA} = \bar{V}_B - \bar{V}_A$$

$$\Rightarrow \quad 5\hat{j} = \bar{V}_B - 5\hat{i}$$

$$\bar{V}_B = 5\hat{j} + 5\hat{i}$$

Velocity of 'B' w.r.to ground.

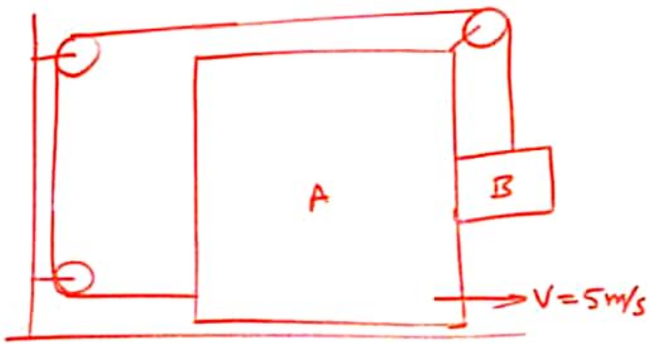
$$V_B = \sqrt{5^2 + 5^2} = 5\sqrt{2} \text{ m/s.}$$



$\tan\alpha = \frac{5}{5}$

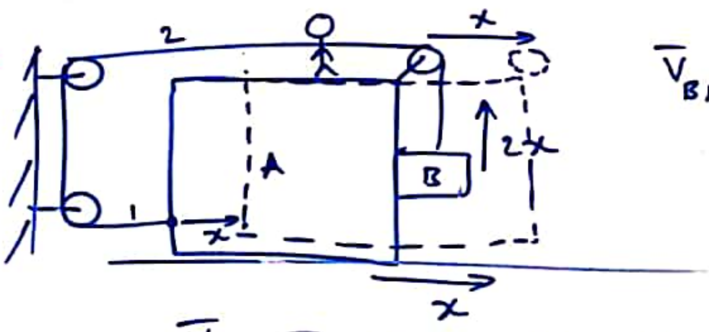
$\alpha = 45^\circ$

Q)



Find velocity of 'B' w.r.to ground.

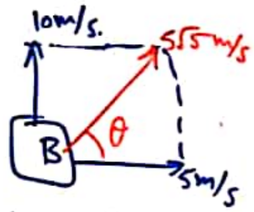
Sol:-



$\bar{V}_{BA} = 2(V_A)\uparrow = 10\ m/s\uparrow$

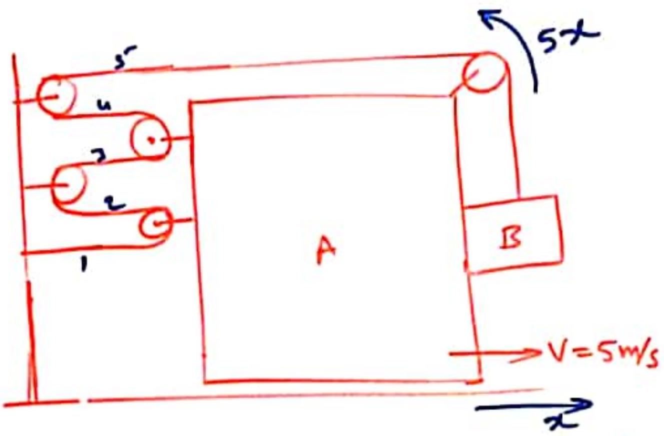
$\bar{V}_B = \bar{V}_{BA} + \bar{V}_A = 10\hat{j} + 5\hat{i}$

$V_B = \sqrt{100+25} = 5\sqrt{5}\ m/s.$



$\tan\theta = \frac{10}{5} = 2$

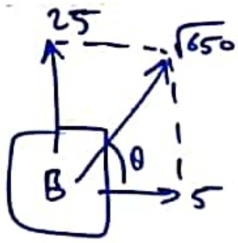
Q)



Find velocity of 'B' w.r.to ground.

