Based on problem #8 we have been promted to find the convergence of, $\sum_{n=2}^{\infty} \frac{(-1)^n n^{\frac{1}{2}}}{2n+1}$,

$\Rightarrow$ Question is? What different or other several convergence tests can we run on the problem to test for the convergence.....Answer would be =several:

Let's look at some of the convergence test

(i)Using the Nth term test for divergence

$\Rightarrow$ Definition: if the series, $\sum a_n$ converges then, $\lim\limits_{n\to\infty} = 0$

$\Rightarrow$ In other words the test implies that if , $\lim\limits_{n\to\infty} \neq 0,$ then the series $\sum a_n$diverges

$$\Rightarrow \lim_{n\to\infty} \frac{\pm 1}{2n^{\frac{1}{2}} + \frac{1}{n^{\frac{1}{2}}}} = \lim_{n\to\infty} \frac{\pm 1}{2n^{\frac{1}{2}} + 0} = 0$$

$\Rightarrow$ This implies that the series converges

(ii)Using the alternating series test

Definition: By altenating series test if a_n is decreasing with terms alternating from from positive to negative values and , $\lim\limits_{n\to\infty} a_n = 0,$ then the series coverges

$$\Rightarrow \sum_{n=2}^{\infty} \frac{(-1)^n\sqrt{n}}{2n+1} = \frac{\sqrt{2}}{5} - \frac{\sqrt{3}}{7} + \frac{\sqrt{4}}{9} - \frac{\sqrt{5}}{11} + \ldots$$

The above terms are decreasing and alternating from positve to negative,next we do the limit,

$$\lim_{n\to\infty} \frac{(-1)^n\sqrt{n}}{2n+1} = \lim_{n\to\infty} \frac{\pm 1}{2\sqrt{n} + \frac{1}{\sqrt{n}}} = 0$$

The series converges by the alternating series test

(iii)Absolute and comparison test for convergence

$$\sum_{n=2}^{\infty} \frac{(-1)^n\sqrt{n}}{2n+1} = \sum_{n=2}^{\infty} \left|\frac{(-1)^n\sqrt{n}}{2n+1}\right| = \sum_{n=2}^{\infty} \frac{\sqrt{n}}{2n+1},$$

Now using the new series and comparison test we have:

Now using the new series and comparison test we have:

Defining the comparison test: If we have a series with non-negative terms ,then this seris will converge if we can find another series larger or equal to the current one and it converges and vice-versa

Let the new series which is larger than the current one be given by:

$$\Rightarrow \sum_{n=2}^{\infty} \frac{\sqrt{n}}{2n}$$

Testing the converge of the above series using the Nth term test we get:

$$\lim_{n\to\infty} \frac{\sqrt{n}}{2n} = \lim_{n\to\infty} \frac{1}{2\sqrt{n}} = 0$$

Thus it converges,Hence the proof

Base on #2 integration by parts is the most viable way to solve the given problem

Question is? When facing a sequence to integrate many times ,which is the most efficient way to do the integration?

Answer...=We can use tabular integration to solve such a lengthy sequence as below:

$$\Rightarrow \int x^3 sin(2x)dx,$$

$$\Rightarrow let, u = x^3, dv = sin(2x)dx$$

$$\begin{vmatrix} u & dv \\ x^3 & sin(2x) \\ 3x^2 & -\frac{1}{2}cos(2x) \\ 6x & -\frac{1}{4}sin(2x) \\ 6 & \frac{1}{8}cos(2x) \\ 0 & \frac{1}{16}sin(2x) \end{vmatrix}$$

$$\Rightarrow -\frac{x^3}{2}cos(2x) + \frac{3x^2}{4}sin(2x) + \frac{6x}{8}cos(2x) - \frac{6}{16}sin(2x) + C$$

THE END