# HYPOTHESIS TESTING APPLIED TO SEGREGATING A AND B STUDENTS

EE5500 Course Project3

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## 1. Derivation of Probability Mass Function & Hypothesis Test for Section A

This section provides sufficient explanation on the steps employed in obtaining the probability mass function (pmf) for the 5 questions with a maximum score of 100 points and hypothesis test to determine scores within the decision region of an A Student and B Student.

### Procedures of PMF for Student A and Student B

I. The piecewise function for the first question was obtained as provided below:

```
fx_A1 = piecewise(s>= 0 & s <=8, 0.01, s >= 9 & s <=19, 0.01*s - 0.07,
s==20, 0.14);
```

II. In order to obtain the pmf for the subsequent 4 questions, taking cognizance of the fact that every question is independent, hence the total PMF is the convolution of the 5 PMF.

```
fx_Aval = eval(subs(fx_A1,s, scores));
fx2a = conv(fx_Aval,fx_Aval);
fx3a = conv(fx2a, fx_Aval);
fx4a = conv(fx3a, fx_Aval);
fx_A = conv(fx4a, fx_Aval)';
```

III. The final PMF of A shows the probability of A Students obtaining scores between 0 – 100

IV. The PMF for B Students was also obtained following steps 1 – 3.

```
fx_B = piecewise(s>= 0 & s <=1, 0.01, s >= 2 & s <=5, 0.02, s>5 & s<11,
0.01*s - 0.03..., s>= 11 & s <=15, 0.08, s>15 & s<=20, 0.23 - 0.01*s);
```

```
fx_Bval = eval(subs(fx_B,s, scores));
fx2b = conv(fx_Bval,fx_Bval);
fx3b = conv(fx2b, fx_Bval);
fx4b = conv(fx3b, fx_Bval);
fx_B = conv(fx4b, fx_Bval);
```

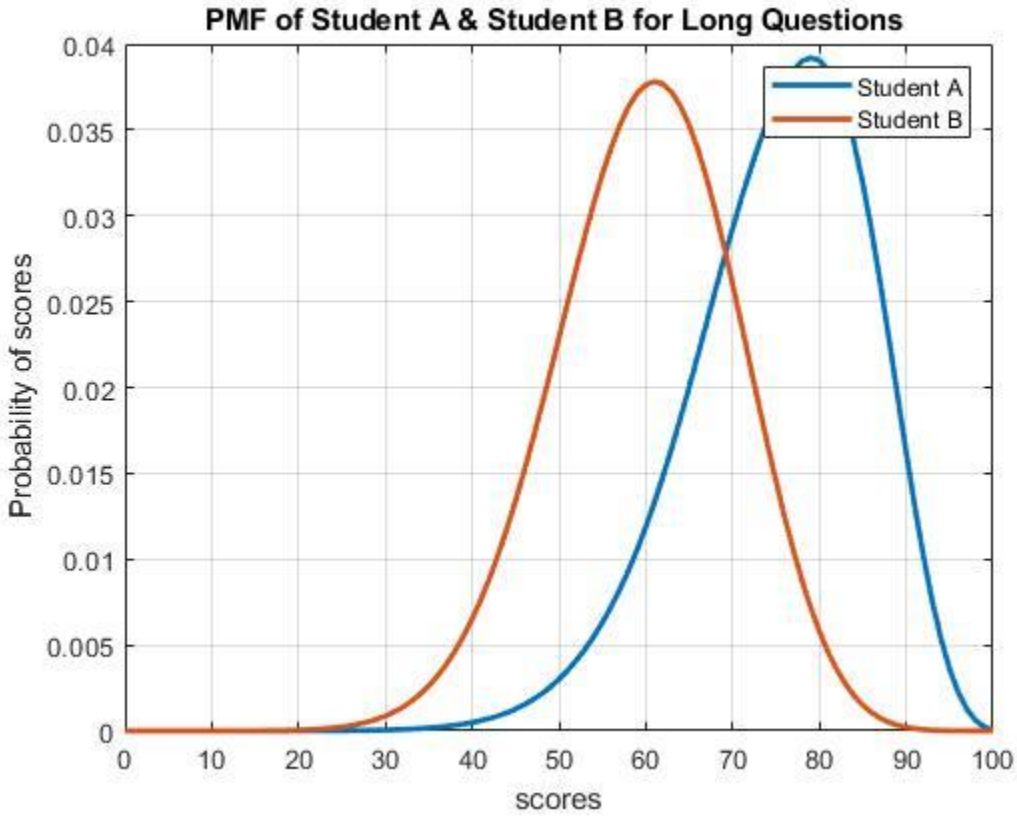
### Procedures for the Hypothesis Test

**I.** Per the formula provided in deriving the hypothesis test, the threshold was initially calculated which is the **Probability (B Students) / Probability (A Students) == 0.6/0.4 = 1.5**