Sol:- velocity of 'B' w.r.to A is

$$\bar{V}_{BA} = 5V_A \uparrow = 25\,m/s \uparrow$$



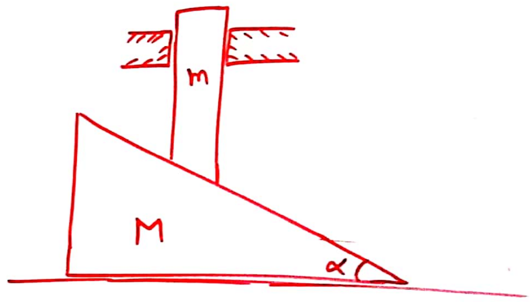
vel. of 'B' w.r.to ground is

$$\bar{V}_B = \bar{V}_{BA} + \bar{V}_A = 25\hat{j} + 5\hat{i}$$

$$\dot{V}_B = \sqrt{25^2 + 5^2} = \sqrt{650}\ m/s$$

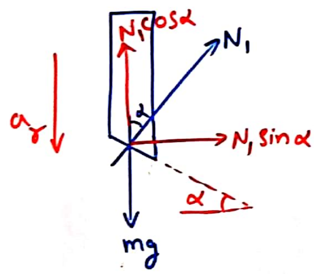
$$\tan\theta = \frac{25}{5} = 5$$

Q) Find acceleration of rod A & wedge B in the arrangement shown in fig. All contact surfaces are smooth.

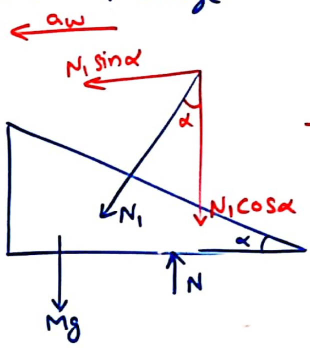


Sol:-

F.B.D of rod



F.B.D of wedge.



$$mg - N_1\cos\alpha = m a_r \quad \text{—(1)}$$

$$N_1\sin\alpha = M a_w \quad \text{—(2)}$$

→ by normal constraint, $a_w \sin\alpha = a_r\cos\alpha \Rightarrow \frac{a_w}{a_r} = \cot\alpha$ —(3)

$$\frac{(1)}{(2)} \Rightarrow \frac{N_1\cos\alpha = mg - ma_r}{N_1\sin\alpha = Ma_w} \Rightarrow \cot\alpha = \frac{mg - ma_r}{Ma_r\cot\alpha} \Rightarrow Ma_r\cot^2\alpha = mg - ma_r$$

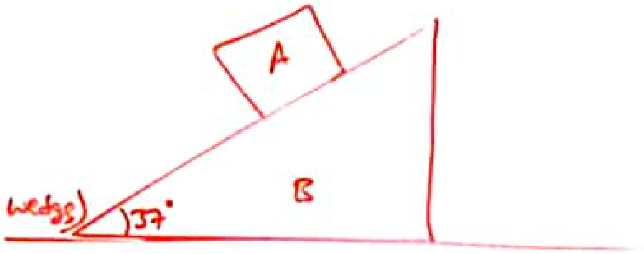
$$\boxed{a_r = \frac{mg}{m + M\cot^2\alpha}} \Rightarrow \boxed{a_w = \frac{mg\cot\alpha}{m + M\cot^2\alpha}}$$

Q)

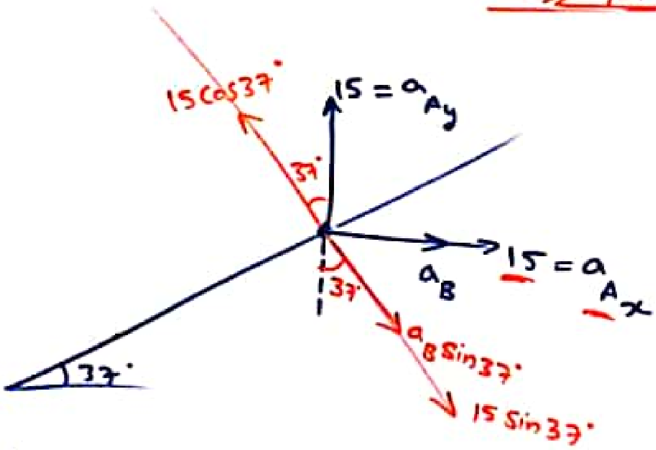
If $\bar{a}_A = 15\hat{i} + 15\hat{j}$

Find $\bar{a}_B$.

(block is always in contact with wedge)



Sol:-



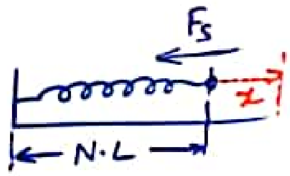
along common normal, components of a_A & a_B must be same.

$$15\sin 37^\circ - 15\cos 37^\circ = a_B \sin 37^\circ$$

$$\Rightarrow 9 - 12 = a_B \frac{3}{5} \Rightarrow a_B = -5 \Rightarrow \bar{a}_B = -5\hat{i}$$

Springs :-

spring force $\bar{F}_s = -k\bar{x}$



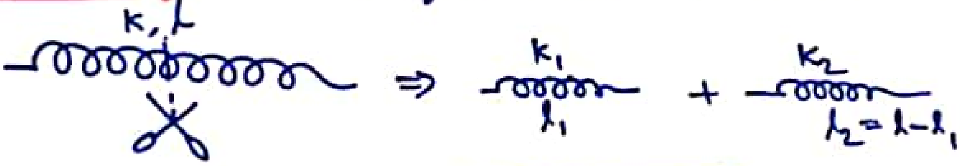
$$F_s = kx$$

where $k \longrightarrow$ spring (or) force constant.

