

Eastern University

Name:Sorozit Boiragi

ID:193400025

Course Code: MAT 201

Course Title: Differential Equations

Section: 6

Final Exam (Assessment), Summer- 2021

---



# Eastern University



## ADMIT CARD Final Examination, Summer 2021

ID : 193400025 - 77F6 Faculty : Engineering &  
Name : SOROZIT BOIRAGI Program : CSE

Course Code	Course Name	Attendance	Credit
CSE 312	Operating System Lab	95.45%	1
CSE 326	Computer Peripherals and Interfacing Lab	85.71%	1
CSE 412	Computer Graphics Laboratory	100%	1
CSE 416	Computer Networks Laboratory	81.82%	1
CSE 477	Thesis/Project	0%	2
EEE 231	Electronics I	81.82%	3
MAT 201	Differential Equations	84.21%	3

### Instruction for Examinees:

- \* Examinees should enter the examination room / hall before 10 minutes of starting of the examination
- \* No examinees shall be allowed to sit for the examination without Admit card
- \* No Examinees shall be allowed to carry any papers except admit card
- \* Cellular phone will not be allowed in the examination room / hall

$$2. 8571 = e^{-0.009328t}$$

$$-0.009328t \ln e = \ln(2.8571)$$

$$-0.009328t = 1.0498$$

$$t = -112.5 \approx 112 \text{ minutes}$$

)      )

$$\begin{array}{rcl} 1\text{hr} & = & 60\text{ min} \\ ? & = & 112\text{ min} \end{array}$$

$$\frac{112}{60} = 1.867 \text{ hr}$$

$$\approx 1\text{hr}, 52 \text{ minutes}$$

! !

From 6am  $\Rightarrow$

~~7am; 52 minutes~~

$\Rightarrow 7:52 \text{ am}$

$$2.8571 = e^{-0.009328t}$$

$$-0.009328t \ln e = \ln(2.8571)$$

$$-0.009328t = 1.0498$$

$$t = -112.5 \approx 112 \text{ minutes}$$

)

$$1 \text{ hr} = 60 \text{ min}$$

$$? = 112 \text{ min}$$

$$\frac{112}{60} = 1.867 \text{ hrs}$$

$$\approx 1 \text{ hr } 52 \text{ minutes}$$

from 6am  $\Rightarrow$

$$i = -i_0 \cos \omega t + c \quad \text{at time } t$$

3(a)

(3a) Resistance = 30 ohms  
Induct. 20 henrys  
To 5000 Volts

$$T = 20 + 7e^{-0.009328t}$$

Now, to find the time at death we have,  
= Thus, we take the temperature difference from time (initial) time  
6 am to when the body died

$$\Rightarrow 27 - 24 = 3^\circ C$$

$$\Rightarrow 3^\circ + 37^\circ C = 40^\circ C$$

$$\Rightarrow 40 = 20 + 7e^{-0.009328t}$$

$$\Rightarrow \frac{20}{7} = e^{-0.009328t}$$

$$T(0) = 27 \quad , \quad T(60) = 24$$

$$\text{so } T_0 = 20^\circ\text{C}$$

$$\Rightarrow T = T_0 + c e^{-kt} \quad \text{using } T(0) = 27$$

$$T = 20 + c e^{-kt}$$

$$\Rightarrow 27 = 20 + c e^{-k(0)} = c = 7$$

$$\Rightarrow T = 20 + 7 e^{-kt}$$

so for  $T(60) = 24$ , we have,

$$24 = 20 + 7 e^{-k(60)}$$

$$\Rightarrow \frac{4}{7} = e^{60k} \Rightarrow 0.5714 = e^{-60k}$$

$$\Rightarrow -60k \ln e = \ln(0.5714)$$

$$= -60k = -0.5597 \Rightarrow k = \underline{\underline{9.328 \times 10^{-3}}}$$

$$T(0) = 27, \quad T(60) = 24$$

$$\text{so } T_0 = 20^\circ\text{C}$$

$$\Rightarrow T = T_0 + c e^{-kt} \quad \text{using } T(0) = 27$$

$$T = 20 + c e^{-kt}$$

$$\Rightarrow 27 = 20 + c e^{-k(0)} = c = 7$$

$$\Rightarrow T = 20 + 7 e^{-kt}$$

$\Rightarrow$  so for  $T(60) = 24$ , we have,

$$24 = 20 + 7 e^{-k(60)}$$

$$\Rightarrow \frac{4}{7} = e^{60k} \Rightarrow 0.5714 = e^{-60k}$$

$$\Rightarrow -60k \ln e = \ln(0.5714)$$

$$= -60k = -0.5597 \Rightarrow k = \underline{\underline{9.328 \times 10^{-6}}}$$

The Complementary

of variation of

Time constant  $\alpha \text{PEA} \cos \omega t$

Thus current  $i = -\frac{1}{G} \cos \omega t + C$  at time  $t$

3(b)

(3b) initial temperature  $= 27^\circ \text{C}$

the room temperature = constant  $= 20^\circ \text{C}$

After 1 hour  $\Rightarrow$  temperature dropped to  $24^\circ \text{C}$

Find the time of death?

Normal body temperature  $37^\circ \text{C}$

Sols

$$\frac{dT}{dt} \propto T - T_0 \Rightarrow \frac{dT}{dt} = -k(T - T_0)$$

The room temperature =  $27^\circ\text{C}$   
 After 1 hour, temperature = constant =  $20^\circ\text{C}$   
 Find the time it takes for temperature dropped to  $24^\circ\text{C}$ .  
 Normal body temperature  $37^\circ\text{C}$

Sols

$$\frac{dT}{dt} \propto T - T_0 \Rightarrow \frac{dT}{dt} = -k(T - T_0)$$

$$T(0) = 27, T(60) = 24$$

$$\text{So } T_0 = 20^\circ\text{C}$$

$$\Rightarrow T = T_0 + C e^{-kt} \quad \text{using } T(0) = 27$$

$$T = 20 + C e^{-kt}$$

$$\Rightarrow 27 = 20 + C e^{-k(0)} = C = 7$$

$$\Rightarrow T = 20 + 7 e^{-kt}$$

Thus current  $i = -\frac{1}{G} \cos 2t + C$  at time  $t$

(3b)

initial temperature  $= 27^\circ C$

The room temperature = constant  $= 20^\circ C$

After 1 hour,  $\Rightarrow$  temperature dropped to  $24^\circ C$

Find the time it takes?

Normal body temperature  $37^\circ C$

Soh

$$\frac{dT}{dt} \propto T - T_0 \Rightarrow \frac{dT}{dt} = -k(T - T_0)$$

$$T(0) = 27, T(t_0) = 24$$

$$\text{So } T_0 = 20^\circ C$$

$$\Rightarrow T = T_0 + C e^{-kt} \quad \text{using } T(0) = 27$$

4.

Solve

$$\frac{dT}{dt} \times T - T_0 \Rightarrow \frac{dT}{dt} = -k(T - T_0)$$

5.

$$T(0) = 27, \quad T(60) = 24$$

$$\text{So } T_0 = 20^\circ\text{C}$$

$$\Rightarrow T = T_0 + c e^{-kt} \quad \text{using } T(0) = 27$$

$$T = 20 + c e^{-kt}$$

$$\Rightarrow 27 = 20 + c e^{-k(0)} = c = 7$$

$$\Rightarrow T = 20 + 7 e^{-kt}$$

$\Rightarrow$  So for  $T(60) = 24$ , we have,

$$24 = 20 + 7 e^{-k(60)}$$

$$\Rightarrow 4 = e^{60k} \Rightarrow 0.5714 = e^{-60k}$$

$$\Rightarrow \frac{di}{dt} = \frac{V}{L} \quad \text{and} \quad L = \frac{RA}{f}$$

$$= \frac{di}{dt} = \frac{V}{RA/f} = \frac{di}{dt} = \frac{fV}{RA}$$

So for a unit area coil with resistivity inductance 20 henries and voltage 50 sin 2t volt we have

$$\frac{di}{dt} = \frac{1}{20} \cdot 50 \sin 2t / 30$$

$$= \frac{di}{dt} = \frac{5}{60} \sin 2t$$

$$di = \frac{5}{60} \sin 2t dt$$

Upon integration

$$\int di = \int \frac{5}{60} \sin 2t dt$$

18  
In case of a unit area coil with resistance  $r$  and inductance  $L$ , we have

$$\frac{di}{dt} = \frac{1}{L} \cdot 50 \sin 2t / 30$$

$$= \frac{di}{dt} = \frac{5}{60} \sin 2t$$

$$di = \frac{5}{60} \sin 2t dt$$

Upon integration

$$\int di = \int \frac{5}{60} \sin 2t dt$$

$$= i = \frac{5}{60} \cdot -2 \cos 2t + C$$

$$= i = -\frac{10}{60} \cos 2t + C$$

$$\frac{di}{dt} \quad , \quad R = \rho \frac{L}{A} \quad , \quad R = \frac{V}{A}$$

$$\Rightarrow V = RA \quad \Rightarrow \rho \frac{V}{L} = \frac{di}{dt}$$

$$\Rightarrow \frac{di}{dt} = \frac{V}{L} \quad \text{and} \quad L = \frac{RA}{\rho}$$

$$= \frac{di}{dt} = \frac{V}{RA/s} = \frac{di}{dt} = \frac{V}{RA}$$

So for a unit area coil with resistivity inductance 20 henrys and voltage 50 mvt vbl we have

$$\frac{di}{dt} = \frac{1}{20} \cdot 50 \sin 2t / 30$$

$$2 \frac{di}{dt} = \frac{5}{60} \sin 2t$$

(39)

