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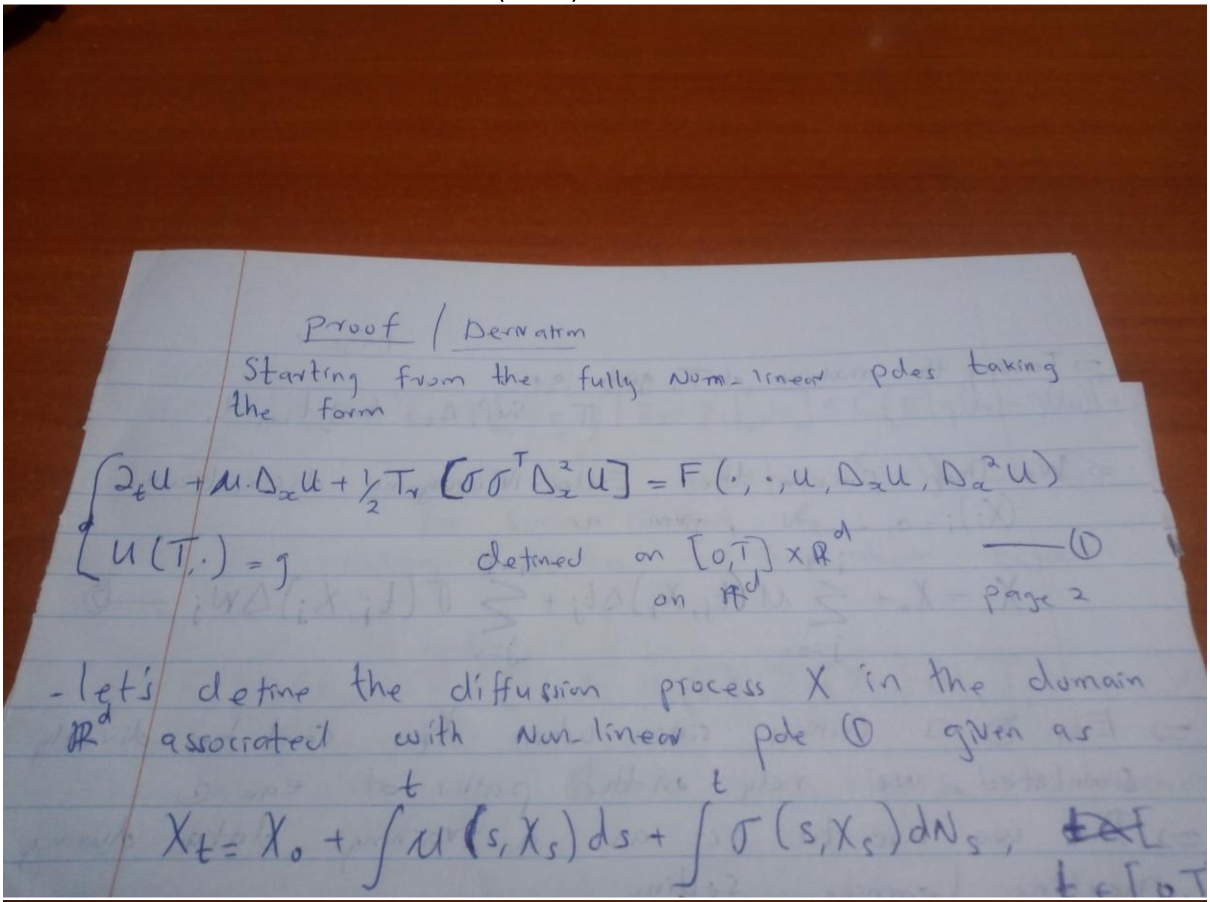
-Revision Materials For a PDE Derivation (Proof)



