Find the domain of the function

$$Z = \arcsin(2x - y)$$

Using a 3D graphics program to generate the graph of the function

=> The domain of the function can be represented by the below coordinates systems domain

A = (-1.67, 2.92, -0.86)

B = point (X Axis) => (-2, 0, 0)

C = Point (Y Axis) => (0, -2.58, 0)

D = (-1.93, 2.69, -1.26)

Find partial derivatives and total derivative of

$$Z = \arcsin(2x^3y)$$

=> We apply the chain rule.

$$\frac{\partial z}{\partial x} = ?$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{1}{\sqrt{1-(2x^3y)^2}} \cdot \frac{\partial}{\partial x}(2x^3y)$$

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$$\Rightarrow \frac{\partial}{\partial x}(2x^3y) = 6x^2y$$

$$\Rightarrow \frac{1}{\sqrt{1-(2x^3y)}} \cdot 6x^2y$$

$$\Rightarrow \frac{6x^2y}{\sqrt{1-4x^6y^2}}$$

$\Rightarrow \frac{\partial z}{\partial y} = ?$ Using chain rule we have.

$$\Rightarrow \frac{\partial z}{\partial y} = \frac{1}{\sqrt{1-(2x^3y)^2}} \cdot \frac{\partial}{\partial y}(2x^3y)$$

$$\Rightarrow \frac{\partial}{\partial y}(2x^3y) = 2x^3$$

$$\Rightarrow \frac{1}{\sqrt{1-(2x^3y)^2}} \cdot 2x^3$$

$$\Rightarrow \frac{2x^3}{\sqrt{1-(2x^3y)^2}}$$

Total derivative

$$dz = \frac{\partial[\arcsin(2x^3y)]}{\partial x}dx + \frac{\partial[\arcsin(2x^3y)]}{\partial y}dy$$

$\Rightarrow$

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$$dz = \frac{6x^2y}{\sqrt{1-4x^6y^2}}\,dx + \frac{2x^3}{\sqrt{1-4x^6y^2}}\,dy$$

Find the value of total derivative of $u = u(x,y)$ if $x = x(t)$, $y = y(t)$ where $t = t_0$

$\Rightarrow$ $u = \arcsin\left(\frac{x^2}{y}\right)$, $x = \sin t$, $y = \cos t$

$$\Rightarrow du = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t}dx + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t}dy$$

$$\Rightarrow \frac{\partial u}{\partial x} = \frac{1}{\sqrt{1-\left(\frac{x^2}{y}\right)^2}} \cdot \frac{\partial}{\partial x}\left(\frac{x^2}{y}\right)$$

$$\Rightarrow \frac{\partial}{\partial x}\left(\frac{x^2}{y}\right) = \frac{2x}{y}$$

$$\Rightarrow \frac{1}{\sqrt{1-\left(\frac{x^2}{y}\right)^2}} \cdot \frac{2x}{y} = \frac{2x}{\sqrt{y^2-x^4}}$$

$$\Rightarrow \frac{\partial u}{\partial y} = \frac{1}{\sqrt{1-\left(\frac{x^2}{y}\right)^2}} \cdot \frac{\partial}{\partial y}\left(\frac{x^2}{y}\right)$$

$$\Rightarrow \frac{\partial}{\partial y}\left(\frac{x^2}{y}\right) = -\frac{x^2}{y^2}$$

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$$\Rightarrow -\frac{x^2}{y^2} \cdot \frac{1}{\sqrt{1-\left(\frac{x^2}{y}\right)^2}} = \frac{-x^2}{y\sqrt{y^2-x^4}}$$

$$\Rightarrow \frac{\partial x}{\partial t} = \cos t \qquad \frac{\partial y}{\partial t} = -\sin t$$

$$du = \frac{2x}{\sqrt{y^2-x^4}} \cdot \cos t \, dx + \frac{x^2}{y\sqrt{y^2-x^4}} \sin t \, dy$$

