시계열분석의 이해 과제 풀이

#1.

$$S(\beta_0) = (\beta_0 - 1.5)^2 + (\beta_0 - 1.7)^2 + (\beta_0 - 2.1)^2 + (\beta_0 - 1.9)^2$$

$$= 4\beta_0^2 - (3 + 3.4 + 4.2 + 3.8)\beta_0 + C$$

$$= 4\beta_0^2 - 14.4\beta_0 + C$$

$$\frac{dS}{d\beta_0} = 8\beta_0 - 14.4 = 0$$

$$\Rightarrow \hat{\beta}_0 = 1.8$$

$\blacksquare$

$-12\beta_0$

-16

-18

-30

-20

-112

-120

-32

-192

-224

#2. $f_x = 2x + 5y$

$f_y = 5x + 12y$

$\blacksquare$

#3.

Sol 1)

$$S(\beta_0, \beta_1) = (\beta_0 - 6)^2 + (\beta_0 + 2\beta_1 - 8)^2 + (\beta_0 + 4\beta_1 - 9)^2 + (\beta_0 + 6\beta_1 - 10)^2$$

$$= 4\beta_0^2 + 56\beta_1^2 - 66\beta_0 - 224\beta_1 + 24\beta_0\beta_1 + C$$

$$\frac{\partial S}{\partial \beta_0} = 8\beta_0 - 66 + 24\beta_1 = 0$$

$$\frac{\partial S}{\partial \beta_1} = 112\beta_1 - 224 + 24\beta_0 = 0$$

$\Rightarrow$

$$112\beta_1 = 224 - 24\beta_0$$

$$= 224 - 198 + 72\beta_1$$

$$40\beta_1 = 26$$

$$\Rightarrow \beta_1 = 0.65$$

$$\Rightarrow 8\beta_0 - 66 + 15.6$$

$$\Rightarrow \beta_0 = 6.3$$

Sol 2) $M = \begin{pmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 4 \\ 1 & 6 \end{pmatrix}$

$$M^T = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 \end{pmatrix}$$

$$M^T M = \begin{pmatrix} 4 & 12 \\ 12 & 56 \end{pmatrix}$$

$$(M^T M)^{-1} M^T \begin{pmatrix} 6 \\ 8 \\ 9 \\ 10 \end{pmatrix} = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{pmatrix}$$