Proof / Derivation

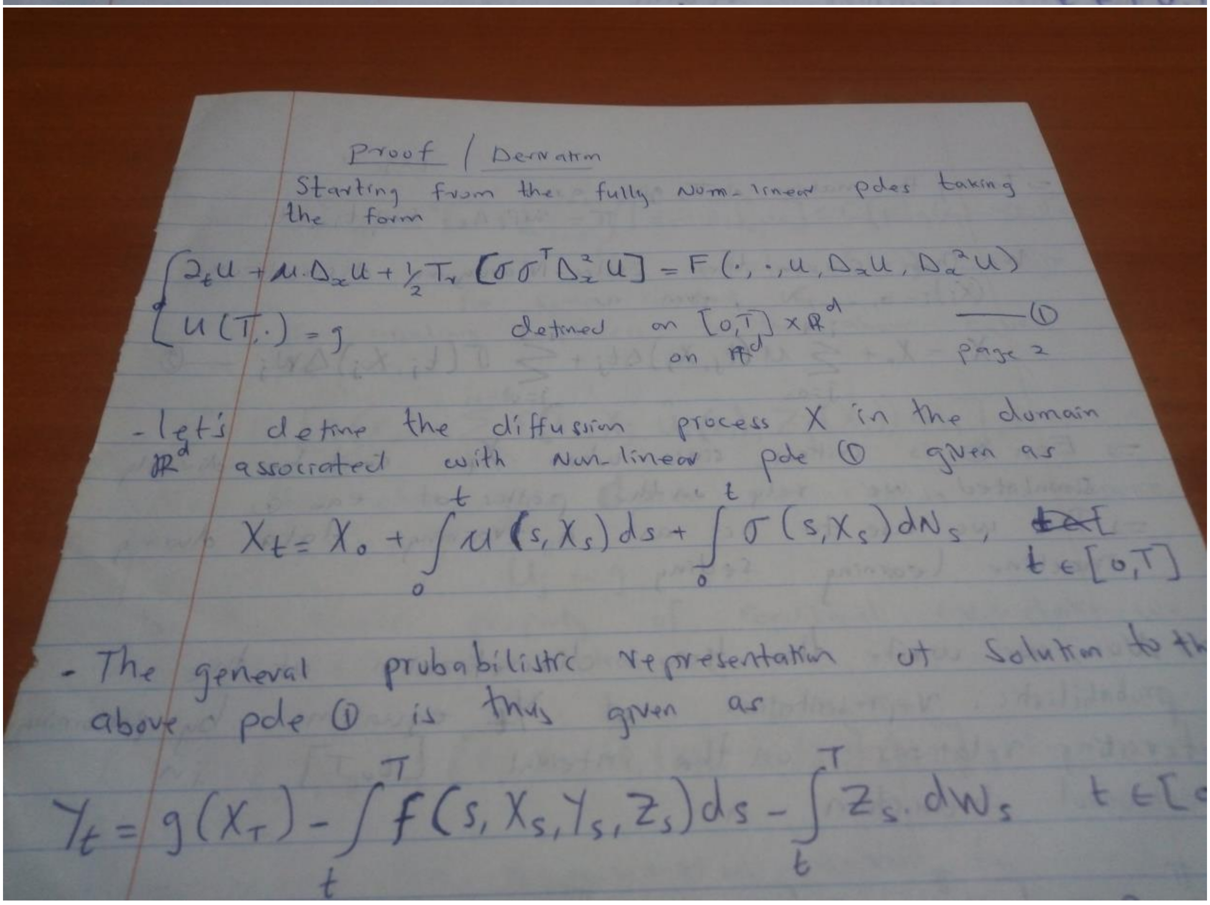
Starting from the fully Non-linear pdes taking the form

$$\begin{cases} \partial_t u + \mu \cdot \Delta_x u + \frac{1}{2} T_r\left[\sigma\sigma^T \Delta_x^2 u\right] = F(\cdot,\cdot,u,\Delta_x u, \Delta_x^2 u) & \text{defined on } [0,T]\times \mathbb{R}^d \\ u(T,.) = g & \text{on } \mathbb{R}^d \end{cases} \quad \text{—— (1)}$$

page 2

- let's define the diffusion process X in the domain $\mathbb{R}^d$ associated with Non-linear pde (1) given as

$$X_t = X_0 + \int^t \mu(s,X_s)\,ds + \int^t \sigma(s,X_s)\,dW_s,$$



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$$X_t = X_0 + \int_0^t \mu(s,X_s)\,ds + \int_0^t \sigma(s,X_s)\,dW_s, \quad t \in [0,T]$$

- The general probabilistic representation of solution to th[e] above pde (1) is thus given as

$$Y_t = g(X_T) - \int_t^T f(s,X_s,Y_s,Z_s)\,ds - \int_t^T Z_s \cdot dW_s \quad t \in [$$