Q19

$$\text{Resistance} = 30 \text{ ohms}$$

$$\text{Inductance} = 20 \text{ henrys}$$

$$\text{Voltage} = 50 \sin 2t \text{ volts}$$

$$\text{Current} = ?$$

$$V = L \frac{di}{dt}$$

$$R = \rho \frac{L}{A}$$

$$R = \frac{V}{A}$$

$$\Rightarrow V = RA \Rightarrow \frac{V}{L} = \frac{di}{dt}$$

$$\Rightarrow \frac{di}{dt} = \frac{V}{L} \quad \text{and} \quad L = \frac{RA}{f}$$

$$= \frac{di}{dt} = \frac{V}{RA/f} = \frac{di}{dt} = \frac{fV}{RA}$$

So for a unit area coil with resistivity induction  
20 henrys and voltage  $50 \sin 2t$  vols we have

$$= -0.5 \bar{e}^{(0.5)} - \bar{e}^{(0.1)} \approx -0.9098$$

2nd iteration.

Now we use

$$\frac{dy}{dx} = x \bar{e}^x + x \bar{e}^x - \bar{e}^x$$

Then for  $x$

$$\int \frac{dy}{dx} dx = \int_0^x -\bar{e}^x dx \quad \text{which gives}$$

$$y - 0 = e^x$$

$$y_2 = e^x \Big|_{x=0.5} = 1.649$$

Third iteration.

Now we use

$$\frac{dy}{dx} = x \bar{e}^x + y_2 = x \bar{e}^x + e^x$$

Then for

$$\int dy dx = \int (x \bar{e}^x + e^x) dx$$

3(a)

(3a) Resistance = 30 ohms

Inductance = 20 henrys

Voltage = 50 sin(2t) Volts

Current = ?

$$V = L \frac{di}{dt}, \quad R = \rho \frac{L}{A}, \quad R = \frac{V}{A}$$

$$\Rightarrow V = RA \Rightarrow R \frac{V}{L} = \frac{di}{dt}$$

$$\Rightarrow \frac{di}{dt} = \frac{V}{L} \quad \text{and} \quad L = \frac{RA}{\rho}$$

Therefore

$$\int \frac{dy}{dx} dx = \int_0^x -e^x dx \quad \text{when } T=0$$

$$y - 0 = e^x$$

$$y_1 = e^x \Big|_{x=0.5} = 1.649$$

Third iteration.

Now we have

$$\frac{dy}{dx} = x e^{-x} + y_1 = x e^{-x} + e^x$$

Therefore

$$\int \frac{dy}{dx} dx = \int_0^x (x e^{-x} + e^x) dx$$

$$y - 0 = x e^{-x} \quad y - 0 = -x e^{-x} - e^x + e^x \Big|_{x=0}$$

$$= 0.7389$$

Hence  $y = 0.7389$  ~~Correct~~ to 4 d.c. at  $x$

$$y - 0 = e^x$$

$$y_0 = e^x \Big|_{x=0.5} = 1.649$$

Third Order

Now we use

$$\frac{dy}{dx} = x e^{-x} + y_0 = x e^{-x} + e^x$$

Now for

$$\int \frac{dy}{dx} dx = \int (x e^{-x} + e^x) dx$$

$$y - 0 = x e^{-x} \quad y - 0 = -x e^{-x} - e^{-x} + e^x \quad \Big|_{x=0.5}$$

$$= 0.7389$$

Hence  $y = 0.7389$ , Correct to 4 d.c at  $x = 0.5$

$$y_1 = \int_0^x x e^{-x} dx$$

Let  $u = x$ ,  $du = dx$   
 $dv = e^{-x}$ ,  $v = -e^{-x}$

$$-x e^{-x} + \int e^{-x} dx = -x e^{-x} - e^{-x} \Big|_{x=0.5}$$

$$= -0.5 e^{(0.5)} - e^{(0.5)} \approx -0.9098$$

2<sup>nd</sup> iteration.

Now we use

$$\frac{dy}{dx} = x e^{-x} + x e^{-x} - e^{-x}$$

Then for  $x$

$$\int_0^x \frac{dy}{dx} dx = \int_0^x -e^{-x} dx \quad \text{which gives}$$

$$y - 0 = e^{-x}$$

$$y_2 = e^{-x} \Big|_{x=0.5} = 1.649$$

0,6065

first iteration

$$y_1 = \int_0^x e^{-x} dx$$

$$\text{let } u = x \quad du = dx \\ dv = e^{-x} \quad v = -e^{-x}$$

$$-x e^{-x} + \int e^{-x} dx = -x e^{-x} - e^{-x} \Big|_{x=0.5}$$

$$= -0.5 e^{-(0.5)} - e^{-(0.5)} \approx -0.9098$$

2nd iteration.

Now we use

$$\frac{dy}{dx} = x e^{-x} \neq x e^{-x} - e^{-x}$$

b) Using picard's method by a program

$$\frac{dy}{dx} = y + xe^{-x}$$

$$y(0) = 1$$

S.h

$$\text{when } x_0 = 0 \quad y = 0$$

at  $x = 0.5$

$$\int_{x_0}^x \frac{dy}{dx} dx = \int_{x_0}^x (y + xe^{-x}) dx \quad \text{where } x_0 = 0$$

$$y - y_0 = \int_{x_0}^x (y + xe^{-x}) dx \quad \text{where } y_0 = 0$$

which becomes

$$y = \int_0^x (y + xe^{-x}) dx$$

1(b)

b) Using picard method b approx

$$\frac{dy}{dx} = y + x e^{-x}, \quad y(0) = 1$$

Soh

when  $x_0 = 0, \quad y = 0$

at  $x = 0.5$

$$\int_{x_0}^x \frac{dy}{dx} dx = \int_{x_0}^x (y + x e^{-x}) dx \quad \text{where } x_0 = 0$$

$$y - y_0 = \int_{x_0}^x (y + x e^{-x}) dx \quad \text{where } y_0 = 0$$

From (Aernot)  $y_p$  we have

$$y_p' = e^x [A \cos 3x + B \sin 3x] + e^x [-3A \sin 3x + 3B \cos 3x] \quad (2)$$

$$\begin{aligned} y_p'' = & e^x [A \cos 3x + B \sin 3x] + e^x [-3A \sin 3x + 3B \cos 3x] + e^x [-3A \sin 3x + 3B \cos 3x] \\ & + e^x [-9A \cos 3x - 9B \sin 3x] + 4e^{2x} \end{aligned} \quad (3)$$

$$\begin{aligned} y_p''' = & e^x [A \cos 3x + B \sin 3x] + e^x [-3A \sin 3x + 3B \cos 3x] \\ & + e^x [-3A \sin 3x + 3B \cos 3x] + e^x [-9A \cos 3x - 9B \sin 3x] \\ & + e^x [-3A \sin 3x + 3B \cos 3x] + e^x [-9A \cos 3x - 9B \sin 3x] \\ & + e^x [-9A \cos 3x - 9B \sin 3x] + e^x [-27A \sin 3x - 27B \cos 3x] \end{aligned}$$

— (4)

