Eastern University

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Course Code: MAT 201

Course Title: Differential Equations

Section: 6

Final Exam (Assessment), Summer- 2021

Eastern University

ADMIT CARD

Final Examination, Summer 2021

ID :	193400025 - 77F6	Faculty :	Engineering &
Name :	SOROZIT BOIRAGI	Program :	CSE

Course Code	Course Name	Attendance	Credit
CSE 312	Operating System Lab	95.45%	1
CSE 326	Computer Peripherals and Interfacing Lab	85.71%	1
CSE 412	Computer Graphics Laboratory	100%	1
CSE 416	Computer Networks Laboratory	81.82%	1
CSE 477	Thesis/Project	0%	2
EEE 231	Electronics I	81.82%	3
MAT 201	Differential Equations	84.21%	3

Instruction for Examinees:

* Examinees should enter the examination room / hall before 10 minutes of starting of the examination
* No examinees shall be allowed to sit for the examination without Admit card
* No Examinees shall be allowed to carry any papers except admit card
* Cellular phone will not be allowed in the examination room / hall

$2.8571 = e^{-0.009328t}$

$-0.009328t \ln e = \ln(2.8571)$

$-0.009328t = 1.0498$

$t = -112.5 \simeq 112$ minutes

1hr — 60 min

? — 112 min

$\frac{112}{60} = 1.867$ hrs

$\simeq$ 1hr : 52 minutes

From 6am =>

~~7am : 52~~ Minutes

=> 7:52 am

$$2.8571 = e^{-0.009328t}$$

$$-0.009328t \ln e = \ln(2.8571)$$

$$-0.009328t = 1.0498$$

$$t = -112.5 \simeq 112 \text{ minutes}$$

1hr – 60 min

? – 112 min

$$\frac{112}{60} = 1.867 \text{ hrs}$$

$$\simeq 1 \text{ hr}; 52 \text{ minutes}$$

From 6am =)

variation of

3(a)

$i = -\frac{1}{\ } \cos 2t + c$ at time t

(3a) Resistance = 30 ohms

induct... 20 henry

50 sin2t volts

$T = 20 + 7e^{-0.009328t}$

Therefore, to find the time at death we have,

Thus, we take the temperature difference from time (initial) time

6 am to when the body died

$\Rightarrow 27 - 24 = 3^\circ C$ $\quad \Rightarrow 3^\circ + 37^\circ C = 40^\circ C$

$\Rightarrow 40 = 20 + 7e^{-0.009328t}$

$\Rightarrow \frac{20}{7} = e^{-0.009328t}$

$T(0) = 27$, $T(60) = 24$

So $T_0 = 20°C$

$\Rightarrow T = T_0 + ce^{-kt}$ using $T(0) = 27$

$T = 20 + ce^{-kt}$

$\Rightarrow 27 = 20 + ce^{-k(0)} = c = 7$

$\Rightarrow T = 20 + 7e^{-kt}$

$\Rightarrow$ so. for $T(60) = 24$, we have,

$24 = 20 + 7e^{-k(60)}$

$\Rightarrow \frac{4}{7} = e^{-60k} \Rightarrow 0.5714 = e^{-60k}$

$\Rightarrow -60k \ln e = \ln(0.5714)$

$= -60k = -0.5597 \Rightarrow k = \underline{\underline{9.328 \times 10^{-3}}}$

$T(0) = 27$, $T(60) = 24$

so $T_0 = 20°C$

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$\Rightarrow -60k \ln e = \ln(0.5714)$

$= -60k = -0.5597 \Rightarrow k = \underline{\underline{9.328 \times 10}}$

Thus current $i = -\frac{1}{6}\cos 2t + c$ at time t

3(b)

(3b) initial temperature $= 27°c$

The room temperature = constant $= 20°c$

After 1 hour ⇒ temperature dropped to $24°c$

Find the time of death?

normal body temperature $37°c$

Soln

$$\frac{dT}{dt} \propto T - T_0 \Rightarrow \frac{dT}{dt} = -k(T - T_0)$$

The [illegible] = 27°c
room Temperature = constant = 20°c
After 1 hour => temperature dropped to 24°c
Find the time of death?
Normal body temperature 37°c

Sol^n

$$\frac{dT}{dt} \propto T - T_0 \quad \Rightarrow \quad \frac{dT}{dt} = -k(T - T_0)$$

$$T(0) = 27 \quad , \quad T(60) = 24$$

So $T_0 = 20°c$

$$\Rightarrow T = T_0 + ce^{-kt} \quad \text{using } T(0) = 27$$

$$T = 20 + ce^{-kt}$$

$$\Rightarrow 27 = 20 + ce^{-k(0)} \quad \Rightarrow \quad c = 7$$

$$\Rightarrow T = 20 + 7e^{-kt}$$

Thus current $i = -\frac{1}{6}\cos 2t + c$ at time t

3(b)

(3b) initial temperature $= 27°c$

The room temperature = constant $= 20°c$

After 1 hour, $\Rightarrow$ temperature dropped to $24°c$

Find the time of death?

Normal body temperature $37°C$

Soln

$$\frac{dT}{dt} \propto T - T_0 \quad \Rightarrow \frac{dT}{dt} = -k(T - T_0)$$

$$T(0) = 27, \quad T(60) = 24$$

So $T_0 = 20°c$

$$\Rightarrow T = T_0 + ce^{-kt} \quad \text{using } T(0) = 27$$

4.

Soln

$\frac{dT}{dt} \propto T - T_0 \quad \Rightarrow \frac{dT}{dt} = -k(T - T_0)$

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$T(0) = 27 \quad , \quad T(60) = 24$

So $T_0 = 20°C$

$\Rightarrow T = T_0 + ce^{-kt}$ using $T(0) = 27$

$T = 20 + ce^{-kt}$

$\Rightarrow 27 = 20 + ce^{-k(0)} \quad = \quad c = 7$

$\Rightarrow T = 20 + 7e^{-kt}$

$\Rightarrow$ So for $T(60) = 24$, we have

$24 = 20 + 7e^{-k(60)}$

$\Rightarrow \frac{4}{7} = e^{-60k} \quad \Rightarrow 0.5714 = e^{-60k}$

$$\Rightarrow \frac{di}{dt} = \frac{V}{L} \quad \text{and} \quad L = \frac{RA}{\rho}$$

$$= \frac{di}{dt} = \frac{V}{RA/\rho} = \frac{di}{dt} = \frac{\rho V}{RA}$$

so for a unit area coil with resistation inductance 20 henries and voltage 50 sin2t volb we have

$$\frac{di}{dt} = \frac{1}{20} \cdot 50 \sin 2t / 30$$

$$= \frac{di}{dt} = \frac{5}{60} \sin 2t$$

$$di = \frac{5}{60} \sin 2t \, dt$$

upon integration

$$\int di = \int \frac{5}{60} \sin 2t \, dt$$

So for a unit area coil with resistance inductance 20 henries, and voltage 50 sin2t volts we have

$$\frac{di}{dt} = \frac{1}{20} \cdot 50 \sin 2t \Big/ 30$$

$$= \frac{di}{dt} = \frac{5}{60} \sin 2t$$

$$di = \frac{5}{60} \sin 2t \, dt$$

upon integration

$$\int di = \int \frac{5}{60} \sin 2t \, dt$$

$$= i = \frac{5}{60} \cdot -2 \cos 2t + C$$

$$= i = -\frac{10}{60} \cos 2t + c$$

$\frac{\ \ }{dt}$, $R = \frac{\rho L}{A}$, $R = \frac{V}{A}$

$\Rightarrow V = RA$ $\Rightarrow \frac{V}{L} = \frac{di}{dt}$

$\Rightarrow \frac{di}{dt} = \frac{V}{L}$ and $L = \frac{RA}{\rho}$

$= \frac{di}{dt} = \frac{V}{RA/\rho}$ $= \frac{di}{dt} = \frac{\rho V}{RA}$

So for a unit area coil with resistation inductance 20 henries and voltage 50 sin2t volt we have

$$\frac{di}{dt} = \frac{1}{20} \cdot 50 \sin 2t / 30$$

$$= \frac{di}{dt} = \frac{5}{60} \sin 2t$$

3(a) Resistance = 30 ohms
Inductance = 20 henry
Voltages = 50 sin2t volts
Current = ?

$$V = L\frac{di}{dt} \quad , \quad R = \frac{\rho L}{A} \quad , \quad R = \frac{V}{A}$$

$$\Rightarrow V = RA \qquad \Rightarrow \frac{V}{L} = \frac{di}{dt}$$

$$\Rightarrow \frac{di}{dt} = \frac{V}{L} \qquad \text{and} \qquad L = \frac{RA}{\rho}$$

$$= \frac{di}{dt} = \frac{V}{RA/\rho} \qquad = \frac{di}{dt} = \frac{\rho V}{RA}$$

So for a unit area coil with resistation inductance 20 henries and voltage 50 sin2t volb we have

3.

$$= -0.5\,e^{-(0.5)} - e^{-(0.5)} \approx -0.9098$$

2nd iteration.