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=> Taking the modulus of π, given as

$$\pi = \sup_i \Delta_t = \sup_i t_{i+1} - t_i$$

=> We then consider the Euler-Maruyama discretization $(X_i)_{i=0,\dots,N}$, given as

$$X_i = X_0 + \sum_{j=0}^{i-1} \mu(t_j, X_j)\Delta t_j + \sum_{j=0}^{i-1} \sigma(t_j, X_j)\Delta W_j \quad — ②$$

=> Eqn ② is critical since when X_t can't be div... simulated, we rely on the paths of eqn ② Th... we use to act as a training data ...

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=> Now, we write the time discretization of the probabilistic representation of Y_t equation by performing iterating relations on the interval $[t_0, T]$ in backward induction as

$$\begin{cases} Y_i^\pi = E_i\left[Y_{i+1}^\pi - f(t_i, X_i, Y_i^\pi, Z_i)\Delta t_i \right. & \text{---} (\\ Z_i = E_i\left[\underline{\Delta W_i}\; Y_{i+1}^\pi \right. , & i = 0, \dots, N-1 \end{cases}$$

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where E_i denotes the $\mathcal{F}_{t_i}$ conditional expectation

=> Assuming we have a terminal relation, the eqn (3) can be written (formulated) iteratively as

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error as

$$\sup_{i\in(0,N)} E\left|Y_{t_i}-Y_i^{\pi}\right|^2 + E\left[\sum_{i=0}^{N-1}\int_{t_i}^{t_{i+1}} |Z_s - Z_i|_2^2\, ds\right] \le C\Big(E|g(X_T)-g(X_N)|^2 + |\pi| + \varepsilon^Z(\pi)\Big) \quad \text{— (4)}$$

for some constant C

=> Thus, the auxilrary process for the above equation is given as

$$Y_i^{\pi} = E_i\Big[g(X_N) + \sum_{j=1}^{N-1} f\big(t_j, X_j, \hat{u}_j(X_j), \hat{Z}_j(X_j)\big)\Delta t_i\Big], \quad i = 0, \ldots$$

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=> Note that when g is C^1, we can choose to initialize the scheme with

$$U_j^{\pi} = g$$

=> By the tower property of conditional expectation, we have the recursive relation as,

$$Y_i^{\pi} = E_i\left[Y_{i+1}^{\pi} + f(t_i, X_i, \hat{u}_i(X_i), \hat{Z}(X_i))\,\Delta t_i\right], \quad i=0,\dots,N$$

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=> Decomposing the approximation error for $i \in [0,N-$

we have

$$E\left|Y_{t_i} - \hat{U}_i(X_i)\right|^2 \le 4\left(E\left|Y_{t_i}-Y_i^{\pi}\right|^2 + E\left|\bar{Y}_i - \hat{Y}_i\right|^2 + E\left|Y_i^{\pi} - Y\right.\right.$$

$$\left.\left. - \hat{U}_i(X_i)\right|^2\right) \quad \text{—— (5)}$$

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⇒ Thus, the auxiliary process is given as

$$Y_i^\pi = E_i\Big[g(X_N) + \sum_{j=i}^{N-1} f(t_j, X_j, \hat{u}_j(x_j), \hat{z}_j(x_j))\Delta t_j\Big], \quad i = 0, \dots, N$$

⇒ Note that when g is C^1, we can choose to initialize the scheme with $u_j^\pi = g$

⇒ By the tower property of conditional expectation, we have the recursive relation as,

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⇒ Decomposing the approximation error for $i \in [0, N-1]$, we have

$$E|Y_{t_i} - \hat{u}_i(x_i)|^2 \le 4\big(E|Y_{t_i} - Y_i^\pi|^2 + E|\bar{Y}_i - \hat{Y}_i|^2 + E|Y_i^\pi - Y_i|^2 + \dots + E|Y_i - \hat{u}_i(x_i)|^2\big) \quad \text{—— (5)}$$

