1. (8pts) Find the Derivative. Do not simplify it.

$$y=\frac{x^3-4x^2+2x}{\sqrt{x}}$$

Solution

$$=> using\ quotient\ rule:$$

$$=> \frac{dy}{dx}=\frac{x^{\frac{1}{2}}[3x^2-8x+2]-\frac{1}{2}*\frac{1}{x^{\frac{1}{2}}}[x^3-4x^2+2x]}{(x^{\frac{1}{2}})^2}$$

$$=> \frac{dy}{dx}=\frac{\sqrt{x}(3x^2-8x+2)-\frac{1}{2\sqrt{x}}(x^3-4x^2+2x)}{x}$$

2. (8pts) Find the Derivative. Do not simplify it.

$$y=\frac{t^3-4t^2-3t}{t^2-2t}$$

Solution

$$=> using\ quotient\ rule:$$

$$=> \frac{dy}{dx}=\frac{(t^2-2t)(3t^2-8t-3)-(t^3-4t^2-3t)(2t-2)}{(t^2-2t)^2}$$

$$=> \frac{dy}{dx}=\frac{(t^2-2t)[3t^2-8t-3]-(2t-2)[t^3-4t^2-3t]}{(t^2-2t)^2}$$

3. (8pts) Find the ***points*** on the curve where the ***tangent line is horizontal***. Show your work without a calculator.

$$y=4x^3+6x^2-24x+1$$

Solution

$$=> \frac{\mathrm{dy}}{\mathrm{d}x} = 12x^2 + 12x - 24$$
$$=> \frac{\mathrm{dy}}{\mathrm{d}x} = 0$$
$$=> 12x^2 + 12x - 24 = 0$$
$$=> a = 12, b = 12, c = -24$$
$$=> x = \frac{-12 \pm \sqrt{12^2 - 4(12)(-24)}}{2(12)}$$
$$=> x = \frac{-12 \pm 36}{24}$$
$$=> x_1 = 1, x_2 = -2$$
$$=> at,\ x_1 = 1:$$
$$=> y = 4(1)^3 + 6(1)^2 - 24(1) + 1 = -13$$

$$=> (0, -13)$$
$$=> at,\ x_2 = -2:$$
$$=> y = 4(-2)^3 + 6(-2)^2 - 24(-2) + 1 = 41$$
$$=> (-2, 41)$$

4. (9pts) Find the ***points*** on the curve at which the ***slope of the tangent line is equal to 3***. Show your work.

$$y = 3x^3 + 6x^2 + 3x$$

Solution

$$=> \frac{\mathrm{dy}}{\mathrm{d}x} = 9x^2 + 12x + 3 = 3$$
$$=> 9x^2 + 12x + 3 = 3$$
$$=> 9x^2 + 12x = 0$$
$$=> 3x(3x + 4) = 0$$
$$=> 3x = 0, 3x + 4 = 0$$
$$=> x_1 = 0, x_2 = -\frac{4}{3}$$
$$=> at, x_1 = 0$$
$$=> y = 3(0)^3 + 6(0)^2 + 3(0) = 0$$
$$=> (0, 0)$$

$$=> at, x_2 = -\frac{4}{3}$$
$$=> y = 3(-\frac{4}{3})^3 + 6(-\frac{4}{3})^2 + 3(-\frac{4}{3}) = -\frac{196}{9}$$
$$=> (-\frac{4}{3}, -\frac{196}{9})$$

5. (10pts) Find an ***equation of the tangent line*** to the curve at the point $(1, e)$.

$$y = x^3 \cdot e^x$$

Solution

$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = x^3e^x + 3x^2e^x, at\ x = 1$$
$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = (1)^3e^1 + 3(1)^2e^1 = e + 3e = 4e$$
$$=> m = 4\mathrm{e}, x_1 = 1, y_1 = e$$
$$=> y - \mathrm{e} = 4\mathrm{e}(x - 1)$$
$$=> y = 4\mathrm{e}x - 4\mathrm{e} + \mathrm{e}$$
$$=> y = 4\mathrm{e}x - 3\mathrm{e}$$

6. (21 pts) Find the derivative of each function.

a. $y = \ln(2x) + 5e^{3x}$

Solution

$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{2x} + 5 * 3 * e^{3x}$$
$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x} + 15e^{3x}$$

b. $y = x^6 - \pi x^4 + 2x + e^x$

Solution

$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = 6x^5 - 4\Pi x^3 + 2 + e^x$$

c. $y = \sqrt{x} - 4x^2$

Solution

$$=> y = x^{\frac{1}{2}} - 4x^2$$

$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2} * x^{\frac{-1}{2}} - 8x$$