Now we use

$$\frac{dy}{dx} = x e^{-x} + x e^{-x} - e^{-x}$$

Therefore

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$$\int_0^x \frac{dy}{dx}\,dx = \int_0^x -e^{-x}\,dx$$ which gives

$$y - 0 = e^{x}$$

$$y_2 = e^{x}\Big|_{x=0.5} = 1.649$$

Third iteration

Now we use

$$\frac{dy}{dx} = x e^{-x} + y_2 = x e^{-x} + e^{x}$$

Therefore

$$\int^x \frac{dy}{dx}\,dx = \int^x \left(x e^{-x} + e^{x}\right)dx$$

3(a)

(3a) Resistance = 30 ohms

Inductance = 20 henry

Voltage = 50 sin2t Volts

Current = ?

$$V = L\frac{di}{dt}, \quad R = \frac{\rho L}{A}, \quad R = \frac{V}{A}$$

$$\Rightarrow V = RA \quad \Rightarrow \frac{V}{L} = \frac{di}{dt}$$

$$\Rightarrow \frac{di}{dt} = \frac{V}{L}$$

and $L = \frac{RA}{\rho}$

Therefore

$$\int_0^x \frac{dy}{dx}\,dx = \int_0^x -\bar{e}^{x}\,dx$$ which gives

$$y - 0 = e^{x}$$

$$y_2 = e^{x}\Big|_{x=0.5} = 1.649$$

Third iteration

Now we use

$$\frac{dy}{dx} = x\bar{e}^{x} + y_2 = x\bar{e}^{x} + e^{x}$$

Therefore

$$\int_0^x \frac{dy}{dx}\,dx = \int_0^x \left(x\bar{e}^{x} + e^{x}\right)dx$$

$$\cancel{y - 0 = x^x} \quad y - 0 = -x\bar{e}^{x} - \bar{e}^{x} + e^{x}\Big|_{x=0.5}$$

$$= 0.7389$$

Hence $y = 0.7389$ correct to 4 d.c at X

$$y - 0 = e^{x}$$

$$y_2 = e^{x}\Big|_{x=0.5} = 1.649$$

Third iteration

Now we use

$$\frac{dy}{dx} = xe^{-x} + y_2 = xe^{-x} + e^{x}$$

Therefore

$$\int_0^x \frac{dy}{dx}\,dx = \int_0^x \left(xe^{-x} + e^{x}\right)dx$$

$$\cancel{y - 0 = x} \quad y - 0 = -xe^{-x} - e^{-x} + e^{x}\Big|_{x=0.5}$$

$$= 0.7389$$

Hence $y = 0.7389$, correct to 4 d.c at $X = 0.5$

iteration

$$y_1 = \int_0^x x\,e^{-x}\,dx \qquad \text{Let} \quad u = x \quad du = dx. \quad dv = e^{-x}. \quad v = -e^{-x}.$$

$$-x e^{-x} + \int e^{-x}\,dx = -x e^{-x} - e^{-x}\Big|_{x=0.5}$$

$$= -0.5\,e^{-(0.5)} - e^{-(0.5)} \approx -0.9098$$

2nd iteration.

Now we use

$$\frac{dy}{dx} = x e^{-x} \cancel{+}\; x e^{-x} - e^{-x}$$

Therefore

$$\int_0^x \frac{dy}{dx}\,dx = \int_0^x -e^{-x}\,dx \qquad \text{which gives}$$

$$y - 0 = e^{x}$$

$$y_2 = e^{x}\Big|_{x=0.5} = 1.649$$

0,6065

first iteration

$$y_1 = \int_0^x x e^{-x}\,dx \qquad \text{let} \quad u = x \quad du = dx. \quad dv = e^{-x} \quad v = -e^{-x}$$

$$-x e^{-x} + \int e^{-x}\,dx = -x e^{-x} - e^{-x} \Big|_{x=0.5}$$

$$= -0.5\, e^{-(0.5)} - e^{-(0.5)} \simeq -0.9098$$

2nd iteration.

Now we use

$$\frac{dy}{dx} = x e^{-x} \cancel{+} x e^{-x} - e^{-x}$$

(b) Using picards method to approximate

$$\frac{dy}{dx} = y + xe^{-x}$$

$, y(0) = 1$

Soln

when $x_0 = 0$. $y = 0$

at $x = 0.5$

$$\int_{x_0}^{x} \frac{dy}{dx}\,dx = \int_{x_0}^{x} (y + xe^{-x})\,dx$$

where $x_0 = 0$

$$y - y_0 = \int_{x_0}^{x} (y + xe^{-x})\,dx$$

where $y_0 = 0$

which became

$$y = \int_{0}^{x} (y + xe^{-x})\,dx$$

1 (b)

b) Using picards method to approximat

$$\frac{dy}{dx} = y + x e^{-x}$$

, $y(0) = 1$

Soln

when $x_0 = 0$. $y = 0$

at $x = 0.5$

$$\int_{x_0}^{x} \frac{dy}{dx} dx = \int_{x_0}^{x} (y + x e^{-x}) dx$$

where $x_0 = 0$

$$\int^{x} (y + x e^{-x}) dx$$

where $y_0 = 0$

Differentiating y_p we have

$$y'_p = e^x\left[A\cos 3x + B\sin 3x\right] + e^x\left[-3A\sin 3x + 3B\cos 3x\right] + 2e^{2x}$$

—— (2)

$$y''_p = e^x\left[A\cos 3x + B\sin 3x\right] + e^x\left[-3A\sin 3x + 3B\cos 3x\right] + e^x\left[-3A\sin 3x + 3B\cos 3x\right]$$
$$+ e^x\left[-9A\cos 3x - 9B\sin 3x\right] + 4e^{2x}$$

—— (3)

$$y'''_p = e^x\left[A\cos 3x + B\sin 3x\right] + e^x\left[-3A\sin 3x + 3B\cos 3x\right]$$
$$+ e^x\left[-3A\sin 3x + 3B\cos 3x\right] + e^x\left[-9A\cos 3x - 9B\sin 3x\right]$$
$$+ e^x\left[-3A\sin 3x + 3B\cos 3x\right] + e^x\left[-9A\cos 3x - 9B\sin 3x\right]$$
$$+ e^x\left[-9A\cos 3x - 9B\sin 3x\right] + e^x\left[27A\sin 3x - 27B\sin 3x\right.$$

—— (4)

Substituting (1), (2) (3) and (4) into (1) we have

$\Rightarrow$ $A = -9$, $B = 3$ $\Rightarrow$ $C = C_3$

$\Rightarrow$

y

$+ e^{x}[-3A\sin 3x + 3B\cos 3x] + e^{x}[-9A\cos 3x - 9B\sin 3x]$

$+ e^{x}[-9A\cos 3x - 9B\sin 3x] + e^{x}[-27A\sin 3x - 27B\sin 3x]$ — (4)

Substituting (1), (2) (3) and (4) into (1) we have

$\Rightarrow A = -9 \quad , B = 3 \qquad \Rightarrow \quad C = C_3$

$\Rightarrow$

$y_p = e^{x}[-9\cos 3x + 3\sin 3x] + Ce^{2x}$

particular soln

$y = y_c + y_p =$

$y = e^{x}[C_1\cos 2x + C_2\sin 2x] + 2C_3e^{2x} + --$

$+ e^{x}[-9\cos 3x + 3\sin 3x]$

$$y''_p = e^x[A\cos 3x + B\sin 3x] + e^x[-3A\sin 3x + 3B\cos 3x] + e^x[\ldots$$
$$+ e^x[-9A\cos 3x - 9B\sin 3x] + 4e^{2x}$$
(3)

$$y'''_p = e^x[A\cos 3x + B\sin 3x] + e^x[-3A\sin 3x + 3B\cos 3x]$$
$$+ e^x[-3A\sin 3x + 3B\cos 3x] + e^x[-9A\cos 3x - 9B\sin 3x]$$
$$+ e^x[-3A\sin 3x + 3B\cos 3x] + e^x[-9A\cos 3x - 9B\sin 3x]$$
$$+ e^x[-9A\cos 3x - 9B\sin 3x] + e^x[27A\sin 3x - 27B\sin 3x$$
(4)