⇒ Using the Martingale representation theorem, there exists a square integrable process $\{Z_s^\pi, t_i \le s \le T\}$ such that

$$g(X_N) + f(t_i, X_i, Y_i, \hat{Z}_i) + \sum_{j=i+1}^{N-1} f(t_j, X_j, \hat{u}_j(X_j), \hat{Z}(X_j))\Delta$$

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⇒ Using the Martingale representation theorem, there exists a square integrable process $\{Z_s^\pi, t_i \le s \le T\}$ such that

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$\Rightarrow$ By Itô's isometry, we have that

$$\hat{Z}_i = E_i \left[\frac{\int_{t_i}^{t_{i+1}} \hat{Z}_s \, ds}{\Delta t_i} \right.$$

$\Rightarrow$ Substituting eqn ⑥ into ~~MAB~~ MDBDP equation

$$J_i^{MB}(U_i, Z_i) = E \Big| g(X_N) - \sum_{j=i+1}^{N-1} f(t, X_j, \hat{U}_j(X_j), \hat{Z}(X_j)) \Delta t_j - \cdots$$

$$- \sum_{j=i+1}^{N-1} \hat{Z}_j(X_j) \cdot \Delta W_j - U_i(X_i) -$$

$$- f(t_i, X_i, U_i(X_i), Z_i(X_i)) \Delta t_i - Z_i(X_i$$

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$$- \sum_{j=i+1}^{N-1} \hat{Z}_j(X_j) \cdot \Delta W_j - U_i(X_i) - \cdots$$

$$- f(t_i, X_i, U_i(X_i), Z_i(X_i)) \Delta t_i - Z_i(X_i) \cdot \Delta W_i \Big|^2$$

We see that the loss function of the MDBDP sche

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⇒ We see that the loss function of the MDBDP scheme can be written as

$$J_i^{MB}(U_i, Z_i) = E\Big| Y_i - U_i(X_i) + \Delta t_i\left[f(t_i, X_i, U_i(X_i), Z_i(X_i)) - f(t_i, X_i, Y_i, \hat{Z}_i)\right] + \sum_{j=i+1}^{N-1}\int_{t_j}^{t_{j+1}}\left[\hat{Z}_s - \hat{Z}_j(X_j)\right] dW_s + \int^{t_{i+1}}\left[\hat{Z}_s - Z_i(X_i)\right] dW_s\Big|^2$$

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$$\hat{Z}_i = \frac{E_i\left[\int_{t_i}^{t_{i+1}} \hat{Z}_s \, ds\right.}{\Delta t_i}$$

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⇒ Minimising over U_i, Z_i, we get the approximation function

network of the regressed

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⇒ Substituting eqn (6) into ...

$$J_i^{MB}(u_i, z_i) = E\Big|g(x_N) - \sum_{j=i+1}^{N-1} f(t, x_j, \hat{u}_j(x_j), \hat{z}(x_j))\Delta t_j - \dots - \sum_{j=i+1}^{N-1} \hat{z}_j(x_j)\cdot \Delta W_j - u_i(x_i) - \dots - f(t_i, x_i, u_i(x_i), z_i(x_i))\Delta t_i - z_i(x_i)\cdot \Delta W_i\Big|^2$$

⇒ We see that the loss function of the MDBDP scheme can be written as

$$J_i^{MB}(u_i, z_i) = E\Big|Y_i - u_i(x_i) + \Delta t_i \big[f(t_i, x_i, u_i(x_i), z_i(x_i)) - \dots - f(t_i, x_i, Y_i, \hat{z}_i) + \sum_{j=i+1}^{N-1} \int_{t_j}^{t_{j+1}} [\hat{z}_s - \hat{z}_j(x_j)]\, dW_s + \dots + \int_{t_i}^{t_{i+1}} [\hat{z}_s - z_i(x_i)]\, dW_s\Big|^2$$

⇒ Minimising over u_i, z_i, we get the approximation error by neural network of the regressed function $Y_i, \hat{Z}_i$

$$E|Y_i - \hat{u}_i(x_i)|^2 + \Delta t_i\, E|\hat{Z}_i - \hat{z}(x_i)|_2^2 \le C(\varepsilon_i^y + \Delta t_i\, \varepsilon_i^z)$$

for some constant c

⇒ Now considering that in MDBDP scheme the function

depended both on the gradient $D_x u$ and the Hessian

$$- \sum_{j=i+1}^{N-1} \hat{z}_j(x_j)\cdot \Delta W_j - u_i(x_i) - \dots - f(t_i, x_i, u_i(x_i), z_i(x_i))\Delta t_i - z_i(x_i)\cdot \Delta W_i\Big|^2$$

