1. (5pts) Find the ***critical number(s)*** of the function. If an answer does not exist, write DNE.

$f(x) = x^4 - 4x^3 + 10$

Solution

$$\Rightarrow f(x) = x^4 - 4x^3 + 10$$
$$\Rightarrow f'(x) = 4x^3 - 12x^2$$
$$\Rightarrow f'(x) = 0$$
$$\Rightarrow 4x^3 - 12x^2 = 0$$
$$\Rightarrow 4x^2(x-3) = 0$$
$$\Rightarrow x = 0, x = 3$$

$$\Rightarrow at, x = 0$$
$$\Rightarrow y = f(x) = 0^4 - 4(0)^3 + 10 = 10$$
$$\Rightarrow (0, 10)$$

$$\Rightarrow at, x = 3$$
$$\Rightarrow y = f(x) = 3^4 - 4(3)^3 + 10 = -17$$
$$\Rightarrow (3, -17)$$

2. (6pts) Find the ***critical number(s)*** of the function. If an answer does not exist, write DNE.

$f(x) = \ln(3x - 9)$

Solution

$$\Rightarrow f(x) = ln(3x-9)$$
$$\Rightarrow f'(x) = \frac{3}{3x-9}$$
$$\Rightarrow f'(x) = 0$$
$$\Rightarrow \frac{3}{3x-9} = 0$$
$$\Rightarrow 3 = 0(Nonsense),$$
$$\Rightarrow \text{ DNE}$$

3. (12pts) Use the ***Closed Interval Method*** to find the ***absolute extrema*** of f on the given interval. Show your work (only use a graphing calculator to check your final answer). Label your answer(s) as either an ***absolute maximum*** or ***absolute minimum.*** Express your final answer(s) as a point (x, y). If an answer does not exist, write DNE.

$f(x) = x^3 - 3x + 3, \quad [-3, 2]$

Solution

$$\Rightarrow f(x) = x^3 - 3x + 3 \qquad [-3, 2]$$
$$\Rightarrow f'(x) = 3x^2 - 3$$
$$\Rightarrow 3x^2 - 3 = 0$$
$$\Rightarrow 3x^2 = 3$$
$$\Rightarrow x = \pm 1$$

$$\Rightarrow at, x = -3$$
$$\Rightarrow f(-3) = (-3)^3 - 3(-3) + 3 = -15$$
$$\Rightarrow at, x = -1$$
$$\Rightarrow f(-1) = (-1)^3 - 3(-1) + 3 = 5$$
$$\Rightarrow at, x = 1$$
$$\Rightarrow f(1) = (1)^3 - 3(1) + 3 = 1$$

$$\Rightarrow at, x = 2$$
$$\Rightarrow f(2) = (2)^3 - 3(2) + 3 = 5$$

$\Rightarrow$ f has an absolute maximum 5 at x=-1,2 and has an absolute minimum -15 at x=-3

$\Rightarrow$ absolute minimum (-3,-15)

$\Rightarrow$ absolute maximum (-1,5) and (2,5)

4. (11pts) Verify that the function satisfies the two ***hypotheses*** of the ***Mean Value Theorem*** on the given interval. Then find all numbers c that satisfy the conclusion of the ***Mean Value Theorem***. Write your final answer in exact form (no decimal approximations).

$f(x) = \frac{x^3}{3} - 3x$, on the interval $[-3,\ 3]$

Solution

$\Rightarrow$ from the mean value theorem,we have

$$\Rightarrow \frac{f(b) - f(a)}{b - a} = f'(c)$$
$$\Rightarrow \text{from } ,f(x) = \frac{x^3}{3} - 3x \qquad [-3, 3]$$
$$\Rightarrow f(3) = \frac{(3)^3}{3} - 3(3) = 0$$
$$\Rightarrow f(-3) = \frac{(-3)^3}{3} - 3(-3) = -18$$
$$\Rightarrow \frac{f(3) - f(-3)}{3 - (-3)} = \frac{0 + 18}{3 + 3} = \frac{18}{6} = 3$$
$$\Rightarrow f'(x) = \frac{3x^2}{3} - 3 = x^2 - 3$$
$$\Rightarrow f'(c) = c^2 - 3$$
$$\Rightarrow c^2 - 3 = 3$$
$$\Rightarrow c^2 = 6$$
$$\Rightarrow c = \pm\sqrt{6}$$
$$\Rightarrow c_1 = \sqrt{6}, c_2 = -\sqrt{6}$$

