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-Revision Materials For PDEs & ODEs

### Question 4

$$36y^2\phi_{xx} - 12\phi_{xy} + \phi_{yy} - 6(1+xy)\phi_x + x\phi_y = 0$$

Soln

a) let  $A = 36y^2$ ,  $B = -12y$ ,  $C = 1$

$$\Delta = B^2 - 4AC = 144y^2 - 144y^2 = 0 \Rightarrow \text{Parabolic}$$

b) Canonical form

The d.e for characteristic

$$\frac{dy}{dx} = \frac{-12y \pm \sqrt{0}}{72y^2} = -\frac{1}{6y}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{6y} \Rightarrow 6y dy + dx = 0$$

$$\int 6y dy + \int dx = \int 0 \Rightarrow 3y^2 + x = c_1$$

choose  $\xi = 3y^2 + x$

choose  $\eta$  "wisely"

$$\begin{vmatrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{vmatrix} \neq 0 \Rightarrow \begin{vmatrix} 1 & 6y \\ 0 & 1 \end{vmatrix} = 1 \neq 0$$

$$\eta = y$$

$$\Rightarrow \xi = 3y^2 + x$$

$$\eta = y$$

We form  $\phi_x, \phi_y, \phi_{xx}, \phi_{xy}, \phi_{yy}$

$$\Rightarrow \phi_x = \frac{\partial \phi}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial \phi}{\partial \eta} \frac{\partial \eta}{\partial x}$$

$$\phi_x = \phi_\xi$$

$$\phi_{xx} = \frac{\partial \phi_\xi}{\partial \xi} \frac{\partial \xi}{\partial x} = \phi_{\xi\xi}$$

$$\phi_{xy} = \frac{\partial \phi_\xi}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial \phi_\eta}{\partial \eta} \frac{\partial \eta}{\partial y} = 6y\phi_{\xi\xi} + \phi_{\xi\eta}$$

$$\phi_y = \frac{\partial \phi}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial \phi}{\partial \eta} \frac{\partial \eta}{\partial y} = 6y\phi_\xi + \phi_\eta$$

$$\phi_{yy} = \frac{\partial}{\partial \xi} [6y\phi_\xi + \phi_\eta] \frac{\partial \xi}{\partial y} + \frac{\partial}{\partial \eta} [6y\phi_\xi + \phi_\eta] \frac{\partial \eta}{\partial y}$$

$$\phi_{yy} = 36y^2\phi_{\xi\xi} + 12\phi_{\xi\eta} + \phi_{\eta\eta}$$

Substitute all into eqn (1)

$$36y^2\phi_{\xi\xi} - 12y[6y\phi_{\xi\xi} + \phi_{\xi\eta}] + \phi_{\eta\eta} - 6(1+xy)\phi_\xi + x\phi_\eta = 0$$

$$\Rightarrow \phi_{\eta\eta} - 6\phi_\xi + x\phi_\eta = 0$$

Since  $\phi$  is a function of both  $\xi$  and  $\eta$  we have

$$\phi_{\eta\eta} + (x-6)\phi_\eta = 0$$

$$\Rightarrow [\phi_{\eta\eta} + x\phi_\eta = 0]$$

c) Solving the canonical form

$$\frac{\partial^2 \phi}{\partial \eta^2} + x\phi_\eta = 0$$

This is a 2nd order ode

$\Rightarrow$  The characteristic is given by

$$\lambda^2 + x = 0 \Rightarrow \lambda^2 = -x$$

$$\Rightarrow \lambda = \pm i\sqrt{x}$$

$$\phi(\xi, \eta) = c_1 \cos \sqrt{x}\eta + c_2 \sin \sqrt{x}\eta$$

$$\Rightarrow \phi(x, y) = c_1 \cos \sqrt{x}y + c_2 \sin \sqrt{x}y$$

d) For  $x = 6$

$$\phi(x, 0) = \phi = 0, \phi_y(x, 0) = x \sin x$$

$$\Rightarrow \phi(x, 0) = c_1 = 0$$

$$\Rightarrow \phi(x, y) = c_2 \sin \sqrt{x}y$$

$$\phi_y(x, y) = \sqrt{x} c_2 \cos \sqrt{x}y$$

$$\phi_y(x, 0) = \sqrt{x} c_2 = x \sin x \Rightarrow c_2 = \frac{x \sin x}{\sqrt{x}}$$

