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-Revision Materials For PDEs & ODEs

Question 4

$$36y^2\phi_{xx} - 12\phi_{xy} + \phi_{yy} - 6(1+xy)\phi_x + x\phi_y = 0 \quad \text{——} (1)$$

Soln

a) let $A = 36y^2$, $B = -12y$, $C = 1$

$\Delta = B^2 - 4AC = 144y^2 - 144y^2 = 0 \Rightarrow$ Parabolic

b) Canonical form

The d.e for characteristic

$$\frac{dy}{dx} = \frac{-12y \pm \sqrt{0}}{72y^2} = -\frac{1}{6y}$$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{6y} \Rightarrow 6y\,dy + dx = 0$

$\int 6y\,dy + \int dx = \int 0 \Rightarrow 3y^2 + x = c_1$

choose $\xi = 3y^2 + x$.

choose η "wisely"

$\begin{vmatrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{vmatrix} \neq 0 \Rightarrow \begin{vmatrix} 1 & 6y \\ 0 & 1 \end{vmatrix} = 1 \neq 0$

$\eta = y$ ✓

$\Rightarrow \xi = 3y^2 + x$

$\eta = y$

We form $\phi_x, \phi_y, \phi_{xx}, \phi_{xy}, \phi_{yy}$

$\Rightarrow \phi_x = \frac{\partial\phi}{\partial\xi}\frac{\partial\xi}{\partial x} + \frac{\partial\phi}{\partial\eta}\frac{\partial\eta}{\partial x}$

$\phi_x = \phi_\xi$

$\phi_{xx} = \frac{\partial\phi_\xi}{\partial\xi}\frac{\partial\xi}{\partial x} = \phi_{\xi\xi}$

$\phi_{xy} = \frac{\partial\phi_\xi}{\partial\xi}\frac{\partial\xi}{\partial y} + \frac{\partial\phi_\xi}{\partial\eta}\frac{\partial\eta}{\partial y}$

$= 6y\phi_{\xi\xi} + \phi_{\xi\eta}$

$\phi_y = \frac{\partial\phi}{\partial\xi}\frac{\partial\xi}{\partial y} + \frac{\partial\phi}{\partial\eta}\frac{\partial\eta}{\partial y}$

$= 6y\phi_\xi + \phi_\eta$

$\phi_{yy} = \frac{\partial[6y\phi_\xi + \phi_\eta]}{\partial\xi}\frac{\partial\xi}{\partial y} + \frac{\partial[6y\phi_\xi + \phi_\eta]}{\partial\eta}\frac{\partial\eta}{\partial y}$

$\phi_{yy} = 36y^2\phi_{\xi\xi} + 12\phi_{\xi\eta} + \phi_{\eta\eta}$

Substitute all into eqn (1)

$36y^2\phi_{\xi\xi} - 12y[6y\phi_{\xi\xi} + \phi_{\xi\eta}] + \cdots$

$+ 36y^2\phi_{\xi\xi} + 12\phi_{\xi\eta} + \phi_{\eta\eta} - 6\phi_\xi - \cdots$

$- 6xy\phi_\xi + 6xy\phi_\xi + x\phi_\eta = 0$

$\Rightarrow \phi_{\eta\eta} - 6\phi_\xi + x\phi_\eta = 0$

Since ϕ is a function of both ξ and η we have

$\phi_{\eta\eta} + (x-6)\phi(\xi,\eta) = 0$

$\Rightarrow \boxed{\phi_{\eta\eta} + x\phi(\xi,\eta) = 0}$

c) Solving the canonical form

$$\frac{\partial^2\phi}{\partial\eta^2} + x\phi(\xi,\eta) = 0$$

This is a 2nd order ode

$\Rightarrow$ The characteristic is given by

$\lambda^2 + x = 0 \Rightarrow \lambda^2 = -x$

$\Rightarrow \lambda = \pm i\sqrt{x}$

$\phi(\xi,\eta) = c_1\cos\sqrt{x}\,\eta + c_2\sin\sqrt{x}\,\eta$

$\Rightarrow$

$\phi(x,y) = c_1\cos\sqrt{x}\,y + c_2\sin\sqrt{x}\,y$

d) For $\alpha = 6$

$\phi(x,0) = 0$, $\phi_y(x,0) = x\sin x$.