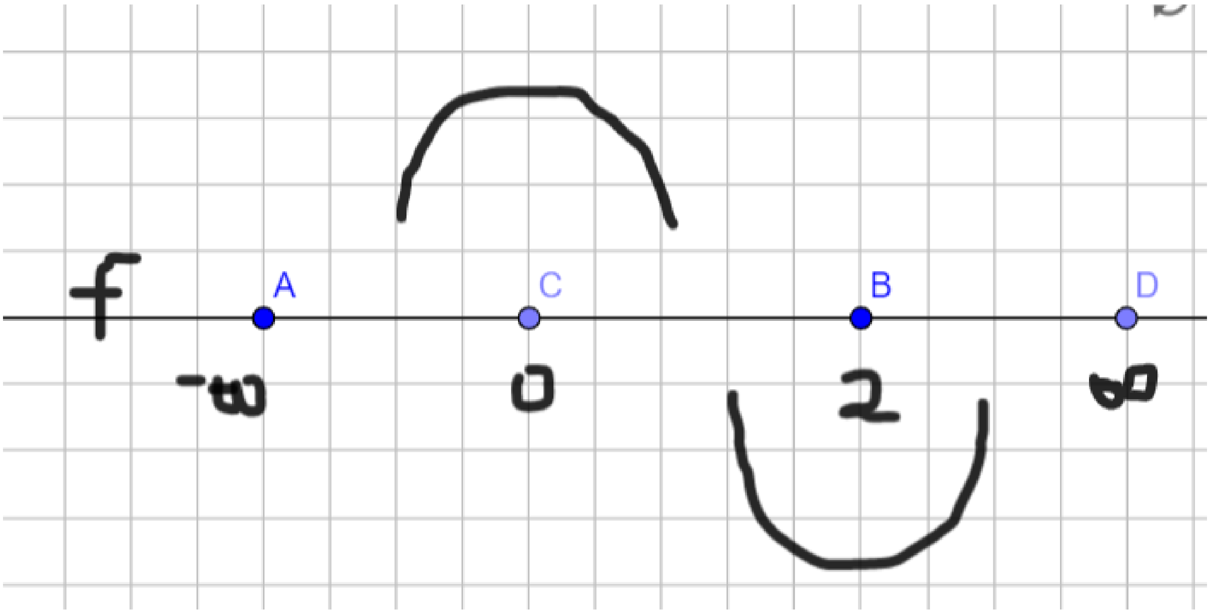
5. (16pts) Answer the following questions based on the function $y = x^3 - 3x^2 + 3$.

a. Use the ***First Derivative Test*** to find any ***local extrema*** of the function. Show your work (only use a graphing calculator to check your final answer). Label your answer(s) as either a ***local maximum*** or ***local minimum.*** Express your final answer(s) as a point (x, y). If an answer does not exist, write DNE.

Solution

$$\Rightarrow y = f(x) = x^3 - 3x^2 + 3$$
$$\Rightarrow f'(x) = 3x^2 - 6x$$
$$\Rightarrow 3x^2 - 6x = 0$$
$$\Rightarrow 3x(x-2) = 0$$
$$\Rightarrow x = 0, x = 2$$

$$\Rightarrow at, x = 0$$
$$\Rightarrow y = 0^3 - 3(0)^2 + 3 = 3$$
$$\Rightarrow (0,3)$$
$$\Rightarrow at, x = 2$$
$$\Rightarrow y = (2)^3 - 3(2)^2 + 3 = -1$$
$$\Rightarrow (2,-1)$$



$\Rightarrow$ since from first derivetive test ,$f(2) = -1 < 0$, we can see from the graph that at x=2 we have a local minimum,thus =(2,-1)

$\Rightarrow f(0) = 3 > 0$, at x=0,we have a local maximum ,thus=(0,3)

b. Find the **interval(s)** on which f is **increasing** and/or **decreasing**.

