

4 (a) Using direct integrals

$$\frac{\partial^2 u}{\partial x \partial y} = e^{2x-3y}$$

Condition

$$u(x,0) = e^{2x}, \quad u(\pi, y) = \cos y$$

Soln

Integrating w.r.t x .

$$\frac{\partial u}{\partial y} = 2 e^{2x-3y} + f(y)$$

Integrating w.r.t y

$$u(x,y) = -6 e^{2x-3y} + F(y) + G(x)$$

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 $u(x,0) = e^{2x}, \quad u(\pi, y) = \cos y$

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Integrating w.r.t x

$$\frac{\partial u}{\partial y} = 2e^{2x-3y} + f(y)$$

Integrating w.r.t y

$$\frac{\partial u}{\partial x} = -6e^{2x-3y} + F(y) + G(x) \quad \text{--- (1)}$$

1st Initial Condition

$$u(x,0) = e^{2x} = -6e^{2x} + F(0) + G(x)$$

$$\Rightarrow G(x) = e^{2x} + 6e^{2x} - F(0)$$

$$\Rightarrow G(x) = 7e^{2x} - F(0) \quad \text{--- (2)}$$

Soln

Integrating w.r.t x

$$\frac{\partial u}{\partial y} = 2e^{2x-3y} + f(y)$$

Integrating w.r.t y

$$\frac{\partial}{\partial y} u(x, y) = -6e^{2x-3y} + F(y) + G(x) \quad \text{--- (1)}$$

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$$u(x, 0) = e^{2x} = -6e^{2x} + F(0) + G(x)$$

$$\Rightarrow G(x) = e^{2x} + 6e^{2x} - F(0)$$

$$\Rightarrow G(x) = 7e^{2x} - F(0) \quad \text{--- (2)}$$

2nd Initial Condition

$$u(\pi, y) = \cos y = -6e^{2\pi-3y} + F(y) + G(\pi)$$

$$\Rightarrow F(y) = \cos y + 6e^{2\pi-3y} - G(\pi)$$

Integrating w.r.t y

$$\frac{\partial}{\partial y} U(x, y) = -6e^{2x-3y} + F(y) + G(x) \quad \text{--- (1)}$$

1st Initial Condition

$$U(x, 0) = e^{2x} = -6e^{2x} + F(0) + G(x)$$

$$\Rightarrow G(x) = e^{2x} + 6e^{2x} - F(0) \quad \text{--- (2)}$$

$$\Rightarrow G(x) = 7e^{2x} - F(0)$$

2nd initial condition

$$U(\pi, y) = \cos y = -6e^{2\pi-3y} + F(y) + G(\pi)$$

$$\Rightarrow F(y) = \cos y + 6e^{2\pi-3y} - G(\pi) \quad \text{--- (3)}$$

Let do it at $F(0)$ we have

$$F(0) = \cos(0) + 6e^{2\pi-3(0)} - G(\pi)$$

$$F(0) = 1 + 6e^{2\pi} - G(\pi) \quad \text{--- (4)}$$

1st Initial Condition

$$u(x,0) = e^{2x} = -6e^{2x} + F(x) + G(x)$$

$$\Rightarrow G(x) = e^{2x} + 6e^{2x} - F(x)$$

$$\Rightarrow G(x) = 7e^{2x} - F(x)$$

— (2)

2nd initial condition

$$u(\pi, y) = \cos y = -6e^{2\pi-3y} + F(y) + G(\pi)$$

$$\Rightarrow F(y) = \cos y + 6e^{2\pi-3y} - G(\pi)$$

— (3)

Let do it at $F(0)$ we have

$$F(0) = \cos(0) + 6e^{2\pi-3(0)} - G(\pi)$$

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— (4)

Take $G(x)$

Taking (4) and substituting in (2)

$$G(y) = 7e^{2y} - 1 - 6e^{2y} + G(\pi) \quad \text{--- (5)}$$

Substituting (3) and (5) into (1) we have

$$U(x, y) = \cancel{0} = -\cancel{6}e^{2x-3y} + \cos y + \cancel{6}e^{2x-3y} - \cancel{G(\pi)} + \\ + 7e^{2y} - 1 - 6e^{2y} + G(\pi)$$

$$= 2x - 1 - 6e^{2y}$$