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4) a) Separation of Variables

$$\frac{\partial u}{\partial t} + \alpha u = \frac{\partial^2 u}{\partial x^2}$$

Soln

let $u(x,t) = X(x)T(t)$

$u_t = XT'$ and $u_{xx} = X''T$

$\Rightarrow XT' + XT = X''T$ —(1)

dividing (1) by XT we have,

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dividing (1) by XT we have.

$$\frac{T'}{T} + 1 = \frac{X''}{X} = -\lambda^2.$$

$$\Rightarrow \frac{X''}{X} = -\lambda^2 \quad \text{and} \quad \frac{T'}{T} + 1 = -\lambda^3$$

dividing ① by XT we have,

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$$\Rightarrow \frac{X''}{X} = -\lambda^2 \quad \text{and} \quad \frac{T'}{T} + 1 = -\lambda^3$$

$$\Rightarrow X'' + \lambda^2 X = 0$$

characteristic

$$a^2 + \lambda^2 = 0$$

$$\Rightarrow a = \pm i\lambda$$

$$X(x) = C_1 \cos \lambda x + C_2 \sin \lambda x$$

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$$u(0,t) = 0 \Rightarrow X(0) = C_1 = 0$$

$$X(x) = C_2 \sin \lambda x$$

$$u(L,t) = 0 \Rightarrow X(L) = C_2 \sin \lambda L = 0$$

For non trivial soln we have.

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$C_2 \neq 0 \Rightarrow \sin \lambda L = 0 \Rightarrow \lambda L = n\pi$, fo

$\Rightarrow \lambda = n\pi/L$

$X(x) = C_n \sin n\pi/ x$

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For non-trivial soln we have,

$C_2 \neq 0 \Rightarrow \sin \lambda L = 0 \Rightarrow \lambda L = n\pi$, for $n = 1, 2, 3, 4, \ldots$

$\Rightarrow \lambda = n\pi / L$

$X_n(x) = C_n \sin \frac{n\pi}{L} x$

$\phi(x,$

$\phi(x$

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$$\frac{dt}{t} = -\frac{\lambda}{3}dy$$

$$\Rightarrow \frac{T'}{T} + 1 = -\lambda^2 \quad \Rightarrow \frac{dT}{dt\,T} = -\lambda^2 - 1$$

$$\Rightarrow \frac{dT}{T} = -(\lambda^2+1)\,dt \quad \Rightarrow \ln T = -(\lambda^2+1)t + c.$$

$$\Rightarrow T_n(t) = e^{-(\lambda^2+1)t + c} \quad \Rightarrow T_n(t) = A_n e^{-(\lambda^2+1)t}$$

General soln

$$U_n(x,t) = C_n \sin \frac{n\pi}{L}x \cdot A_n e^{-(\lambda^2+1)t}$$

$$U(x,t) = B_n \sin \frac{n\pi}{L}x \cdot e^{-(\lambda^2+1)t}$$

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$\Rightarrow \frac{T'}{T} + 1 = -\lambda^2 \quad \Rightarrow \frac{dT}{dt\,T} = -\lambda^2 - 1$

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(b) Given the initial condition

$U(x,0) = \sin \frac{6\pi x}{L}$ we have

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$$U(x,0) = B_n \sin \frac{n\pi x}{L} \cdot e^{-(\lambda^2+1)\cdot 0} = B_n \sin \frac{n\pi x}{L} = \sin$$

$\Rightarrow$ $B_n = 1$ and $n = 6$

Thus For the particular Solution we have

$(\lambda^2+1)t$

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General soln

$$U_n(x,t) = C_n \sin \frac{n\pi}{L} x \cdot A_n e^{-(\lambda^2+1)t}$$

$$U(x,t) = B_n \sin \frac{n\pi}{L} x \cdot e^{-(\lambda^2+1)t}$$

(b) Given the initial condition

$$U(x,0) = \sin \frac{6\pi x}{L} \quad \text{we have}$$

$$U(x,0) = B_n \sin \frac{n\pi x}{L} \cdot e^{-(\lambda^2+1)\cdot 0} = B_n \sin \frac{n\pi x}{L} = \sin \frac{6\pi x}{L}$$

$$\Rightarrow B_n = 1 \quad \text{and} \quad n = 6$$

Thus for the particular solution we have

$$U(x,t) = 1 \cdot \sin \frac{6\pi x}{L} \cdot e^{-(\lambda^2+1)t}$$

$$-(\lambda^2+1)t$$

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3(a) $\hat{n} = \frac{1}{\sqrt{4+\alpha^2}}\left[2\hat{i} - \alpha\hat{j}\right]$

$\Rightarrow \frac{1}{\sqrt{4+\alpha^2}}\left[2\frac{\partial\phi}{\partial x} - \alpha\frac{\partial\phi}{\partial y}\right] = 0$

$\Rightarrow 2\frac{\partial\phi}{\partial x} - \alpha\frac{\partial\phi}{\partial y} = 0\cdot\sqrt{4+\alpha^2}$

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Using the separation of variables we h

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Using the separation of variables we have.

