$$\frac{dy}{dt} = y^3 - y$$

Soln

We first solve the first order ode to obtain $y(t)$ function

=) This is a bernouli equation so we proceed as follows

=) multiply the equation by y^{-3}

$$\Rightarrow \frac{dy}{dt} + y = y^3$$

$$\Rightarrow y^{-3}\frac{dy}{dt} + y \cdot y^{-3} = y^3 \cdot y^{-3}$$

$$\Rightarrow y^{-3}\frac{dy}{dt} + y^{-2} = 1 \quad \text{——— (1)}$$

let $z = y^{-2}$ $\Rightarrow \frac{dz}{dt} = -2y^{-3}\frac{dy}{dt}$

=) Substitute into eqn (1) we have

$$y^{-3}\frac{dy}{dt} = -\frac{1}{2}\frac{dz}{dt}$$

$$\Rightarrow -\frac{1}{2}\frac{dz}{dt} + z = 1 \quad \Rightarrow \frac{dz}{dt} - 2z = -2$$

We define the Integrating factor as

$$I.F = e^{\int -2dt} = e^{-2t}$$

$$\Rightarrow z \cdot e^{-2t} = \int -2 \cdot e^{-2t} dt + C$$

$\Rightarrow z e^{-2t} = e^{-2t} + c$ but $z = y^{-2}$

$\Rightarrow 1/y^2 = 1 + c e^{2t}$ $\Rightarrow y = \frac{1}{\sqrt{1+ce^{2t}}}$

$\Rightarrow y(0) = \frac{1}{\sqrt{1+c}} = 1/2$ $\Rightarrow c = 3$

$\Rightarrow y(t) = \frac{1}{\sqrt{1+3e^{2t}}}$

$\Rightarrow$ Taking $\lim_{t \to \infty} y(t) = \lim_{t \to \infty} \frac{1}{\sqrt{1+3e^{2t}}} = \lim_{t \to \infty} \frac{1}{\sqrt{1+\infty}} = \frac{1}{\infty} = 0$

$= \underline{0}$

2

$(\ln t) y' + \frac{1}{t+3} y = t$

<u>Soln</u>

We first find the value of $y(t)$ as follows

$\Rightarrow y'(t) + \frac{1}{\ln t (t-3)} y = \frac{t}{\ln t}$

$\Rightarrow \frac{dy}{dt} + \frac{1}{\ln t (t-3)} y = \frac{t}{\ln t}$

$\Rightarrow$ We use the Integrating factor method

$$I.F = e^{\int \frac{1}{(t-3)\ln t} dt}$$

$$\Rightarrow e^{\int \frac{1}{(t-3)\ln t} dt} \cdot \frac{dy}{dt} + \frac{1}{(t-3)\ln t} y = \frac{t}{\ln t} \cdot e^{\int \frac{1}{(t-3)\ln t} dt}$$

$$\Rightarrow y(t) = \frac{t}{\ln t} \cdot e^{\int \frac{1}{(t-3)\ln t} dt} + c$$

$$\Rightarrow y(t) = \frac{\int \frac{t e^{\int \frac{1}{\ln t (t-3)} dt}}{\ln t} dt}{e^{\int \frac{1}{\ln t (t-3)} dt}} + c$$

$$y(2) = \frac{\int \frac{2 e^{\int -\frac{1}{\ln 2} dt}}{\ln 2} dt}{e^{\int -\frac{1}{\ln 2} dt}} = 0$$

with $\ln t (t-3) \Big|_{t=0}$ we have interval

$t \in [-3, 3]$

$\Rightarrow$ Thus solution exists in the interval : $(0, 3)$

Determine 3^{rd} order non-linear ODE

Soln

First to determine the order of an ode we look at the highest ordered derivative

$\Rightarrow$ so we have (b) and (d) with highest ordered derivative being (3) are non-linear since $q(a)$ is not equal to zero