IV Find mixed partial derivative and prove equality

$$Z = \arcsin(x-2y)$$

$$\Rightarrow \frac{\partial z}{\partial x} = ?$$ Using chain rule

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{1}{\sqrt{1-(x-2y)^2}} \cdot \frac{\partial}{\partial x}(x-2y)$$

$$\Rightarrow \frac{\partial}{\partial x}(x-2y) = 1$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{1}{\sqrt{1-x^2+4xy-4y^2}} \quad \text{--- (1)}$$

$\Rightarrow$ Differentiating (1) w.r.t y we have

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = ?$$

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$$\Rightarrow \frac{\partial}{\partial y}\left(\frac{1}{\sqrt{1-x^2+4xy-4y^2}}\right)$$

$$\Rightarrow \frac{-2(x-2y)}{(1-x^2+4xy-4y^2)^{\frac{1}{2}}} \quad \text{——}(*)$$

$\Rightarrow$ Differiating z w.r.t y, we get

$$\frac{\partial z}{\partial y} = \frac{1}{\sqrt{1-(x-2y)^2}} \cdot \frac{\partial}{\partial y}(x-2y)$$

$$\Rightarrow \frac{\partial}{\partial y}(x-2y) = -2$$

$$\Rightarrow \frac{1}{\sqrt{1-(x-2y)^2}} \cdot -2$$

$$\Rightarrow \frac{-2}{\sqrt{1-x^2+4xy-4y^2}}$$

Differiating the above w.r.t x.

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = ?$$

$$= \frac{\partial}{\partial x}\left[\frac{-2}{(1-x^2+4xy-4y^2)^{\frac{1}{2}}}\right]$$

$$\Rightarrow \frac{-2x+4y}{(1-x^2+4xy-4y^2)^{\frac{1}{2}}}$$ which equals (*)

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V Locate all relative extrema points of

$Z = xy(12 - x - y)$

$\Rightarrow \frac{\partial z}{\partial x} = ?$

$\Rightarrow Z = 12xy - x^2y - xy^2$

$\Rightarrow \frac{\partial z}{\partial x} =$

$\Rightarrow \frac{\partial z}{\partial x} = 12y - 2xy - y^2 = 0$

$\Rightarrow \frac{\partial z}{\partial y} = 12x - x^2 - 2xy = 0$

$\Rightarrow y(12 - 2x - y) = 0$ ——— ①

$\Rightarrow x(12 - x - 2y) = 0$ ——— ②

$\Rightarrow$ from ① $y = 0$
and $12 - 2x - y = 0$

$\Rightarrow$ from ② $x = 0$ and
$12 - x - 2y = 0$

$\Rightarrow$ For $x = 0$ $\Rightarrow y = 12$

$\Rightarrow$ The critical points are $(0,0)$, $(0,12)$ and $(4,4)$

$\Rightarrow$ With 2nd derivative

$\left.\frac{\partial^2 z}{\partial x^2}\right|_{(0,0)} = -2y\Big|_{(0,0)} \Rightarrow$

$(0,0) \Rightarrow$ minimum point
$(0,12) \Rightarrow$ Saddle point
$(4,4) \Rightarrow$ Maximum point

VI Find Maximum and minimum Values of

$$z(x,y) = x^2 + y^2$$

Subject to $x - y = k$.

$\Rightarrow \dfrac{\partial z}{\partial x} = 2x = 0 \Rightarrow x = 0$

$\Rightarrow \dfrac{\partial z}{\partial y} = 2y = 0 \Rightarrow y = 0$

at any given $x = 0$

$-y = k \Rightarrow$ implying that

minimum point is at $(0,0)$

$\Rightarrow \dfrac{\partial^2 f}{\partial x^2} = 2$, $D(x,y) = 4$

$\Rightarrow (0,4) \Rightarrow$ Maximum point

VII Apply minimum squares method to establish dependence between y and x

$x_0 = 9$

x	1	2	3	4	5	6
y	2	4.9	7.9	11.1	17	14.1

=> Using the normal equations

$f(x) = y = m(x) + b$

$\Rightarrow nb + m\sum x_i = \sum y_i$

$\Rightarrow \sum x_i b + m\sum x_i^2 = \sum x_i y_i$

x_i	y_i	x_i^2	$x_i y_i$
1	2	1	2
2	4.9	4	9.8
3	7.9	9	23.7
4	11.1	16	44.4
5	17	25	85
6	14.1	36	84.6
totals 21	57	91	249.5

