For 1 asset

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Merton dimension (d=1) and OneAsset dimension(d=2)

The reason is Merton class of Pdes is said to be a degenerate case where the factor V , volatility σ and the risk premium λ are constant.

Note that this equation dependents only on x and but not v,since factor V is constant

So during the construction of the Bellman equation these components are treated as a constant thus the explicit solution u(t,x) is only dependent on 2 variables x and t,

But during the implementation of the Merton Pde time t is set to (T=1) or can be changed accordingly,this implies that our explicit solution to our Merton Pde will only vary depending on x only (thus its variation can only occur in 1-dimension i.e in x-direction)

The other dimensions (variations) which will come about due to the λ(v) factor are set as constants due to Mertons degenerate behaviour page.36

But in the case of OneAsset ,notice that we rewrite its Bellman equation in a such a way that it is dependent on the 3 components x,t,v

The λ here is not constant but it is treated as a function of v i.e λ(v) –this implies that this component can be varied in more than 2 dimensions

During the implementation of OneAsset Pde,time t is set (T=1) ,so this means our explicit solution u(t,x,v) is now only dependent on 2 variables x and v unlike in Merton Pde which only had x as the only variable

The choice of dimension will dependent on how many variables the explicit solution is dependent on

Like in the case on Merton Pde,u(x,t) depends on x and t ,buh since t is set to (T=1) we can only vary x,thus its dimension can only be 1

In the case of OneAsset Pde ,u(x,t,v) depends on x,t and v ,buh since t is set maybe(T-1) ,we can only vary this Pde in x and v directions(2-dimensions)

You will note that OneAsset can also be done by setting the dimension to more that 2 e.g d=5 , the reason for this is because the explicit solution u(x,t,v) is dependent on 3 parameters x,v,t, but remember v is also a component of λ i.e λ(v) ---implies that this v variable can take several dimensions on its own since its treated also as variable of the function λ

Conclusion : Merton Pde has a degenerate case behaviour where λ,V are all treated as constants thus limiting the the dimension to vary in x and t directions ,(note T=1) thus we only remain with x to vary its dimension(thus d=1)

In OneAsset Pde , λ, V are not constants thus λ is taken as a function of v (λ(v)) –thus the variable v here can take several dimensions(directions) on its own, adding this to x and t variables in the explicit solution u(x,t,v) ,we see that its dimension can only begin from 2

Reason being that time t is set(T=1) ,so we remain with x and v variables to vary their directions (thus limiting its dimension to begin from d=2)

Note that with OneAsset ,No leverage Pdes ..these Pdes takes the same explicit type of solution of the Bellmans equation u(x,t,v) –thus their dimensions can begin from 2 ,3….

FYI ,we get the 3^{rd} ,4^{th} …dimensions as a result of λ(v) function which on its own alters the v component to vary in different directions(hence different dimensions)