$$\boxed{\phi(x, y) = \sqrt{x} \sin x \cdot \sin \sqrt{x}y}$$

$36y^2 \phi_{xx} - 12 \phi_{xy} + \phi_{yy} - 6(1+xy) \phi_x + x \phi_y = 0$

Soln

a) let  $A = 36y^2$ ,  $B = -12y$ ,  $C = 1$   
 $\Delta = B^2 - 4AC = 144y^2 - 144y^2 = 0 \Rightarrow$  Parabolic

b) Canonical form  
 The d.e for characteristic

$$\frac{dy}{dx} = \frac{-12y \pm \sqrt{0}}{72y^2} = -\frac{1}{6y}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{6y} \Rightarrow 6y dy + dx = 0$$

$$\int 6y dy + \int dx = \int 0 \Rightarrow 3y^2 + x = c_1$$

choose  $\xi = 3y^2 + x$   
 choose  $\eta$  "wisely"

$$\begin{vmatrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{vmatrix} \neq 0 \Rightarrow \begin{vmatrix} 1 & 6y \\ 0 & 1 \end{vmatrix} = 1 \neq 0$$

$$\eta = y$$

$$\Rightarrow \xi = 3y^2 + x$$

$$\eta = y$$

we form  $\phi_x, \phi_y, \phi_{xx}, \phi_{xy}, \phi_{yy}$

$$\phi_x = \frac{\partial \phi}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial \phi}{\partial \eta} \frac{\partial \eta}{\partial x}$$

$$\phi_x = \phi_\xi$$

$$\phi_{xx} = \frac{\partial \phi_\xi}{\partial \xi} \frac{\partial \xi}{\partial x} = \phi_{\xi\xi}$$

$$\phi_{xy} = \frac{\partial \phi_\xi}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial \phi_\eta}{\partial \eta} \frac{\partial \eta}{\partial y}$$

$$= 6y \phi_{\xi\xi} + \phi_{\xi\eta}$$

$$\phi_y = \frac{\partial \phi}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial \phi}{\partial \eta} \frac{\partial \eta}{\partial y}$$

$$= 6y \phi_\xi + \phi_\eta$$

$$\phi_{yy} = 2 \left[ 6y \phi_\xi + \phi_\eta \right] \frac{\partial \xi}{\partial y} + 2 \left[ 6y \phi_\xi + \phi_\eta \right] \frac{\partial \eta}{\partial y}$$

$$\phi_{yy} = 36y^2 \phi_{\xi\xi} + 12 \phi_{\xi\eta} + \phi_{\eta\eta}$$

Substitute all into eqn (1)

$$36y^2 \phi_{\xi\xi} - 12y [6y \phi_{\xi\xi} + \phi_{\xi\eta}] + \phi_{\eta\eta} - 6(1+xy) \phi_\xi + x \phi_\eta = 0$$

$$+ 36y^2 \phi_{\xi\xi} + 12 \phi_{\xi\eta} + \phi_{\eta\eta} - 6 \phi_\xi - 6xy \phi_\xi + 6xy \phi_\xi + x \phi_\eta = 0$$