Substituting (1), (2), (3) and (4) into (1) we have

$$\Rightarrow A = -9, B = 3$$

$$\Rightarrow C = C_3$$

$\Rightarrow$

$$+ e^{2x} \{-3A \sin 3x + 3B \cos 3x\} + e^{2x} \{-9A \cos 3x - 9B \sin 3x\}$$

$$+ e^{2x} \{-9A \cos 3x - 9B \sin 3x\} + e^{2x} [27A \sin 3x - 27B \cos 3x]$$

Substituting (1), (2), (3) and (4) into (1) we have

$$\Rightarrow A = -9, B = 3 \quad \Rightarrow C = C_3$$

$$\Rightarrow y_p = e^{2x} [-9 \cos 3x + 3 \sin 3x] + C e^{2x}$$

particular sln

$$y = y_c + y_p =$$

$$y = e^{2x} [C_1 \cos 2x + C_2 \sin 2x] + 2C_3 e^{2x} + \underline{\underline{+ e^{2x} [-9 \cos 3x + 3 \sin 3x]}}$$

$$y_p = e^x [A \cos 3x + B \sin 3x] + e^{2x} [-3A \sin 3x + 3B \cos 3x] + e^x [-9A \cos 3x - 9B \sin 3x] + 4e^{2x}$$

$$y'''_p = e^x [A \cos 3x + B \sin 3x] + e^{2x} [-3A \sin 3x + 3B \cos 3x]$$

$$+ e^x [-3A \sin 3x + 3B \cos 3x] + e^{2x} [-9A \cos 3x - 9B \sin 3x]$$

$$+ e^x [-3A \sin 3x + 3B \cos 3x] + e^{2x} [-9A \cos 3x - 9B \sin 3x]$$

$$+ e^x [-9A \cos 3x - 9B \sin 3x] + e^{2x} [-27A \sin 3x - 27B \cos 3x]$$

Substituting (1), (2), (3) and (4) into (1) we have

$$\Rightarrow A = -9, B = 3 \Rightarrow C = C_3$$

$\Rightarrow$

$$y_p = e^x [-9 \cos 3x + 3 \sin 3x] + C e^{2x}$$

particular sln

$$y = y_c + y_p =$$

$$+ e^{3x} [-3A \sin 3x + 3B \cos 3x] + e^{3x} [-9A \cos 3x - 9B \sin 3x]$$

$$+ e^{3x} [-3A \sin 3x + 3B \cos 3x] + e^{3x} [-9A \cos 3x - 9B \sin 3x]$$

$$+ e^{3x} [-9A \cos 3x - 9B \sin 3x] + e^{3x} [27A \sin 3x - 27B \cos 3x]$$

Substituting (1), (2), (3) and (4) into (1) we have

$$\Rightarrow A = -9, B = 3 \Rightarrow C = C_3$$

$\Rightarrow$

$$y_p = e^{3x} [-9 \cos 3x + 3 \sin 3x] + C_3 e^{3x}$$

particular sln

$$y = y_c + y_p =$$

$$y = e^{3x} [C_1 \cos 3x + C_2 \sin 3x] + 2C_3 e^{3x} + \dots$$

$$+ e^{3x} [-9 \cos 3x + 3 \sin 3x]$$

$$y_p = e^x [A \cos 3x + B \sin 3x] + C e^{2x} \quad (1)$$

By differentiating  $y_p$  we have

$$y'_p = e^x [A \cos 3x + B \sin 3x] + e^x [-3A \sin 3x + 3B \cos 3x] + 2C e^{2x} \quad (2)$$

$$\begin{aligned} y''_p &= e^x [A \cos 3x + B \sin 3x] + e^x [-3A \sin 3x + 3B \cos 3x] + e^x [-3A \sin 3x + 3B \cos 3x] \\ &\quad + e^x [-9A \cos 3x - 9B \sin 3x] + 4e^{2x} \end{aligned} \quad (3)$$

$$\begin{aligned} y'''_p &= e^x [A \cos 3x + B \sin 3x] + e^x [-3A \sin 3x + 3B \cos 3x] \\ &\quad + e^x [-3A \sin 3x + 3B \cos 3x] + e^x [-9A \cos 3x - 9B \sin 3x] \\ &\quad + e^x [-3A \sin 3x + 3B \cos 3x] + e^x [-9A \cos 3x - 9B \sin 3x] \\ &\quad + e^x [-9A \cos 3x - 9B \sin 3x] + e^x [-27A \sin 3x - 27B \cos 3x] \end{aligned}$$

$$\begin{array}{r}
 -2\lambda^2 + 9\lambda \\
 -2\lambda^2 + 4\lambda \\
 \hline
 5\lambda - 10 \\
 5\lambda - 10 \\
 \hline
 0
 \end{array}$$

$$(\lambda - 2)(\lambda^2 - 2\lambda + 5) = 0$$

$$\Rightarrow \lambda - 2 = 0 \quad \text{and} \quad \lambda^2 - 2\lambda + 5 = 0$$

$$a = 1, b = -2, c = 5$$

$$\lambda = \frac{2 \pm \sqrt{4 - 20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2}$$

$$\lambda_1 = 1 + 2i, \lambda_2 = 1 - 2i, \lambda_3 = 2$$

$$y_c = e^{2x} [c_1 \cos 2x + c_2 \sin 2x] + c_3 e^{2x}$$

using the method of undetermined coefficients

$e^{px} A \cos 3x + B \sin 3x$

Variation of

$$y_p = e^x [A \cos 3x + B \sin 3x] + C e^{2x} \quad (1)$$

By differentiating  $y_p$  we have

$$y'_p = e^x [A \cos 3x + B \sin 3x] + e^x [-3A \sin 3x + 3B \cos 3x] + 2C e^{2x} \quad (2)$$

$$\begin{aligned} y''_p &= e^x [A \cos 3x + B \sin 3x] + e^x [-3A \sin 3x + 3B \cos 3x] + e^x [-3A \sin 3x + 3B \cos 3x] \\ &\quad + e^x [-9A \cos 3x - 9B \sin 3x] + 4C e^{2x} \end{aligned} \quad (3)$$

$$y''_p = e^x [A \cos 3x + B \sin 3x] + e^x [-3A \sin 3x + 3B \cos 3x]$$

$$\begin{array}{r} \lambda - 2 \mid \lambda^3 - 4\lambda^2 + 9\lambda - 10 \\ \lambda^3 - 2\lambda^2 \\ \hline -2\lambda^2 + 9\lambda \\ -2\lambda^2 + 4\lambda \\ \hline 5\lambda - 10 \\ 5\lambda - 10 \\ \hline 0 \end{array}$$

$$(\lambda - 2)(\lambda^2 - 2\lambda + 5) = 0$$

$$\Rightarrow \lambda - 2 = 0 \quad \text{and} \quad \lambda^2 - 2\lambda + 5 = 0$$

$$a = 1, b = -2, c = 5$$

$$\lambda = \frac{2 \pm \sqrt{4-20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2}$$

$$\lambda_1 = 1+2i, \lambda_2 = 1-2i, \lambda_3 = 2$$

$$y_c = e^{2x} [c_1 \cos 2x + c_2 \sin 2x] + c_3 e^{2x}$$

using the method of undetermined coefficient

$$\begin{array}{r} \overline{-2} \\ \lambda - 2 \end{array} \left| \begin{array}{r} \lambda^2 - 2\lambda + 5 \\ \lambda^3 - 4\lambda^2 + 9\lambda - 10 \\ \lambda^3 - 2\lambda^2 \\ \hline -2\lambda^2 + 9\lambda \\ -2\lambda^2 + 4\lambda \\ \hline 5\lambda - 10 \\ 5\lambda - 10 \\ \hline 0 \end{array} \right.$$

$$(\lambda - 2)(\lambda^2 - 2\lambda + 5) = 0$$

$$\Rightarrow \lambda - 2 = 0 \quad \text{and} \quad \lambda^2 - 2\lambda + 5 = 0$$

$$a = 1, b = -2, c = 5$$

$$\lambda = \frac{2 \pm \sqrt{4 - 20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2}$$

$$\lambda_1 = 1 + 2i, \lambda_2 = 1 - 2i, \lambda_3 = 2$$

$$y_c = e^{2x} [c_1 \cos 2x + c_2 \sin 2x] + c_3 e^{2x}$$

using the method of undetermined coefficients

For the complete soln

$$\text{Q} \quad \lambda^3 - 4\lambda^2 + 9\lambda - 10 = 0$$

$$\begin{array}{r} \lambda = 2 \\ \underline{\lambda - 2} \end{array} \left\{ \begin{array}{r} \lambda^2 - 2\lambda + 5 \\ \lambda^3 - 4\lambda^2 + 9\lambda - 10 \\ \hline \lambda^3 - 2\lambda^2 \\ \hline -2\lambda^2 + 9\lambda \\ -2\lambda^2 + 4\lambda \\ \hline 5\lambda - 10 \\ 5\lambda - 10 \\ \hline 0 \end{array} \right.$$

$$(\lambda - 2)(\lambda^2 - 2\lambda + 5) = 0$$

$$\Rightarrow \lambda - 2 = 0 \quad \text{and} \quad \lambda^2 - 2\lambda + 5 = 0$$

$$a = 1, b = -2, c = 5$$

$$\lambda = \frac{2 \pm \sqrt{4 - 20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2}$$

