NAME:

1) The Lifetime of the new TRACC battery is normally distributed. It has a mean of 100 hours and a standard deviation of 6.25 hours. If a battery is selected at random, determine the following:

(a) P (X > 105) [3]

Solution

$$\Rightarrow \mu = 100 \text{ --mean, } \sigma = 6.25\text{--standard deviation}$$

$$\Rightarrow P(X > 105),$$
$$\Rightarrow \text{ Let X=105}$$

$$\Rightarrow Z = \frac{X-\mu}{\sigma} = \frac{105-100}{6.25} = 0.8$$

$$\Rightarrow P(X > 105) = P(Z > 0.8) = 1.0 - P(Z \leq 0.8)$$

$$\Rightarrow 1.0 - 0.7881 = 0.2119$$

(b) P (115 < X < 120) [4]

Solution

$$\Rightarrow \mu = 100 \text{ --mean}, \sigma = 6.25\text{--standard deviation}$$

$$\Rightarrow P(115 < X < 120),$$
$$\Rightarrow \text{ Let X=115}$$

$$\Rightarrow Z = \frac{X - \mu}{\sigma} = \frac{115 - 100}{6.25} = 2.4$$

$$\Rightarrow \text{ Let X=120}$$

$$\Rightarrow Z = \frac{X - \mu}{\sigma} = \frac{120 - 100}{6.25} = 3.2$$

$$\Rightarrow P(2.4 < Z < 3.2) = P(Z < 3.2) - P(Z \leq 2.4)$$

$$\Rightarrow 0.99931 - 0.9918 = 0.00751$$

(c) P (X < 110) [2]

Solution

$\Rightarrow \mu = 100$ --mean, σ =6.25--standard deviation

$$\Rightarrow P(X < 110),$$
$$\Rightarrow \text{ Let X=110}$$

$$\Rightarrow Z = \frac{X-\mu}{\sigma} = \frac{110-100}{6.25} = 1.6$$

$$\Rightarrow P(X < 110) = P(Z < 1.6)$$

$$\Rightarrow = 0.9452$$

2) The probability that John, a cricketer will strike a ball for 6 in a game is 0.4. Assume that in a particular game he only faces 12 deliveries (balls). Determine that the number of deliveries he will hit is: [1]

(a) exactly seven sixes

Solution

$$\Rightarrow p = 0.4 = \frac{4}{10}, q = 0.6 = \frac{6}{10}$$

$$\Rightarrow n = 12, r = 7$$

$$\Rightarrow P(X=7) = \binom{12}{7} \times 0.4^7 \times 0.6^5$$

$$\Rightarrow 792 \times 0.4^7 \times 0.6^5 = 0.1009$$

(b) at most one six

Solution

$$\Rightarrow p = 0.4 = \frac{4}{10}, q = 0.6 = \frac{6}{10}$$

$$\Rightarrow n = 12$$

$$\Rightarrow P(X \leq 1) = P(X = 0) + P(X = 1)$$

$$\Rightarrow P(X = 0) = \binom{12}{0} \times 0.4^0 \times 0.6^{12}$$

$$\Rightarrow 1 \times 0.4^0 \times 0.6^{12} = 0.002177$$

$$\Rightarrow P(X = 1) = \binom{12}{1} \times 0.4^1 \times 0.6^{11}$$

$$\Rightarrow 12 \times 0.4^1 \times 0.6^{11} = 0.01741$$

$$\Rightarrow P(X \leq 1) = 0.002177 + 0.01741$$

(c) between 3 and 6 sixes

Solution

$$\Rightarrow p = 0.4 = \frac{4}{10}, q = 0.6 = \frac{6}{10}$$

$$\Rightarrow n = 12$$

$$\Rightarrow P(3 \leq X \leq 6) = P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$$

$$\Rightarrow P(X = 3) = \binom{12}{3} \times 0.4^3 \times 0.6^9$$

$$\Rightarrow 220 \times 0.4^3 \times 0.6^9 = 0.1419$$

$$\Rightarrow P(X = 4) = \binom{12}{4} \times 0.4^4 \times 0.6^8$$

$$\Rightarrow 495 \times 0.4^4 \times 0.6^8 = 0.2128$$

$$\Rightarrow P(X = 5) = \binom{12}{5} \times 0.4^5 \times 0.6^7$$

$$\Rightarrow 792 \times 0.4^5 \times 0.6^7 = 0.2270$$

$$\Rightarrow P(X = 6) = \binom{12}{6} \times 0.4^6 \times 0.6^6$$

$$\Rightarrow 924 \times 0.4^6 \times 0.6^6 = 0.1766$$

$$\Rightarrow P(3 \leq X \leq 6) = 0.1419 + 0.2128 + 0.2270 + 0.1766 = 0.7583$$

(d) How many balls do we expect him to hit for six?

Solution

$$\Rightarrow \text{number of balls we expect him to hit for six}$$

$$\Rightarrow 0.4 = \frac{4}{10} \times 12$$

$$\Rightarrow 4.8 \text{ balls } \simeq 5 \text{ balls}$$

THE END