$\Rightarrow \phi(x,0) = c_1 = 0$

$\Rightarrow \phi(x,y) = c_2\sin\sqrt{x}\,y$

$\phi_y(x,y) = \sqrt{x}\,c_2\cos\sqrt{x}\,y$

$\phi_y(x,0) = \sqrt{x}\,c_2 = x\sin x \Rightarrow c_2 = \sqrt{x}\sin x$

$\boxed{\phi(x,y) = \sqrt{x}\sin x \cdot \sin\sqrt{x}\,y}$

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$36y^2\phi_{xx} - 12\phi_{xy} + \phi_{yy} - 6(1+xy)\phi_x + x\phi_y = 0$ ①

Soln

a) let $A = 36y^2$, $B = -12y$, $C = 1$

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$\int 6y\,dy + \int dx = \int 0 \Rightarrow 3y^2 + x = c_1$

choose $\xi = 3y^2 + x$.

choose η "wisely"

$\begin{vmatrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{vmatrix} \neq 0 \Rightarrow \begin{vmatrix} 1 & 6y \\ 0 & 1 \end{vmatrix} = 1 \neq 0$

$\eta = y$

$\Rightarrow \xi = 3y^2 + x$

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we form $\phi_x, \phi_y, \phi_{xx}, \phi_{xy}, \phi_{yy}$

$\Rightarrow \phi_x = \frac{\partial\phi}{\partial\xi}\frac{\partial\xi}{\partial x} + \frac{\partial\phi}{\partial\eta}\frac{\partial\eta}{\partial x}$

$\phi_x = \phi_\xi$

$\phi_{xx} = \frac{\partial\phi_\xi}{\partial\xi}\frac{\partial\xi}{\partial x} = \phi_{\xi\xi}$

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$\phi_y = \frac{\partial\phi}{\partial\xi}\frac{\partial\xi}{\partial y} + \frac{\partial\phi}{\partial\eta}\frac{\partial\eta}{\partial y}$

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$\phi_{yy} = 36y^2\phi_{\xi\xi} + 12\phi_{\xi\eta} + \phi_{\eta\eta}$

Substitute all into eqn ①

$36y^2\phi_{\xi\xi} - 12y[6y\phi_{\xi\xi} + \phi_{\xi\eta}] + \ldots$

$+ 36y^2\phi_{\xi\xi} + 12\phi_{\xi\eta} + \phi_{\eta\eta} - 6\phi_\xi - \ldots$

$- 6xy\phi_\xi + 6xy\phi_\xi + x\phi_\eta = 0$

$\Rightarrow \phi_{\eta\eta} - 6\phi_\xi + x\phi_\eta = 0$

Since ϕ is a function of both ξ and η we have

$\phi_{\eta\eta} + (\alpha - 6)\phi(\xi,\eta) = 0$

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c) Solving the Canonical form

$\frac{\partial^2\phi}{\partial\eta^2} + \alpha\phi(\xi,\eta) = 0$

This is a 2nd order ode

$\Rightarrow$ The characteristic is given by

$\lambda^2 + \alpha = 0 \Rightarrow \lambda^2 = -\alpha$

$\Rightarrow \lambda = \pm i\sqrt{\alpha}$

$\phi(\xi,\eta) = c_1\cos\sqrt{\alpha}\eta + c_2\sin\sqrt{\alpha}\eta$

$\Rightarrow$

$\phi(x,y) = c_1\cos\sqrt{\alpha}y + c_2\sin\sqrt{\alpha}y$

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$\phi(x,0) = 0$, $\phi_y(x,0) = x\sin x$.