(1a) Silu the differential eqn

$$\frac{d^3y}{dx^3} - 4 \frac{d^2y}{dx^2} + 9 \frac{dy}{dx} - 10y = 2 \cos^2 x + e^{2x} \quad \text{--- (1)}$$

Silu  
3rd order eqns

$$y''' - 4y'' + 9y' - 10y = 2 \cos^2 x + e^{2x}$$

B)  $\lambda^3 - 4\lambda^2 + 9\lambda - 10 = 0$  For the complete soln

$$\begin{array}{r} \lambda = 2. \quad \lambda^2 - 2\lambda + 5 \\ \lambda - 2 \left\{ \begin{array}{r} \lambda^3 - 4\lambda^2 + 9\lambda - 10 \\ \lambda^3 - 2\lambda^2 \\ \hline -2\lambda^2 + 9\lambda \\ -2\lambda^2 + 4\lambda \\ \hline 5\lambda - 10 \\ 5\lambda - 10 \\ \hline 0 \end{array} \right. \end{array}$$

$$(\lambda - 2)(\lambda^2 - 2\lambda + 5) = 0$$

1(a)

(1a) Solve the differential eqn

$$\frac{d^3y}{dx^3} - 4 \frac{d^2y}{dx^2} + 9 \frac{dy}{dx} - 10y = 2 \cos^2 x + e^{-2x} \quad \text{---(1)}$$

Solving  
3<sup>rd</sup> order eqns

$$y''' - 4y'' + 9y' - 10y = 2 \cos^2 x + e^{-2x}$$

④  $\lambda^3 - 4\lambda^2 + 9\lambda - 10 = 0$  For the complete soln

$$\begin{array}{r} \lambda = 2. \quad \frac{\lambda^3 - 2\lambda + 3}{\lambda - 2} \\ \underline{-} \quad \begin{array}{r} \lambda^3 - 4\lambda^2 + 9\lambda - 10 \\ \underline{\lambda^3 - 2\lambda^2} \\ -2\lambda^2 + 9\lambda \\ -2\lambda^2 + 4\lambda \end{array} \end{array}$$

1(a)

(1a) Solve the differential eqn.

$$\frac{d^3y}{dx^3} - 4 \frac{d^2y}{dx^2} + 9 \frac{dy}{dx} - 10y = 2 \cos^2 x + \bar{e}^{2x} \quad \text{---(1)}$$

Solu

3rd order eqns

$$y''' - 4y'' + 9y' - 10y = 2 \cos^2 x + \bar{e}^{2x}$$

∴  $x^3 - 4x^2 + 9x - 10 = 0$  For the complete

$$x^3 - 3x + 8$$

Up part integrand w.r.t  $v_1 = \frac{3 \ln(\tan 3x)}{3} \cdot \cos 3x$

$$v_1 = -\cos 3x \cdot \ln(\tan 3x)$$

Solving for  $v_2$

$$v_2' = \begin{vmatrix} \cos 3x & 0 \\ -3 \sin 3x & \cot 3x \end{vmatrix} = \frac{\cot 3x \cdot \cos 3x}{3}$$

Upon integration,

$$v_2 = \frac{1}{3} \ln(\tan 3x) \cdot \sin 3x$$

$$v_2 = \sin 3x \ln(\tan 3x)$$

$$y_p = -\cos 3x \ln(\tan 3x) \cos 3x + \sin 3x \ln(\tan 3x) \sin 3x.$$

Sum

$$y = y_c + y_p = c_1 \cos 3x + c_2 \sin 3x + \dots +$$

$$\dots + \underline{\sin 3x \ln(\tan 3x) \sin 3x - \cos 3x \ln(\tan 3x) \cos 3x}$$

Upon integration we have

$$v_1 = \frac{3 \ln(\tan 3x)}{3} \cdot \cos 3x$$

$$v_1 = -\cos 3x \cdot \ln(\tan 3x)$$

Solving for  $v_2$

$$v_2' = \begin{vmatrix} \cos 3x & 0 \\ -3 \sin 3x & \cot 3x \end{vmatrix} = \frac{\cot 3x \cdot \cos 3x}{3}$$

Upon integration

$$v_2 = \frac{\beta \ln(\tan 3x) \cdot \sin 3x}{\beta}$$

$$v_2 = \sin 3x \ln(\tan 3x)$$

$$y_p = -\cos 3x \ln(\tan 3x) \cos 3x + \sin 3x \ln(\tan 3x) \sin 3x$$

SCM

$$y = y_c + y_p = c_1 \cos 3x + c_2 \sin 3x + \dots +$$

$$\dots + \sin 3x \ln(1 - \dots)$$

$$3. \quad v_1' = \begin{vmatrix} 0 & \sin 3x \\ \cot 3x & 3\cos 3x \end{vmatrix} = \frac{\cot 3x}{3\cos^2 3x + 3\sin^2 3x}$$

$$\begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix} = \frac{\cot 3x - \sin 3x}{3}$$

Upon integration we have

$$v_1 = \frac{3\ln(\tan 3x) + \cos 3x}{3}$$

$$v_1 = \frac{1}{3} \cos 3x \cdot \ln(\tan 3x)$$

Solving for  $v_2$

$$v_2' = \begin{vmatrix} \cos 3x & 0 \\ -3\sin 3x & \cot 3x \end{vmatrix} = \frac{\cot 3x \cdot \cos 3x}{3}$$

Upon integration,

$$v_2 = \frac{\beta \ln(\tan 3x) \cdot \sin 3x}{\beta}$$

$$X(t) = -\frac{1}{2} \left( \frac{1}{2} e^t - \frac{1}{2} \left( \frac{1}{2} \right) e^{-t} + \left[ \frac{1}{2} t + \frac{1}{2} e^t + \frac{1}{2} t e^t \left( \frac{1}{2} \right) e^{-t} \right] + \left[ \frac{1}{2} e^t - \frac{1}{2} t e^t - \frac{1}{4} e^{2t} \left( \frac{1}{2} \right) e^{-t} \right] \right)$$

b) Method of variation of parameter

$$\frac{d^2y}{dx^2} + qy = \cot 3x$$

Sch.

$$y'' + qy = \cot 3x$$

Complete sch.

$$y'' + qy = 0$$

$$\lambda^2 + q = 0 \quad \lambda^2 = -q \quad \lambda = \pm 3i$$

$$y_c = c_1 \cos 3x + c_2 \sin 3x \Rightarrow y_c = c_1 y_1 + c_2 y_2$$

$$y_p = v_1 y_1 + v_2 y_2$$

$$y_p = v_1 \cdot \cos 3x + v_2 \sin 3x$$

$$v_1^T y_1 + v_2^T y_2 = 0 \Rightarrow v_1^T \cos 3x + v_2^T \sin 3x = 0$$

$$v_1^T y_1' + v_2^T y_2' = g(x) \quad -3v_1^T \sin 3x + v_2^T \cos 3x = \cot 3x$$

using Grammer rule.

$$v_1^T = \begin{vmatrix} 0 & \sin 3x \\ \cot 3x & 3 \cos 3x \end{vmatrix} = \frac{\cot 3x \sin 3x}{3 \cos^2 3x + 3 \sin^2 3x}$$

$$\begin{vmatrix} \cos 3x & \sin 3x \\ -3 \sin 3x & 3 \cos 3x \end{vmatrix} = \frac{\cot 3x - \sin 3x}{3}$$

Upant integration w.r.t.

$$v_1 = \frac{3 \ln(\tan 3x)}{3} \cos 3x$$

$$v_1 = -\cos 3x \cdot \ln(\tan 3x)$$

Solving for  $v_2$

$$v_2^T = \begin{vmatrix} \cos 3x & 0 \\ -3 \sin 3x & \cot 3x \end{vmatrix} = \cot 3x \cdot \cos 3x$$

$$W \neq \text{Silv} \neq g(\omega)$$

$$v_1^T y_1 + v_2^T y_2 = 0 \Rightarrow v_1^T \cos 3x + v_2^T \sin 3x = 0$$

$$v_1^T y_1^T + v_2^T y_2^T = g(\omega) \quad -3v_1^T \sin 3x + 3v_2^T \cos 3x = \cot 3x$$

-wing Grammars rule.

