(i) Converting ode into system of coupled odes

$$y''''(t) + 4y(t) = \cos 2t, \quad 0 < t < 1$$

Boundary conditions $y(0) = y''(0) = 0$

$y(1) = y''(1) = 0$

<u>Soln</u>

We let the d.e be written as

$$d^{4}y + 4y = \cos 2t \Rightarrow d^{4}y = -4y + \cos 2t$$

$$y''''(t) + 4y(t) = \cos 2t \quad , \quad 0 < t < 1$$

Boundary conditions $y(0) = y''(0) = 0$

$y(1) = y''(1) = 0$

<u>Soln</u>

We let the d.e be written as

$$\frac{d^4y}{dt^2} + 4y = \cos 2t \quad \Rightarrow \quad \frac{d^4y}{dt^2} = -4y + \cos 2t$$

Let $x_1 = y$, $x_2 = y'$, $x_3 = y''$, $x_4 = y'''$

$x_1' = x_2$, $x_2' = x_3$, $x_3' = x_4'$, $x_4' = y'''' = -4x_1 + \cos 2t$

We let the d.e be written as

$$\frac{d^4 y}{dt^2} + 4y = \cos 2t \Rightarrow \frac{d^4 y}{dt^0} = -4y + \cos 2t$$

let $x_1 = y$, $x_2 = y'$, $x_3 = y''$, $x_4 = y'''$

$x_1' = x_2$, $x_2' = x_3$, $x_3' = x_4'$, $x_4' = y'''' = -4x_1 + \cos 2t$

$$\Rightarrow \frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = x_3$$

$$\frac{dx_3}{dt} = x_4$$

$$\frac{dx_4}{dt} = -4x_1 + \cos 2t$$

$\Rightarrow$ let $x_1 = A$, $x_2 = B$, $x_3 = C$, $x_4 = D$

$\Rightarrow$ For Boundary Conditions

$x_1(0) = x_3(0) = 0$

$x_1(1) = x_3(1) = 0$

$$\overline{\quad dt \quad}$$

$$\frac{dx_3}{dt} = x_4$$

$$\frac{dx_4}{dt} = -4x_1 + \cos 2t$$

⇒ For Boundary Conditions

$x_1(0) = x_3(0) = 0$

$x_1(1) = x_3(1) = 0$

$$\Rightarrow \begin{pmatrix} dx_1/dt \\ dx_2/dt \\ dx_3/dt \\ dx_4/dt \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \cos 2t \end{pmatrix}$$

We simplify the above equation by breaking it into 2-sets of coupled d.e based on the boundary conditions at $t=0$ and $t=1$

At time $t=0$, the cosine function becomes

$$\cos 2t\theta = \cos(2 \cdot 0) \quad — (1)$$

$$\Rightarrow t = 1$$

$$\cos 2t1 = \cos(2 \cdot 1) \quad — (2)$$

$\Rightarrow$ So we add an element of $\cos(2t)$ to the coupled d.e as below

$$\frac{dA_0}{dt} = B + \cos 2t\theta \qquad \frac{dA_1}{dt} = B + \cos 2t1$$

$$\frac{dB_0}{dt} = C + \cos 2t\theta \qquad \frac{dB_1}{dt} = C + \cos 2t1$$

$$\frac{dC_0}{dt} = D + \cos 2t\theta \qquad \frac{dC_1}{dt} = D + \cos 2t1$$

At time $t=0$, the cosine function become

$\cos 2t_0 = \cos(2\cdot 0)$ — (1)

$\Rightarrow t = 1$

$\cos 2t_1 = \cos(2\cdot 1)$ — (2)

$\Rightarrow$ So we add an element of $\cos(2t)$ to the coupled d.e as below

$\frac{dA_0}{dt} = B + \cos 2t_0$ $\qquad$ $\frac{dA_1}{dt} = B + \cos 2t_1$

$\frac{dB_0}{dt} = C + \cos 2t_0$ $\qquad$ $\frac{dB_1}{dt} = C + \cos 2t_1$

$\frac{dC_0}{dt} = D + \cos 2t_0$ $\qquad$ $\frac{dC_1}{dt} = D + \cos 2t_1$

$\frac{dD_0}{dt} = -4A + \cos 2t_0$ $\qquad$ $\frac{dD_1}{dt} = -4A + \cos 2t_1$

B.C $\Rightarrow$ $A(0) = C(0) = 0$

$A(1) = C(1) = 0$