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4. Direction fields to ODE

$$\frac{dy}{dx} = 2y - y^2$$

Soln

=> First we need to solve for $y(x)$ as below

$$\frac{dy}{dx} + 2y = -y^2$$

=> Multiply by y^{-2}, we have

$$y^{-2}\frac{dy}{dx} - 2y \cdot y^{-2} = -1$$

, let $z = y^{-1}$

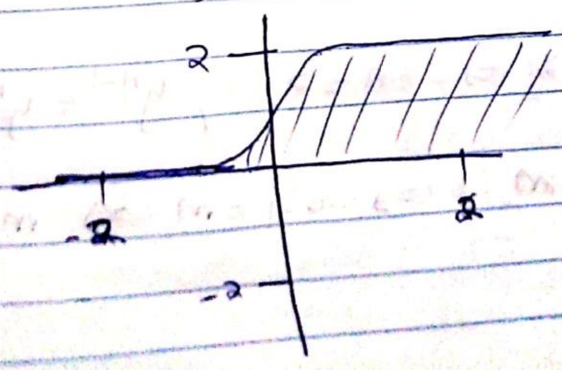
$$\Rightarrow \frac{dz}{dx} = -y^{-2}\frac{dy}{dx} \quad \Rightarrow y^{-2}\frac{dy}{dx} = -\frac{dz}{dx}$$

$$\Rightarrow \frac{dz}{dx} + 2z = 1 \quad \Rightarrow I.F = e^{\int 2\,dx} = e^{2x}$$

$$\Rightarrow z \cdot e^{2x} = \int e^{2x}\,dx + c \quad \Rightarrow \frac{1}{2}e^{2x} + c$$

$$\Rightarrow \frac{1}{y} = \frac{1}{2}e^{2x} + c$$

=> Sketching the solution above above we ha



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Implying that (a) and (c) are the direction fields since the values of $y(x)$ are diverging from at y being 0 and 2

Find value of n and m such that below equation is exact

$$(xy^n + x^2) + (x^2y^m + y^3)\frac{dy}{dx} = 0$$

Soln

Multiply the eqn by dx to obtain

$$(xy^n + x^2)\,dx + (x^2y^m + y^3)\,dy = 0$$

$\Rightarrow$ For an exact D.E the following condition must be met

i.e.

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\Rightarrow$

$$M(x,y) = xy^n + x^2 \quad , \quad N(x,y) = x^2y^m + y^3$$

$$\Rightarrow \frac{\partial M}{\partial y} = nxy^{n-1} \quad , \quad \frac{\partial N}{\partial x} = 2xy^m$$

$$\Rightarrow nx = 2x \Rightarrow n = 2 \quad , \quad y^{n-1} = y^m$$

$$\Rightarrow n-1 = m \Rightarrow 2-1 = m \Rightarrow m = 1$$

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$\Rightarrow$ Verifying the value

$n = 2, \quad m = 1$

$$\frac{\partial M}{\partial y} = nxy^{n-1} = 2xy \quad \text{and} \quad \frac{\partial N}{\partial x} = 2xy^{m} = 2xy \quad \checkmark$$

6 Use Euler method to solve

$$\frac{dy}{dt} = 2t + y \quad , y(0) = 3 \quad , h = 0.2$$

at $y(0.6) = ?$

Soln

$\Rightarrow$ The Euler method is given by

$$y_{n+1} = y_n + h f(x_n, y_n)$$

$\Rightarrow$ Thus, we have

$t_0 = 0, \quad y_0 = 3 \quad , h = 0.2$

$0 \quad 0.2 \quad 0.4 \quad 0.6$

t_0

$y_0 = 3 \quad y_1 = ?$

$$y_1 = y_0 + 0.2 f(t_0, y_0) = 3 + 0.2 [2 \cdot 0 + 3]$$
$$= 3 + 0.2(3) = 3.6$$

$$y_2 = y_1 + 0.2 [2 \cdot 0.2 + 3.6]$$
$$= 3.6 + 0.2 [2 \cdot 0.2 + 3.6] = 4.4$$

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$y_3(0.6) = y_2 + 0.2[2 \cdot 0.4 + 4.4]$

$= 4.4 + 0.2[2 \cdot 0.4 + 4.4] = 5.44$

=> Thus the soln of IVP at $y(0.6)$ is given by

$y(0.6) = 5.44$

Determine and classify critical point of the system

$y_1'(t) = -5y_1(t) - 8y_2 t$

$y_2'(t) = 4y_1(t) + 3y_2(t)$

soln

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{bmatrix} -5 & -8 \\ 4 & 3 \end{bmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

=> Finding the critical point of the above we have

$-5y_1 - 8y_2 = 0$

$4y_1 + 3y_2 = 0$

=> $0 - 17y_2 = 0$ => $y_2 = 0$,

at $y_2 = 0$, $y_1 = 0$

=> $-5y_1 - 8y_2 = 0$ => $y_1 = 8$, $y_2 = -5$

=> $4y_1 + 3y_2 = 0$ => $y_1 = -3$, $y_2 = 4$

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$\Rightarrow$ critical points are