$$\Rightarrow \phi_{\eta\eta} - 6 \phi_\xi + x \phi_\eta = 0$$

Since  $\phi$  is a function of both  $\xi$  and  $\eta$  we have

$$\phi_\eta + (x-6) \phi_\xi = 0$$

$$\Rightarrow \phi_\eta + x \phi_\xi = 0$$

c) Solving the canonical form

$$\frac{\partial^2 \phi}{\partial \eta^2} + x \phi_\xi = 0$$

This is a 2nd order ode

$$\Rightarrow$$
 The characteristic is given by
 
$$\lambda^2 + x = 0 \Rightarrow \lambda^2 = -x$$

$$\Rightarrow \lambda = \pm i\sqrt{x}$$

$$\phi(\xi, \eta) = c_1 \cos \sqrt{x} \eta + c_2 \sin \sqrt{x} \eta$$

$$\Rightarrow \phi(x, y) = c_1 \cos \sqrt{x} y + c_2 \sin \sqrt{x} y$$

d) For  $x = 6$

$$\phi(x, 0) = 0 = 0, \phi_y(x, 0) = x \sin x$$

$$\Rightarrow \phi(x, 0) = c_1 = 0$$

$$\Rightarrow \phi(x, y) = c_2 \sin \sqrt{x} y$$

$$\phi_y(x, y) = \sqrt{x} c_2 \cos \sqrt{x} y$$

$$\phi_y(x, 0) = \sqrt{x} c_2 = x \sin x \Rightarrow c_2 = \frac{x \sin x}{\sqrt{x}}$$

$$\phi(x, y) = \sqrt{x} \sin x \cdot \sin \sqrt{x} y$$



-Revision Materials For PDEs &amp; ODEs

$$3\epsilon y^2 \phi_{xx} - 12\phi_{xy} + 4y y - 6(1+xy)\phi_x + x\phi_y = 0$$

a)  $\underline{A = 36 \mu^2}$ ,  $B = -12 \mu$ ,  $C = 1$

$$\Delta = B^2 - 4AC = 144y^2 - 144y^2 = 0 \Rightarrow \text{Parabel}$$

b) Canonical form

The d.e for characteristic

$$\frac{dy}{dx} = -\frac{12y\sqrt{6}}{7x^2} = -\frac{1}{6y}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{ay} \Rightarrow ay dy + dx = 0$$

$$\int \delta y dy + \int dx = \int 0 \Rightarrow 2y^2 + x = C$$

Chair  $E = 319^2 + x$

chase 2 "wily"

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \neq 0$$

$$g = 2$$

①

Substrate all info enr ①

$$3G\gamma^2\phi_{ee} - 12\gamma\gamma[\phi_{ey}\phi_{ez} + \phi_{ez}^2] + \dots$$

$$\Rightarrow \psi_n - e \psi_e + x \psi_n = 0$$

Since  $\phi$  is a function of

$\epsilon$  and  $\eta$

$$\phi_n + (\alpha - \beta) \phi(\epsilon_n) = 0$$

$$\Rightarrow \sqrt{\psi^* \psi} = 0$$

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-Revision Materials For PDEs & ODEs

b) Canonical form  
The de for characteristic

$$\frac{dy}{dx} = -\frac{12y \pm \sqrt{0}}{72y^2} = -\frac{1}{6y}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{6y} \Rightarrow 6y dy + dx = 0$$

$$\int 6y dy + \int dx = \int 0 \Rightarrow 3y^2 + x = c_1$$

Choose  $\xi = 3y^2 + x$   
choose  $\eta$  "arbitrary"

$$\left| \begin{matrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{matrix} \right| \neq 0 \Rightarrow \left| \begin{matrix} 1 & 6y \\ 0 & 1 \end{matrix} \right| = 1 \neq 0$$

$$\Rightarrow \xi = 3y^2 + x$$

$$\eta = y$$

Now form  $\phi_x, \phi_y, \phi_{xx}, \phi_{xy}, \phi_{yy}$

$$\phi_x = \frac{\partial \phi}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial \phi}{\partial \eta} \frac{\partial \eta}{\partial x}$$

$$\phi_x = \phi_\xi$$

$$\phi_{xx} = \phi_{\xi\xi}$$

Substitute all into eqn

$$36y^2 \phi_{\xi\xi} - 12y [6y \phi_{\xi\xi} + \phi_{\xi\eta}] + \dots$$

$$+ 36y^2 \phi_{\xi\xi} + 12 \phi_{\xi\eta} + \phi_{\eta\eta} - 6 \phi_{\xi\xi} -$$