II. The decision region was obtained by comparing (PMF of A Students / PMF of B Students) against the threshold

III. An empty array was created for both decision region of Student A and Student B, every time either of the condition is met, the array is populated with the score which is the index of the division **of fx_A (i) /fx_B (i)** greater or less than 1.5 for A Student and B Student respectively

IV. In order to compute the missed detection probability which is the probability that a student who deserves an A grade is allotted a B grade and vice versa, based on the hypothesis test and the threshold, any Student who scores 72 and above falls within the category of Student A. In this case, all the scores of Student B where checked against scores in the decision region of A, the probability of every term that fulfilled the defined condition is summed to obtain the probability of an instructor awarding a B student grade A and vice versa



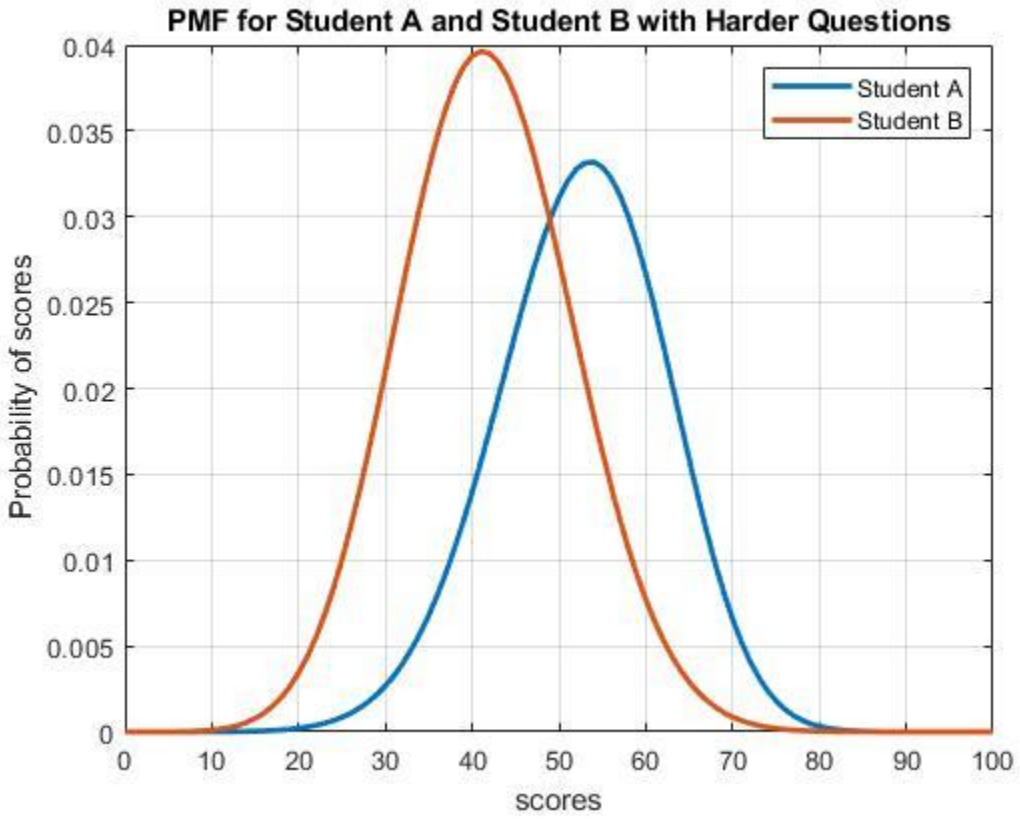

Moreover, the plot of both PMF for Student A and B are shown above. Further to the hypothesis test derived in Section A, the need to verify our results required statistical approximation of both probabilities taking a random sample of 10000. In order to achieve this objective, a for loop was used in randomly generating a score value using the PMF and checking the obtained value against each decision region. Anytime the condition is fulfilled, the value is stored in an array. A relative frequency computation is done after 10000 samples to determine the probability.

```
N = 10000;
yk = [];
q = length(DecB);                          %Length of Decision region of B
 for i = 1:1:N
    random_number = randsample(counts,1,true,fx_A);%Random value using PMF A
     for j =1:1:q

        if random_number == DecB(j) %Check value against Student B region
            yk = [yk random_number];
            break;
        else
            continue;
        end

    end

 end
 p = length(yk)/N;                    %Probability using relative frequency
```

It is important to note that very similar results were obtained with the hypothesis test which shows a good level of confidence in the approximation.