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$\boxed{\phi(x,y) = \sqrt{\alpha}\sin x.\sin\sqrt{\alpha}y}$

Question 4

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$$- 6xy\phi_\xi + 6xy\phi_\xi + x\phi_\eta = 0$$

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$$\phi_{\eta\eta} + (\cancel{x} - 6)\phi(\xi,\eta) = 0$$

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2nd order ode

b) Canonical form

The d.e for characteristic

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We form $\phi_x, \phi_y, \phi_{xx}, \phi_{xy}, \phi_{yy}$

$$\phi_x = \frac{\partial \phi}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial \phi}{\partial \eta}\frac{\partial \eta}{\partial x}$$

$$\phi_x = \phi_\xi$$

$$\frac{\partial \phi_\xi}{\partial \xi}\frac{\partial \xi}{\partial x} = \phi_{\xi\xi}$$

Substitute all into eqn (1)

$$36y^2\phi_{\xi\xi} - 12y\left[6y\phi_{\xi\xi} + \phi_{\xi\eta}\right] + \cdots$$
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$$\phi(\xi,\eta) = c_1 \cos\sqrt{x}\,\eta + c_2 \sin$$

$$\Rightarrow \quad c_1 \cos\sqrt{x}\,y + c_2 \sin$$

$)e^2$

$? = ($

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$\begin{vmatrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{vmatrix} \neq 0 \Rightarrow \begin{vmatrix} 1 & 6y \\ 0 & 1 \end{vmatrix} = 1 \neq 0$ ✓

$\Rightarrow \xi = 3y^2 + x, \quad \eta = y$

$\eta = y$

We form $\Phi_x, \Phi_y, \Phi_{xx}, \Phi_{xy}, \Phi_{yy}$

$\Rightarrow \Phi_x = \frac{\partial \Phi}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial \Phi}{\partial \eta}\frac{\partial \eta}{\partial x}$

$\Phi_x = \Phi_\xi$

$\Phi_{xx} = \frac{\partial \Phi_\xi}{\partial \xi}\frac{\partial \xi}{\partial x} = \Phi_{\xi\xi}$

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$= 6y\Phi_{\xi\xi} + \Phi_{\xi\eta}$

$\Phi_y = \frac{\partial \Phi}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial \Phi}{\partial \eta}\frac{\partial \eta}{\partial y}$

$= 6y\Phi_\xi + \Phi_\eta$

$\Phi_{yy} = \frac{\partial [6y\Phi_\xi + \Phi_\eta]}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial [6y\Phi_\xi + \Phi_\eta]}{\partial \eta}\frac{\partial \eta}{\partial y}$

$= 36y^2\Phi_{\xi\xi} + 12\Phi_{\xi\eta} + \Phi_{\eta\eta}$

Φ is a function of both ξ and η we have

$\Phi_{\eta\eta} + (\alpha - 6)\Phi(\xi,\eta) = 0$

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$\frac{\partial^2 \Phi}{\partial \eta^2} + \alpha\Phi(\xi,\eta) = 0$

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d) For $\alpha = 6$

$\Phi(x,0) = 0 = 0, \quad \Phi_y(x,0) = x\sin x$

$\Rightarrow \Phi(x,0) = c_1 = 0$

$\Rightarrow \Phi(x,y) = c_2 \sin\sqrt{\alpha}y$

$\Phi_y(x,y) = \sqrt{\alpha}\, c_2 \cos\sqrt{\alpha}y$

$\Phi_y(x,0) = \sqrt{\alpha}\, c_2 = x\sin x \Rightarrow c_2 =$

$\boxed{\Phi(x,y) = \sqrt{x}\sin x \cdot \sin\sqrt{x}y}$

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$\Rightarrow \frac{dy}{dx} = -\frac{1}{6y} \Rightarrow 6y\,dy + dx = 0$

$\int 6y\,dy + \int dx = \int 0 \Rightarrow 3y^2 + x = c_1$

choose $\xi = 3y^2 + x$

choose η "arbitrarily"

$\begin{vmatrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{vmatrix} \neq 0 \Rightarrow \begin{vmatrix} 1 & 6y \\ 0 & 1 \end{vmatrix} = 1 \neq 0$