Solution

$$\Rightarrow f'(x) = x^2 - 6x$$

$$\Rightarrow \text{we take a point on the left side of zero,e.g x=-10}$$

$$\Rightarrow f'(-10) = 3(-10)^2 - 6(-10) = 360$$

$$\Rightarrow \text{on the graph we get an increasing interval}$$

$$\Rightarrow \text{we take a point on between zero and 2 ,e.g x=-1}$$

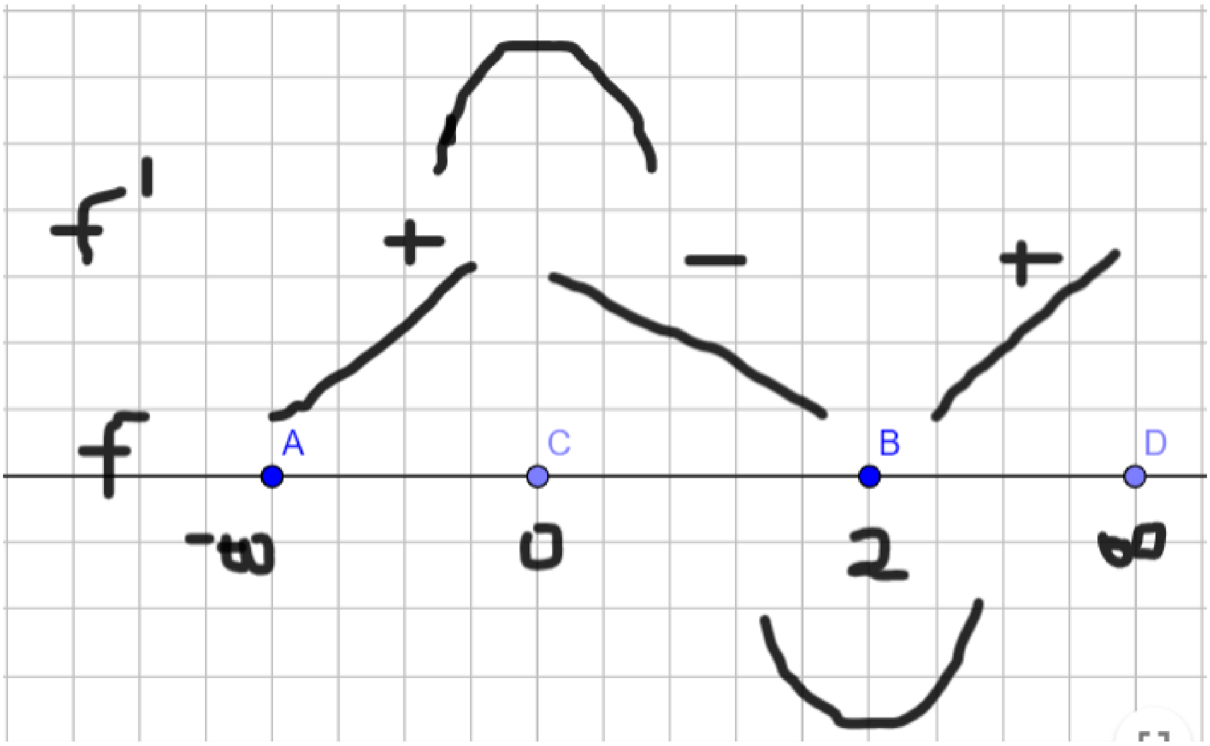
$$\Rightarrow f'(-1) = 3(-1)^2 - 6(-1) = -3$$