$\Rightarrow 6b + 21m = 57$

$21b + 91m = 249.5$

$$\Rightarrow b = \frac{57 - 21m}{6}$$

$$\Rightarrow 21\left(\frac{57-21m}{6}\right)+91m=249.5$$

$$\Rightarrow 1197-441m+546=1497$$

$$\Rightarrow m=1.793$$

$$\Rightarrow b=\frac{57-21(1.793)}{6}$$

$$=3.226$$

$$\Rightarrow f(x)=y=1.793x+3.226$$

VIII Solve the following differential equation

$$\Rightarrow y'=y^4$$

$$\Rightarrow \frac{dy}{dx}=y^4$$

$$\Rightarrow \frac{1}{y^4}\frac{dy}{dx}=1$$

$$\Rightarrow -\frac{1}{3y^3}=t+b$$

$$\Rightarrow y=-\left(\frac{1}{3(t+b)}\right)^{1/3}$$

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IX $xy' = x + 4y$

$$\Rightarrow x\frac{dy}{dx} = x + 4y$$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{4}{x}y$$

$$\Rightarrow \frac{dy}{dx} - \frac{4}{x}y = 1$$

This is a bernouli equation we can solve using Integrating factor method.

$$I.F = \mu(x) = e^{-\int \frac{4}{x}dx} = \frac{1}{x^4}$$

$$\Rightarrow y.\frac{1}{x^4} = \frac{1}{x^4}\left(\frac{dy}{dx} - \frac{4}{x}y\right) = \frac{1}{x^4}$$

$$\Rightarrow (\mu(x).y)' = \mu(x).q(x) = \left(\frac{1}{x^4}y\right)' = \frac{1}{x^4}$$

$$\Rightarrow y = -\frac{x}{3} + bx^4$$

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X $y' - \frac{y}{x} = -\frac{2}{x^2}$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = -\frac{2}{x^2}$

This is a linear eqn, we solve by Integrating factor.

$\frac{dy}{dx} + P(x)y = Q(x)$

$I.F = \mu(x) = e^{\int -\frac{1}{x}dx} \quad 7 - \frac{2}{x^2} = \frac{1}{x}$

$\Rightarrow (\mu(x) \cdot y)' = \mu(x) \cdot q(x) = \left(\frac{1}{x}y\right)' = -\frac{2}{x^2}$

$= y \cdot \frac{1}{x} = \int \frac{1}{x} \cdot \frac{-2}{x^2} + b$

$\Rightarrow y = \frac{1}{x} + bx$

XI $y'' = 2 - x$

This is a 2nd order ode

$\Rightarrow \frac{d^2y}{dx^2} = 2 - x$

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$\Rightarrow$ Integrating w.r.t x once we have.

$$\frac{dy}{dx} = 2x - \frac{x^2}{2} + b$$

$\Rightarrow$ Integrating w.r.t x once more we got

$$y = x^2 - \frac{x^3}{6} + bx + c$$

$$\Rightarrow y = -\frac{x^3}{6} + x^2 + bx + c$$

XII $y'' + 36y = 36 + 66x - 36x^3$

$\Rightarrow$ We will use the method of undetermined. Coefficients.

$$y = y_c + y_p.$$

$\Rightarrow$ Homogenous part of eqn.