$(0,0)$, $(8,-5)$, $(-3,4)$

$\Rightarrow$ Classification

$$\Rightarrow \begin{pmatrix} -5 & -8 \\ 4 & 3 \end{pmatrix}\Bigg|_{(0,0)} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} -5 & -8 \\ 4 & 3 \end{pmatrix} = \begin{vmatrix} -5-\lambda & -8 \\ 4 & 3-\lambda \end{vmatrix}$$

$$\Rightarrow (3-\lambda)(-5-\lambda) + 32 = 0$$

$$\Rightarrow 15 - 3\lambda + 5\lambda + \lambda^2 + 32 = 0 \Rightarrow \lambda^2 + 2\lambda + 17 = 0$$

$$\lambda = \frac{-2 \pm \sqrt{4-68}}{2} = \frac{-2 \pm \sqrt{-64}}{2}$$

$$\Rightarrow \lambda = -1 \pm 4i$$

$\Rightarrow$ Thus these critical points are spiral sink and are assymptotically stable

8. Construct 1^{st} order linear differential system of

$$y'' + t^2 y' + 3y = \cos t \quad \text{with} \quad y(0) = 1 , \ y'(0) = 3.$$

Soln

let $x_1 = y$ and $x_2 = y'$

$\Rightarrow$ Differentiating the above we have

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$\Rightarrow x_1' = y'$ and $x_2' = y''$

$\Rightarrow x_1' = y' = x_2$ —— (1)

$\Rightarrow x_2' = y'' = ?$

$\Rightarrow$ From $y'' + t^2 y' + 3y = \cos t$
we have
$$y'' = -t^2 y' - 3y + \cos t$$

$\Rightarrow x_2' = y'' = -t^2 x_1' - 3x_1 + \cos t$ —— (2)

$\Rightarrow$ In matrix form we have from eqn (1) and (2)

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -3 & -t^2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \cos t \end{pmatrix}$$

and Condition
$$x_1(0) = 0$$
$$x_2(0) = 3 \quad \Rightarrow \underline{\underline{X' = AX + B}}$$

9. Compute resolvent matrix and matrix exponential e^{At}

$$A = \begin{pmatrix} 3 & 5 \\ -1 & -1 \end{pmatrix}$$

$\Rightarrow$ To find the resolvent matrix we have
$$R(\lambda) = (\lambda I - A)^{-1}$$

$$\Rightarrow \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 3 & 5 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} \lambda - 3 & -5 \\ 1 & \lambda + 1 \end{pmatrix}^{-1}$$

=) To find its inverse we have

$(\lambda+1)(\lambda-3)+5 \Rightarrow \lambda^2-2\lambda+2$

$$\Rightarrow \frac{1}{\lambda^2-2\lambda+2}\begin{pmatrix}\lambda+1 & 5\\ -1 & \lambda-3\end{pmatrix}$$

=) Matrix exponential e^{At}

$$\Rightarrow e^{At} = I + At + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \cdots$$

$$\Rightarrow \sum_{k=0}^{\infty} \frac{A^k t^k}{k!} = I\sum_{k=0}^{\infty}\frac{t^k}{k!} c_{0,k} + A\sum_{k=0}^{\infty}\frac{t^k}{k!} c_{1,k}$$

$e^{\lambda t} = b_0 t + b_1(t)\lambda$ with λ being eigenvalues of A

$\lambda^2-2\lambda+2$

$$\Rightarrow \lambda = \frac{2 \pm 2\sqrt{2}}{2}$$

$\lambda_1 = 1+\sqrt{3}$, $\lambda_2 = 1-\sqrt{3}$

$$\Rightarrow e^{(1+\sqrt{3})t} = b_0(t) + b_1(t)(1+\sqrt{3}) \quad -(1)$$

$$e^{(1-\sqrt{3})t} = b_0(t) + b_1(t)(1-\sqrt{3}) \quad -(2)$$

=) Solving for $b_0(t)$ and $b_1(t)$ we have

$$e^{At} = e^{(1-\sqrt{3})t} - \left[\frac{e^{(1+\sqrt{3})t} - e^{(1-\sqrt{3})t}}{2\sqrt{3}}\right]\cdot(1-\sqrt{3})\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \cdots$$

$$\cdots + \left[\frac{e^{(1+\sqrt{3})t} - e^{(1-\sqrt{3})t}}{2\sqrt{3}}\right]\cdot\begin{bmatrix}3 & 5\\ -1 & -1\end{bmatrix}$$