$$- 6xy \phi_{\xi\xi} + 6xy \phi_{\xi\eta} + x \phi_{\eta\eta} = 0$$

$$\Rightarrow \phi_{\eta\eta} - 6 \phi_{\xi\xi} + x \phi_{\eta\eta} = 0$$

Since  $\phi$  is a function of both  $\xi$  and  $\eta$  we have

$$\phi_{\eta\eta} + (x-6) \phi_{\eta\eta} = 0$$

$$\Rightarrow \phi_{\eta\eta} + x \phi_{\eta\eta} = 0$$

c) Solving the canonical form

$$\frac{\partial^2 \phi}{\partial \eta^2} + x \phi_{\eta\eta} = 0$$

$\eta$  is a 2nd order ode  
characteristic is given by

$$\Rightarrow \lambda^2 + x = 0 \Rightarrow \lambda^2 = -x$$

$$\Rightarrow \lambda = \pm i\sqrt{x}$$

$$\phi(\xi, \eta) = C_1 \cos(\sqrt{x}\eta) + C_2 \sin(\sqrt{x}\eta)$$



-Revision Materials For PDEs &amp; ODEs

## -Revision Materials For PDEs & ODEs

$$dy = \frac{\partial y}{\partial x} dx + \frac{\partial y}{\partial t} dt$$

$$f(x) = \frac{1}{x^2} = x^{-2}$$

2/2



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-Revision Materials For PDEs & ODEs

Ques 1

$$t^2 x'' - (x+2)t x' + (x+2)x = 0$$

Soln

Reducing the ode into a system of odes

Let  $x_1 = x$ ,  $x_2 = x'$ ,  $x_3 = x''$

$$\Rightarrow \frac{dx_1}{dt} = \frac{dx}{dt} = x_1' = x' = x_2$$

$$\frac{dx_2}{dt} = \frac{dx'}{dt} = x'' = x_3$$

$$\Rightarrow \frac{dx_1}{dt} = x_2 \quad \text{--- (1)}$$

$$\frac{dx_2}{dt} = \frac{(x+2)t x_2 - (x+2)x_1}{t^2} \quad \text{--- (2)}$$

In matrix form we have

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{x+2}{t^2} & \frac{x+2}{t} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

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-Revision Materials For PDEs & ODEs

Reducing the order with a system

Let  $x_1 = x$ ,  $x_2 = x'$ ,  $x_3 = x''$

$$\Rightarrow \frac{dx_1}{dt} = \frac{dx_2}{dt} = x_1' = x_2 = x_3$$

$$\frac{dx_2}{dt} = \frac{dx_1'}{dt} = x_1'' = x_3$$

$$\Rightarrow \frac{dx_1}{dt} = x_2 \quad \text{--- (1)}$$

$$\frac{dx_2}{dt} = \frac{(x+2)x_2}{t^2} - \frac{(x+2)}{t^2} x_1 \quad \text{--- (2)}$$

In matrix form we have

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{x+2}{t^2} & \frac{x+2}{t} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

For  $x = 6$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{8}{t^2} & \frac{8}{t} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Associated homogeneous eqn

$$\begin{pmatrix} 0 & 1 \\ -\frac{8}{t^2} & \frac{8}{t} \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} -\lambda & 1 \\ -\frac{8}{t^2} & \frac{8}{t} - \lambda \end{pmatrix}$$

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{8}{t^2} & \frac{8}{t} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

For  $x = 6$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{8}{t^2} & \frac{8}{t} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Associated homogeneous eqn

$$\begin{pmatrix} 0 & 1 \\ -\frac{8}{t^2} & \frac{8}{t} \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} -\lambda & 1 \\ -\frac{8}{t^2} & \frac{8}{t} - \lambda \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & 1 \\ -\frac{8}{t^2} & \frac{8}{t} - \lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - \frac{8}{t}\lambda + \frac{8}{t^2} = 0$$

