

Inorder to solve this system of coupled differential equations we need to first reduce it into a system of first order differential equations

$\Rightarrow$  we then proceed as follows:

$\Rightarrow$  we first rewrite the coupled system as follows:

$$\Rightarrow \frac{d^3 f}{dn^3} = -\frac{f}{2} \frac{d^2 f}{dn^2}$$

$$\Rightarrow \frac{\partial^2 g}{\partial \eta^2} = \frac{\sigma}{2} \left( -f \frac{\partial g}{\partial \eta} + g \frac{\partial f}{\partial \eta} + 2x \frac{\partial f}{\partial \eta} \frac{\partial g}{\partial x} \right)$$

$$\Rightarrow \frac{\partial^2 \Phi}{\partial \eta^2} = \frac{S_c}{2} \left( -f \frac{\partial \Phi}{\partial \eta} + 2x \frac{\partial f}{\partial \eta} \frac{\partial \Phi}{\partial x} \right)$$

$$\Rightarrow \text{Let, } x_1 = f, x_2 = f', x_3 = f'', x_4 = f'''$$

$\Rightarrow$  differentiating the above wrt  $\eta$  we obtain;

$$\Rightarrow x'_1 = x_2, x'_2 = x_3, x'_3 = x_4$$

$\Rightarrow$  This implies that,

$$x'_1 = x_2 \text{ --- (1)}$$

$$x'_2 = x_3 \text{ --- (2)}$$

$$x'_3 = -\frac{x_1 x_3}{2} \text{ --- (3)}$$

Similarly we let,  $x_5 = g, x_6 = \Phi, \Longleftrightarrow x_7 = g', x_8 = \Phi', x_9 = g'', x_{10} = \Phi''$

Differentiating the above wrt  $\eta$  we obtain the following;

$$x'_5 = g' = x_7 \text{ --- (4)}$$

$$x'_6 = \Phi' = x_8 \text{ --- (5)}$$

From the boundary conditions we can obtain the values for  $g$  and  $\Phi$  with their derivatives wrt  $\eta$

Thus ,the last 2 equations simplifies into;

$$x_7' = \frac{\sigma}{2}(-x_1x_7 + x_5x_2 + 2x_1x_2[\frac{-x_6\alpha\eta e^{\frac{x_5\sqrt{x_1}}{1+\epsilon x_5\sqrt{x_1}}}}{2\sqrt{x_1}(1+\epsilon x_5\sqrt{x_1})^2}]) - - - (6)$$

$$x_8' = \frac{S_c}{2}(-x_1x_8 + 2x_1x_2[\frac{x_6e^{\frac{x_5\sqrt{x_1}}{1+\epsilon x_5\sqrt{x_1}}}}{2\sqrt{x_1}(1+\epsilon x_5\sqrt{x_1})^2}]) - - - (7)$$

⇒ The system of first order Differential Equations (1)--(7) is what we will now solve

⇒ using the solve\_ivp inbuilt solver in python to obtain the values for g and Φ

\*\*\* Now onto python: \*\*\* ⇒⇒⇒ ...