$\eta = y$

$\Rightarrow \xi = 3y^2 + x$

$\eta = y$

We form $\phi_x, \phi_y, \phi_{xx}, \phi_{xy}, \phi_{yy}$

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$= 6y\phi_{\xi\xi} + \phi_{\xi\eta}$

$\phi_y = \frac{\partial \phi}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial \phi}{\partial \eta}\frac{\partial \eta}{\partial y}$

$= 6y\phi_\xi + \phi_\eta$

$= \ldots \frac{\partial[6y\phi_\xi + \phi_\eta]}{\partial \eta}\frac{\partial \eta}{\partial y}$

$\Rightarrow \phi_{\eta\eta} - 6\phi_\xi + x\phi_\eta = 0$

Since ϕ is a function of both ξ and η we have

$\phi_{\eta\eta} + (x-6)\phi(\xi,\eta) = 0$

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$\Rightarrow$

$\phi(x,y) = c_1 \cos\sqrt{x}\,y + c_2 \sin\sqrt{x}\,y$

d) For $x = 6$

$\phi(x,0) = 0, \quad \phi_y(x,0) = x$

$\Rightarrow \phi(x,0) = c_1 = 0$

$\Rightarrow \phi(x,y) = c_2 \sin\sqrt{x}\,y$

$\phi_y(x,y) = \sqrt{x}\,c_2 \cos\sqrt{x}\,y$

$\phi_y(x,0) = \sqrt{x}\,c_2 = x \ldots$

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-Revision Materials For PDEs & ODEs

Question 1

1) $t^2x'' - (\alpha+2)tx' + (\alpha+2)x = 0$

Soln

Reducing the ode into a system of odes

let $x_1 = x$, $x_2 = x'$, $x_3 = x''$

$\Rightarrow \frac{dx_1}{dt} = \frac{dx}{dt} = x_1' = x' = x_2$

$\frac{dx_2}{dt} = \frac{dx'}{dt} = x'' = x_3$

$\Rightarrow \frac{dx_1}{dt} = x_2$ —(1)

$\frac{dx_2}{dt} = \frac{(\alpha+2)t x_2}{t^2} - \frac{(\alpha+2)}{t^2}x_1$ —(2)

In matrix form we have

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{\alpha+2}{t^2} & \frac{\alpha+2}{t} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$-\frac{6}{t}v_1 + v_2 = 0$

$-\frac{8}{t^2}v_1 - \frac{2}{t}v_2 = 0 \Rightarrow v_2 = \frac{6}{t}v_1$

for $v_1 = 1$

$v_2 = 6/t$

$h_1(t) = \begin{pmatrix} 1 \\ 6/t \end{pmatrix} e^{(6/t)t} = \begin{pmatrix} 1 \\ 6/t \end{pmatrix} e^6$

Eigenvector associated with λ_2

$\begin{pmatrix} -2/t & 1 \\ -8/t^3 & 6/t \end{pmatrix}\begin{pmatrix} v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow v_4 = 2/t\, v_3 \Rightarrow v_3 = 1$

$v_4 = 2/t$

$h_2(t) = \begin{pmatrix} 1 \\ 2/t \end{pmatrix} e^{(2/t)t} = \begin{pmatrix} 1 \\ 2/t \end{pmatrix} e^2$

$X_h(t) = c_1\begin{pmatrix} 1 \\ 6/t \end{pmatrix} e^6 + c_2\begin{pmatrix} 1 \\ 2/t \end{pmatrix} e^2$

(ii) $\frac{dx_1}{dt} = x_2$

$\frac{dx_2}{dt} = 8/t\, x_2 - 8/t^2\, x_1 + 8$

form

$\Rightarrow \frac{dx_1}{dt} = x_2$ —(1)

$\frac{dx_2}{dt} = \frac{(\alpha+2)t x_2}{t^2} - \frac{(\alpha+2)}{t^2}x_1$ —(2)

In matrix form we have

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{\alpha+2}{t^2} & \frac{\alpha+2}{t} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

For $\alpha = 6$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{8}{t^2} & 8/t \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Associated homogenous eqn

$$\begin{pmatrix} 0 & 1 \\ -8/t^2 & 8/t \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} -\lambda & 1 \\ -8/t^2 & 8/t - \lambda \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & 1 \\ -8/t^2 & 8/t - \lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 8/t\,\lambda + 8/t^2 = 0$$