$$\Rightarrow \text{on the graph we get a decreasing interval}$$

$$\Rightarrow \text{we take a point on right side of 2 ,e.g x=10}$$

$$\Rightarrow f'(10) = 3(10)^2 - 6(10) = 240$$

$$\Rightarrow \text{on the graph we get an increasing interval}$$



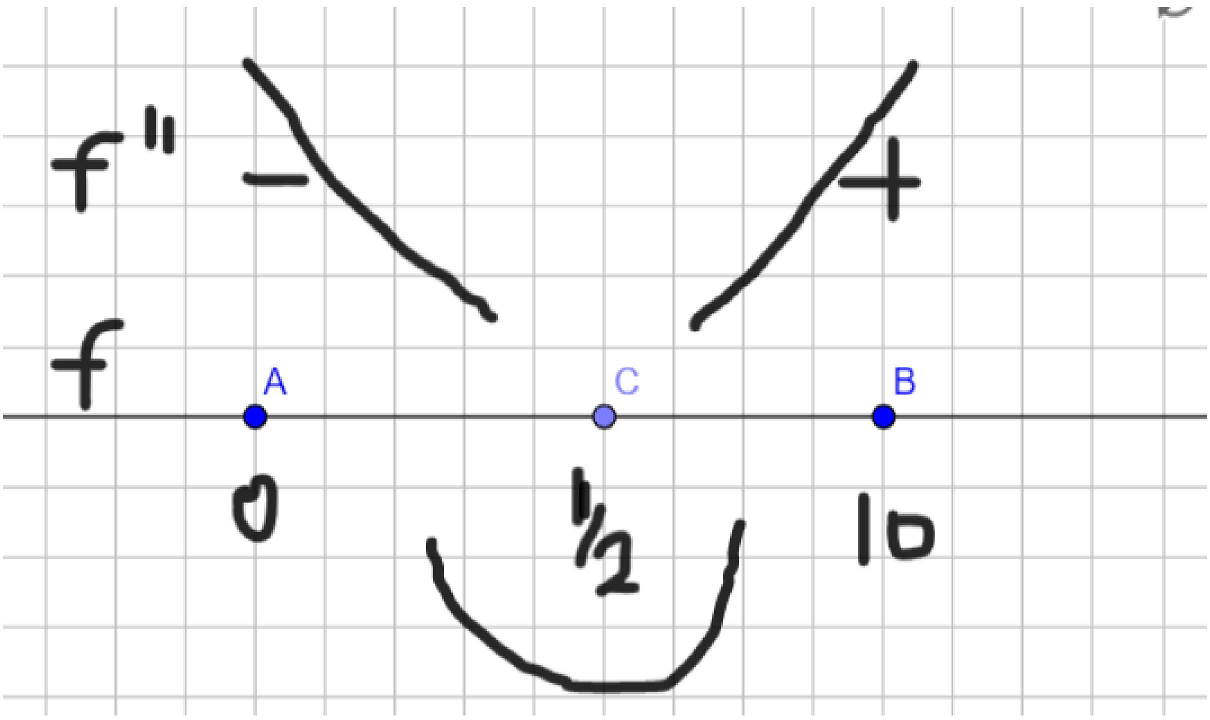
$$\Rightarrow \text{intervals}$$
$$\Rightarrow \text{increasing } = -\infty < x < 0$$
$$\Rightarrow \text{decreasing } = 0 < x < 2$$
$$\Rightarrow \text{increasing } = 2 < x < \infty$$

6. (16pts) Answer the following questions based on the function $f(x) = \frac{2}{3}x^3 - x^2 - 4x + 6.$

a. Find the ***inflection point(s)*** of the function. Show your work (only use a graphing calculator to check your final answer). Express your final answer(s) as a point (x, y). If an answer does not exist, write DNE.

