# Muon Decay and Charge-Exchange Simulation Report

## 1. Introduction

This report describes a Monte Carlo simulation developed in Python to model the decay of muons and their transitions between charge states in matter. The model focuses on the exponential decay behavior of muons, stochastic charge-exchange processes between Mu⁺ and Mu⁰ states, spin phase evolution due to magnetic precession, and beam profile effects. The simulation is intended for research and educational purposes, offering insights into muon dynamics and the use of statistical physics methods in computational modeling.

## 2. Simulation Overview

The simulation consists of two major components:  
1. Muon decay and charge exchange modeled as a time-ordered stochastic process.  
2. A beam model estimating muon stopping depth from energy and angle distributions.  
  
The muon spin dynamics are modeled through accumulation of phase during precession, affected by whether the muon is in the Mu⁺ or Mu⁰ state. Decay times are sampled from an exponential distribution, and events are weighted by their spin asymmetry signal.

## 3. Methodology

Muon decay times are generated using inverse transform sampling, a common technique in Monte Carlo methods. The simulation includes random transitions between Mu⁺ and Mu⁰ states based on predefined Poisson process rates. Phase accumulation is computed across each interval until the muon decays. The simulation logs each event's decay time and spin phase, which is used to compute a time-weighted histogram representing the experimental signal observed in muon spin rotation (μSR) experiments.

## 4. Monte Carlo Techniques Used

The following Monte Carlo methods are applied in this simulation:  
- Inverse transform sampling for muon decay times.  
- Poisson-distributed waiting times for state transitions (Senba's method).  
- Random selection of precession frequencies during Mu⁰ states.  
- Weighted histogramming of decay events with (1 + A·cos(φ)) weighting.

## 5. Beam Profile Simulation

A simple beam simulation module is included to demonstrate how muon stopping depths depend on initial energy and incident angle. Energy is sampled from a Gaussian distribution centered at 4 MeV, and angle from a narrow Gaussian distribution. Stopping range is estimated using a power-law approximation based on continuous slowing down approximation (CSDA) principles.

## 6. References

1. Particle Data Group (PDG), "Monte Carlo Techniques," https://pdg.lbl.gov  
2. M. Senba, "Monte Carlo Simulation of Muonium Formation and Destruction in Gases," Hyperfine Interactions, 1991.  
3. J.H. Brewer, "Muon Spin Rotation/Relaxation/Resonance," Encyclopedia of Applied Physics, 1994.  
4. D.E. Groom et al., "Muon Stopping Powers and Ranges in Materials," Atomic Data and Nuclear Data Tables, 2001.  
5. Geant4 Collaboration, "Physics Reference Manual," CERN, 2020.