3) Given that a jar contains 20 candies such that 11 are red and 9 are green. John selects a candy notes its colour and returns it to the jar. He then selects another candy from the jar.

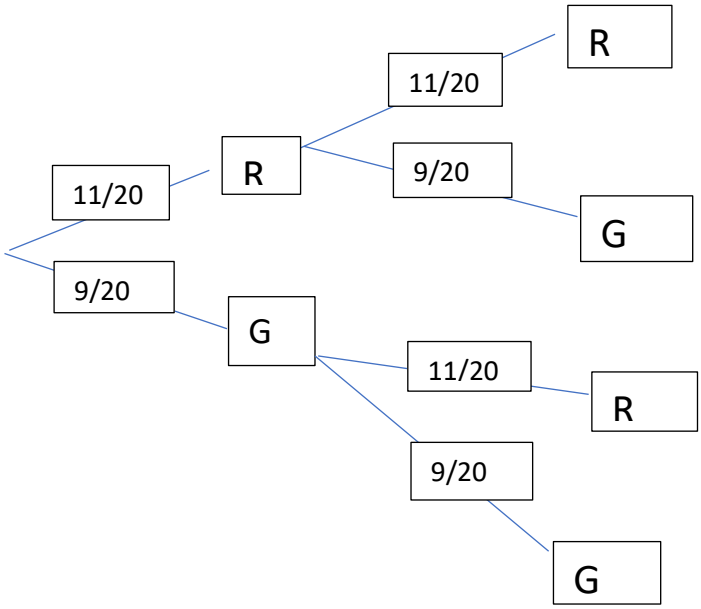
(a) Draw a tree diagram showing the respective probabilities for each of his selection(s) [5]

Solution

With replacement

P (R)=11/20

P(G) =9/20



(b) Determine the probability that (i) John selected a red candy followed by a green candy [2]

(ii) John selects 2 green candies [2]

Solution

(i)

$$=> P(RG) = \frac{11}{20} * \frac{9}{20} = \frac{99}{400} = 0.2475$$

(ii)

$$=> P(GG) = \frac{9}{20} * \frac{9}{20} = \frac{81}{400} = 0.2025$$

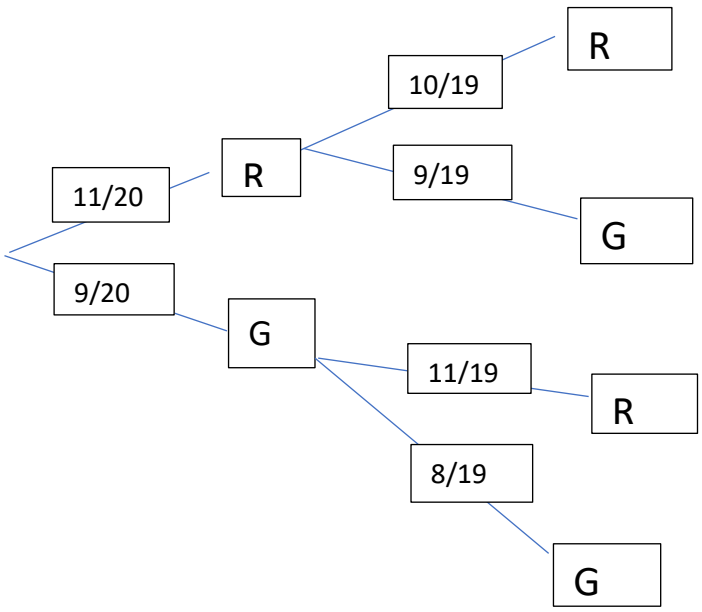
(c) Draw a tree diagram showing all the respective probabilities if John did NOT return the first candy from his selection, and then selected another candy. [6]

Solution

Without replacement

P (R)=11/20

P(G) =9/20



(d) Using your diagram from part (c), determine the probability that John got at least one (1) red candy. [4]

Solution

$$=> P(atleast\ 1\ red\ candy) = P(RR) + P(RG) + P(GR)$$
$$=> (\tfrac{11}{20} * \tfrac{10}{19}) + (\tfrac{11}{20} * \tfrac{9}{19}) + (\tfrac{9}{20} * \tfrac{11}{19})$$
$$=> \tfrac{11}{38} + \tfrac{99}{380} + \tfrac{99}{380} = \tfrac{77}{95}$$

1) The table shows the number of college students who prefer a given core course.

Toppings	Year 1	Year 2	Year 3
Stat 120	10	13	21
Libs 130	20	19	13
Comm 117	13	10	20

(a) Find the probability that a randomly selected student prefers Stat 120 [3]

Solution

Toppings	Year 1	Year 2	Year 3	**Total**
Stat 120	10	13	21	**44**
Libs 130	20	19	13	**52**
Comm 117	13	10	20	**43**
Total	**43**	**42**	**54**	**139**

$$=> P(Stat\ 120) = \frac{44}{139}$$

(b) Find the probability that a student prefers Comm 117 given that the student is in year 2 [4]

Solution

$$=> P(Comm\ 117|\ Year\ 2) = \frac{\frac{10}{139}}{\frac{42}{139}} = \frac{10}{42} = \frac{5}{21}$$

2) Suppose that the PDF for the number of years it takes to earn a Bachelor of Science (B.Sc.) degree is given below.

X	P(X = x)
3	0.05
4	0.40
5	X
6	0.15
7	0.10

(a) Find the value of x, $P(X = 5)$. [1]

Solution

$$=> 0.05 + 0.40 + x + 0.15 + 0.10 = 1$$
$$=> x + 0.70 = 1$$
$$=> x = 0.30$$

(b) On average, how many years do you expect it to take for an individual to earn a B.Sc.? [4]

Solution

$$=> E(X) = \sum_{k=1}^{n} x_k \cdot p_k$$
$$=> E(X) = (3 * 0.05) + (4 * 0.40) + (5 * 0.30) + (6 * 0.15) + (7 * 0.10) = 4.85$$

(c) Find the standard deviation of the Random Variable, X. [4]

Solution

$$=> \sigma^2(X) = \sum_{k=1}^{n} x_k^2 . p_k$$
$$=> \sigma^2(X) = (3 - 4.85)^2 * (0.05) + (4 - 4.85)^2 * (0.40) + (5 - 4.85)^2 * (0.30) + \ldots$$
$$=> +(6 - 4.85)^2 * (0.15) + (7 - 4.85)^2 * (0.10) = 1.1275$$
$$=> \sigma = \sqrt{1.1275} = 1.0618$$

5. Massy stores recently opened a new branch in Chaguanas. A survey shows that the probability that a shopper chooses the new branch is 0.29

From a sample of 10 shoppers chosen at random, find the probability that:

(a) three shoppers will choose the Chaguanas branch [4]

Solution

This is a binomial distribution with parameters, p=0.29 and n=10

$$=> P(X = k) = \binom{n}{k} . p^k . (1 - p)^{n-k} \, , for \; k = 0, 1, 2...$$
$$=> P(X = 3) = \binom{10}{3} . (0.29)^3 . (1 - 0.29)^7 = 0.2662$$

(b) at least seven will choose the Chaguanas branch [8]

Solution

$$=> P(X \geq 7) = P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)$$

$$=> \binom{10}{7} * (0.29)^7 * (0.71)^3 + \binom{10}{8} * (0.29)^8 * (0.71)^2 + \ldots$$

$$=> + \binom{10}{9} * (0.29)^9 * (0.71)^1 + \binom{10}{10} * (0.29)^{10} * (0.71)^0 = 0.008651$$

THE END