→ k is independent on F_s & x.

→ k depends on nature of material of spring & length of spring.

Cutting of spring :-



$$kl = \text{const} \Rightarrow \boxed{kl = k_1 l_1 = k_2 l_2}$$

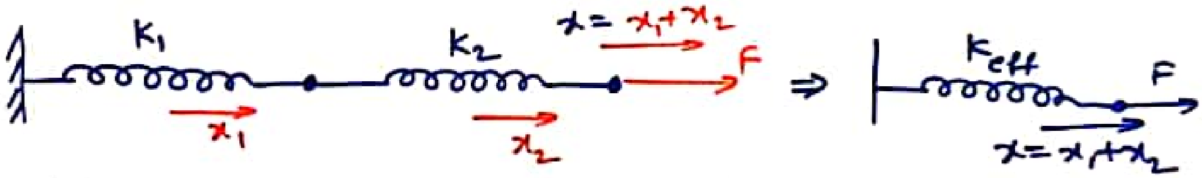
$$\boxed{k_1 = \frac{kl}{l_1}} \quad \& \quad \boxed{k_2 = \frac{kl}{l_2}}$$

→ If a spring of spring constant K, is cut into two equal parts, then spring constant of each part becomes $2K$

$$K l = K' \cdot \frac{l}{2} \Rightarrow K' = 2K.$$

(ideal spring has no mass)

Series combination of springs:-



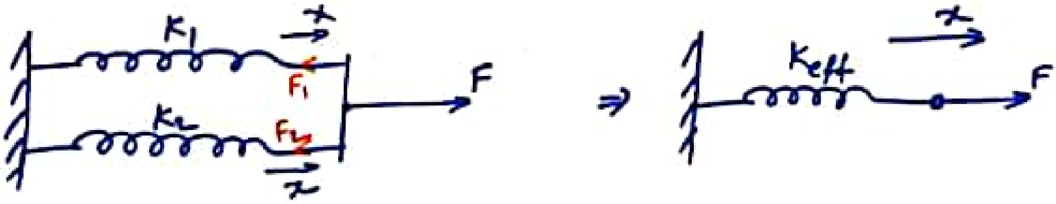
As the springs are ideal, in both the springs, same amount of restoring force will develop.

$$k_1 x_1 = k_2 x_2 = k_{eff}\, x = F$$

→ $x = x_1 + x_2$

$$\Rightarrow \frac{F}{k_{eff}} = \frac{F}{k_1} + \frac{F}{k_2} \Rightarrow \boxed{\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}}$$

Parallel combination of Springs :-



In this both springs are extend by same amount but restoring forces developed in springs are different.

$$F = F_1 + F_2$$

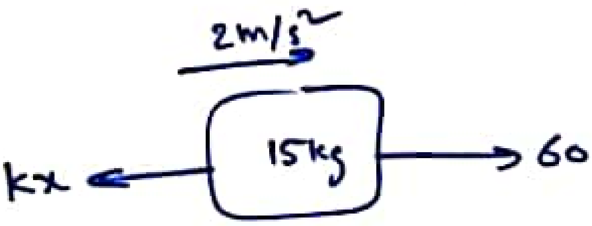
$$\Rightarrow \quad k_{eff} \cdot x = k_1 x + k_2 x$$

$$\Rightarrow \quad \boxed{k_{eff} = k_1 + k_2}$$

Q) If acceleration of 15 kg mass at an instant is $2 m/s^2$, find acceleration of 10 kg at that instant.