For $t > 0$

$a = 1$, $b = -8/t$, $c = 8/t^2$

$$\lambda = 8/t \pm \sqrt{\frac{64}{t^2} - 32/t^2} \Big/ 2$$

$h_2(t) = \begin{pmatrix} 1 \\ 2/t \end{pmatrix} e^{(2/t)t} = \begin{pmatrix} 1 \\ 2/t \end{pmatrix} e^2$

$X_h(t) = c_1\begin{pmatrix} 1 \\ 6/t \end{pmatrix} e^6 + c_2\begin{pmatrix} 1 \\ 2/t \end{pmatrix} e^2$

(ii) $\frac{dx_1}{dt} = x_2$

$\frac{dx_2}{dt} = 8/t\, x_2 - 8/t^2\, x_1 + 8$

In matrix form

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -8/t^2 & 8/t \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 8 \end{pmatrix}$$

General soln is given by

$X_h(t) = c_1\begin{pmatrix} 1 \\ 6/t \end{pmatrix} e^6 + c_2\begin{pmatrix} 1 \\ 2/t \end{pmatrix} e^2$

For the particular soln

$X_p(t) = c_1(t)\begin{pmatrix} 1 \\ 6/t \end{pmatrix} e^6 + c_2(t)\begin{pmatrix} 1 \\ 2/t \end{pmatrix} e^2$

To solve for $c_1(t)$ & $c_2(t)$

$c_1'(t)\begin{pmatrix} 1 \\ 6/t \end{pmatrix} e^6 + c_2'(t)\begin{pmatrix} 1 \\ 2/t \end{pmatrix} e^2 = ($

$$c_1'(t) = \frac{\begin{vmatrix} 0 & e^2 \\ 8 & 2/t\, e^2 \end{vmatrix}}{\begin{vmatrix} e^6 & e^2 \\ 6/t\, e^6 & 2/t\, e^2 \end{vmatrix}} = 2te^{-6}$$

$c_1(t) = t^2$

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Reducing the ode into a system of ...

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$$\frac{dx_2}{dt} = \frac{dx'}{dt} = x'' = x_3$$

$$\Rightarrow \frac{dx_1}{dt} = x_2 \quad \text{—(1)}$$

$$\frac{dx_2}{dt} = \frac{(\alpha+2)}{t}x_2 - \frac{(\alpha+2)}{t^2}x_1 \quad \text{—(2)}$$

In matrix form we have

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{\alpha+2}{t^2} & \frac{\alpha+2}{t} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

For $\alpha = 6$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{8}{t^2} & \frac{8}{t} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Associated homogenous eqn

$$\begin{pmatrix} 0 & 1 \\ -8/t^2 & 8/t \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} -\lambda & 1 \\ -8/t^2 & 8/t-\lambda \end{pmatrix}$$

$$h_1(t) = \begin{pmatrix} 1 \\ 6/t \end{pmatrix} e^{(6/t)t} = \begin{pmatrix} 1 \\ 6/t \end{pmatrix} e^6$$

Eigenvector associated with λ_2

$$\begin{pmatrix} -\frac{2}{t} & 1 \\ -8/t^3 & 6/t \end{pmatrix}\begin{pmatrix} v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\Rightarrow v_4 = 2/t\, v_3 \Rightarrow v_3 = 1$, $v_4 = 2/t$

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$$X_h(t) = c_1\begin{pmatrix} 1 \\ 6/t \end{pmatrix}e^6 + c_2\begin{pmatrix} 1 \\ 2/t \end{pmatrix}e^2$$

(ii) $\frac{dx_1}{dt} = x_2$

$$\frac{dx_2}{dt} = 8/t\, x_2 - 8/t^2\, x_1 + 8$$

In matrix form

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -8/t^2 & 8/t \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 8 \end{pmatrix}$$