Solution

$$\Rightarrow f'(x) = \frac{3(2x^2)}{3} - 2x - 4$$
$$\Rightarrow f'(x) = 2x^2 - 2x - 4$$
$$\Rightarrow f''(x) = 4x - 2$$
$$\Rightarrow 4x - 2 = 0$$
$$\Rightarrow 4x = 2, x = \frac{1}{2}$$
$$\Rightarrow f''(0) = 4(0) - 2 = -2$$
$$\Rightarrow f''(10) = 4(10) - 2 = 38$$
$$\Rightarrow f''(0.5) = 4(0.5) - 2 = 0$$



$$\Rightarrow \text{inflection point is at , } x = \frac{1}{2}$$

$$\Rightarrow f(\frac{1}{2}) = \frac{2}{3}(\frac{1}{2})^3 - (\frac{1}{2})^2 - 4(\frac{1}{2}) + 6 = \frac{23}{6}$$

$$\Rightarrow (\frac{1}{2}, \frac{23}{6})$$

b. Use the ***Concavity Test*** to find the **interval(s)** on which f is ***concave up*** and/or ***concave down***.

Solution

$$\Rightarrow \text{using the concavity test,we proceed as below}$$

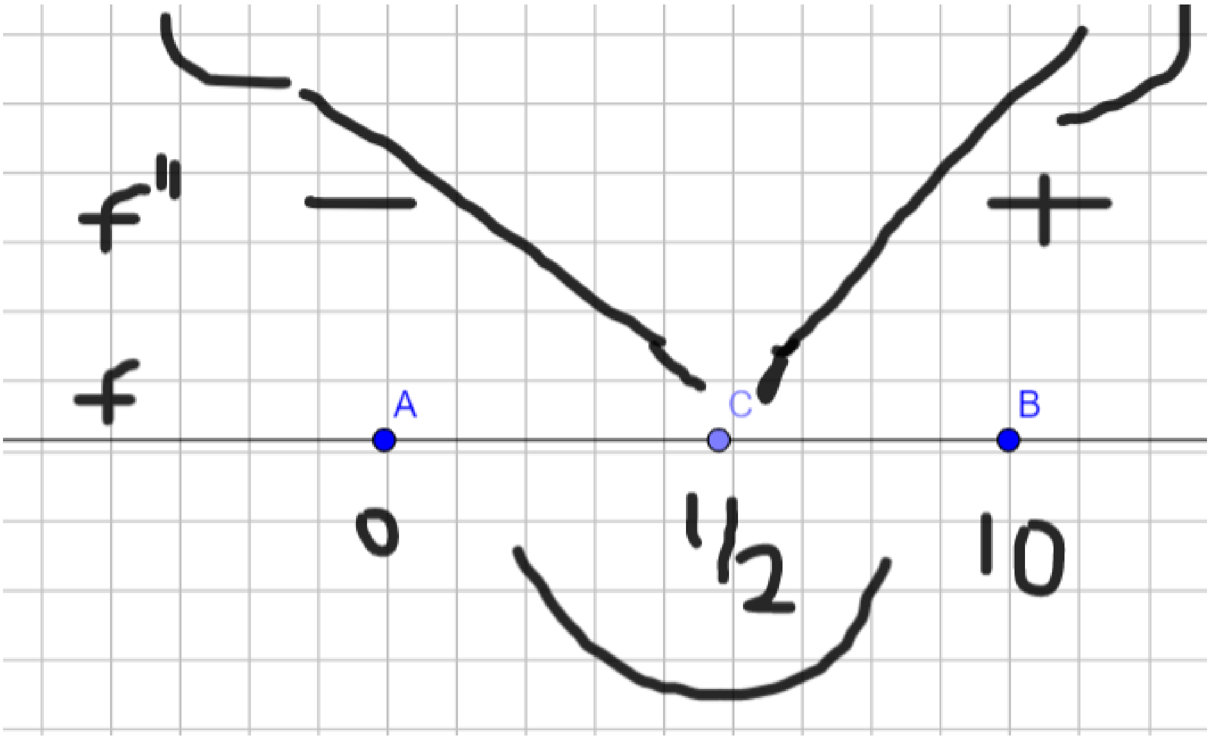
$$\Rightarrow f''(0) = 4(0) - 2 = -2, \text{ gives a negative result}$$