For  $t > 0$

$$a = 1, b = -\frac{8}{t}, c = \frac{8}{t^2}$$

$$\lambda = \frac{8}{t} \pm \sqrt{\frac{64}{t^2} - \frac{32}{t^2}} \cdot \frac{1}{2}$$

$$\lambda_1 = \frac{6}{t}, \lambda_2 = \frac{2}{t}$$

Eigen vector associated with  $\lambda_1$

$$\begin{pmatrix} -\frac{6}{t} & 1 \\ -\frac{8}{t^2} & -\frac{2}{t} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$h_1(t) = \begin{pmatrix} \frac{1}{6t} \\ \frac{1}{6t} \end{pmatrix} e^{\frac{6}{t}t} = \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \end{pmatrix} e$$

Eigen vector associated with  $\lambda_2$

$$\begin{pmatrix} -\frac{2}{t} & 1 \\ -\frac{8}{t^2} & \frac{6}{t} \end{pmatrix} \begin{pmatrix} v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow v_4 = \frac{2}{t} v_3 \Rightarrow v_3 = 1$$

$$v_4 = \frac{2}{t}$$

$$h_2(t) = \begin{pmatrix} \frac{1}{2t} \\ \frac{2}{t} \end{pmatrix} e^{\frac{2}{t}t} = \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix} e^2$$

$$X_h(t) = c_1 \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \end{pmatrix} e^6 + c_2 \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix} e^2$$

$$(ii) \frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = \frac{8}{t} x_2 - \frac{8}{t^2} x_1 + 8$$

In matrix form

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{8}{t^2} & \frac{8}{t} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 8 \end{pmatrix}$$

General soln is given by

$$X_h(t) = c_1 \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \end{pmatrix} e^6 + c_2 \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix} e^2$$

For the particular soln

$$X_p(t) = c_1(t) \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \end{pmatrix} e^6 + c_2(t) \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix} e^2$$

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{8}{t^2} & \frac{8}{t} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 8 \end{pmatrix}$$

In matrix form

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{8}{t^2} & \frac{8}{t} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 8 \end{pmatrix}$$

General soln is given by

$$X_h(t) = c_1 \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \end{pmatrix} e^6 + c_2 \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix} e^2$$

For the particular soln

$$X_p(t) = c_1(t) \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \end{pmatrix} e^6 + c_2(t) \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix} e^2$$

To solve for  $c_1(t)$  &  $c_2(t)$

$$c_1'(t) \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \end{pmatrix} e^6 + c_2'(t) \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix} e^2 = \begin{pmatrix} 0 \\ 8 \end{pmatrix}$$

$$c_1'(t) = \frac{\begin{vmatrix} 0 & e^2 \\ 8 & \frac{1}{2}e^2 \end{vmatrix}}{\begin{vmatrix} \frac{1}{6}e^6 & e^6 \\ \frac{1}{6}e^6 & \frac{1}{2}e^6 \end{vmatrix}} = 2te^{10}$$

$$c_1(t) = \frac{t^2}{e^{10}}$$

$$c_2'(t) = \frac{\begin{vmatrix} e^6 & 0 \\ \frac{1}{6}e^6 & 8 \end{vmatrix}}{-\frac{4}{t}e^{12}} = c_2(t) = -\frac{t^2}{e^4}$$

G.S

$$X(t) = c_1 \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \end{pmatrix} e^6 + c_2 \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix} e^2 + \frac{t^2}{e^{10}} \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \end{pmatrix} e^6 - \frac{t^2}{e^4} \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix} e^2$$



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-Revision Materials For PDEs & ODEs

$$b) \frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = \frac{8}{t} x_2 - \frac{8}{t^2} \left(1 - \frac{\epsilon^2}{t}\right) x_1 + 8\epsilon$$

The homogenous part of eqn

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{8}{t^2} - \frac{8\epsilon^2}{t^3} & \frac{8}{t} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Eigen ~~value~~ <sup>N. val.</sup> system with A

$$\begin{vmatrix} -\lambda & 1 \\ \frac{8}{t^2} - \frac{8\epsilon^2}{t^3} & \frac{8}{t} - \lambda \end{vmatrix} = 0$$

$$\lambda^2 = \frac{8}{t} \lambda - \frac{8}{t^2} + \frac{8\epsilon^2}{t^3} = 0$$

$$a = 1, \quad b = -\frac{8}{t}, \quad c = -\frac{8}{t^2} + \frac{8\epsilon^2}{t^3}$$