$$y'' + 36y = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} + 36y = 0$$

$\Rightarrow$ The Characteristic eqn is given as

$$\lambda^2 + 36 = 0$$

$$\Rightarrow \lambda^2 = -36$$

$\lambda = \pm 6i$

$\Rightarrow \lambda = \alpha + \beta i$

$\Rightarrow \alpha = 0, \quad \beta = \pm 6$

Complete solution

$$y_c = [c_1 \cos 6x + c_2 \sin 6x] e^{0x}$$

For the particular solution we have

$$y_p = c_1(x) \cos 6x + c_2(x) \sin 6x$$

$$y'_p = -6c_1(x) \sin 6x + 6c_2(x) \cos 6x.$$

$$y''_p = -36c_1(x) \cos - 36 c_2(x) \sin 6x.$$

Substituting into $y'' + 36y = 0$

and solve for y_p we have

$$y_p = -x^2 + 2x + 1$$

$$y = y_c + y_p$$

$$y = c_1 \cos 6x + c_2 \sin 6x - x^2 + 2x + 1$$

XIII Find all admissible basic solution

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 = b_3$$

$$a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 = b_4$$

$\Rightarrow$

$$x_1 + 2x_2 - 3x_3 + 2x_4 + 0x_5 = 2$$

$$x_1 - x_2 - 3x_3 - 4x_4 - 3x_5 = -4$$

$$2x_1 + 3x_2 + x_3 + 5x_4 + 2x_5 = 1$$

$$x_1 - 2x_2 - 2x_3 - 3x_4 - 5x_5 = -7$$

$\Rightarrow$ for $n - m = 0$

Basic variable

$\Rightarrow x_1, x_2, e_1, e_3 \geq 0$

$\Rightarrow e_1 = 2, \quad e_3 = 1$

$\Rightarrow x_3, x_4, e_2, e_4 \leq 0$

$\Rightarrow e_2 = -4, \quad e_4 = -7$

In matrix notation we obtain $c^T x$ s.t

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$Ax \geq b \quad , x \geq 0$

where $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$, $b = \begin{pmatrix} 2 \\ -4 \\ 1 \\ -7 \end{pmatrix}$, $A = \begin{pmatrix} 1 & 2 & -3 & 2 & 0 \\ 1 & -1 & -3 & -4 & -3 \\ 2 & 3 & 1 & 5 & 2 \\ 1 & -2 & -2 & -3 & -5 \end{pmatrix}$

$\Rightarrow A_B^{-1} b \Rightarrow A_B^{-1} A_N$

$\Rightarrow x_B = \begin{pmatrix} 1 & 0 & -1 & 0 & 2 \\ -5 & -3 & -1 & 0 & -1 \\ -1 & 3 & 0 & -4 & -1 \\ -8 & -5 & -5 & -4 & -12 \end{pmatrix}$

XII) Use graphical Method and simplex method to determine maximum Value of the function

$$Z(X) = c_1 X_1 + c_2 X_2$$

$\Rightarrow a_{11} x_1 + a_{12} x_2 \leq b_1$

$a_{21} x_1 + a_{22} x_2 \leq b_2$

$a_{31} x_1 + a_{32} x_2 \leq b_3 \quad , x_1 \geq 0$

$a_{41} x_1 + a_{42} x_2 \leq b_4 \quad\quad x_2 \geq 0$

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$\Rightarrow \quad -2x_1 - 2x_2 \leq 16$

$-4x_1 + 6x_2 \leq 24$

$-4x_1 + 6x_2 \leq 0$

$2x_1 + 2x_2 \leq 10$

We write the problem in standard form as

$$C = \begin{pmatrix} c_1 \\ c_2 \\ 0 \\ 0 \end{pmatrix}, \quad b = \begin{pmatrix} 16 \\ 24 \\ 0 \\ 10 \end{pmatrix}, \quad A = \begin{pmatrix} -2 & -2 \\ -4 & 6 \\ -4 & 6 \\ 2 & 2 \end{pmatrix}$$