$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2\sqrt{x}} - 8x$$

d. $y = \cos\left(x^4\right)$

Solution

$$=> using\ chain\ rule:$$

$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} * \frac{\mathrm{d}u}{\mathrm{d}x}$$

$$=> let, u = x^4, and\ , y = cos(u)$$

$$=> \frac{\mathrm{d}u}{\mathrm{d}x} = 4x^3, \frac{\mathrm{d}y}{\mathrm{d}u} = -sin(u)$$

$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = -sin(u) * 4x^3 = -4x^3 sin(x^4)$$

e. $y = \ln(\sin x) - \frac{1}{2}\sin^2 x$

Solution

$$=> sin^2(x) = \frac{1}{2}(1 - cos(2x))$$

$$=> y = ln(sin(x)) - \frac{1}{2}(\frac{1}{2} - \frac{cos(2x)}{2})$$

$$=> y = ln(sin(x)) - \frac{1}{4} + \frac{cos(2x)}{4}$$

$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{cos(x)}{sin(x)} + 0 + \frac{1}{4} * 2(-sin(2x))$$

$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{tan(x)} - \frac{1}{2}sin(2x) = cot(x) - \frac{1}{2}sin(2x)$$

f. $y = 6\csc(x) + 4\cot(x)$

Solution

$$=> y = \frac{6}{sin(x)} + \frac{4}{tan(x)}$$

$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{6cos(x)}{sin^2(x)} - \frac{4sec^2(x)}{tan^2(x)}$$

$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-6}{sin(x)tan(x)} - \frac{4sec^2(x)}{tan^2(x)}$$

$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = -6cot(x)csc(x) - 4csc^2(x)$$

7. (8pts) Find $f'(1)$ when $f(x) = \dfrac{x^4}{x^3 - 3}$.

Solution

$$= f'(x) = \frac{(x^3-3)*4x^3-x^4(3x^2)}{(x^3-3)^2}$$

$$=> f'(x) = \frac{4x^6-12x^3-3x^6}{(x^3-3)^2}$$

$$=> f'(x) = \frac{x^6-12x^3}{(x^3-3)^2}$$

$$=> f'(1) = \frac{(1)^6-12(1)^3}{((1)^3-3)^2} = -\frac{11}{4}$$

8. (8pts) Use ***implicit differentiation*** to find $\frac{dy}{dx}$ of the equation.

$$3x^6 + 4y^3 = 12$$

Solution

$$=> 3*6*x^5\frac{\mathrm{d}x}{\mathrm{d}x} + 4*3*y^2\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

$$=> 18x^5 + 12y^2\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

$$=> 12y^2\frac{\mathrm{d}y}{\mathrm{d}x} = -18x^5$$

$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{18x^5}{12y^2} = -\frac{3}{2}x^5y^{-2}$$

9. (10pts) Use ***logarithmic differentiation*** to find the derivative of the function.

$$y = (x)^x$$

Solution

$$=> ln(y) = ln(x)^x$$
$$=> ln(y) = xln(x)$$
$$=> \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = x * \frac{1}{x} + ln(x)$$
$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = y(1 + ln(x)), but\ y = x^x\ :$$
$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = x^x(1 + ln(x))$$

10. (10pts) The volume of a cube is increasing at a rate of 10 cm^3/min.
How fast is the surface area increasing when the length of an edge is 30 cm?

Solution

$$=> V(volume) = l^3, A(surface\ area) = 6l^2$$
$$=> differentiating\ V\ wrt\ t\ explicitly, we\ get :$$
$$=> \frac{\mathrm{d}V}{\mathrm{d}t} = 3l^2\frac{\mathrm{d}l}{\mathrm{d}t}, but\ we\ are\ given,\ \frac{\mathrm{d}V}{\mathrm{d}t} = 10cm^3/min$$
$$=> 10 = 3l^2\frac{\mathrm{d}l}{\mathrm{d}t}$$
$$=> \frac{\mathrm{d}l}{\mathrm{d}t} = \frac{10}{3l^2}$$
$$=> differentiating\ A\ wrt\ t\ explicitly, we\ get :$$
$$=> \frac{\mathrm{d}A}{\mathrm{d}t} = 12l\frac{\mathrm{d}l}{\mathrm{d}t}$$
$$=> \frac{\mathrm{d}A}{\mathrm{d}t} = 12l(\frac{10}{3l^2}) = \frac{40}{l}, at,\ l = 30cm$$
$$=> \frac{\mathrm{d}A}{\mathrm{d}t} = \frac{40}{30} = \frac{4}{3}cm^2/min$$