General soln is given by

$$X_h(t) = c_1\begin{pmatrix} 1 \\ 6/t \end{pmatrix}e^6 + c_2\begin{pmatrix} 1 \\ 2/t \end{pmatrix}e^2$$

For the particular soln

$$X_p(t) = C_1(t)\begin{pmatrix} 1 \\ 6/t \end{pmatrix}e^6 + C_2(t)\begin{pmatrix} 1 \\ 2/t \end{pmatrix}e^2$$

For $\alpha = 6$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{8}{t^2} & \frac{8}{t} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Associated homogenous eqn

$$\begin{pmatrix} 0 & 1 \\ -8/t^2 & 8/t \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} -\lambda & 1 \\ -8/t^2 & 8/t-\lambda \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & 1 \\ -8/t^2 & 8/t-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 8/t\,\lambda + 8/t^2 = 0$$

For $t > 0$

$a = 1$, $b = -8/t$, $c = 8/t^2$

$$\lambda = 8/t \pm \sqrt{\frac{64}{t^2} - 32/t^2} \Big/ 2$$

$$\lambda_1 = \frac{6}{t}, \quad \lambda_2 = 2/t$$

Eigenvector associated with λ_1

$$\begin{pmatrix} -6/t & 1 \\ -8/t^2 & -2/t \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

In matrix form

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -8/t^2 & 8/t \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 8 \end{pmatrix}$$

General soln is given by

$$X_h(t) = c_1\begin{pmatrix} 1 \\ 6/t \end{pmatrix}e^6 + c_2\begin{pmatrix} 1 \\ 2/t \end{pmatrix}e^2$$

For the particular soln

$$X_p(t) = C_1(t)\begin{pmatrix} 1 \\ 6/t \end{pmatrix}e^6 + C_2(t)\begin{pmatrix} 1 \\ 2/t \end{pmatrix}e^2$$

To solve for $C_1(t)$ & $C_2(t)$

$$C_1'(t)\begin{pmatrix} 1 \\ 6/t \end{pmatrix}e^6 + C_2'(t)\begin{pmatrix} 1 \\ 2/t \end{pmatrix}e^2 = \begin{pmatrix} 0 \\ 8 \end{pmatrix}$$

$$C_1'(t) = \frac{\begin{vmatrix} 0 & e^2 \\ 8 & 2/t\, e^2 \end{vmatrix}}{\begin{vmatrix} e^6 & e^6 \\ 6/t\, e^6 & 2/t\, e^6 \end{vmatrix}} = 2te^{10}$$

$$C_1(t) = \frac{t^2}{e^{10}}$$

$$C_2'(t) = \frac{\begin{vmatrix} e^6 & 0 \\ 6/t\, e^6 & 8 \end{vmatrix}}{-4/t\, e^{12}} = \quad C_2(t) = -t^2/e^6$$

G.S

$$X(t) = c_1\begin{pmatrix} 1 \\ 6/t \end{pmatrix}e^6 + c_2\begin{pmatrix} 1 \\ 2/t \end{pmatrix}e^2 + t^2/e^{10}\begin{pmatrix} 1 \\ 6/t \end{pmatrix}e^6 - t^2/e^6\begin{pmatrix} 1 \\ 2/t \end{pmatrix}e^2$$

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b) $\frac{dx_1}{dt} = x_2$

$\frac{dx_2}{dt} = \frac{8}{t}x_2 - \frac{8}{t^2}\left(1 - \frac{\varepsilon^2}{t}\right)x_1 + 8\varepsilon$

The homogenen part of eqn

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{8}{t^2} - \frac{8\varepsilon^2}{t^3} & \frac{8}{t} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Eigen ~~vector~~ value assocm with A

$$\begin{vmatrix} -\lambda & 1 \\ \frac{8}{t^2} - \frac{8\varepsilon^2}{t^3} & \frac{8}{t} - \lambda \end{vmatrix} = 0$$

$\lambda^2 - \frac{8}{t}\lambda - \frac{8}{t^2} + \frac{8\varepsilon^2}{t^3} = 0$

$a = 1, \quad b = -\frac{8}{t}, \quad c = -\frac{8}{t^2} + \frac{8\varepsilon^2}{t^3}$