$\Rightarrow$ The basic change leads to $B = \{10, 24\}$ and $N = \{1, 3\}$

$\Rightarrow$ We interchange the pivot column and the pivot row to find

$$A_B = \begin{pmatrix} -2 & -2 \\ -4 & 6 \\ -4 & 6 \\ 2 & 3 \end{pmatrix}, \quad A_N = \begin{pmatrix} 2 & 1 \\ 4 & 0 \\ -4 & -1 \\ 1 & 0 \end{pmatrix}, \quad C_B = \begin{pmatrix} -4 \\ 0 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \\ 1 \\ 4 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} -4 \\ 0 \\ 1 \\ 2 \end{pmatrix}, \quad \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{pmatrix} = \begin{pmatrix} -5 \\ -4 \\ 2 \\ 3 \end{pmatrix}$$

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$\Rightarrow$ The optimality criterion is satisfied, the optimal solution is

$$x = (0, 8, 0, 2)^T$$ with objective function value being

$$C^T x = -32$$

XX The distribution of the random variable X is given by

$$X = \begin{pmatrix} x_1 & x_2 & x_3 \\ P_1 & P_2 & P_3 \end{pmatrix}$$

Calculate.

a) $M(x)$, $M(2aX + M(x))$

$$X = \begin{pmatrix} -1 & 0 & 1 \\ 0.5 & 0.25 & 0. \end{pmatrix}.$$

$$M(X) = M(0.5) + M(0.25) + M(0)$$

$$= 0.5 + 0.25 + 0 = 0.75$$

$\Rightarrow M(2aX + M(x))$

$$\Rightarrow M(2aX) + M(0.75)$$

$$\Rightarrow M(-2) + M(0) + M(1) + M(0.75)$$

$$= -2 + 1 + 0.75 = -0.25 = 0.25$$

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$D(X) \Rightarrow \mu = \sum X\, M(X)$

$\Rightarrow (-1)(0.5) + 0(0.25) + 1(0)$

$= -0.5 = 0.5$

$D(2aX + M(X) + b)$

$\mu = \sum(2aX + M(X) + b)$

$\Rightarrow 2(-1) + 0.75 + (-0.25)$

$= -1.5$

$\sigma(X) \Rightarrow \sigma^2 = \sum(X - D(X))^2\, M(X)$

$= [(-1+0.5)^2 + (0-0.5)^2 + (1-0.5)^2]\, 0.75$

$= 0.5625$

XVII. The probability that customers will enter the bank in one minute

$\Rightarrow n = 1000$

$p = 0.002$

$m = 3$

a) $\Rightarrow P_1 = \frac{1}{2} P_0$

$\Rightarrow P_n = \frac{1}{2} P_{n-1}$ for $1 \le n \le 3$

$$\sum_{i=1}^{3} P_1 = 1$$

$\Rightarrow$

$$\Rightarrow \sum_{i=1}^{3} \tfrac{1}{2} \cdot 0.02 = \sum_{i=1}^{3} P_1 = 0.01$$

$$\Rightarrow \frac{2^{3-n}}{1000} \quad \text{for} \quad 0 \leq n \leq 4$$

$$\text{probability} = \sum_{n=0}^{3} \frac{3}{n.pn} = 3/10 = 0.3$$

b) less than 3 $m = 3$

$$\Rightarrow P_1 = \tfrac{1}{2} P_0 \quad \text{for} \quad 1 \leq n \leq 3$$

$$\Rightarrow \sum_{i=1}^{2} P_1 = 1, \quad \frac{2^{2-n}}{1000}, \quad \text{for} \quad 0 \leq n < 4$$

$$\text{probability} = \sum_{n=0}^{2} \frac{2}{n.pn} = 2/10 = 0.2$$

c) Atleast 1 customer

$\rightarrow$ This equates to the probability of $P(1)$ 1-customer, plus probability of 2-customer and probability of 3-customer

$\Rightarrow$

$$\text{Probability of 1-customer} \Rightarrow \frac{1}{n.pn} = 1/10 = 0.1$$

$$= 0.3 + 0.2 + 0.1 = 5/10$$