Substituting (1), (2) (3) and (4) into (1) we hav

$\Rightarrow$ $A = -9$, $B = 3$ $\Rightarrow$ $C = C_3$

$\Rightarrow$
$$y_p = e^x[-9\cos 3x + 3\sin 3x] + Ce^{2x}$$

particular soln
$$y = y_c + y_p =$$

$+ e^{x}[-3A\sin 3x + 3B\cos 3x] + e^{x}[-9A\cos 3x - 9B\sin 3x]$

$+ e^{x}\{-3A\sin 3x + 3B\cos 3x] + e^{x}[-9A\cos 3x - 9B\sin 3x]$

$+ e^{x}[-9A\cos 3x - 9B\sin 3x] + e^{x}[27A\sin 3x - 27B\sin 3x$ ——— (4)

Substituting (1), (2) (3) and (4) into (1) we have

$\Rightarrow A = -9$, $B = 3$ $\Rightarrow$ $C = C_3$

$\Rightarrow$

$$y_p = e^{x}[-9\cos 3x + 3\sin 3x] + Ce^{2x}$$

particular soln

$$y = y_c + y_p =$$

$$y = e^{x}[C_1\cos 2x + C_2\sin 2x] + 2C_3e^{2x} + \text{--}$$

$$+ e^{x}\{-9\cos 3x + 3\sin 3x]$$

$$y_p = e^{x}\left[A\cos 3x + B\sin 3x\right] + Ce^{2x} \qquad (1)$$

Differentiating y_p we have

$$y'_p = e^{x}\left[A\cos 3x + B\sin 3x\right] + e^{x}\left[-3A\sin 3x + 3B\cos 3x\right] + 2e^{2x} \qquad (2)$$

$$y''_p = e^{x}\left[A\cos 3x + B\sin 3x\right] + e^{x}\left[-3A\sin 3x + 3B\cos 3x\right] + e^{x}\left[-3A\sin 3x + 3B\cos 3x\right]$$
$$+ e^{x}\left[-9A\cos 3x - 9B\sin 3x\right] + 4e^{2x} \qquad (3)$$

$$y'''_p = e^{x}\left[A\cos 3x + B\sin 3x\right] + e^{x}\left[-3A\sin 3x + 3B\cos 3x\right]$$

$$+ e^{x}\left[-3A\sin 3x + 3B\cos 3x\right] + e^{x}\left[-9A\cos 3x - 9B\sin 3x\right]$$

$$+ e^{x}\left[-3A\sin 3x + 3B\cos 3x\right] + e^{x}\left[-9A\cos 3x - 9B\sin 3x\right]$$

$$+ e^{x}\left[-9A\cos 3x - 9B\sin 3x\right] + e^{x}\left[27A\sin 3x - 27B\ldots\right.$$

$$-2\lambda^2+9\lambda$$
$$-2\lambda^2+4\lambda$$
$$5\lambda-10$$
$$5\lambda-10$$
$$0$$

$(\lambda-2)(\lambda^2-2\lambda+5)=0$

$\Rightarrow \lambda-2=0$ and $\lambda^2-2\lambda+5=0$

$a=1$, $b=-2$ $c=5$

$\lambda=\dfrac{2\pm\sqrt{4-20}}{2}=\dfrac{2\pm\sqrt{-16}}{2}=\dfrac{2\pm4i}{2}$

$\lambda_1=1+2i$, $\lambda_2=1-2i$, $\lambda_3=2$

$y_c=e^{x}\left[c_1\cos2x+c_2\sin2x\right]+c_3e^{2x}$

using the method of undetermined coefficient

$$y_p = e^{x}\left[A\cos 3x + B\sin 3x\right] + Ce^{2x} \quad \text{——} ①$$

Differentiating y_p we have

$$y'_p = e^{x}\left[A\cos 3x + B\sin 3x\right] + e^{x}\left[-3A\sin 3x + 3B\cos 3x\right] + 2e^{2x} \quad \text{——} ②$$

$$y''_p = e^{x}\left[A\cos 3x + B\sin 3x\right] + e^{x}\left[-3A\sin 3x + 3B\cos 3x\right] + e^{x}\left[-3A\sin 3x + 3B\cos 3x\right]$$
$$+ e^{x}\left[-9A\cos 3x - 9B\sin 3x\right] + 4e^{2x} \quad \text{——} ③$$

$$y''_p = e^{x}\left[A\cos 3x + B\sin 3x\right] + e^{x}\left[-3A\sin 3x + 3B\cos 3x\right]$$

$$\lambda - 2 \,\big)\, \lambda^3 - 4\lambda^2 + 9\lambda - 10$$

$$\lambda^3 - 2\lambda^2$$

$$-2\lambda^2 + 9\lambda$$

$$-2\lambda^2 + 4\lambda$$

$$5\lambda - 10$$

$$5\lambda - 10$$

$$0$$

$(\lambda - 2)(\lambda^2 - 2\lambda + 5) = 0$

$\Rightarrow \lambda - 2 = 0$ and $\lambda^2 - 2\lambda + 5 = 0$

$a = 1$, $b = -2$ $c = 5$

$\lambda = \frac{2 \pm \sqrt{4-20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2}$

$\lambda_1 = 1 + 2i$, $\lambda_2 = 1 - 2i$, $\lambda_3 = 2$

$$y_c = e^{x}\left[c_1 \cos 2x + c_2 \sin 2x\right] + c_3 e^{2x}$$

Using the method of undetermined coefficients

$$\lambda^2 - 2\lambda + 5$$
$$\lambda - 2 \overline{\big) \lambda^3 - 4\lambda^2 + 9\lambda - 10}$$
$$\lambda^3 - 2\lambda^2$$
$$-2\lambda^2 + 9\lambda$$
$$-2\lambda^2 + 4\lambda$$
$$5\lambda - 10$$
$$5\lambda - 10$$
$$0$$

$$(\lambda - 2)(\lambda^2 - 2\lambda + 5) = 0$$

$\Rightarrow \lambda - 2 = 0$ and $\lambda^2 - 2\lambda + 5 = 0$

$a = 1, \quad b = -2 \quad c = 5$

$$\lambda = \frac{2 \pm \sqrt{4-20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2}$$

$$\lambda_1 = 1 + 2i, \quad \lambda_2 = 1 - 2i, \quad \lambda_3 = 2$$

$$y_c = e^{x}\left[c_1 \cos 2x + c_2 \sin 2x\right] + c_3 e^{2x}$$

using the method of undetermined coefficients

$\lambda^3 - 4\lambda^2 + 9\lambda - 10 = 0$ For the complete sols

$\lambda = 2$

$$\begin{array}{r|l} & \lambda^2 - 2\lambda + 5 \\ \lambda - 2 & \lambda^3 - 4\lambda^2 + 9\lambda - 10 \\ & \lambda^3 - 2\lambda^2 \\ \hline & -2\lambda^2 + 9\lambda \\ & -2\lambda^2 + 4\lambda \\ \hline & 5\lambda - 10 \\ & 5\lambda - 10 \\ \hline & 0 \end{array}$$

$(\lambda - 2)(\lambda^2 - 2\lambda + 5) = 0$

$\Rightarrow \lambda - 2 = 0$ and $\lambda^2 - 2\lambda + 5 = 0$

$a = 1$, $b = -2$ $c = 5$

$\lambda = \dfrac{2 \pm \sqrt{4-20}}{2} = \dfrac{2 \pm \sqrt{-16}}{2} = \dfrac{2 \pm 4i}{2}$

(1a) Solve the differential eqn

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 9\frac{dy}{dx} - 10y = 2\cos^2 x + e^{2x} \quad \text{--- (1)}$$

Soln

3rd order ode

$$y''' - 4y'' + 9y' - 10y = 2\cos^2 x + e^{2x}$$

$\lambda^3 - 4\lambda^2 + 9\lambda - 10 = 0$ For the complete soln

$\lambda = 2.$

$$\begin{array}{r|l} & \lambda^2 - 2\lambda + 5 \\ \lambda - 2 & \lambda^3 - 4\lambda^2 + 9\lambda - 10 \\ & \lambda^3 - 2\lambda^2 \\ \hline & -2\lambda^2 + 9\lambda \\ & -2\lambda^2 + 4\lambda \\ \hline & 5\lambda - 10 \\ & 5\lambda - 10 \\ \hline & 0 \end{array}$$

$$(\lambda - 2)(\lambda^2 - 2\lambda + 5) = 0$$

1(a)

(1a) Solve the differential eqn

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 9\frac{dy}{dx} - 10y = 2\cos^2 x + e^{-2x} \quad -(1)$$

Soln

3rd order roots

$$y''' - 4y'' + 9y' - 10y = 2\cos^2 x + e^{-2x}$$

$\lambda^3 - 4\lambda^2 + 9\lambda - 10 = 0$ For the complete soln

$\lambda = 2$

$$\begin{array}{r|l} & \lambda^2 - 2\lambda + 5 \\ \lambda - 2 & \lambda^3 - 4\lambda^2 + 9\lambda - 10 \\ & \lambda^3 - 2\lambda^2 \\ \hline & -2\lambda^2 + 9\lambda \\ & -2\lambda^2 + 4\lambda \end{array}$$

1(a)

(1a) Solve the differential eqn.