$$\Rightarrow f''(10) = 4(10) - 2 = 38, \text{gives a positive result}$$

$$\Rightarrow f''(0.5) = 4(0.5) - 2 = 0, \text{ gives the inflection point for change in concavity}$$

$$\Rightarrow \text{this implies that we have a concave down on the interval , } -\infty < x < \frac{1}{2} \text{ and concave up on,} \frac{1}{2} < x < \infty$$

$$\Rightarrow \text{now we look at the graph}$$



7. (14pts) Find each limit. Use ***L'Hospital's Rule*** when appropriate. Show that the rule applies before using it. If the rule does not apply, explain why.

a. $\lim_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x}}$

Solution

$$\Rightarrow \lim_{x\to 0^+} \frac{ln(0)}{\frac{1}{0}} = \frac{ln(0)\times 0}{1} = undefined$$

$$\Rightarrow \text{so we use L'Hospitals rule}$$

$$\Rightarrow \text{let, } f(x) = ln(x), g(x) = \frac{1}{x} = x^{-1}$$

$$\Rightarrow f'(x) = \frac{1}{x}, g'(x) = -x^{-2} = -\frac{1}{x^2}$$

$$\Rightarrow \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = -x = 0$$

b. $\lim_{x \to 0} \frac{3-3\cos x}{e^x - x - 1}$

Solution

$$\Rightarrow \lim_{x\to 0} \frac{3-3cos(0)}{e^0-0-1} = \frac{0}{0} = undefined$$

$$\Rightarrow \text{we use L'Hospitals rule}$$
$$\Rightarrow \text{Let, } f(x) = 3-3cos(x), g(x) = e^x - x - 1$$

$$\Rightarrow f'(x) = 0-3(-sin(x)) = 3sin(x), g'(x) = e^x - 1$$
$$\Rightarrow \lim_{x\to 0} \frac{3sin(x)}{e^x-1} = \lim_{x\to 0} \frac{3sin(0)}{e^0-1} = \frac{0}{0} = undefined$$

$$\Rightarrow \text{Let, } f(x) = 3sin(x), g(x) = e^x - 1$$
$$\Rightarrow f'(x) = 3cos(x), g'(x) = e^x$$

$$\Rightarrow \lim_{x\to 0} \frac{3cos(x)}{e^x} = \lim_{x\to 0} \frac{3cos(0)}{e^0} = \frac{3}{1} = 3$$

8. (10pts) Find the ***antiderivative*** of each function.

a. $f(x) = 3x^5 - 2x^4 + 4x^2 - 6$

Solution

$$\Rightarrow f(x) = 3x^5 - 2x^4 + 4x^2 - 6$$
$$\Rightarrow \int f(x)dx = \int (3x^5 - 2x^4 + 4x^2 - 6)dx$$
$$\Rightarrow \frac{3}{6}x^6 - \frac{2}{5}x^5 + \frac{4}{3}x^3 - 6x + C$$
$$\Rightarrow \frac{1}{2}x^6 - \frac{2}{5}x^5 + \frac{4}{3}x^3 - 6x + C$$

b. $f(x) = \csc x \cot x - 3e^{x}$

Solution

$$\Rightarrow \int f(x)dx = ?$$

$$\Rightarrow \int csc(x)cot(x)dx - \int 3e^{x}dx$$

$$\Rightarrow \int csc(x)cot(x)dx =$$

$$\Rightarrow \text{Let,} u = csc(x)$$

$$\Rightarrow du = -cot(x)csc(x)dx$$

$$\Rightarrow dx = -\frac{du}{cot(x)csc(x)}$$

$$\Rightarrow \int \frac{u \times cot(x) \times -du}{cot(x)csc(x)} = \int -du = -u + C$$