## 2. Derivation of Probability Mass Function & Hypothesis Test for Harder Questions

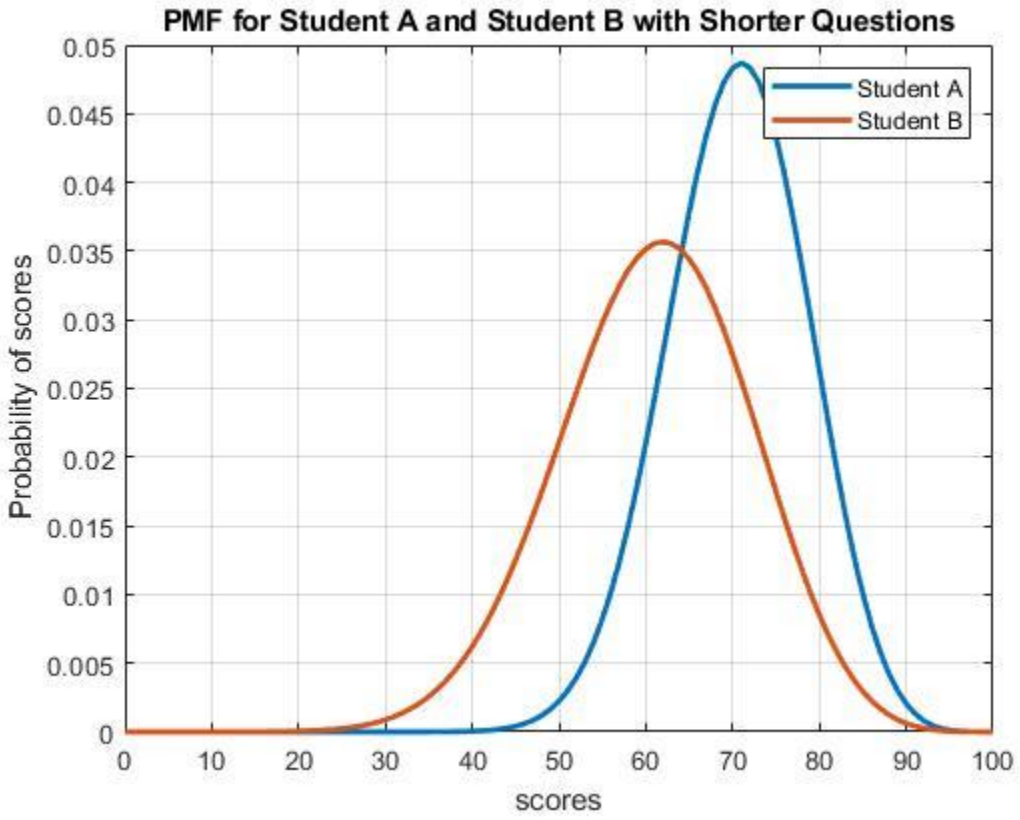
The outlined procedures used in obtaining the Probability Mass Function for section A was also put to use in this section with the assumption that the instructor makes the questions harder. The plot of both PMF as against the scores for both Student A and B are shown below:




Apparently, it could be observed from the plot that the threshold score for both A Student and B reduced as a result of leftward shift in the plot. As against a score of 72 in the previous section, the threshold value is 53.

### 3. Derivation of Probability Mass Function & Hypothesis Test for Shorter Questions

Similar concept and methods were also applied to this section. The only difference is each question has more points has compared to the previous sections. A 4 stage convolution was done so as to obtain a maximum score of 100 points.




From the plot above, both plot for Students A and B experienced a shift relative to the first section. The threshold score for A Students is 69.

## 4. Observations of Hypothesis Test

Based on the hypothesis tests for the three different examination format adopted by the instructor, the probability of an A student getting a B grade and a B student obtaining an A grade are tabulated below:

| | Long Questions | Long Questions (10,000 Samples) | Hard Questions | Shorter Questions |
|---|---|---|---|---|
| **Probability of Grade A for B Student (False Alarm)** | 0.134 | 0.1358 | 0.142 | 0.256 |
| **Probability of Grade B for A Student (Missed Detection)** | 0.3313 | 0.33 | 0.395 | 0.3967 |
| **Probability of Error (Average Cost)** | 0.2129 | 0.2099 | 0.2434 | 0.3123 |
| **Decision Region for A Student (Scores)** | 72-100 | 72-100 | 53-97 | 69-98 |
| **Decision Region of B Student (Scores)** | 0-71 | 0-71 | 0-52, 98-100 | 0-68, 99-100 |

In the light of the results obtained above, the likelihood of a student obtaining a B instead of an A if the instructor gives harder questions is higher than the long question format which is evident from the difficulty in the questions and external conditions cited which could hinder good performance. The value in the exam with shorter questions of an A student getting B was also higher which could be as a result of the high score allocation of each question placing higher risk on the Student if a question is done incorrectly.

## 5. CONCLUSION

After evaluating the results obtained in both the hypothesis test, statistical approximation and also considering the behavioral pattern of students, undoubtedly every student would prefer to be classified as an A student by the instructor if deserved. Also, instructors would want a system that minimizes incorrect grading. My preference is for the first examination with longer questions as it has a well-defined score range for an A student and a B student. Also, the error probability for this examination format is least comparable to the test with harder and shorter questions which reflect minimal error in score allocation.