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 9\frac{dy}{dx} - 10y = 2\cos^2 x + e^{-2x} \quad \text{—}(1)$$

Soln

3rd order roots

$$y''' - 4y'' + 9y' - 10y = 2\cos^2 x + e^{-2x}$$

$$\lambda^3 - 4\lambda^2 + 9\lambda - 10 = 0$$ For the complete

$\lambda^2 \; 2\lambda + 5$

Upon integration we have

$$v_1 = \frac{3\ln(\tan 3x)\cdot \cos 3x}{3}$$

$$v_1 = -\cos 3x \cdot \ln(\tan 3x)$$

Solving for v_2

$$v_2' = \frac{\begin{vmatrix} \cos 3x & 0 \\ -3\sin 3x & \cot 3x \end{vmatrix}}{3} = \frac{\cot 3x \cdot \cos 3x}{3}$$

Upon integration

$$v_2 = \frac{\cancel{3}\ln(\tan 3x)\cdot \sin 3x}{\cancel{3}}$$

$$v_2 = \sin 3x \ln(\tan 3x)$$

$$y_p = -\cos 3x \ln(\tan 3x)\cos 3x + \sin 3x \ln(\tan 3x)\sin 3x.$$

Soln

$$y = y_c + y_p = c_1\cos 3x + c_2 \sin 3x + \ldots +$$

$$\ldots + \sin 3x \ln(\tan 3x)\sin 3x - \cos 3x \ln(\tan 3x)\cos 3x$$

upon integration we have

$$v_1 = \frac{-3\ln(\tan 3x)\cos 3x}{3}$$

$$v_1 = -\cos 3x \cdot \ln(\tan 3x)$$

Solving for v_2

$$v_2' = \frac{\begin{vmatrix} \cos 3x & 0 \\ -3\sin 3x & \cot 3x \end{vmatrix}}{3} = \frac{\cot 3x \cdot \cos 3x}{3}.$$

Upon integration

$$v_2 = \frac{\cancel{3}\ln(\tan 3x) \cdot \sin 3x}{\cancel{3}}$$

$$v_2 = \sin 3x \ln(\tan 3x)$$

$$y_p = -\cos 3x \ln(\tan 3x)\cos 3x + \sin 3x \ln(\tan 3x)\sin 3x.$$

Sol

$$y = y_c + y_p = \underline{c_1\cos 3x + c_2 \sin 3x +} \quad - - - \quad +$$

$$- - + \sin 3x \ln(\tan \ldots$$

3\.

$$v_1' = \frac{\begin{vmatrix} 0 & \sin 3x \\ \cot 3x & 3\cos 3x \end{vmatrix}}{\begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix}} = \frac{\cot 3x \ldots}{3\cos^2 3x + 3\sin^2 3x}$$

$$= \frac{\cot 3x \cdot \sin 3x}{3}$$

upont integratin we ha

$$v_1 = \frac{3\ln(\tan 3x) \cos 3x}{3}$$

$$v_1 = \cos 3x \cdot \ln(\tan 3x)$$

Solving for v_2

$$v_2' = \frac{\begin{vmatrix} \cos 3x & 0 \\ -3\sin 3x & \cot 3x \end{vmatrix}}{3} = \frac{\cot 3x \cdot \cos 3x}{3}$$

Upon integration

$$v_2 = \frac{\cancel{3} \ln(\tan 3x) \cdot \sin 3x}{\cancel{3}}$$

$$X(t) = -\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}e^{t} - \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}e^{-t} + \left[\frac{1}{2}t + \frac{1}{2}e^{t} + \frac{1}{2}te^{t}\begin{pmatrix}1\\1\end{pmatrix}e^{t}\right] + \left[\frac{1}{2}e^{t} - \frac{1}{2}te^{t} - \frac{1}{4}e^{2t}\begin{pmatrix}1\\-1\end{pmatrix}e^{-t}\right]$$

2(b)

b) Method of variation of parameter

$$\frac{d^2y}{dx^2} + 9y = \cot 3x$$

Soln

$$y'' + 9y = \cot 3x$$

Complete soln.

$$y'' + 9y = 0$$

$$\lambda^2 + 9 = 0 \qquad \lambda^2 = -9 \qquad \lambda = \pm 3i$$

$$y_c = c_1 \cos 3x + c_2 \sin 3x \Rightarrow y_c = c_1 y_1 + c_2 y_2$$

$$y_p = v_1 y_1 + v_2 y_2.$$

$$y_p = v_1 \cdot \cos 3x + v_2 \sin 3x$$

$v_1' y_1 + v_2' y_2 = 0$ $\Rightarrow v_1' \cos 3x + v_2' \sin 3x = 0$

$v_1' y_1' + v_2' y_2' = g(x)$ $-3v_1' \sin 3x + 3v_2' \cos 3x = \cot 3x$

using Crammers rule

$$v_1' = \frac{\begin{vmatrix} 0 & \sin 3x \\ \cot 3x & 3\cos 3x \end{vmatrix}}{\begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix}} = \frac{\cot 3x \sin 3x}{3\cos^2 3x + 3\sin^2 3x}$$

$$= \frac{\cot 3x \cdot \sin 3x}{3}$$

upon integration we have

$$v_1 = \frac{3\ln(\tan 3x)}{3} \cos 3x$$

$$v_1 = \cos 3x \cdot \ln(\tan 3x)$$

Solving for v_2

$$v_2' = \begin{vmatrix} \cos 3x & 0 \\ -3\sin 3x & \cot 3x \end{vmatrix} = \frac{\cot 3x \cdot \cos 3x}{}$$

We solve for

$v_1' y_1 + v_2' y_2 = 0$ $\Rightarrow$ $v_1' \cos 3x + v_2' \sin 3x = 0$

$v_1' y_1' + v_2' y_2' = g(x)$ $\quad -3v_1' \sin 3x + 3v_2' \cos 3x = \cot 3x$

Using Crammers rule

$$v_1' = \frac{\begin{vmatrix} 0 & \sin 3x \\ \cot 3x & 3\cos 3x \end{vmatrix}}{\begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix}} = \frac{\cot 3x \sin 3x}{3\cos^2 3x + 3\sin^2 3x}$$

We solve for

$v_1' y_1 + v_2' y_2 = 0$ $\Rightarrow v_1' \cos 3x + v_2' \sin 3x = 0$

$v_1' y_1' + v_2' y_2' = g(x)$ $-3 v_1' \sin 3x + 3 v_2' \cos 3x = \cot 3x$

Using Crammers rule

$$v_1' = \frac{\begin{vmatrix} 0 & \sin 3x \\ \cot 3x & 3\cos 3x \end{vmatrix}}{\begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix}} = \frac{\cot 3x \sin 3x}{3\cos^2 3x + 3\sin^2 3x}$$

$$= \frac{\cot 3x \cdot \sin 3x}{3}$$

upon integration we have

$$v_1 = \frac{3\ln(\tan 3x) - \cos 3x}{2}$$

$3x = 0$

$x = \cot 3x$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} + c_1 + c_2 = 0 \quad - (1)$$

$$\frac{1}{2} - \frac{1}{2} + c_1 - c_2 = 0 \quad - (2)$$

$$\Rightarrow \frac{1}{4} + c_1 + c_2 = 0 \qquad \text{solving simultaneously}$$

$$\underline{0 + c_1 - c_2 = 0}$$

$$\frac{1}{4} + 2c_1 \quad 0 = 0$$

$$\Rightarrow 2c_1 = -\frac{1}{4} \qquad \Rightarrow c_1 = -\frac{1}{2}$$

$$\cancel{\text{since}} \quad c_1 = c_2 \qquad c_2 = -\frac{1}{2}$$

particular solut.