⇒ We see that the loss function of the MDBDP scheme can be written as

$$J_i^{MB}(u_i, z_i) = E\Big|Y_i - u_i(x_i) + \Delta t_i \big[f(t_i, x_i, u_i(x_i), z_i(x_i)) - \dots - f(t_i, x_i, Y_i, \hat{z}_i) + \sum_{j=i+1}^{N-1} \int_{t_j}^{t_{j+1}} [\hat{z}_s - \hat{z}_j(x_j)]\, dW_s + \dots + \int_{t_i}^{t_{i+1}} [\hat{z}_s - z_i(x_i)]\, dW_s\Big|^2$$

⇒ Minimising over u_i, z_i, we get the approximation error by neural network of the regressed function $Y_i, \hat{Z}_i$

$$E|Y_i - \hat{u}_i(x_i)|^2 + \Delta t_i\, E|\hat{Z}_i - \hat{z}(x_i)|_2^2 \le C(\varepsilon_i^y + \Delta t_i\, \varepsilon_i^z)$$

for some constant c

⇒ Now considering that in MDBDP scheme the function F is depended both on the gradient $D_x u$ and the Hessian $D_x^2 u$

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$$- \sum_{j=i+1}^{N-1} \hat{Z}_j(x_j) \cdot \Delta W_j - u_i(x_i)$$

$$- f(t_i, x_i, u_i(x_i), z_i(x_i)) \Delta t_i - z_i(x_i) \cdot \Delta W_i \Big|^2$$

$\Rightarrow$ We see that the loss function of the MDBDP scheme can be written as

$$J_i^{MB}(u_i, z_i) = E \Big| Y_i - u_i(x_i) + \Delta t_i \big[f(t_i, x_i, u_i(x_i), z_i(x_i)) - \ldots$$

$$- f(t_i, x_i, Y_i, \hat{z}_i) + \sum_{j=i+1}^{N-1} \int_{t_j}^{t_{j+1}} [\hat{z}_s - \hat{z}_j(x_j)] \, dW_s + \ldots$$

$$+ \int_{t_i}^{t_{i+1}} [\hat{z}_s - z_i(x_i)] \, dW_s \Big|^2$$

$\Rightarrow$ Minimising over u_i, z_i, we get the approximation error by neural network of the regressed function $Y_i, \hat{Z}_i$

$$E \big| Y_i - \hat{u}_i(x_i) \big|^2 + \Delta t_i \, E \big| \hat{z}_i - \hat{z}(x_i) \big|_2^2 \leq C \left(\varepsilon_i^y + \Delta t_i \, \varepsilon_i^z \right)$$

for some constant c

$\Rightarrow$ Now considering that in MDBDP scheme the function F is depended both on the gradient $D_x u$ and the Hessian $D_x^2 u$

$\Rightarrow$ and with the assumption that the general solution u to the pde is smooth, then we shall denote our minimized approximation error solution over Y, Z, Γ valued in $\mathbb{R} \times \mathbb{R}^d \times S^d$ by

$$Y_t = u(t, X_t) \qquad Z_t = D_x u(t, X_t) \qquad \Gamma_t = D_x^2 u(t, X_t)$$

over the interval $t \in [0, T]$

$\Rightarrow$

Using the above hypothesis procedure and applying the Itô's formula to Y_t function, then the error approximation of (Y, Z, Γ) satisfying the back

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⇒ Using the above hypothesis procedure and applying the Itô's formula to Y_t function, then the error approximation of (Y, Z, Γ) satisfying the backward equation ⇓

$$Y_t = g(X_T) - \int_t^T F(s, X_s, Y_s, Z_s, \Gamma_s)\, ds - \int_t^T Z_s^T \sigma(s, X_s)\, dW_s$$

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$\boxtimes$

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$$Y_t = u(t, X_t) \qquad Z_t = \Delta_x u(t, X_t) \qquad \Gamma_t = \Delta_x^2 u(t, X_t)$$

over the interval $t \in [0,T]$

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