$$v_1^T = \begin{vmatrix} 0 & \sin 3x \\ \cot 3x & 3 \cos 3x \end{vmatrix} = \frac{\cot 3x \sin 3x}{3 \cos^2 3x + 3 \sin^2 3x}$$

$$\begin{vmatrix} \cos 3x & \sin 3x \\ -3 \sin 3x & 2 \cos 3x \end{vmatrix}$$

$$\text{We solve for } g(x)$$

$$v_1^T y_1 + v_2^T y_2 = 0 \Rightarrow v_1^T \cos 3x + v_2^T \sin 3x = 0$$

$$v_1^T y_1^T + v_2^T y_2^T = g(x) \quad -3v_1^T \sin 3x + 3v_2^T \cos 3x = \cot 3x$$

Using Grammer rule.

$$v_1^T = \begin{vmatrix} 0 & \sin 3x \\ \cot 3x & 3 \cos 3x \end{vmatrix} = \frac{\cot 3x \sin 3x}{3 \cos^2 3x + 3 \sin^2 3x}$$

$$\begin{vmatrix} \cos 3x & \sin 3x \\ -3 \sin 3x & 3 \cos 3x \end{vmatrix} = \frac{\cot 3x \sin 3x}{3}$$

Upant integration we have

$$v_1 = \frac{3 \ln(\tan 3x)}{2} \cos 3x$$

$$-\frac{1}{2} + \frac{1}{2} + c_1 + c_2 = 0 \quad \text{--- (1)}$$

$$\frac{1}{2} - \frac{1}{2} + c_1 - c_2 = 0 \quad \text{--- (2)}$$

$$\Rightarrow \frac{1}{4} + c_1 + c_2 = 0 \quad \text{solving}$$

$$\frac{1}{4} + 2c_1 = 0$$

$$\Rightarrow 2c_1 = -\frac{1}{4} \Rightarrow c_1 = -\frac{1}{8}$$

$$\text{case-2} \quad c_1 = c_2 \quad c_2 = -\frac{1}{8}$$

Particular soln.

$$X(t) = -\frac{1}{2}(1)e^t - \frac{1}{2}\left(\frac{1}{-1}\right)\bar{e}^t + \left[\frac{1}{2}t + \frac{1}{2}\bar{e}^t + \frac{1}{2}t\bar{e}^t(1)e^t\right] + \left[\frac{1}{2}e^t - \frac{1}{2}t\bar{e}^t - \frac{1}{4}\bar{e}^{2t}\left(\frac{1}{-1}\right)^t\right]$$

2(b)

b) Method of variation of parameter

$$\frac{d^2y}{dx^2} + qy = \cot 3x$$

Schm

$$y'' + qy = \cot 3x$$

Complete soln.

$$y_1 + y_2 = 0$$

$$\begin{aligned} c_1 + c_2 - c_1 &= 0 \\ c_1 + 2c_1 &= 0 \Rightarrow c_1 = -\frac{1}{2} \\ c_1 &= c_2 \quad c_2 = -\frac{1}{2} \end{aligned}$$

Partialan soln.

$$X(t) = -\frac{1}{2}(1)e^t - \frac{1}{2}\left(\frac{1}{-1}\right)\bar{e}^t + \left[\frac{1}{2}t + \frac{1}{2}\bar{e}^t + \frac{1}{2}te^t(-1)^t\right] + \left[\frac{1}{2}t^2 - \frac{1}{2}t\bar{e}^t - \frac{1}{4}e^{2t}(-1)^t\right]$$

2(b)

b) Method of variation of parameters

$$\frac{d^2y}{dx^2} + 9y = \cot 3x$$

Sch

$$y'' + 9y = \cot 3x$$

Complete soln.

$$y'' + 9y = 0$$

$$\lambda^2 + 9 = 0 \quad \lambda^2 = -9 \quad \lambda = \pm 3i$$

$$y_c = c_1 \cos 3x + c_2 \sin 3x \Rightarrow y_c = c_1 y_1 +$$

$$\cot = \frac{1}{\tan}$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} + c_1 + c_2 = 0 \quad \text{--- (1)}$$

$$\frac{1}{2} - \frac{1}{2} + 4 - c_2 = 0 \quad \text{--- (2)}$$

$$\Rightarrow \frac{1}{4} + c_1 + c_2 = 0 \quad \text{solving simultaneously}$$

$$\frac{1}{4} + 4 - c_2 = 0$$

$$\frac{1}{4} + 2c_1 = 0$$

$$\Rightarrow 2c_1 = -\frac{1}{4} \Rightarrow c_1 = -\frac{1}{8}$$

$$c_1 = c_2 \quad c_2 = -\frac{1}{8}$$

Particular soln.

$$X(t) = -\frac{1}{2}(1)e^t - \frac{1}{2}(-1)\bar{e}^t + \left[ \frac{1}{2}t + \frac{1}{2}\bar{e}^t + \frac{1}{2}t\bar{e}^t (1)e^t \right] + \left[ \frac{1}{2}e^t - \frac{1}{2}t\bar{e}^t - \frac{1}{4}\bar{e}^{2t} (-1)^t \right]$$

2(b)

b) Method of variation of parameters

$$\frac{d^2y}{dx^2} + qy = \cot 3x$$

Sch

$$C_0 t = \frac{1}{t_{\text{ans}}}$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} + C_1 + C_2 = 0 \quad - (1)$$

$$\frac{1}{2} - \frac{1}{2} + 4 - C_2 = 0 \quad - (2)$$

$$\Rightarrow \frac{1}{4} + C_1 + C_2 = 0$$

solving simultaneously

$$\frac{1}{4} + 4 - C_2 = 0$$

$$\frac{1}{4} + 2C_1 - 0 = 0$$

$$\Rightarrow 2C_1 = -\frac{1}{4} \Rightarrow C_1 = -\frac{1}{8}$$

$$\text{so } C_1 = C_2 \quad C_2 = -\frac{1}{8}$$

particular soln.

$$X(t) = -\frac{1}{2}(1)e^t - \frac{1}{2}\left(\frac{1}{-1}\right)\bar{e}^t + \left[\frac{1}{2}t + \frac{1}{2}\bar{e}^t + \frac{1}{2}t\bar{e}^t (1)e^t\right] + \left[\frac{1}{2}t - \frac{1}{2}\bar{e}^t - \frac{1}{4}e^{2t}\right]$$

Since for  $c_1(t)$  and  $c_2(t)$  we have

$$c_1'(t) \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t + c_2'(t) \begin{pmatrix} -1 \\ 1 \end{pmatrix} \bar{e}^t = \begin{pmatrix} -t \\ e^t \end{pmatrix}$$

$$c_1'(t) = \frac{\begin{vmatrix} -t & \bar{e}^t \\ e^t & -\bar{e}^t \end{vmatrix}}{\begin{vmatrix} 1 & \bar{e}^t \\ e^t & -\bar{e}^t \end{vmatrix}} = \frac{t\bar{e}^t - e^t \cdot \bar{e}^t}{-1 - 1} = \frac{t\bar{e}^t - 1}{-2} \\ = \frac{1}{2}t - t\bar{e}^t$$

$$c(t) = \frac{1}{2}t - \left[ -t\bar{e}^t - \bar{e}^t \right] + e^t$$

$$= c(t) = \frac{1}{2}t + \frac{1}{2}\bar{e}^t + \frac{1}{2}t\bar{e}^t$$

$$c_2'(t) = \frac{\begin{vmatrix} e^t & -t \\ e^t & \bar{e}^t \end{vmatrix}}{-2} = \frac{e^{2t} + t\bar{e}^t}{-2}$$

$$c_2(t) = -\frac{1}{4}e^{2t} - \frac{1}{2}\left[t\bar{e}^t - e^t\right]$$

$$= -\frac{1}{4}e^{2t} - \frac{1}{2}t\bar{e}^t + \frac{1}{2}e^t$$

Solve

$$X_0(t) = \left( \frac{1}{2}t + \frac{1}{2}\bar{e}^t + \frac{1}{2}t\bar{e}^t \right) \left( 1 \right) e^t + \left( -\frac{1}{4}e^{2t} - \frac{1}{2}t\bar{e}^t + \frac{1}{2}e^t \right) \left( 1 \right) \bar{e}^t$$

$$\cot = \frac{1}{\tan}$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} + c_1 + c_2 = 0 \quad - (1)$$

