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1 Matrices A and B are defined as

$$A = \begin{pmatrix} -1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & 1 \\ 2 & 1 \\ 1 & 1 \end{pmatrix}$$

a) Briefly explain dimensional compatibility in multiplication context

<u>Soln</u>

(i) Dimensional - compatibility means that for 2 matrices to be compatible, then the number of columns in the matrix A must be the same as the number of rows in matrix B

=) In addition, for dimensional compatibility to be viable, then the dimensions of both matrices A and B must also be compatible

(ii) Explain why $A \times B$ cannot be computed whilst $A \times B^T$ can

<u>Soln</u>

In our case above $A \times B$ cannot be computed because in both matrices dimensions are not compatible

=) This means that A has a dimension of 3×2, same as B =) further implying that A has 2 number of column and B has 3 number of rows

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which makes the multiplication of the 2 matrices incompatible

=> However, in $A \times B^T$ => this multiplication is possible because we have both matrices dimensions compatible with A having 2 columns and B^T with 2 rows.

Hence compatible.

Calculate matrix C such that
$$C = A \times B^T$$

Soln

We have that $A = \begin{pmatrix} -1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix}$ and

$$B = \begin{pmatrix} 2 & 1 \\ 2 & 1 \\ 1 & 1 \end{pmatrix}$$

=> let B^T be given by $B^T = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

=> then $A \times B^T = \begin{pmatrix} -1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 2 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} (-1\times2)+(1\times1) & (-1\times2)+(1\times1) & (-1\times1)+(1\times1) \\ (1\times2)+(2\times1) & (1\times2)+(2\times1) & (1\times1)+(2\times1) \\ (1\times2)+(3\times1) & (1\times2)+(3\times1) & (1\times1)+(3\times1) \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -1 & 0 \\ 4 & 4 & 3 \\ 5 & 5 & 4 \end{pmatrix}$$

(C) By expanding the 1^{st} column compute determinant of C and show that $\det(C) = 0$

Soln

We have C given by =)

$$\Rightarrow \quad C = \begin{pmatrix} -1 & -1 & 0 \\ 4 & 4 & 3 \\ 5 & 5 & 4 \end{pmatrix}$$

=) Expanding the 1^{st} column

$$\Rightarrow -1\begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix} +1\begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix} +0\begin{pmatrix} 4 & 4 \\ 5 & 5 \end{pmatrix}$$

$$\Rightarrow -1(16-15) +1(16-15) +0(20-20)$$

$$= -1(1) +1(1) +0$$

$$\Rightarrow -1+1+0 = 0$$

Thus determinant of c =) $\underline{\det(C)} = 0$

Demonstrate linearly dependence and explain
$\det(C)=0 \iff \text{rank } C < 3$

Soln

=> From matrix C we have

$$C = \begin{pmatrix} -1 & -1 & 0 \\ 4 & 4 & 3 \\ 5 & 5 & 4 \end{pmatrix}$$

=>

$$c_1 \begin{pmatrix} -1 \\ 4 \\ 5 \end{pmatrix} + c_2 \begin{pmatrix} -1 \\ 4 \\ 5 \end{pmatrix} + c_3 \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix} = 0$$

=> $-c_1 - c_2 = 0$

=> $4c_1 + 4c_2 + 3c_3 = 0$

$5c_1 + 5c_2 + 4c_3 = 0$

=> $-c_2 = 0$ => $c_2 = 0$

$c_1 = 0$

$c_3 = 0$

=> So the span of the columns of matrix C will be the set of all linear combination of the corresponding vectors

=> Meaning that a set of these vectors will be linearly independent if the only solution for

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$ is $c_i = 0 \ \forall i$

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=> Thus from our case above our C_i Solution is zero => implying of a linearly independent Set of Column vectors

(iii) Statement explanation.

$\det(C) = 0 \iff rank(C) < 3$

Soln

=> This simply implies that the rank of Matrix C i.e rank(C) is viewed as 3 where 3 is the size of the largest non-zero 3 x 3 submatrix with non-zero determinant

=> In other words, we have ~~now~~ $\det(C) = 0$ if and only if, the columns are linearly independent with rank (C) < 3

(e) Based on (C) and (d), will C have inverse. Explain.

Soln

No. C will not have an inverse

=> This is simply because the determinant of matrix C is zero, since we need to divide all entries of the invertible matrix with the determinant

=> Implying that we end up with an UNDEFINED VALUE.

$\boxed{X}$

THANK YOU!!!