$$X(t) = -\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}e^{t} - \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}e^{-t} + \left[\frac{1}{2}t + \frac{1}{2}e^{-t} + \frac{1}{2}te^{-t}\begin{pmatrix}1\\1\end{pmatrix}e^{t}\right] + \left[\frac{1}{2}e^{t} - \frac{1}{2}te^{t} - \frac{1}{4}e^{2t}\begin{pmatrix}1\\-1\end{pmatrix}e^{-t}\right.$$

2(b)

b) Method of variation of parameter

$$\frac{d^2y}{dx^2} + 9y = \cot 3x$$

Soln

$$y'' + 9y = \cot 3x$$

Complete soln

$$y'' + 9y = 0$$

$\frac{1}{4} + 2c_1 \quad 0 = 0$

$\Rightarrow 2c_1 = -\frac{1}{4} \qquad \Rightarrow c_1 = -\frac{1}{2}$

$\cancel{\ldots} \qquad c_1 = c_2 \qquad c_2 = -\frac{1}{2}$

particular solun.

$$X(t) = -\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}e^{t} - \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}e^{-t} + \left[\frac{1}{2}t + \frac{1}{2}e^{-t} + \frac{1}{2}te^{-t}\begin{pmatrix}1\\1\end{pmatrix}e^{t}\right] + \left[\frac{1}{2}e^{t} - \frac{1}{2}te^{t} - \frac{1}{4}e^{2t}\begin{pmatrix}1\\-1\end{pmatrix}e^{-t}\right.$$

2 (b)

b) Method of variation of parameter

$$\frac{d^2y}{dx^2} + 9y = \cot 3x$$

Soln

$y'' + 9y = \cot 3x$

Complete soln.

$y'' + 9y = 0$

$\lambda^2 + 9 = 0 \qquad \lambda^2 = -9 \qquad \lambda = \pm 3i$

$y_c = c_1 \cos 3x + c_2 \sin 3x \qquad \Rightarrow y_c = c_1 y_1 +$

$= 0$

$= \cot 2x.$

$\cot = \frac{1}{\tan}$

$\Rightarrow \frac{1}{2} + \frac{1}{2} + c_1 + c_2 = 0 \quad -(1)$

$\frac{1}{2} - \frac{1}{2} + c_1 - c_2 = 0 \quad -(2)$

$\Rightarrow \frac{1}{4} + c_1 + c_2 = 0$ solving simultaneously

$\underline{0 + c_1 - c_2 = 0}$

$\frac{1}{4} + 2c_1 \quad 0 = 0$

$\Rightarrow 2c_1 = -\frac{1}{4} \quad \Rightarrow c_1 = -\frac{1}{2}$

~~since~~ $\because c_1 = c_2 \quad c_2 = -\frac{1}{2}$

particular solur.

$$X(t) = -\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}e^{t} - \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}e^{-t} + \left[\frac{1}{2}t + \frac{1}{2}e^{-t} + \frac{1}{2}te^{-t}\begin{pmatrix}1\\1\end{pmatrix}e^{t}\right] + \left[\frac{1}{2}e^{t} - \frac{1}{2}te^{t} - \frac{1}{4}e^{2t}\begin{pmatrix}1\\-1\end{pmatrix}e^{-t}\right]$$

2(b)

b) Method of variation of parameter

$$\frac{d^2y}{dx^2} + 9y = \cot 3x$$

Soln

$\cot = \frac{1}{\tan}$

$\Rightarrow \frac{1}{2} + \frac{1}{2} + c_1 + c_2 = 0 \quad -(1)$

$\frac{1}{2} - \frac{1}{2} + c_1 - c_2 = 0 \quad -(2)$

$\Rightarrow \frac{1}{4} + c_1 + c_2 = 0$ solving simultaneously

$0 + c_1 - c_2 = 0$

$\frac{1}{4} + 2c_1 \quad 0 = 0$

$\Rightarrow 2c_1 = -\frac{1}{4} \Rightarrow c_1 = -\frac{1}{2}$

$\cancel{\text{from eq.}}(2) \quad c_1 = c_2 \quad c_2 = -\frac{1}{2}$

particular solution

$$X(t) = -\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}e^{t} - \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}e^{-t} + \left[\frac{1}{2}t + \frac{1}{2}e^{-t} + \frac{1}{2}te^{-t}\begin{pmatrix}1\\1\end{pmatrix}e^{t}\right] + \left[\frac{1}{2}e^{t} - \frac{1}{2}te^{t} - \frac{1}{4}e^{2t}\right.$$

2\. Solve for $c_1(t)$ and $c_2(t)$ we have

3\. $c_1'(t)\begin{pmatrix}1\\1\end{pmatrix}e^{t} + c_2'(t)\begin{pmatrix}1\\-1\end{pmatrix}e^{-t} = \begin{pmatrix}-t\\e^{t}\end{pmatrix}$

$$c_1'(t) = \frac{\begin{vmatrix}-t & e^{-t}\\ e^{t} & -e^{-t}\end{vmatrix}}{\begin{vmatrix}e^{t} & e^{-t}\\ e^{t} & -e^{-t}\end{vmatrix}} = \frac{te^{-t} - e^{t}\cdot e^{-t}}{-1-1} = \frac{te^{-t}-1}{-2}$$

$$= \frac{1}{2} - \frac{te^{-t}}{2}$$

$$c(t) = \frac{1}{2}t - \left[-te^{-t} - e^{-t}\right] \qquad te^{t}$$

$$= c(t) = \frac{1}{2}t + \frac{1}{2}e^{-t} + \frac{1}{2}te^{-t}$$

$$c_2'(t) = \frac{\begin{vmatrix}e^{t} & -t\\ e^{t} & e^{t}\end{vmatrix}}{-2} = \cancel{\frac{e^{2t}+te^{t}}{-2}}$$

$$c_2(t) = -\frac{1}{4}e^{2t} - \frac{1}{2}\left[te^{t} - e^{t}\right]$$

$$= -\frac{1}{4}e^{2t} - \frac{1}{2}te^{t} + \frac{1}{2}e^{t}$$

Soln

$$X_p(t) = \left(\frac{1}{2}t + \frac{1}{2}e^{-t} + \frac{1}{2}te^{-t}\right)\begin{pmatrix}1\\1\end{pmatrix}e^{t} + \left(\frac{1}{2}e^{t} - \frac{1}{2}te^{t} - \frac{1}{4}e^{2t}\right)\begin{pmatrix}1\\ \ \end{pmatrix}e^{-t}$$

$\cot = \frac{1}{\tan\theta}$

$\Rightarrow \frac{1}{2} + \frac{1}{2} + c_1 + c_2 = 0 \quad -①$

$\frac{1}{2} - \frac{1}{2} + c_1 - c_2 = 0 \quad -②$

$\Rightarrow \frac{1}{4} + c_1 + c_2 = 0$ solving simultaneously

$0 + c_1 - c_2 = 0$

$\frac{1}{4} + 2c_1 \quad 0 = 0$

$\Rightarrow 2c_1 = -\frac{1}{4} \quad \Rightarrow c_1 = -\frac{1}{8}$

$\therefore c_1 = c_2 \quad c_2 = -\frac{1}{8}$

$$c_1'(t) = \frac{\begin{vmatrix} -t & e^{-t} \\ e^{t} & -e^{-t} \end{vmatrix}}{\begin{vmatrix} e^{t} & e^{-t} \\ e^{t} & -e^{-t} \end{vmatrix}} = \frac{te^{-t} - e^{t} \cdot e^{-t}}{-1-1} = \frac{te^{-t} - 1}{-2}$$

$$= \frac{1}{2} - \frac{te^{-t}}{2}$$

te^{t}

$$c(t) = \frac{1}{2}t - \left[-te^{-t} - e^{-t}\right]$$

$$= c(t) = \frac{1}{2}t + \frac{1}{2}e^{-t} + \frac{1}{2}te^{-t}$$

$$c_2'(t) = \frac{\begin{vmatrix} e^{t} & -t \\ e^{t} & e^{t} \end{vmatrix}}{-2} = \frac{e^{2t} + te^{t}}{-2}$$

$$c_2(t) = -\frac{1}{4}e^{2t} - \frac{1}{2}\left[te^{t} - e^{t}\right]$$

$$= -\frac{1}{4}e^{2t} - \frac{1}{2}te^{t} + \frac{1}{2}e^{t}$$

Soln

$$X_p(t) = \left(\frac{1}{2}t + \frac{1}{2}e^{-t} + \frac{1}{2}te^{-t}\right)\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{t} + \left(\frac{1}{2}e^{t} - \frac{1}{2}te^{t} - \frac{1}{4}e^{2t}\right)\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-t} +$$

$$+ c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{t} + c_2\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-t}$$

initial Condition

$c_1(t) = \frac{1}{2}t - [-te^{-t} - e^{-t}]$

$= c_1(t) = \frac{1}{2}t + \frac{1}{2}e^{-t} + \frac{1}{2}te^{-t}$

$c_2'(t) = \dfrac{\begin{vmatrix} e^{t} & -t \\ e^{t} & e^{t} \end{vmatrix}}{-2} = \dfrac{e^{2t} + t\,\cancel{e^{t}}}{\cancel{-2}}$