Eigen ~~vector~~ value assocm with A

$$\begin{vmatrix} -\lambda & 1 \\ \frac{8}{t^2} - \frac{8\varepsilon^2}{t^3} & \frac{8}{t} - \lambda \end{vmatrix} = 0$$

$\lambda^2 - \frac{8}{t}\lambda - \frac{8}{t^2} + \frac{8\varepsilon^2}{t^3} = 0$

$a = 1, \quad b = -\frac{8}{t}, \quad c = -\frac{8}{t^2} + \frac{8\varepsilon^2}{t^3}$

$\lambda = \frac{4}{t} \pm \sqrt{\frac{2}{t^2} - \frac{2\varepsilon^2}{t^3}}$

Using soln in (a) we have

Eigen vector assocm with λ_1

$$\begin{pmatrix} \frac{4}{t} + \sqrt{\frac{2}{t^2} - \frac{2\varepsilon^2}{t^3}} & 1 \\ \frac{8}{t^2} - \frac{8\varepsilon^2}{t^3} & \frac{8}{t} - \frac{4}{t} - \sqrt{\frac{2}{t} - \frac{2\varepsilon^2}{t^3}} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$= \begin{matrix} v_1 = 1 \\ v_2 = -\frac{4}{t} + \sqrt{\frac{2}{t^2} - \frac{2\varepsilon^2}{t^3}} \end{matrix} \quad \frac{4}{t} + \sqrt{\frac{2}{t^2} - \frac{2\varepsilon^2}{t^3}}$

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$$\begin{pmatrix} 4/t + \sqrt{2/t^2 - 2\varepsilon^2/t^3} & \\ 8/t^2 - 8\varepsilon^2/t^3 & 8/t - 4/t - \sqrt{2/t - 2\varepsilon^2/t^3} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$= \begin{matrix} v_1 = 1 \\ v_2 = -4/t + \sqrt{2/t^2 - 2\varepsilon^2/t^3} \end{matrix}$$

$$h_1(t) = \begin{pmatrix} 1 \\ -4/t + \sqrt{2/t^2 - 2\varepsilon^2/t^3} \end{pmatrix} e^{4/t + \sqrt{2/t^2 - 2\varepsilon^2/t^3}}$$

$$h_2(t) = \begin{pmatrix} 1 \\ -4/t + \sqrt{2/t^2 - 2\varepsilon^2/t^3} \end{pmatrix} e^{\frac{4}{t} - \sqrt{2/t^2 - 2\varepsilon^2/t^3}}$$

Using the partition ~~form~~ concept for $\varepsilon \in [0,1]$ we choose $\varepsilon \in 0$

$$\Rightarrow X_h(t) = c_1 \begin{pmatrix} 1 \\ -5/t \end{pmatrix} e^{(5/t)t} + c_2 \begin{pmatrix} 1 \\ -3/t \end{pmatrix} e^{(3/t)t}$$

As for the particular soln $X_p(t)$

$$c_1(t) = c_2(t) = 0$$

$\Rightarrow$ Approximate Soln

$$\Rightarrow \underset{\sim}{X}(t) = c_1 \begin{pmatrix} 1 \\ -5/t \end{pmatrix} e^{5} + c_2 \begin{pmatrix} 1 \\ -3/t \end{pmatrix} e^{3}$$

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$36y^2\phi_{xx} - 12\phi_{xy} + \phi_{yy} - 6(1+xy)\phi_x + x\phi_y = 0$ — (1)

Soln

a) let $A = 36y^2$, $B = -12y$, $C = 1$

$\Delta = B^2 - 4AC = 144y^2 - 144y^2 = 0 \Rightarrow$ Parabolic

b) Canonical form

The d.e for characteristic

$$\frac{dy}{dx} = \frac{-12y \pm \sqrt{0}}{72y^2} = -\frac{1}{6y}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{6y} \Rightarrow 6y\,dy + dx = 0$$