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1 Finding the system of differential equation

Soln

=> Given tank 1, we have 1 gallon of brine equivalent of $y(t)$ and 4 pounds of salt which will be dissolved into tank 2

=> Thus, we have $4y_2(t)$ for 1st d.E,

=> As for tank 2, with only 2 gallons of pure water, this will be represented by the amount from the 1st tank $2y_1(t)$

=> Note that all tanks will have a rate of 2 gal/min corresponding to $2t$

=> Thus we have

$$\begin{cases} y_1'(t) = 2t + 4y_2(t) + y_1(t) \\ y_2'(t) = 2y_1(t) + 2t \end{cases}$$

Solve the D.E

$$A = \begin{pmatrix} 2 & -3 & 2 \\ 0 & -1 & 3/2 \\ 0 & 0 & 2 \end{pmatrix} \text{ and } y(0) = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

Soln

=> We first find the eigenvalues of the system as below

$$\begin{vmatrix} 2-\lambda & -3 & 2 \\ 0 & -1-\lambda & 3/2 \\ 0 & 0 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow 2-\lambda \begin{bmatrix} -1-\lambda & 3/2 \\ 0 & 2-\lambda \end{bmatrix} = 0 \quad \Rightarrow 2-\lambda\left[\lambda^2-\lambda-2\right]=0$$

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$\Rightarrow \lambda_1 = 2$

and $\lambda^2 - \lambda - 2 = 0$

$\Rightarrow \lambda_2 = 2$ and $\lambda_3 = -1$

$\Rightarrow$ Eigenvector associated with $\lambda_1 = 2$, we have

$$\begin{pmatrix} 0 & -3 & 2 \\ 0 & -3 & 3/2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow 0v_1 - 3v_2 + 2v_3 = 0$ $\Rightarrow v_2 = 1$

$\Rightarrow v_3 = 3/2$

$\Rightarrow v_1 = 0$

$$X_1(t) = \begin{pmatrix} 0 \\ 1 \\ 3/2 \end{pmatrix} e^{2t}$$

$$\Rightarrow X_2(t) = \begin{pmatrix} 0 \\ 1 \\ 3/2 \end{pmatrix} t e^{2t} + \begin{pmatrix} c_3 \\ c_4 \\ c_5 \end{pmatrix} e^{2t}$$

$$\Rightarrow \begin{pmatrix} 0 & -3 & 2 \\ 0 & -3 & 3/2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} c_3 \\ c_4 \\ c_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3/2 \end{pmatrix}$$

$$\Rightarrow X_2(t) = \begin{pmatrix} 0 \\ 1 \\ 3/2 \end{pmatrix} t e^{2t} + \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} e^{2t}$$

$X_3(t) \Rightarrow$ $\lambda_3 = -1$

$$\begin{pmatrix} 3 & -3 & 2 \\ 0 & 0 & 3/2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow 3v_1 - 3v_2 + 2v_3 = 0 \quad \Rightarrow v_1 = 1$

$v_2 = 1$

$v_3 = 0$

$$X_3(t) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} e^{-t}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = c_1 \begin{pmatrix} 0 \\ 1 \\ 3/2 \end{pmatrix} e^{2t} + c_2 \left[\begin{pmatrix} 0 \\ 1 \\ 3/2 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} e^{2t} \right] + c_3 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} e^{-t}$$

at $y(0) = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ we have

$\Rightarrow \quad 0 = 0c_1 + 0c_2 + c_3$

$1 = 1c_1 + 1c_2 + c_3 \quad \Rightarrow c_3 = 0$

$2 = \frac{3}{2}c_1 + 3c_2 = c_3 \quad c_1 = 0$

$c_2 = 1$

$\Rightarrow$ solution,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3/2 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} e^{2t}$$

12. Determine and classify singular points of

$$\begin{cases} y_1' = y_2 - 3 \\ y_2' = -2y_1 + y_1^2 - y_2 \end{cases}$$

Soln

$\Rightarrow$ First we need to rewrite the system of eqn into a 2nd order linear ode as below

$\Rightarrow$ let $x_1 = y$ and $x_2 = y'$

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=) differentiating the above we have

$\Rightarrow x_1' = y' \quad$ and $\quad x_2' = y''$

$\Rightarrow y_1' = x_1' = y' \quad$ and $\quad y_2' = y'' = -2y_1 + 2y_1^2 - y_2$

$\Rightarrow y_2'' = -2x_2 + x_1^2 - y_2$

=) But for a given y function we have

$\Rightarrow y'' - 2y' + y^2 - y$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{2dy}{dx} + y^2 - y = 0$$

$\Rightarrow P(x) = -2 \quad$ and $\quad q(x) = -1$

=) The singular point is at $x=0$ for the above o.d.e.