$$\frac{1}{2} - \frac{1}{2} + 4 - c_2 = 0 \quad - (2)$$

$$\Rightarrow \frac{1}{4} + c_1 + c_2 = 0 \quad \text{Solving simultaneously}$$

$$\underline{0 + 4 - c_2 = 0}$$

$$\frac{1}{4} + 2c_1 = 0$$

$$\Rightarrow 2c_1 = -\frac{1}{4} \Rightarrow c_1 = -\frac{1}{8}$$

therefore

$$c_1 = c_2$$

$$c_2 = -\frac{1}{8}$$

$$c_1(t) = \frac{\begin{vmatrix} -t & e^t \\ e^t & -e^t \end{vmatrix}}{\begin{vmatrix} e^t & -e^t \\ e^t & -e^t \end{vmatrix}} = \frac{te^t - e^t \cdot e^t}{-1 - 1} = \frac{te^t - e^{2t}}{-2}$$

$$c_1(t) = \frac{1}{2}t - \left[ -t\bar{e}^t - \bar{e}^{2t} \right]$$

$$= c_1(t) = \frac{1}{2}t + \frac{1}{2}\bar{e}^t + \frac{1}{2}t\bar{e}^t$$

$$c_2'(t) = \frac{\begin{vmatrix} e^t & -t \\ e^t & e^t \end{vmatrix}}{-2} = \frac{e^{2t} + t\bar{e}^t}{-2}$$

$$c_2(t) = -\frac{1}{4}e^{2t} - \frac{1}{2}\left[t\bar{e}^t - e^t\right]$$

$$= -\frac{1}{4}e^{2t} - \frac{1}{2}t\bar{e}^t + \frac{1}{2}e^t$$

Soln

$$X_1(t) = \left( \frac{1}{2}t + \frac{1}{2}\bar{e}^t + \frac{1}{2}t\bar{e}^t \right) \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + \left( \frac{1}{2}e^t - \frac{1}{2}t\bar{e}^t - \frac{1}{4}e^{2t} \right) \begin{pmatrix} 1 \\ -1 \end{pmatrix} \bar{e}^t +$$

$$+ c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} \bar{e}^t$$

initial

Condition

$$c(0) = \frac{1}{2}t - \left[ -6e^t - e^{-t} \right]$$

$$= c(0) = \frac{1}{2}t + \frac{1}{2}e^t + \frac{1}{2}t e^{-t}$$

$$c'_1(t) = \begin{vmatrix} e^t & -t \\ e^{-t} & t \end{vmatrix} = e^{2t} + t e^{-t}$$

$$c_1(t) = -\frac{1}{4}e^{2t} - \frac{1}{2}[6e^t - e^{-t}]$$

$$= -\frac{1}{4}e^{2t} - \frac{1}{2}t e^{-t} + \frac{1}{2}e^t$$

Solve

$$X_1(t) = \left( \frac{1}{2}t + \frac{1}{2}e^t + \frac{1}{2}t e^{-t} \right) \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + \left( \frac{1}{2}e^t - \frac{1}{2}t e^{-t} - \frac{1}{4}e^{2t} \right) \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} +$$

$$+ c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$$

Initial Condition

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = X(0) = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

For the particular soln

$$X_p(t) = c_1(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2(t) \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$$

To solve for  $c_1(t)$  and  $c_2(t)$  we have

$$c_1(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2(t) \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} = \begin{pmatrix} -t \\ e^t \end{pmatrix}$$

$$c_1(t) = \frac{\begin{vmatrix} -t & e^{-t} \\ e^t & -e^{-t} \end{vmatrix}}{\begin{vmatrix} e^t & e^{-t} \\ e^t & -e^{-t} \end{vmatrix}} = \frac{te^{-t} - e^t \cdot e^{-t}}{-1 - 1} = t$$

$$X_p(t) = c_1(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2(t) \begin{pmatrix} 1 \\ -1 \end{pmatrix} \bar{e}^{-t}$$

To solve for  $c_1(t)$  and  $c_2(t)$  we have

$$c_1'(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2'(t) \begin{pmatrix} 1 \\ -1 \end{pmatrix} \bar{e}^{-t} = \begin{pmatrix} -t \\ \bar{e}^t \end{pmatrix}$$

$$c_1'(t) = \frac{\begin{vmatrix} -t & \bar{e}^t \\ \bar{e}^t & -\bar{e}^{-t} \end{vmatrix}}{\begin{vmatrix} 1 & \bar{e}^t \\ \bar{e}^t & -\bar{e}^{-t} \end{vmatrix}} = \frac{\bar{t}\bar{e}^t - \bar{e}^t \cdot \bar{e}^{-t}}{-1 - 1} = \frac{\bar{t}\bar{e}^t - 1}{-2} = \frac{1}{2} - t\bar{e}^t$$

$$c_1(t) = \frac{1}{2}t - \left[ -t\bar{e}^t - \bar{e}^t \right] e^t$$

$$= c_1(t) = \frac{1}{2}t + \frac{1}{2}\bar{e}^t + \frac{1}{2}t\bar{e}^t$$

$$c_2'(t) = \frac{\begin{vmatrix} e^t & -t \\ \bar{e}^t & \bar{e}^{-t} \end{vmatrix}}{-2} = \frac{e^{2t} + t\bar{e}^t}{-2}$$

$$c_2(t) = -\frac{1}{4}e^{2t} - \frac{1}{2} \left[ t\bar{e}^t - \bar{e}^t \right]$$

For the particular soln

$$\int e^t \frac{du}{dt} dt = \int e^t du \quad u = -e^{-t} \quad v = -e^{-t} \quad t e^t + \int e^t dt$$

$$X_p(t) = c_1(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2(t) \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$$

To solve for  $c_1(t)$  and  $c_2(t)$  we have

$$c_1(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2(t) \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} = \begin{pmatrix} -t \\ e^t \end{pmatrix}$$

$$c_1(t) = \frac{\begin{vmatrix} -t & e^t \\ e^t & -e^{-t} \end{vmatrix}}{\begin{vmatrix} e^t & e^t \\ e^t & -e^{-t} \end{vmatrix}} = \frac{te^{-t} - e^t \cdot e^t}{-1 - 1} = \frac{te^{-t} - 1}{-2} = \frac{1}{2}t - \frac{te^t}{2}$$

$$c_2(t) = \frac{1}{2}t - \left[ -te^t - e^t \right]$$

$$= c_2(t) = \frac{1}{2}t + \frac{1}{2}e^t + \frac{1}{2}te^t$$

Eigen vector associated with  $\lambda_1 = 1$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = -v_1 + v_2 = 0$$

$$v_2 = v_1$$

$$(choose v_1 = 1)$$

$$v_2 = 1$$

Forsl Eigen vector =  $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Solv to the system

$$h(t) = \begin{pmatrix} 1 \end{pmatrix} e^t$$

Eigenvector associated with  $\lambda_2 = -1$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = v_3 + v_4 = 0$$

$$\Rightarrow v_4 = -v_3 = \text{choose } v_3 = 1 \text{ and } v_4 = -1$$

D<sup>nd</sup> sln to the system

$$h(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$$

$$X_{ht} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$$

$$\lambda^2 - 1$$

Find the eigen values  $(A - \lambda I) = 0$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} -\lambda & 1 \\ 1 & -\lambda \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda^2 = 1$$

$$\lambda = \pm 1$$

$$\lambda_1 = 1 \quad \text{and} \quad \lambda_2 = -1$$

Eigen vector associated with  $\lambda_1 = 1$

$$\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = 0 \Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda^2 = 1$$

$$\lambda = \pm 1$$

$$\lambda_1 = 1 \text{ and } \lambda_2 = -1$$

Eigen vector associated with  $\lambda_1 = 1$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = -v_1 + v_2 = 0$$

$$v_2 = v_1$$

$$(choose v_1 = 1)$$

$$\therefore v_2 = 1$$

$$\text{Eigen vector} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Subt to the system

$$h(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t$$

Eigen vector associated with  $\lambda_2 = -1$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = v_3 + v_4 = 0$$

$$(choose v_3 = 1)$$

$A - \lambda I = 0$

$$\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} 2-\lambda & 1 \\ 1 & 3-\lambda \end{pmatrix}$$