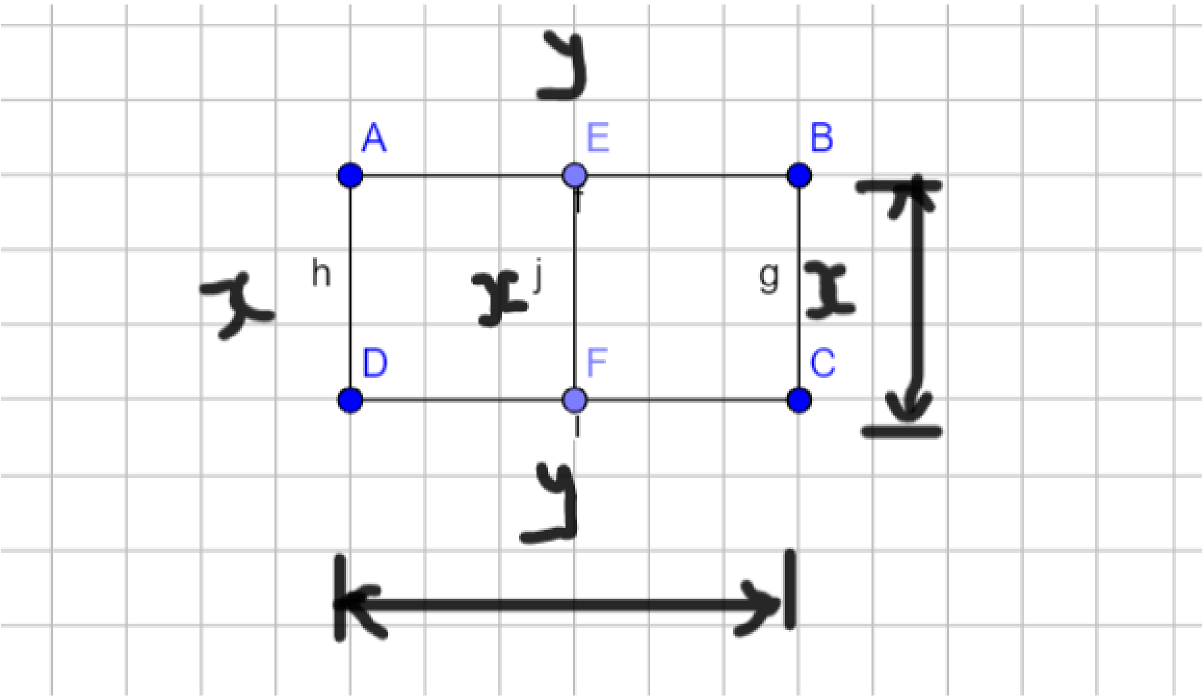
$$\Rightarrow -csc(x) - \int 3e^{x}dx$$

$$\Rightarrow -csc(x) - 3e^{x} + C$$

9. (10pts) You are working on a class project to design a "tiny home." You want to create a 216 sq ft rectangular floor plan that is divided into two rooms. The dividing wall will be parallel to one side of the rectangular sides. Of all the 216 sq ft rectangles, what dimensions require the least amount of wall paneling?

a. Label the provided diagram illustrating the general situation.
Let x denote the length of each of two sides and one wall divider.
Let y denote the length of the other two sides.

Solution



b. Use the labeled diagram from part (a) to write an expression for the total amount of wall paneling P in terms of both x and y. [**Hint:** This is the primary equation].

Solution

$$\Rightarrow p(x, y) = y + x + x + y + x = 3x + 2y$$

c. Use the given information to write an equation that relates the variables. [**Hint:** This is the secondary equation].

Solution

$$\Rightarrow \text{We are Area as , } 216 sq\ ft$$
$$\Rightarrow A = 216 = x \times y$$
$$\Rightarrow y = \frac{216}{x} sq\ ft$$

d. Use the equation from part (c) to write the total amount of wall paneling as a function of one variable, $P(x)$.

Solution

$$\Rightarrow P(x) = 3x + 2y = 3x + 2(\frac{216}{x}) = \frac{3x^2 + 432}{x}$$

e. Finish solving the problem by finding the rectangular dimensions that use the least amount of wall paneling. Show your work and be sure to verify that the value you found is a minimum.