=) Classification

$$P(x)\Big|_{x=0} = -2$$

$$\text{and} \quad q(x)\Big|_{x=0} = -1$$

=) Thus this $x=0$ is a regular singular point

13. Solve the system

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 8e^t \\ 18e^t \end{pmatrix} \quad \text{and} \quad y(0) = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

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Soln

$$A = \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix} \Rightarrow (\lambda I - A) \Rightarrow \begin{vmatrix} 1-\lambda & 4 \\ 1 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)(1-\lambda) - 4 = 0$$

$$\Rightarrow (1-\lambda)^2 = 4 \Rightarrow \lambda_1 = 3 \text{ and } \lambda_2 = -1$$

$\Rightarrow$ Eigenvectors corresponding to $\lambda_1 = 3$

$$\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow -2v_1 + 4v_2 = 0$$

$$\Rightarrow v_1 = 1, \ v_2 = \tfrac{1}{2}$$

$$\Rightarrow X_1(t) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t}$$

$$\Rightarrow \lambda_1 = -1 \Rightarrow \begin{pmatrix} 0 & 4 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow 0v_3 + 4v_4 = 0$$

$$\Rightarrow v_4 = 0, \quad v_3 = 1$$

$$X_2(t) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t}$$

$\Rightarrow$ Complete solution $\begin{pmatrix} x \\ y \end{pmatrix} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t}$

$\Rightarrow$ For the particular soln

$$\Rightarrow c_1(t)\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t} + c_2(t)\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t} = \begin{pmatrix} 8e^{t} \\ 18e^{t} \end{pmatrix}$$

$$c_1'(t) = \frac{\begin{vmatrix} 8e^{t} & e^{-t} \\ 18e^{t} & 0 \end{vmatrix}}{\begin{vmatrix} 2e^{3t} & e^{-t} \\ e^{3t} & 0 \end{vmatrix}} = \frac{18}{e^{2t}} = 18e^{-2t}$$

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$\Rightarrow c_1(t) = \int 18 e^{-2t} dt = -9 e^{-2t}$

$$\Rightarrow c_2'(t) = \frac{\begin{vmatrix} 2e^{3t} & 8e^{t} \\ e^{-2t} & 18 e^{t} \end{vmatrix}}{e^{2t}} = 28 e^{2t}$$

$$\Rightarrow c_2(t) = \int 28 e^{2t} dt = 14 e^{2t}$$

$\Rightarrow$ Solution

$$\begin{pmatrix} x \\ y \end{pmatrix} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t} - 9 e^{-3t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t} + 14 e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t}$$

$\Rightarrow$ Applying Condition at $y(0) = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$\Rightarrow 4 = 2c_1 + c_2 - 18 + 14 \qquad \Rightarrow c_1 = 9$

$0 = c_1 + 0c_2 - 9 = 0 \qquad c_2 = -10$

$\Rightarrow$ particular soln

$$\begin{pmatrix} x \\ y \end{pmatrix} = 9 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t} - 10 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t} - 9 e^{-3t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t} + 14 e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t}$$

14. Particular solution of $\frac{d^2y}{dt^2} + y = \sin t$

Soln

$\Rightarrow$ For the complete solution we have

$$\Rightarrow \frac{d^2y}{dt^2} + y = 0 \Rightarrow \lambda^2 + 1 = 0 \Rightarrow \lambda^2 = -1$$

$$\Rightarrow \lambda = \pm i$$

$$\Rightarrow y_c = c_1 \cos t + c_2 \sin t$$

$\Rightarrow$ For the particular soln

$$y_p = A \cos t + B \sin t$$

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$\Rightarrow y'_p = -A\sin t + B\cos t$

$\Rightarrow y''_p = -A^2\cos t - B^2\sin t$

$\Rightarrow (-A^2\cos t - B^2\sin t) + A\cos t + B\sin t = \sin t$

$\Rightarrow A = -t/2 \quad , \quad B = 0$

$\Rightarrow y_p(t) = t/2 \cos t$

Solution

$y = y_c + y_p$

$\Rightarrow y(t) = c_1\cos t + c_2\sin t - t/2 \cos t$

THANK YOU !!!