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = 0 \Rightarrow (2-\lambda)(3-\lambda) - 1 = 0 \Rightarrow \lambda^2 - 5\lambda + 5 = 0$$

$$\lambda = \pm 1$$

$$\lambda_1 = 1 \quad \text{and} \quad \lambda_2 = -1$$

Eigen vector associated with  $\lambda_1 = 1$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow -v_1 + v_2 = 0$$

$$v_2 = v_1$$

$$(\text{choose } v_1 = 1)$$

$$v_2 = 1$$

Eigen vector =  $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

clnt b the system

$$\lambda = \pm 1$$

$$\lambda_1 = 1 \quad \text{and} \quad \lambda_2 = -1$$

Eigen vector associated with  $\lambda_1 = 1$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow -v_1 + v_2 = 0$$

$$v_2 = v_1$$

(choose  $v_1 = 1$ )

$$v_2 = 1$$

Eigen vector =  $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

From slnt to the system

$$h(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t$$

Eigen vector associated with  $\lambda_2 = -1$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow v_3 + v_4 = 0$$

$$\Rightarrow v_4 = -v_3 \quad \text{choose } v_3 = 1$$

and  $v_4 = -1$

1st slnt to the system

$$h(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t$$

$$\text{Sln } U(x+t) = 2 e^{-7x - \frac{7t}{2} + \frac{5t}{2}}$$

2(a)

$$\text{Ques) } \frac{di}{dt} = v - t \quad \text{and} \quad \frac{dv}{dt} = e^t + i \quad \text{Condition}$$

$$i(0) = 0 \quad v(0) = 0 \quad \left| \begin{array}{l} x_1(0) = 0 \\ x_2(0) = 0 \end{array} \right.$$

$$\text{Let } x_1 = i \quad \text{and} \quad x_2 = v$$

$$x_1' = i' = \frac{di}{dt} = x_2 - t$$

$$x_2' = v' = \frac{dv}{dt} = x_1 + e^t$$

$$\frac{di}{dt} = x_2 - t$$

$$\frac{dv}{dt} = x_1 + e^t$$

In matrix form.

$$\begin{pmatrix} \frac{di}{dt} \\ \frac{dv}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} -t \\ e^t \end{pmatrix}$$

$$\Rightarrow D = 2 \quad \text{and} \quad \lambda = -7$$

Sln  
 $U(x,t) = 2 e^{-7x - 7\frac{t}{2} + \frac{5k}{2}}$

2(a)

(Q)  $\frac{di}{dt} = v - t \quad \text{and} \quad \frac{dv}{dt} = e^t + i \quad \text{Condition}$

$$i(0) = 0 \quad v(0) = 0$$

$$\begin{cases} x_1(0) = 0 \\ x_2(0) = 0 \end{cases}$$

Let  $x_1 = i$  and  $x_2 = v$

$$x'_1 = i' = \frac{di}{dt} = x_2 - t$$

$$x'_2 = v' = \frac{dv}{dt} = x_1 + e^t$$

$$\frac{di}{dt} = x_2 - t$$

$$\frac{dv}{dt} = x_1 + e^t$$

In matrix form.

1. F

$$U(G_1, t) = D e^{\lambda x + \frac{1}{2} \lambda^2 t + \frac{5t}{2}}$$

2. I

$$U(x_0) = 2 e^{-7x} = D e^{\lambda x}$$

3.

$$\Rightarrow D = 2 \quad \text{and} \quad \lambda = -7$$

Sln

$$U(x+t) = 2 e^{-7x - 7\frac{t}{2} + \frac{5t}{2}}$$

2(a)

RQ)  $\frac{di}{dt} = v - t$  and  $\frac{dv}{dt} = e^t + i$  Condition

$$i(0) = 0 \quad v(0) = 0$$

$$x_1(0) = 0 \\ x_2(0) = 0$$

Let  $x_1 = i$  and  $x_2 = v$

$$x_1' = i' = \frac{di}{dt} = x_2 - t$$

$$x_2' = v' = \frac{dv}{dt} = x_1 + e^t$$

$$U(x,t) = A e^{\lambda x} \cdot e^{\frac{5t}{2} + \frac{7t}{2}}$$

Applying the condition.

$$U(x,t) = D e^{\lambda x + \frac{15t}{2} + \frac{5t}{2}}$$

$$U(x_0) = 2 e^{7x} = D e^{7x}$$

$$\Rightarrow D = 2 \quad \text{and} \quad \lambda = -7$$

Soln

$$U(x,t) = 2 e^{-7x - \frac{7t}{2} + \frac{5t}{2}}$$

2(a)

Q9)  $\frac{di}{dt} = v - t$  and  $\frac{dv}{dt} = e^t + i$  Condition

$$i(0) = 0 \quad v(0) = 0$$

$$x_1(0) = 0 \\ x_2(0) = 0$$

After writing the variables

$$\frac{x'}{x} = \lambda \quad \text{(1) and} \quad s - \frac{2T'}{T} = \lambda \quad \text{(2)}$$

Solving (1) we have

$$\frac{dx}{dx} = \lambda \Rightarrow \frac{dx}{x} = \lambda dx$$

Upon integration we have

$$\ln x = \lambda x + c \quad X_0 = e^{\lambda x + c} = X_0 - A e^{\lambda x}$$

Solving (2) we have

$$s - \frac{2T'}{T} = \lambda \Rightarrow s - \lambda = \frac{2T'}{T} \Rightarrow \frac{T'}{T} = \frac{s - \lambda}{2}$$

$$\Rightarrow \frac{dT}{dT} = \frac{s - \lambda}{2} \Rightarrow \frac{dT}{T} = \left( \frac{s}{2} - \frac{\lambda}{2} \right) dt$$

Upon integration we have

$$u(x,t) = A e^{\lambda x} \cdot e^{\frac{5t}{2} + \frac{xt}{2}}$$

Applying the condition.

$$u(0,t) = D e^{\lambda x + \frac{xt}{2} + \frac{5t}{2}}$$

$$u(x_0) = 2 e^{-7x} = D e^{\lambda x}$$

$$\Rightarrow D = 2 \quad \text{and} \quad \lambda = -7$$

-610

$$-7x - 7t + 5t$$

$$\frac{dx}{dX} = \lambda \Rightarrow \frac{dx}{X} = \lambda dt$$

Upon integration we have

$$\ln X = \lambda t + c \quad X_0 = e^{\lambda t + c} = X_0 = A e^{\lambda t}$$

Solving (2) we have

$$5 - \frac{2T^1}{T} = \lambda \Rightarrow 5 - \lambda = \frac{2T^1}{T} \Rightarrow \frac{T^1}{T} = \frac{5 - \lambda}{2}$$

$$\Rightarrow \frac{dT}{T} = \frac{5 - \lambda}{2} dt \Rightarrow \frac{dT}{T} = \left( \frac{5}{2} - \frac{\lambda}{2} \right) dt$$

Upon integration we have

$$\ln T = \frac{5t}{2} + \frac{\lambda t}{2} + B$$

$$\Rightarrow T(t) = C e^{\frac{5t}{2} + \frac{\lambda t}{2}}$$

$$\text{or } I(t) = X_0 \bar{I}(t)$$

$$\ln X = \lambda x + c \quad X_0 = e^{\lambda x + c} = X_0 = A e^{\lambda x}$$

Given (2) we have

$$5 - \frac{2T'}{T} = \lambda \Rightarrow 5 - \lambda = \frac{2T'}{T} \Rightarrow \frac{T'}{T} = \frac{5 - \lambda}{2}$$

$$\Rightarrow \frac{dT}{dT} = \frac{5 - \lambda}{2} \Rightarrow \frac{dT}{T} = \left( \frac{5 - \lambda}{2} \right) dt$$

Upon integration we have

$$\ln T = \frac{5t}{2} + \frac{\lambda t}{2} + B$$

$$\Rightarrow T(t) = C e^{\frac{5t + \lambda t}{2}}$$

$$\text{So, } \underline{\text{Soln}} \quad U(x,t) = X_0 T(t)$$

$$+ \frac{2 - \frac{\gamma}{2} \lambda}{\lambda} x \quad \text{or other ways}$$

$$\Rightarrow \frac{x'}{x} = s - \frac{2\gamma^1}{T} = \lambda$$

Separating the variables

$$\frac{x'}{x} = \lambda \quad \text{--- (1) and} \quad s - \frac{2\gamma^1}{T} = \lambda \quad \text{--- (2)}$$

Solving (1) we have

$$\frac{dx}{dx} = \lambda \Rightarrow \frac{dx}{x} = \lambda dx$$

Upon integration we have

$$\ln x = \lambda x + c \quad X_{(2)} = e^{\lambda x + c} = X_0 = A e^{\lambda x}$$

Solving (2) we have

$$\frac{s - \lambda}{\lambda} = \frac{2\gamma^1}{T} \Rightarrow \frac{\gamma^1}{T} = \frac{s - \lambda}{2}$$

$$\frac{I^1}{T} = \frac{S}{2} - \frac{1}{2} \frac{x^1}{x}$$

or other words

$$\Rightarrow \frac{x^1}{x} = S - \frac{2I^1}{T} = \lambda$$

Separating the variables

$$\frac{x^1}{x} = \lambda \quad \text{--- (1) and} \quad S - \frac{2I^1}{T} = \lambda \quad \text{--- (2)}$$

