PARTIAL DIFFERENTIATION

$\Rightarrow$ Given a function lets say, $z = x^4y^2$, then we are to find $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$

$\Rightarrow$ when differentiating wrt x we keep y as a constant, $\dfrac{\partial z}{\partial x} = 4x^3y^2$

$\Rightarrow$ differentiating wrt y we keep x as a constant, $\dfrac{\partial z}{\partial y} = 2x^4y$

Product rule

$\Rightarrow$ We let this be given by f'g+g'f ,lets look at an example: $z = y^2 sinx$

$\Rightarrow$ we now let : $f = y^2, and$ g=sinx,

$\Rightarrow$ differentiating this function wrt x, $f' = 0$ and $g' = cosx$

$\Rightarrow$ this becomes, $0 + y^2cosx = y^2cosx$

Quotient rule

$\Rightarrow$ to use quotient rule we us , $\dfrac{vu' - uv'}{v^2}$

$\Rightarrow$ Let a function be given by; $z = \dfrac{x+y}{x-y}$,we differentiate this wrt x

$\Rightarrow$ now we let u=x+y and,v=x-y

$\Rightarrow$ this implies that u'=1,and v'=1

$$\Rightarrow \frac{(x-y)-(x+y)}{(x-y)^2} = \frac{-2y}{(x-y)^2}$$

$\Rightarrow$ differentiating wrt y , u'=1, and v'=-1

$$\Rightarrow \frac{(x-y)-(x+y)(-1)}{(x-y)^2} = \frac{2x}{(x-y)^2}$$

Chain rule

$\Rightarrow$ let z=$(x^2+y)^2$,differentiating wrt x

$$\Rightarrow \frac{\partial z}{\partial x} = 2(x^2+y)(2x) \Rightarrow 4x(x^2+y)$$

$\Rightarrow$ differentiating wrt y we have, $\frac{\partial z}{\partial x} = 2(x^2+y)(1) \Rightarrow 2(x^2+y)$

$\Rightarrow$ lets look at another example ; $z = sin(x^2)$

$\Rightarrow$ let $u = x^2$ and differentiating u wrt x we get 2x

$\Rightarrow$ now we need to find; $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x}$

$\Rightarrow$ with $\frac{\partial z}{\partial u} = cos(u)$,and $\frac{\partial u}{\partial x} = 2x \Rightarrow \frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} = cos(u)(2x) = 2xcos(x^2)$

More examples

Given for instance the example ; $y = ae^{-\frac{(x-b^2)}{c^2}}$

$\Rightarrow$ differentiating the above function wrt a,we use the product rule that f'g+g'f

$\Rightarrow$ in this case we set f=a and $g = e^{-\frac{(x-b)^2}{c^2}}$,we obtain f' and g' wrt a

$$\Rightarrow f' = 1 \ ,g' = 0 \Rightarrow 0 + 1 * e^{-\frac{(x-b)^2}{c^2}} \implies \frac{\partial y}{\partial a} = e^{-\frac{(x-b)^2}{c^2}}$$

differentiating wrt b,we set f=a,and $g = e^{-\frac{(x-b)^2}{c^2}}$,we now find f' and g' wrt b

$$\Rightarrow f' = 0, g' = \frac{c^2(2)(-1)(x-b)^2(-1)e^{-\frac{(x-b)^2}{c^2}}}{(c^2)^2} - 0$$

$$\Rightarrow \frac{\partial y}{\partial b} = \frac{0 + a * 2c^2(x-b)e^{-\frac{(x-b)^2}{c^2}}}{c^4} = \frac{2a(x-b)e^{-\frac{(x-b)^2}{c^2}}}{c^2}$$

$\Rightarrow$ differentiating wrt c,we set f=a and $g = e^{-\frac{(x-b)^2}{c^2}}$,we find f' and g' wrt c;

$$\Rightarrow f' = 0, g' = \frac{0 - (-(x-b)^2) * 2c * e^{-\frac{(x-b)^2}{c^2}}}{(c^2)^2} = \frac{2c(x-b)^2 e^{-\frac{(x-b)^2}{c^2}}}{c^4}$$

$$\Rightarrow \frac{\partial y}{\partial c} = a * \frac{2c(x-b)^2 e^{-\frac{(x-b)^2}{c^2}}}{c^4} = \frac{2ac(x-b)^2 e^{-\frac{(x-b)^2}{c^2}}}{c^4} = \frac{2a(x-b)^2 e^{-\frac{(x-b)^2}{c^2}}}{c^3}$$

Let's have a look at the cost function

$$\Rightarrow \text{Given the function}, mse = \frac{1}{2n} \sum_{i=1}^{n} (y-t)^2$$

$$\Rightarrow \text{differentiating wrt y we get;} \Rightarrow \frac{1}{2n} * 2 * \sum_{i=1}^{n} (y-t)(1)$$

$$\Rightarrow \frac{1}{2n} \sum_{i=1}^{n} (y-t)$$

THE END