$c_2(t) = -\frac{1}{4}e^{2t} - \frac{1}{2}[te^{t} - e^{t}]$

$= -\frac{1}{4}e^{2t} - \frac{1}{2}te^{t} + \frac{1}{2}e^{t}$

Soln

$X_p(t) = \left(\frac{1}{2}t + \frac{1}{2}e^{-t} + \frac{1}{2}te^{-t}\right)\begin{pmatrix}1\\1\end{pmatrix}e^{t} + \left(\frac{1}{2}e^{t} - \frac{1}{2}te^{t} - \frac{1}{4}e^{2t}\right)\begin{pmatrix}1\\-1\end{pmatrix}e^{-t} +$

$+ c_1\begin{pmatrix}1\\1\end{pmatrix}e^{t} + c_2\begin{pmatrix}1\\-1\end{pmatrix}e^{-t}$

initial Condition

$\begin{pmatrix}x_1\\x_2\end{pmatrix} = X(0) = \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix} + c_1\begin{pmatrix}1\\1\end{pmatrix} + c_2\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

For the particular soln

$$X_p(t) = c_1(t)\begin{pmatrix}1\\1\end{pmatrix}e^{t} + c_2(t)\, e\begin{pmatrix}1\\-1\end{pmatrix}e^{-t}$$

To solve for $c_1(t)$ and $c_2(t)$ we have

$$c_1'(t)\begin{pmatrix}1\\1\end{pmatrix}e^{t} + c_2'(t)\begin{pmatrix}1\\-1\end{pmatrix}e^{-t} = \begin{pmatrix}-t\\e^{t}\end{pmatrix}$$

$$c_1'(t) = \frac{\begin{vmatrix}-t & e^{-t}\\ e^{t} & -e^{-t}\end{vmatrix}}{\begin{vmatrix}e^{t} & e^{-t}\\ e^{t} & -e^{-t}\end{vmatrix}} = \frac{te^{-t} - e^{t}.e^{-t}}{-1-1} = t$$

let $u = t$ $du = dt$
$dv = e^{-t}$ $v = -e^{-t}$

$-te^{-t} + \int e^{-t}dt$

e^{t} $e^{t}.e^{-t}$

$-e^{-t}.e^{t}$

$-1-$

Fundamental Soln

$X_p(t) = c_1(t)\begin{pmatrix}1\\1\end{pmatrix}e^{t} + c_2(t)\, e\begin{pmatrix}1\\-1\end{pmatrix}e^{-t}$

To solve for $c_1(t)$ and $c_2(t)$ we have

$c_1'(t)\begin{pmatrix}1\\1\end{pmatrix}e^{t} + c_2'(t)\begin{pmatrix}1\\-1\end{pmatrix}e^{-t} = \begin{pmatrix}-t\\e^{t}\end{pmatrix}$

$c_1'(t) = \dfrac{\begin{vmatrix}-t & e^{-t}\\ e^{t} & -e^{-t}\end{vmatrix}}{\begin{vmatrix}e^{t} & e^{-t}\\ e^{t} & -e^{-t}\end{vmatrix}} = \dfrac{te^{-t} - e^{t}\cdot e^{-t}}{-1-1} = \dfrac{te^{-t}-1}{-2}$

$= \frac{1}{2} - \frac{te^{-t}}{2}$

$c(t) = \frac{1}{2}t - \left[-te^{-t} - e^{-t}\right]$

$= c(t) = \frac{1}{2}t + \frac{1}{2}e^{-t} + \frac{1}{2}te^{-t}$

$c_2'(t) = \dfrac{\begin{vmatrix}e^{t} & -t\\ e^{t} & e^{t}\end{vmatrix}}{-2} = \cancel{\dfrac{e^{2t}+te^{t}}{-2}}$

$c_2(t) = -\frac{1}{4}e^{2t} - \frac{1}{2}\left[te^{t} - e^{t}\right]$

te^{t}

$e^{t} \quad e^{t}\cdot e^{-t}$

$-e^{t}\cdot e^{-t}$

$-1-$

For the particular soln

$$X_p(t) = c_1(t)\begin{pmatrix}1\\1\end{pmatrix}e^{t} + c_2(t)\, e\begin{pmatrix}1\\-1\end{pmatrix}e^{-t}$$

let $u = t$ $\quad du = dt$

$dv = e^{-t}$ $\quad v = -e^{-t}$

$-te^{-t} + \int e^{-t}\,dt$

e^{-t} $\quad e^{t}\cdot e^{-t}$

$-e^{t}\cdot e^{-t}$

-1

To solve for $c_1(t)$ and $c_2(t)$ we have

$$c_1'(t)\begin{pmatrix}1\\1\end{pmatrix}e^{t} + c_2'(t)\begin{pmatrix}1\\-1\end{pmatrix}e^{-t} = \begin{pmatrix}-t\\e^{t}\end{pmatrix}$$

$$c_1'(t) = \frac{\begin{vmatrix}-t & e^{-t}\\ e^{t} & -e^{-t}\end{vmatrix}}{\begin{vmatrix}e^{t} & e^{-t}\\ e^{t} & -e^{-t}\end{vmatrix}} = \frac{te^{-t} - e^{t}\cdot e^{-t}}{-1-1} = \frac{te^{-t} - 1}{-2}$$

$$= \tfrac{1}{2} - \tfrac{te^{-t}}{2}$$

$$c(t) = \tfrac{1}{2}t - \left[-te^{-t} - e^{-t}\right]$$

te^{t}

$$= c(t) = \tfrac{1}{2}t + \tfrac{1}{2}e^{-t} + \tfrac{1}{2}te^{-t}$$

Eigen vector associated with $\lambda_1 = 1$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = -v_1 + v_2 = 0$$

$v_2 = v_1$

choose $v_1 = 1$

$v_2 = 1$

Eigen vector $= \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

First solut to the system

$$h_1(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t}$$

Eigenvector associated with $\lambda_2 = -1$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = v_3 + v_4 = 0$$

$=$

$\Rightarrow v_4 = -v_3$ = choose $v_3 = 1$

and $v_4 = -1$

2nd solution to the system

$$h_2(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$$

$$X_h(t) = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$$

$\lambda^2 = 1$

Find the eigen value $|A - \lambda I| = 0$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} -\lambda & 1 \\ 1 & -\lambda \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda^2 = 1$$

$$\lambda = \pm 1$$

$\lambda_1 = 1$ and $\lambda_2 = -1$

Eigen vector associated with $\lambda_1 = 1$

$$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda^2 = 1$$

$$\lambda = \pm 1$$

$\lambda_1 = 1$ and $\lambda_2 = -1$

Eigen vector associated with $\lambda_1 = 1$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = -v_1 + v_2 = 0$$

$v_2 = v_1$

choose $v_1 = 1$

$v_2 = 1$

Eigen vector $= \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

First soln to the system

$$h_1(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t}$$

Eigenvector associated with $\lambda_2 = -1$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = v_3 + v_4 = 0$$

$=$

choose $v_3 = 1$

$\lambda^2 - 1$

Find the eigen value $|A - \lambda I| = 0$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} -\lambda & 1 \\ 1 & -\lambda \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda^2 = 1$$

$$\lambda = \pm 1$$

$\lambda_1 = 1$ and $\lambda_2 = -1$

Eigen vector associated with $\lambda_1 = 1$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = -v_1 + v_2 = 0$$