$\int 6y\,dy + \int dx = \int 0 \Rightarrow 3y^2 + x = c_1$

choose $\xi = 3y^2 + x$

choose η "wisely"

$$\begin{vmatrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{vmatrix} \neq 0 \Rightarrow \begin{vmatrix} 1 & 6y \\ 0 & 1 \end{vmatrix} = 1 \neq 0$$

$\eta = y$

$\Rightarrow \xi = 3y^2 + x$

$\eta = y$

we form $\phi_x, \phi_y, \phi_{xx}, \phi_{xy}, \phi_{yy}$

$$\Rightarrow \phi_x = \frac{\partial\phi}{\partial\xi}\frac{\partial\xi}{\partial x} + \frac{\partial\phi}{\partial\eta}\frac{\partial\eta}{\partial x}$$

$\phi_x = \phi_\xi$

$$\phi_{xx} = \frac{\partial\phi_\xi}{\partial\xi}\frac{\partial\xi}{\partial x} = \phi_{\xi\xi}$$

$$\phi_{xy} = \frac{\partial\phi_\xi}{\partial\xi}\frac{\partial\xi}{\partial y} + \frac{\partial\phi_\xi}{\partial\eta}\frac{\partial\eta}{\partial y} = 6y\phi_{\xi\xi} + \phi_{\xi\eta}$$

$$\phi_y = \frac{\partial\phi}{\partial\xi}\frac{\partial\xi}{\partial y} + \frac{\partial\phi}{\partial\eta}\frac{\partial\eta}{\partial y} = 6y\phi_\xi + \phi_\eta$$

$$\phi_{yy} = \frac{\partial[6y\phi_\xi + \phi_\eta]}{\partial\xi}\frac{\partial\xi}{\partial y} + \frac{\partial[6y\phi_\xi + \phi_\eta]}{\partial\eta}\frac{\partial\eta}{\partial y}$$

$\phi_{yy} = 36y^2\phi_{\xi\xi} + 12\phi_{\xi\eta} + \phi_{\eta\eta}$

Substitute all into eqn (1)

$36y^2\phi_{\xi\xi} - 12y[6y\phi_{\xi\xi} + \phi_{\xi\eta}] + \cdots$

$+ 36y^2\phi_{\xi\xi} + 12\phi_{\xi\eta} + \phi_{\eta\eta} - 6\phi_\xi - \cdots$

$- 6xy\phi_\xi + 6xy\phi_\xi + x\phi_\eta = 0$

$\Rightarrow \phi_{\eta\eta} - 6\phi_\xi + x\phi_\eta = 0$

Since ϕ is a function of both ξ and η we have

$\phi_{\eta\eta} + (x-6)\phi(\xi,\eta) = 0$

$\Rightarrow \boxed{\phi_{\eta\eta} + \alpha\phi(\xi,\eta) = 0}$

c) Solving the Canonical form

$$\frac{\partial^2\phi}{\partial\eta^2} + \alpha\phi(\xi,\eta) = 0$$

This is a 2nd order ode

$\Rightarrow$ The characteristic is given by

$\lambda^2 + \alpha = 0 \Rightarrow \lambda^2 = -\alpha$

$\Rightarrow \lambda = \pm i\sqrt{\alpha}$

$\phi(\xi,\eta) = c_1\cos\sqrt{\alpha}\eta + c_2\sin\sqrt{\alpha}\eta$

$\Rightarrow$

$\phi(x,y) = c_1\cos\sqrt{\alpha}y + c_2\sin\sqrt{\alpha}y$

d) For $\alpha = 6$

$\phi(x,0) = 0$, $\phi_y(x,0) = x\sin x$.

$\Rightarrow \phi(x,0) = c_1 = 0$

$\Rightarrow \phi(x,y) = c_2\sin\sqrt{\alpha}y$

$\phi_y(x,y) = \sqrt{\alpha}\,c_2\cos\sqrt{\alpha}y$

$\phi_y(x,0) = \sqrt{\alpha}\,c_2 = x\sin x \Rightarrow c_2 = \sqrt{x}\sin x$

$\boxed{\phi(x,y) = \sqrt{x}\sin x.\sin\sqrt{x}y}$