Let $\phi(x,y) = X(x)\,Y(y)$

$\phi_x = X'Y$ and $\phi_y = XY'$

$X'Y - \alpha XY' = 0$ — (1)

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$$\Rightarrow 2\frac{\partial \phi}{\partial x} - x\frac{\partial \phi}{\partial y} = 0 \cdot \sqrt{4+x^2}$$

$$\Rightarrow 2\frac{\partial \phi}{\partial x} - x\frac{\partial \phi}{\partial y} = 0$$

Using the separation of variables we have

Let $\phi(x,y) = X(x)\, Y(y)$

$\phi_x = X'Y$ and $\phi_y = XY'$

$\Rightarrow 2 \cdot X'Y - \alpha XY' = 0 \quad -(I)$

dividing eqn (I) by XY we have

$2\frac{X'}{X} - \alpha\frac{Y'}{Y} = 0 \quad \Rightarrow \frac{X'}{X} = \frac{\alpha}{2}\frac{Y'}{Y}$

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dividing eqn (I) by XY we have

$2\frac{X'}{X} - \alpha\frac{Y'}{Y} = 0 \quad \Rightarrow \frac{X'}{X} = \frac{\alpha}{2}\frac{Y'}{Y} = -\lambda$ (lambda)

$\frac{X'}{X} = -\lambda \quad -(II)$ and $\frac{\alpha}{2}\frac{Y'}{Y} = -\lambda \quad -(III)$

$\Rightarrow$ Solving (II) we have

$\frac{dX}{X} = -\lambda dx \quad \Rightarrow \ln X = -\lambda x + c$

$\Rightarrow X(x) = e^{-\lambda x + c} \quad \Rightarrow X(x) = Ae^{-\lambda x}$

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$\Rightarrow 2 \cdot X'Y - \alpha XY' = 0 \quad -(i)$

dividing eqn (i) by XY we have

$$2\frac{X'}{X} - \alpha\frac{Y'}{Y} = 0 \Rightarrow \frac{X'}{X} = \frac{\alpha}{2}\frac{Y'}{Y} = -\lambda \text{ (lambda)}$$

$$\frac{X'}{X} = -\lambda \quad -(ii) \text{ and } \frac{\alpha}{2}\frac{Y'}{Y} = -\lambda \quad -(iii)$$

$\Rightarrow$ Solving (ii) we have

$$\frac{dX}{X} = -\lambda dx \Rightarrow \ln X = -\lambda x + c$$

$$\Rightarrow X(x) = e^{-\lambda x + c} \Rightarrow X(x) = A e^{-\lambda x}$$

Solving (iii)

$$\Rightarrow \frac{6}{2}\frac{Y'}{Y} = -\lambda \Rightarrow \frac{dY}{Y} = -\frac{\lambda}{3}dy \Rightarrow$$

$$\Rightarrow \ln Y = -\frac{\lambda}{3}y + c$$

$$\Rightarrow Y = e^{-\frac{\lambda}{3}y + c}$$

$$\Rightarrow Y = B e^{-\frac{\lambda}{3}y}$$

General soln.

$$\phi(x,y) = X \cdot Y = A e^{-\lambda x} \cdot B e^{-\frac{\lambda}{3}y}$$

$$= C \cdot e^{-(\lambda x + \frac{\lambda y}{3})} = C e^{-(\lambda x + \frac{\lambda y}{3})}$$

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Solving (ii)

$$\Rightarrow \frac{6}{9}\frac{Y'}{Y} = -\lambda \Rightarrow \frac{dY}{Y} = -\frac{\lambda}{3}dy \Rightarrow$$

$$\Rightarrow \ln Y = -\lambda/_3\, y + c$$

$$\Rightarrow Y = e^{-\lambda/_3 y + c}$$

$$\Rightarrow Y = Be^{-\lambda/_3 y}$$

General soln.

$$\phi(x,y) = X.Y = Ae^{-\lambda x} . Be^{-\lambda/_3 y}$$

$$= C.e^{-(\lambda x + \frac{\lambda y}{3})} = \underline{\underline{Ce^{-(\lambda x + \frac{\lambda y}{3})}}}$$

Applying the boundary condition

$$\phi(1,y) = e^{-y}, \quad \text{we have } x = 1$$

$\Rightarrow Y = Be$

General soln.

$$\phi(x,y) = X.Y = Ae^{-\lambda x} . Be^{-\lambda/_3 y}$$

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$\cancel{\phi(1,y)} =$

$$\phi(1,y) = Ce^{-(\lambda + \frac{\lambda y}{3})} = e^{-y}$$

$$\Rightarrow C = 1 \quad \text{and} \quad -\lambda/_3 = -1 \Rightarrow \lambda = 3$$

Particular Solution

$$-(3x + 3/_3\, y)$$

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Applying the boundary condition

$\phi(1,y) = e^{-y}$, we have
$x = 1$

$\cancel{\phi(1,y) =}$

$$\phi(1,y) = C e^{-(\lambda + \frac{\lambda y}{3})} = e^{-y}$$

$\Rightarrow \quad C = 1 \quad$ and $\quad -\lambda/3 = -1 \quad \Rightarrow \quad \lambda = 3$

Particular solution

$$\phi(x,y) = 1 \cdot e^{-(3x + \frac{3}{3}y)}$$

$$\Rightarrow \phi(x,y) = 1 \cdot e^{-(3x+y)}$$

$$\Rightarrow \underline{\underline{\phi(x,y) = e^{-(3x+y)}}}$$