Eigen ~~value~~ <sup>N. val.</sup> system with A

$$\begin{vmatrix} -\lambda & 1 \\ \frac{8}{t^2} - \frac{8\epsilon^2}{t^3} & \frac{8}{t} - \lambda \end{vmatrix} = 0$$

$$\lambda^2 = \frac{8}{t} \lambda - \frac{8}{t^2} + \frac{8\epsilon^2}{t^3} = 0$$

$$a = 1, \quad b = -\frac{8}{t}, \quad c = -\frac{8}{t^2} + \frac{8\epsilon^2}{t^3}$$

$$\lambda = \frac{4}{t} \pm \sqrt{\frac{2}{t^2} - \frac{2\epsilon^2}{t^3}}$$

Using soln in (a) we have

Eigen vector system with  $\lambda$ ,

$$\begin{pmatrix} \frac{4}{t} + \sqrt{\frac{2}{t^2} - \frac{2\epsilon^2}{t^3}} & 1 \\ \frac{8}{t^2} - \frac{8\epsilon^2}{t^3} & \frac{8}{t} - \frac{4}{t} - \sqrt{\frac{2}{t^2} - \frac{2\epsilon^2}{t^3}} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} v_1 = 1 \\ v_2 = -\frac{4}{t} + \sqrt{\frac{2}{t^2} - \frac{2\epsilon^2}{t^3}} \end{pmatrix}$$

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-Revision Materials For PDEs & ODEs

$$\begin{pmatrix} \frac{4}{t} + \sqrt{\frac{2}{t^2} - \frac{2t^2}{t^3}} & \frac{8}{t^2} - \frac{4}{t} - \sqrt{\frac{2}{t^2} - \frac{2t^2}{t^3}} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} v_1 = 1 \\ v_2 = -\frac{4}{t} + \sqrt{\frac{2}{t^2} - \frac{2t^2}{t^3}} \end{pmatrix}$$

$$h_1(t) = \left( -\frac{4}{t} + \sqrt{\frac{2}{t^2} - \frac{2t^2}{t^3}} \right) e^{\frac{4}{t} - \sqrt{\frac{2}{t^2} - \frac{2t^2}{t^3}}}$$

$$h_2(t) = \left( -\frac{4}{t} + \sqrt{\frac{2}{t^2} - \frac{2t^2}{t^3}} \right) e^{\frac{4}{t} - \sqrt{\frac{2}{t^2} - \frac{2t^2}{t^3}}}$$

Using the partition ~~from~~ concept for  $a \in [0, 1]$   
 we have  $a \in 0$

$$\Rightarrow X_h(t) = c_1 \begin{pmatrix} 1 \\ -\frac{1}{t} \end{pmatrix} e^{\left(\frac{5}{t}\right)t} + c_2 \begin{pmatrix} 1 \\ -\frac{1}{t} \end{pmatrix} e^{\left(\frac{3}{t}\right)t}$$

As for the particular soln  $X_p(t)$

$$c_1(t) = c_2(t) = 0$$

$$\Rightarrow \text{Approximate Soln}$$

$$\Rightarrow \underline{X(t) = c_1 \begin{pmatrix} 1 \\ -\frac{1}{t} \end{pmatrix} e^{\frac{5}{t}} + c_2 \begin{pmatrix} 1 \\ -\frac{1}{t} \end{pmatrix} e^{\frac{3}{t}}}$$



$36y^2 \phi_{xx} - 12 \phi_{xy} + \phi_{yy} - 6(1+xy) \phi_x + x \phi_y = 0$

Soln

a) let  $A = 36y^2$ ,  $B = -12y$ ,  $C = 1$   
 $\Delta = B^2 - 4AC = 144y^2 - 144y^2 = 0 \Rightarrow$  Parabolic

b) Canonical form  
 The d.e for characteristic

$$\frac{dy}{dx} = \frac{-12y \pm \sqrt{0}}{72y^2} = -\frac{1}{6y}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{6y} \Rightarrow 6y dy + dx = 0$$