Solution

$$\Rightarrow P(x) = 3x + 2y = 3x + 2(\frac{216}{x})$$

$$\Rightarrow \frac{3x^2 + 432}{x} = 0$$

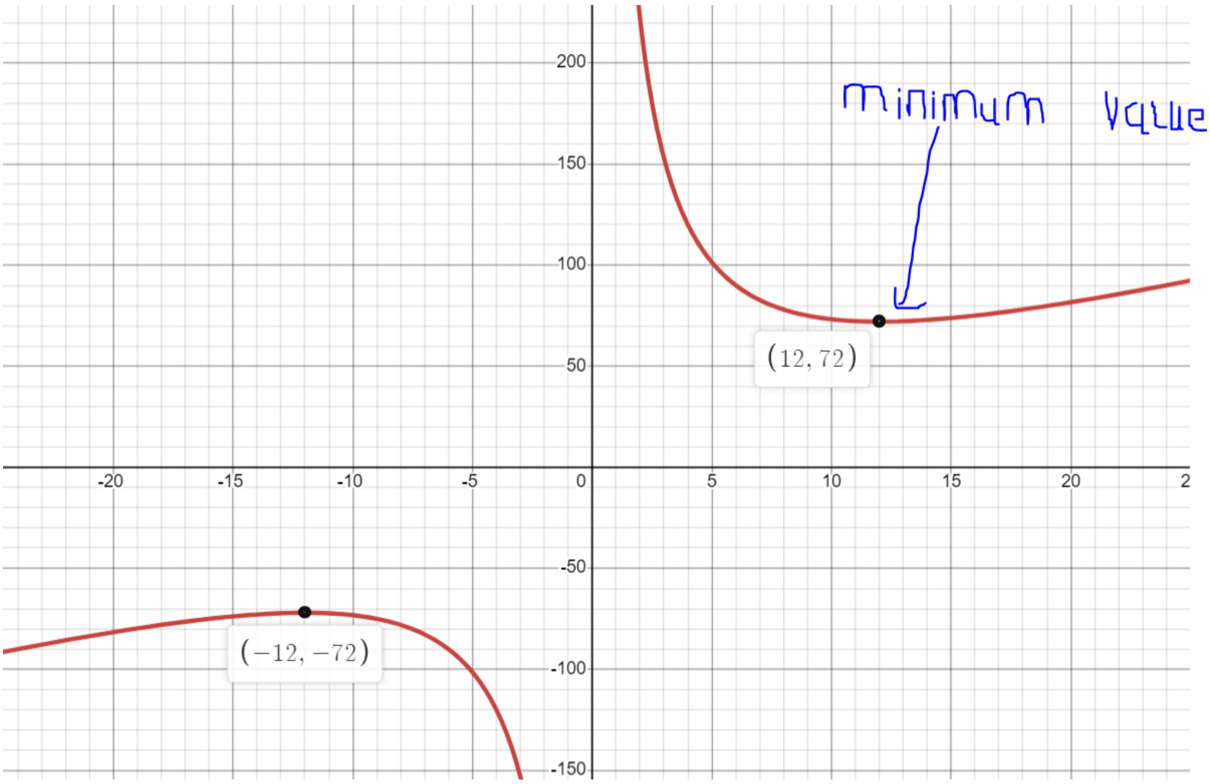
$$\Rightarrow 3x^2 = -432$$

$$\Rightarrow x^2 = \pm 12$$

$$\Rightarrow x = 12, \text{- lowest value}, x = -12, - \text{maximum value}$$

$$\Rightarrow P(12) = \frac{3(12)^2 + 432}{12} = 72$$

$$\Rightarrow y = \frac{216}{12} = 18$$



$$\Rightarrow \text{dimensions are}, x = 12 \text{ and } y = 18$$
$$\Rightarrow \text{dimensions } = 12 \times 18$$

THANK YOU!!!

THE END