Solving (1) we have

$$\frac{dx}{dx} = \lambda \Rightarrow \frac{dx}{x} = \lambda dx$$

Upon integration we have

$$\ln x = \lambda x + c \quad X_{(2)} = e^{\lambda x + c} = X$$

$$\frac{T'}{T} = \frac{S}{2} - \frac{1}{2} \frac{x'}{x}$$

or other words

$$\Rightarrow \frac{x'}{x} = S - \frac{2T'}{T} = \lambda$$

Separating the variables

$$\frac{x'}{x} = \lambda - \textcircled{1} \quad \text{and} \quad S - \frac{2T'}{T} = \lambda -$$

Solving \textcircled{1} we have

$$2 \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 5u$$

$$\text{Condition } u(x, 0) = 2e^{-7x}$$

5d<sub>u</sub>

$$(eL) \quad u(x, t) = X(t) T(x)$$

$$2X\bar{T}' + X' \bar{T} = 5X\bar{T}, \quad \text{divide by } X\bar{T}$$

$$\Rightarrow \frac{2X\bar{T}'}{X\bar{T}} + \frac{X' \bar{T}}{X\bar{T}} = \frac{5X\bar{T}}{X\bar{T}}$$

$$\Rightarrow 2\frac{\bar{T}'}{\bar{T}} + \frac{X'}{X} = 5$$

$$\Rightarrow 2\frac{\bar{T}'}{\bar{T}} = 5 - \frac{X'}{X}$$

(4(b))

4(b)

Separation of variables

$$2 \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 5u$$

Condition  $u(x, 0) = 2e^{-tx}$

Solve

$$(eL) \quad u(x, t) = X(x) T(t)$$

$$2X'T' + X'T = 5XT, \text{ divide by } XT \text{ we have}$$

$$\Rightarrow \frac{2X'T'}{XT} + \frac{X'T}{XT} = \frac{5XT}{XT}$$

$$\Rightarrow 2\frac{T'}{T} + \frac{X'}{X} = 5$$

$$\Rightarrow 2\frac{T'}{T} = 5 - \frac{X'}{X}$$

$$+ 7e^{2x} - 1 - 6e^{-x} + 4y^n$$

$$u(x,y) = \underline{\cos y + 7e^{2x} - 1 - 6e^{-x}}$$

(4(b)) Separation of Variables

$$2 \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 5u \quad \text{Condition } u(x,0) = 2e^{-7x}$$

Solve

$$(e^L \quad u(x,t) = X(x) T(t))$$

$$2XT' + X'T = 5XT, \quad \text{divide by } XT \text{ we have}$$

$$\Rightarrow \frac{2XT'}{XT} + \frac{X'T}{XT} = \frac{5XT}{XT}$$

$$\Rightarrow 2T' + X' = 5$$

$$U(x,y) = \underline{\underline{0}} = -6e^{2x+3y} + \cos y + 6e^{2x-3y} - 6\pi + - \\ + 7e^{2x} - 1 - 6e^{2x} + g(\pi)$$

$$\underline{\underline{U(x,y)}} = \underline{\underline{\cos y + 7e^{2x} - 1 - 6e^{2x}}} \quad \square$$

4(b)

(4b) Separation of Variable

$$2 \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 5u$$

$$\text{Condition } U(x,0) = 2e^{-7x}$$

Solution

$$(eL) \quad U(x,t) = X(x)T(t)$$

$$2X'T' + X'T = 5XT, \quad \text{dividing by } XT$$

$$G(x) = 7e^{2x} - 1 - 6e^{2\pi} + G(\pi) \quad \text{--- (5)}$$

Substituting (3) and (5) into (1) we have

$$U(x,y) = \cancel{-6e^{2x+3y}} + \cos y + \cancel{6e^{2x+3y}} - G(\pi) + - \\ + 7e^{2x} - 1 - 6e^{2\pi} + G(\pi)$$

$$U(x,y) = \underline{\cos y + 7e^{2x} - 1 - 6e^{2\pi}}$$

◻

4(b)

4(b) Separation of Variables

Condition  $U(x,0) =$

$$G(x) = 7e^{2x} - 1 - 6e^{2x\pi} + G(\pi) \quad \longrightarrow \quad (3)$$

Substituting (3) and (5) into (1) we have

$$U(x,y) = \cancel{0} = -6e^{2x-3y} + \cos y + 6e^{2x-3y} - G(\pi) + \\ + 7e^{2x} - 1 - 6e^{2x\pi} + G(\pi)$$

(1)<sup>st</sup> initial condition

$$u(x, 0) = e^{2x} = -6e^{2x} + F(0) + G(x)$$

$$\Rightarrow G(0) = e^{2 \cdot 0} + 6e^{2 \cdot 0} - F(0)$$

$$\Rightarrow G(0) = 7e^{2 \cdot 0} - F(0) \quad \text{--- (2)}$$

2<sup>nd</sup> initial condn

$$u(\pi, y) = \cos y = -6e^{2\pi - 3y} + F(y) + G(\pi)$$

$$\Rightarrow F(y) = \cos y + 6e^{2\pi - 3y} - G(\pi) \quad \text{--- (3)}$$

Let us if at  $F(0)$  we have

$$F(0) = \cos(0) + 6e^{2\pi - 3(0)} - G(\pi)$$

$$F(0) = 1 + 6e^{2\pi} - G(\pi) \quad \text{--- (4)}$$

Take

$G(x)$

Taking (4) and substituting in (2)

$\partial y$

Integrating w.r.t  $y$

$$\text{Q} \quad u(x,y) = -6e^{2x-3y} + F(y) + G(x) \quad \text{--- (1)}$$

1st Initial Condition

$$u(x,0) = e^{2x} = -6e^{2x} + F(0) + G(x)$$

$$\Rightarrow G(x) = e^{2x} + 6e^{2x} - F(0) \quad \text{--- (2)}$$

$$\Rightarrow G(x) = 7e^{2x} - F(0)$$

2nd Initial Condition

$$u(\pi, y) = \cos y = -6e^{2\pi-3y} + F(y) + G(\pi)$$

$$\Rightarrow F(y) = \cos y + 6e^{2\pi-3y} - G(\pi) \quad \text{--- (3)}$$

(as if at  $F(0)$  we have

$$F(0) = \cos(0) + 6e^{2\pi-3(0)} - G(\pi)$$

$$F(0) = 1 + 6e^{2\pi} - G(\pi) \quad \text{--- (4)}$$

Solv

Integrating w.r.t x.

$$\frac{\partial u}{\partial y} = 2e^{2x-3y} + f(y)$$

Integrating w.r.t y

$$u(x,y) = -6e^{2x-3y} + F(y) + G(x) \quad \text{--- (1)}$$

1<sup>st</sup> Initial Condition

$$u(x,0) = e^{2x} = -6e^{2x} + F(0) + G(x)$$

$$\Rightarrow G(x) = e^{2x} + 6e^{2x} - F(0)$$

$$\Rightarrow G(x) = 7e^{2x} - F(0) \quad \text{--- (2)}$$

2<sup>nd</sup> Initial Condition

$$u(\pi, y) = \cos y = -6e^{2\pi-3y} + F(y) + G(\pi)$$

$$\Rightarrow F(y) = \cos y + 6e^{2\pi-3y} - G(\pi)$$

(4a) Using 4(a)

direct integrat

$$\frac{\partial^3 u}{\partial x^3} = e^{2x-3y} \quad \text{Condition: } u(x,0) = e^{2x}, u(\pi,y) = \cos y$$

Soln

Integrating w.r.t x.

$$\frac{\partial u}{\partial y} = 2e^{2x-3y} + f(y)$$

Integrating w.r.t y

$$u(x,y) = -6e^{2x-3y} + F(y) + g(x) \quad \text{--- (1)}$$

Initial Condition

$$u(x,0) = e^{2x} = -6e^{2x} + F(0) + g(x)$$

$$\Rightarrow G(0) = e^{2x} + 6e^{2x} - F(0)$$

$$\Rightarrow G(x) = 7e^{2x} - F(0) \quad \text{--- (2)}$$

4(a)

(4a) Using direct integral

$$\frac{\partial^2 u}{\partial x^2} = e^{2x-3y} \quad \text{Condition: } u(x,0) = e^{2x}, u(\pi, y) = \cos y$$

Soln

Integrating w.r.t x.

$$\frac{\partial u}{\partial y} = 2e^{2x-3y} + f(y)$$

Integrating w.r.t y

$$u(x,y) = -6e^{2x-3y} + F(y) + G(x)$$