$v_2 = v_1$

Choose $v_1 = 1$

$v_2 = 1$

Eigen vector $= \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Solut to the system

$\lambda = \pm 1$

$\lambda_1 = 1$ and $\lambda_2 = -1$

Eigenvector associated with $\lambda_1 = 1$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = -v_1 + v_2 = 0$$

$v_2 = v_1$

choose $v_1 = 1$

$v_2 = 1$

Eigen vector $= \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

First solution to the system

$$h_1(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t}$$

Eigenvector associated with $\lambda_2 = -1$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = v_3 + v_4 = 0$$

$=$

$\Rightarrow v_4 = -v_3$ = choose $v_3 = 1$

and $v_4 = -1$

2nd solution to the system

$$h_2(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$$

Soln

$U(x,t) = 2e^{-7x - \frac{7t}{2} + \frac{5t}{2}}$

2 (a)

2 a) $\frac{di}{dt} = v - t$ and $\frac{dv}{dt} = e^{t} + i$ Condition

$i(0) = 0 \quad v(0) = 0$ | $x_1(0) = 0$, $x_2(0) = 0$

Let $x_1 = i$ and $x_2 = v$

$x_1' = i' = \frac{di}{dt} = x_2 - t$

$x_2' = v' = \frac{dv}{dt} = x_1 + e^{t}$

$\frac{di}{dt} = x_2 - t$

$\frac{dv}{dt} = x_1 + e^{t}$

In matrix form

$$\begin{pmatrix} \frac{di}{dt} \\ \frac{dv}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} -t \\ e^{t} \end{pmatrix}$$

3.

$\Rightarrow D = 2$ and $\lambda = -7$

Soln

$$u(x,t) = 2e^{-7x - \frac{7t}{2} + \frac{5t}{2}}$$

2(a)

2a) $\frac{di}{dt} = v - t$ and $\frac{dv}{dt} = e^{t} + i$

Condition

$i(0) = 0 \quad v(0) = 0$

$x_1(0) = 0$

$x_2(0) = 0$

Let $x_1 = i$ and $x_2 = v$

$$x_1' = i' = \frac{di}{dt} = x_2 - t$$

$$x_2' = v' = \frac{dv}{dt} = x_1 + e^{t}$$

$$\frac{di}{dt} = x_2 - t$$

$$\frac{dv}{dt} = x_1 + e^{t}$$

In matrix form

1. F
(

$$u(x,t) = D e^{\lambda x + \lambda t/2 + 5t/2}$$

2. I

$$u(x,0) = 2e^{-7x} = D e^{\lambda x}$$

3.

$$\Rightarrow D = 2 \quad \text{and} \quad \lambda = -7$$

Soln

$$\underline{\underline{u(x,t) = 2 e^{-7x - 7t/2 + 5t/2}}}.$$

2 (a)

2a) $\frac{di}{dt} = v - t$ and $\frac{dv}{dt} = e^{t} + i$ Condition

$i(0) = 0 \quad v(0) = 0$ | $x_1(0) = 0$, $x_2(0) = 0$

Let $x_1 = i$ and $x_2 = v$

$$x_1' = i' = \frac{di}{dt} = x_2 - t$$

$$x_2' = v' = \frac{dv}{dt} = x_1 + e^{t}$$

$$u(x,t) = Ae^{\lambda x} \cdot C e^{\frac{5t}{2} + \frac{\lambda t}{2}}$$

Applying the condition

$$u(x,t) = D e^{\lambda x + \frac{\lambda t}{2} + \frac{5t}{2}}$$

$$u(x,0) = 2e^{-7x} = D e^{\lambda x}$$

$$\Rightarrow D = 2 \quad \text{and} \quad \lambda = -7$$

Soln

$$u(x,t) = 2 e^{-7x - \frac{7t}{2} + \frac{5t}{2}}$$

2(a)

Q2a) $\frac{di}{dt} = v - t$ and $\frac{dv}{dt} = e^{t} + i$ Condition

$i(0) = 0$ $\quad v(0) = 0$ $\quad x_1(0) = 0$

$x_2(0) = 0$

separating the variables

$\frac{X'}{X} = \lambda$ — (1) and $5 - \frac{2T'}{T} = \lambda$ — (2)

solving (1) we have

$\frac{dX}{dx X} = \lambda \Rightarrow \frac{dX}{X} = \lambda dx$

upon integration we have

$\ln X = \lambda x + c \quad X(x) = e^{\lambda x + c} = X(x) = A e^{\lambda x}$

solving (2) we have

$5 - \frac{2T'}{T} = \lambda \Rightarrow 5 - \lambda = \frac{2T'}{T} \Rightarrow \frac{T'}{T} = 5/2 - \lambda/2$

$\Rightarrow \frac{dT}{dt\, T} = 5/2 - \lambda/2 \Rightarrow \frac{dT}{T} = (5/2 - \lambda/2)\, dt$

upon integration we have

$$u(x,t) = A e^{\lambda x} \cdot B e^{\frac{5}{2}t + \frac{\lambda t}{2}}$$

Applying the condition

$$u(x,t) = D e^{\lambda x + \frac{\lambda t}{2} + \frac{5t}{2}}$$

$$u(x,0) = 2e^{-7x} = D e^{\lambda x}$$

$$\Rightarrow D = 2 \quad \text{and} \quad \lambda = -7$$

Sln $\quad 7x - 7t + 5t$

$$\frac{dX}{dxX} = \lambda \Rightarrow \frac{dX}{X} = \lambda dx$$

upon integration we have

$$\ln X = \lambda x + c \qquad X_{(x)} = e^{\lambda x + c} = X_{(x)} = A e^{\lambda x}$$

solving (2) we have

$$5 - \frac{2T'}{T} = \lambda \Rightarrow 5 - \lambda = \frac{2T'}{T} \Rightarrow \frac{T'}{T} = \frac{5}{2} - \frac{\lambda}{2}$$

$$\Rightarrow \frac{dT}{dtT} = \frac{5}{2} - \frac{\lambda}{2} \Rightarrow \frac{dT}{T} = \left(\frac{5}{2} - \frac{\lambda}{2}\right) dt$$

upon integration we have

$$\ln T = \frac{5t}{2} + \frac{\lambda t}{2} + B$$

$$\Rightarrow T_{(t)} = C e^{\frac{5}{2}t + \frac{\lambda t}{2}}$$

$$U(x,t) = X_{(x)} T_{(t)}$$

$\ln X = \lambda x + c$ $X_{(x)} = e^{\lambda x + c}$ $= X_{(x)} = A e^{\lambda x}$

solving (2) we have

$5 - \frac{2T'}{T} = \lambda$ $\Rightarrow$ $5 - \lambda = \frac{2T'}{T}$ $\Rightarrow$ $\frac{T'}{T} = \frac{5}{2} - \frac{\lambda}{2}$

$\Rightarrow$ $\frac{dT}{dt\, T} = \frac{5}{2} - \frac{\lambda}{2}$ $\Rightarrow$ $\frac{dT}{T} = \left(\frac{5}{2} - \frac{\lambda}{2}\right) dt$

upon integration we have

$\ln T = \frac{5}{2}t + \frac{\lambda t}{2} + B$

$\Rightarrow$ $T_{(t)} = C e^{\frac{5}{2}t + \frac{\lambda t}{2}}$

Gen Soln $U(x,t) = X_{(x)} T_{(t)}$

$+ \quad \frac{}{2} \quad -\frac{}{2}\frac{\lambda}{X}$ or other words

$\Rightarrow \frac{X'}{X} = 5 - \frac{2T'}{T} = \lambda$

separating the variables

$\frac{X'}{X} = \lambda$ — (1) and $5 - \frac{2T'}{T} = \lambda$ — (2)

solving (1) we have

$\frac{dX}{dx X} = \lambda \Rightarrow \frac{dX}{X} = \lambda dx$

Upon integration we have

$\ln X = \lambda x + c \qquad X(x) = e^{\lambda x + c} = X(x) = A e^{\lambda x}$

solving (2) we have

$5 - \frac{2T'}{T} = \lambda \Rightarrow 5 - \lambda = \frac{2T'}{T} \Rightarrow \frac{T'}{T} = \frac{5-\lambda}{2}$

$$\frac{T'}{T} = \frac{5}{2} - \frac{1}{2}\frac{X'}{X}$$

or other words

$$\Rightarrow \frac{X'}{X} = 5 - \frac{2T'}{T} = \lambda$$

separating the variables

$$\frac{X'}{X} = \lambda \quad \text{—(1)} \quad \text{and} \quad 5 - \frac{2T'}{T} = \lambda \quad \text{—(2)}$$

solving (1) we have

$$\frac{dX}{dx X} = \lambda \Rightarrow \frac{dX}{X} = \lambda dx$$

Upon integration we have

$$\ln X = \lambda x + c \qquad X(x) = e^{\lambda x + c} = X_0$$

$$\frac{T'}{T} = \frac{5}{2} - \frac{1}{2}\frac{X'}{X}$$ or other words

$$\Rightarrow \frac{X'}{X} = 5 - \frac{2T'}{T} = \lambda$$

separating the variable

$$\frac{X'}{X} = \lambda \quad -(1)$$ and $$5 - \frac{2T'}{T} = \lambda \quad -$$

solving (1) we have

Separation of Variables

$$2\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 5u \qquad \text{Condition} \quad u(x,0) = 2e^{-7x}$$