$$\int 6y dy + \int dx = \int 0 \Rightarrow 3y^2 + x = c_1$$

choose  $\xi = 3y^2 + x$   
 choose  $\eta$  "wisely"

$$\begin{vmatrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{vmatrix} \neq 0 \Rightarrow \begin{vmatrix} 1 & 6y \\ 0 & 1 \end{vmatrix} = 1 \neq 0$$

$$\eta = y$$

$$\Rightarrow \xi = 3y^2 + x$$

$$\eta = y$$

we form  $\phi_x, \phi_y, \phi_{xx}, \phi_{xy}, \phi_{yy}$

$$\phi_x = \frac{\partial \phi}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial \phi}{\partial \eta} \frac{\partial \eta}{\partial x}$$

$$\phi_x = \phi_\xi$$

$$\phi_{xx} = \frac{\partial \phi_\xi}{\partial \xi} \frac{\partial \xi}{\partial x} = \phi_{\xi\xi}$$

$$\phi_{xy} = \frac{\partial \phi_\xi}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial \phi_\eta}{\partial \eta} \frac{\partial \eta}{\partial y}$$

$$= 6y \phi_{\xi\xi} + \phi_{\xi\eta}$$

$$\phi_y = \frac{\partial \phi}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial \phi}{\partial \eta} \frac{\partial \eta}{\partial y}$$

$$= 6y \phi_\xi + \phi_\eta$$

$$\phi_{yy} = \frac{\partial}{\partial \xi} [6y \phi_\xi + \phi_\eta] \frac{\partial \xi}{\partial y} + \frac{\partial}{\partial \eta} [6y \phi_\xi + \phi_\eta] \frac{\partial \eta}{\partial y}$$

$$\phi_{yy} = 36y^2 \phi_{\xi\xi} + 12 \phi_{\xi\eta} + \phi_{\eta\eta}$$

Substitute all into eqn (1)

$$36y^2 \phi_{\xi\xi} - 12y [6y \phi_{\xi\xi} + \phi_{\xi\eta}] + \phi_{\eta\eta} - 6(1+xy) \phi_\xi - x \phi_\eta = 0$$

$$+ 36y^2 \phi_{\xi\xi} + 12 \phi_{\xi\eta} + \phi_{\eta\eta} - 6 \phi_\xi - 6xy \phi_\xi + 6xy \phi_\xi + x \phi_\eta = 0$$

$$\Rightarrow \phi_{\eta\eta} - 6 \phi_\xi + x \phi_\eta = 0$$

Since  $\phi$  is a function of both  $\xi$  and  $\eta$  we have

$$\phi_{\eta\eta} + (x-6) \phi_\eta = 0$$

$$\Rightarrow [\phi_{\eta\eta} + x \phi_\eta = 0]$$

c) Solving the canonical form

$$\frac{\partial^2 \phi}{\partial \eta^2} + x \phi_\eta = 0$$

This is a 2nd order ode

$$\Rightarrow$$
 The characteristic is given by
 
$$\lambda^2 + x = 0 \Rightarrow \lambda^2 = -x$$

$$\Rightarrow \lambda = \pm i\sqrt{x}$$

$$\phi(\xi, \eta) = c_1 \cos \sqrt{x} \eta + c_2 \sin \sqrt{x} \eta$$

$$\Rightarrow \phi(x, y) = c_1 \cos \sqrt{x} y + c_2 \sin \sqrt{x} y$$

d) For  $x = 6$

$$\phi(x, 0) = 0 = 0, \phi_y(x, 0) = x \sin x$$

$$\Rightarrow \phi(x, 0) = c_1 = 0$$

$$\Rightarrow \phi(x, y) = c_2 \sin \sqrt{x} y$$

$$\phi_y(x, y) = \sqrt{x} c_2 \cos \sqrt{x} y$$

$$\phi_y(x, 0) = \sqrt{x} c_2 = x \sin x \Rightarrow c_2 = \frac{x \sin x}{\sqrt{x}}$$

$$\boxed{\phi(x, y) = \sqrt{x} \sin x \cdot \sin \sqrt{x} y}$$