Sol :-

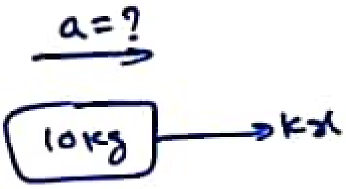
F.B.D of 15 kg



$$60 - kx = 15(2)$$

$$\Rightarrow kx = 30$$

F.B.D of 10 kg

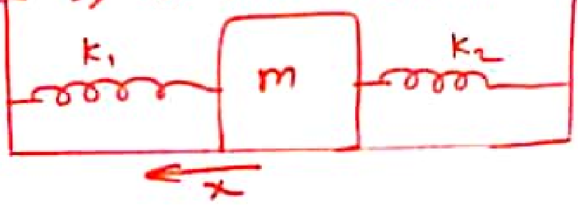


$$kx = 10a$$

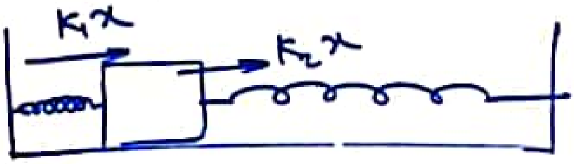
$$\Rightarrow 30 = 10a$$

$$\Rightarrow a = 3 m/s^2$$

Q) Initial springs are in natural length. Now block is displaced by x from mean position & released. Find acceleration of block just after released.



Sol:-



$$F_{net} = (k_1 + k_2)x$$

$$\Rightarrow ma = (k_1 + k_2)x.$$

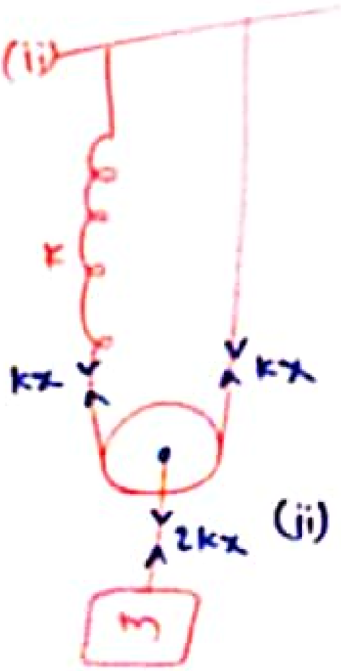
$$a = \left(\frac{k_1 + k_2}{m}\right)x.$$

Note:- here both spring are connected in Parallel combination.

Q) Block 'm' is in equilibrium, Find elongation in spring

(i)

(ii)

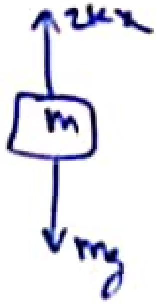


Sol:- (i)

$kx = mg$

$x = \frac{mg}{k}$

(ii)

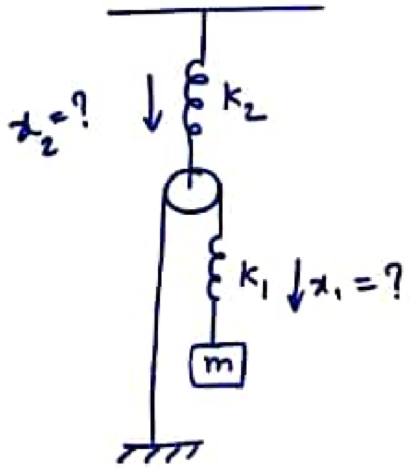


$2kx = mg$

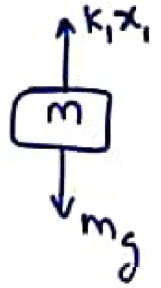
$x = \frac{mg}{2k}$

Q) Find extension in springs.

block m is in equilibrium.

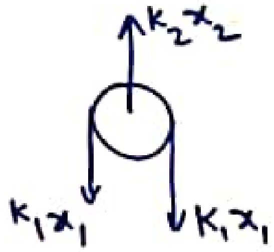


Sol:-



$$k_1 x_1 = mg$$

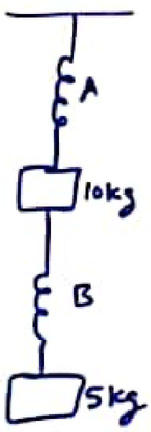
$$x_1 = \frac{mg}{k_1}$$



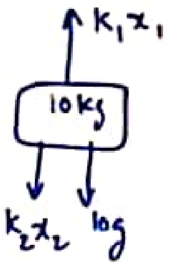
$$k_2 x_2 = 2 k_1 x_1$$

$$x_2 = \frac{2k_1}{k_2} \cdot \frac{mg}{k_1} = \frac{2mg}{k_2}$$

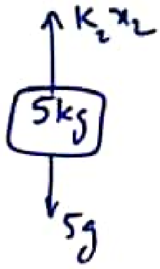
Q) System is in equilibrium. If spring 'A' breaks then find acceleration of each block just after spring A breaks.



Sol:-



$K_1x_1 = 150$



$K_2x_2 = 50$

As spring 'A' breaks

$$a_{10kg} = \frac{K_1x_1}{10} \downarrow$$

$$= \frac{150}{10} = 15\,m/s^2 \downarrow$$

$$a_{5kg} = \frac{K_2x_2 - 5g}{5} = 0$$