<u>Soln</u>

let $u(x,t) = X(x)\,T(t)$

$2XT' + X'T = 5XT$, dividing by XT we h

$$\Rightarrow \frac{2XT'}{XT} + \frac{X'T}{XT} = \frac{5\cancel{XT}}{\cancel{XT}}$$

$$\Rightarrow 2\frac{T'}{T} + \frac{X'}{X} = 5$$

$$\Rightarrow 2\frac{T'}{T} = 5 - \frac{X'}{X}$$

4(b)

(4(b)) Separation of Variable

$2\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 5u$ Condition $u(x,0) = 2e^{-7x}$

Soln

let $u(x,t) = X(x)T(t)$

$2XT' + X'T = 5XT$, dividing by XT we have

$\Rightarrow \frac{2XT'}{XT} + \frac{X'T}{XT} = \frac{5\cancel{XT}}{\cancel{XT}}$

$\Rightarrow 2\frac{T'}{T} + \frac{X'}{X} = 5$

$\Rightarrow 2\frac{T'}{} = 5 - \frac{X'}{X}$

$+ 7e^{2x} - 1 - 6e^{2y} + \cos(y\ldots)$

$$\underline{u(x,y) = \cos y + 7e^{2x} - 1 - 6e^{2y}}$$

4(b)

4(b) Separation of Variable

$$2\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 5u \qquad \text{Condition } u(x,0) = 2e^{-7x}$$

Soln

Let $u(x,t) = X(x)\,T(t)$

$2XT' + X'T = 5XT$, dividing by XT we have

$$\Rightarrow \frac{2XT'}{XT} + \frac{X'T}{XT} = \frac{5\cancel{XT}}{\cancel{XT}}$$

$$\Rightarrow 2\frac{T'}{T} + \frac{X'}{X} = 5$$

$$u(x,y) = -6e^{2x-3y} + \cos y + 6e^{2\pi-3y} - \cancel{c(\pi)} + \cdots$$

$$+ 7e^{2x} - 1 - 6e^{2\pi} + \cancel{c(\pi)}$$

$$\underline{u(x,y) = \cos y + 7e^{2x} - 1 - 6e^{2\pi}}$$ $\boxed{\checkmark}$

4(b)

(4(b)) Separation of Variable

$$2\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 5u$$ Condition $u(x,0) = 2e^{-7x}$

Soln

Let $u(x,t) = X(x)\,T(t)$

$$2XT' + X'T = 5XT$$, dividing by XT

$$G(x) = 7e^{2x} - 1 - 6e^{2\pi} + G(\pi) \longrightarrow (5)$$

Substituting (3) and (5) into (1) we have

$$u(x,y) = -\cancel{6e^{2x-3y}} + \cos y + \cancel{6e^{2\pi-3y}} - \cancel{G(\pi)} + \cdots$$

$$+ 7e^{2x} - 1 - 6e^{2\pi} + \cancel{G(\pi)}$$

$$\underline{u(x,y) = \cos y + 7e^{2x} - 1 - 6e^{2\pi}}$$

4(b)

4(b) Separation of Variable

Condition $u(x,y) =$

$G(x) = 7e^{2x} - 1 - 6e^{2\pi} + G(\pi)$ ⟶ (5)

Substituting (3) and (5) into (1) we have

$u(x,y) = \cancel{} = -\cancel{6e^{2x-3y}} + \cos y + \cancel{6e^{2\pi-3y}} - \cancel{G(\pi)} +$

$+ 7e^{2x} - 1 - 6e^{2\pi} + \cancel{G(\pi)}$

$\quad e^{2x} \quad 1 - 6e^{2\pi}$

1st initial condition

$$u(x,0) = e^{2x} = -6e^{2x} + F(0) + G(x)$$

$$\Rightarrow G(x) = e^{2x} + 6e^{2x} - F(0) \quad \text{—— (2)}$$

$$\Rightarrow G(x) = 7e^{2x} - F(0)$$

2nd initial condit.

$$u(\pi, y) = \cos y = -6e^{2\pi - 3y} + F(y) + G(\pi)$$

$$\Rightarrow F(y) = \cos y + 6e^{2\pi - 3y} - G(\pi) \quad \text{—— (3)}$$

let do it at $F(0)$ we have

$$F(0) = \cos(0) + 6e^{2\pi - 3(0)} - G(\pi)$$

$$F(0) = 1 + 6e^{2\pi} - G(\pi) \quad \text{—— (4)}$$

~~Take~~ $G(x)$

Taking (4) and substituting in (2)

∂y

Integrating w.r.t y

$$U(x,y) = -6e^{2x-3y} + F(y) + G(x) \quad \text{—(1)}$$

1st Initial Condition

$$U(x,0) = e^{2x} = -6e^{2x} + F(0) + G(x)$$

$$\Rightarrow G(x) = e^{2x} + 6e^{2x} - F(0) \quad \text{—— (2)}$$

$$\Rightarrow G(x) = 7e^{2x} - F(0)$$

2nd initial condition

$$U(\pi, y) = \cos y = -6e^{2\pi-3y} + F(y) + G(\pi)$$

$$\Rightarrow F(y) = \cos y + 6e^{2\pi-3y} - G(\pi) \quad \text{—(3)}$$

let do it at $F(0)$ we have

$$F(0) = \cos(0) + 6e^{2\pi-3(0)} - G(\pi)$$

$$F(0) = 1 + 6e^{2\pi} - G(\pi) \quad \text{—(4)}$$

Soln

Integrating w.r.t x.

$$\frac{\partial u}{\partial y} = 2e^{2x-3y} + f(y)$$

Integrating w.r.t y

$$u(x,y) = -6e^{2x-3y} + F(y) + G(x) \quad \text{--- (1)}$$

1st Initial Condition

$$u(x,0) = e^{2x} = -6e^{2x} + F(0) + G(x)$$

$$\Rightarrow G(x) = e^{2x} + 6e^{2x} - F(0)$$

$$\Rightarrow G(x) = 7e^{2x} - F(0) \quad \text{--- (2)}$$

2nd initial condition

$$u(\pi, y) = \cos y = -6e^{2\pi-3y} + F(y) + G(\pi)$$

$$\Rightarrow F(y) = \cos y + 6e^{2\pi-3y} - G(\pi)$$

(4a) 4 (a) Using direct integration

$$\frac{\partial^2 u}{\partial x \partial y} = e^{2x-3y}$$

Condition
$u(x,0) = e^{2x}$, $u(\pi, y) = \cos y$

Soln
Integrating w.r.t x.

$$\frac{\partial u}{\partial y} = 2e^{2x-3y} + f(y)$$

Integrating w.r.t y

$$U(x,y) = -6e^{2x-3y} + F(y) + G(x) \quad \text{--- (1)}$$

1^{st} Initial Condition

$$U(x,0) = e^{2x} = -6e^{2x} + F(0) + G(x)$$

$$\Rightarrow G(x) = e^{2x} + 6e^{2x} - F(0)$$

$$\Rightarrow G(x) = 7e^{2x} - F(0) \quad \text{--- (2)}$$

4 (a)

(4)(a) Using direct integration

$$\frac{\partial^2 u}{\partial x \partial y} = e^{2x-3y}$$

Condition

$u(x,0) = e^{2x}$, $u(\pi, y) = \cos y$

Soln

Integrating w.r.t x.

$$\frac{\partial u}{\partial y} = 2e^{2x-3y} + f(y)$$

Integrating w.r.t y

$$U(x,y) = -6e^{2x-3y} + F(y) + G(x)$$