1. Determine the derivative $\frac{dy}{dx}$ of $y = 5^{-2x+7}$. [2 marks]

Solution

$$ln(y) = (-2x+7)ln(5)$$
$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 2ln(5)$$
$$\Longrightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -2yln(5)$$
$$ans \Longrightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -2(5^{-2x+7})ln(5) = -2ln(5)5^{-2x+7}$$

2. Determine the minimum value of the function $f(x) = 4^x - 8$. [4 marks]

Solution

$$f'(x) = 4^x ln4 - 0$$
$$=> f'(x) = 2^{2x} ln2^2 = 2^{2x}.2.\,ln(2) = 2^{2x+1}ln(2)$$
$$=> f'(x) = 2^{2x+1}ln(2) = 0$$
$$=> 2^{2x+1} = 0$$
$$Finding\ critical\ points$$
$$=> The\ function\ doesn't\ have\ solution\ for\ critical\ points\ hence\ no\ minimum\ value$$

3. Determine the derivative $\frac{dy}{dx}$ for $y = \tan x \cos x$. [2 marks]

Solution

$$=> using\ product\ rule$$
$$=> \frac{\mathrm{d}y}{\mathrm{d}x} = tan(x).-sin(x) + cos(x).\,sec^2(x)$$
$$=> -sin(x)tan(x) + cos(x)sec^2(x)$$
$$=> -sin(x)\frac{sin(x)}{cos(x)} + cos(x)\frac{1}{cos(x)cos(x)}$$
$$=> \frac{-sin^2x}{cosx} + \frac{1}{cosx} = -\frac{sin^2x+sin^2x+cos^2x}{cosx} = \frac{cos^2x}{cosx} = cosx$$

4. The growth in the population of a group of rabbits is given by $P(t) = 800e^{0.08t}$, where P is the population at time t measured in weeks.

a. What is the initial population of rabbits? [2 marks]

Solution

$$=> \textit{Initial population will be at } t = 0$$
$$=> p(t) = 800e^{0.08*0} = 800e^0 = 800$$

b. How many rabbits are there after 14 days? [3 marks]

Solution

$$=> 14 \textit{ days totals } 2 \textit{ weeeks}$$
$$=> P(t) = 800e^{0.08*2} = 800 * 1.1735 = 938.808 = 939$$

c. What is the rate of change of rabbits after 14 days? [3 marks]

Solution

$$=> \textit{Given } => P(t) = 800e^{0.08*t} \textit{ we have}$$
$$P'(t) = 0.08 * 800e^{0.08*t}$$
$$=> \textit{at}, t = 2 \textit{ we obtain} :$$
$$=> 0.08 * 800e^{0.08*2} = 75 \textit{ rabits per week}$$

5. Consider the function $f(x) = \sin^4(4x)$.

a. Determine $f'(x)$. [2 marks]

Solution

$$=> \textit{using chain rule}$$
$$=> f'(x) = ?, \textit{ we differentiate } sin^4 u \textit{ wrt } u, \textit{where } u = 4x$$
$$=> 4sin^3 u => 4sin^3 4x$$
$$\textit{Then we differentiate } sin4x \textit{ wrt } x, \textit{to get } = 4cos4x$$
$$=> f'(x) = 4sin^3 4x.4cos4x = 16sin^3(4x)cos(4x)$$

b. Determine $f''(x)$.[2 marks]

Solution

$$=> f''(x) = \frac{\mathrm{d}(16sin^3(4x)cos(4x))}{\mathrm{d}x}$$

$$=> 16[4 * 3sin^2 4x * cos4x * cos4x + sin^3 4x * -4sin4x]$$

$$=> 16[12sin^2(4x)cos^2(4x) - 4sin^4(4x)]$$

1. Consider the function $f(x) = -\tan(x + \frac{\pi}{2})$.

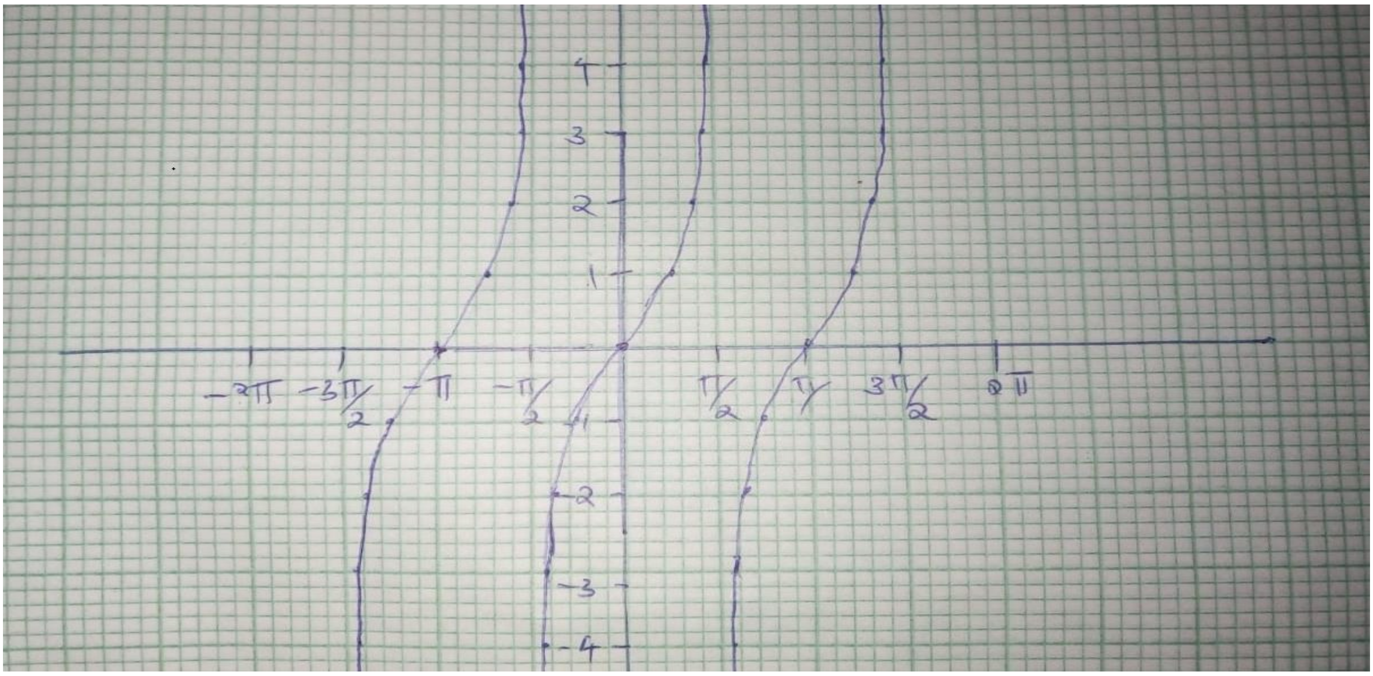
a. Determine all the transformations from the graph of the parent function, tan x. [4 marks]

Solution

$f(x) = -\tan(x + \pi/2)$

$\Rightarrow$ Period $= \pi$

$\Rightarrow$ Horizontal shift - graph is shifted left $\pi/2$



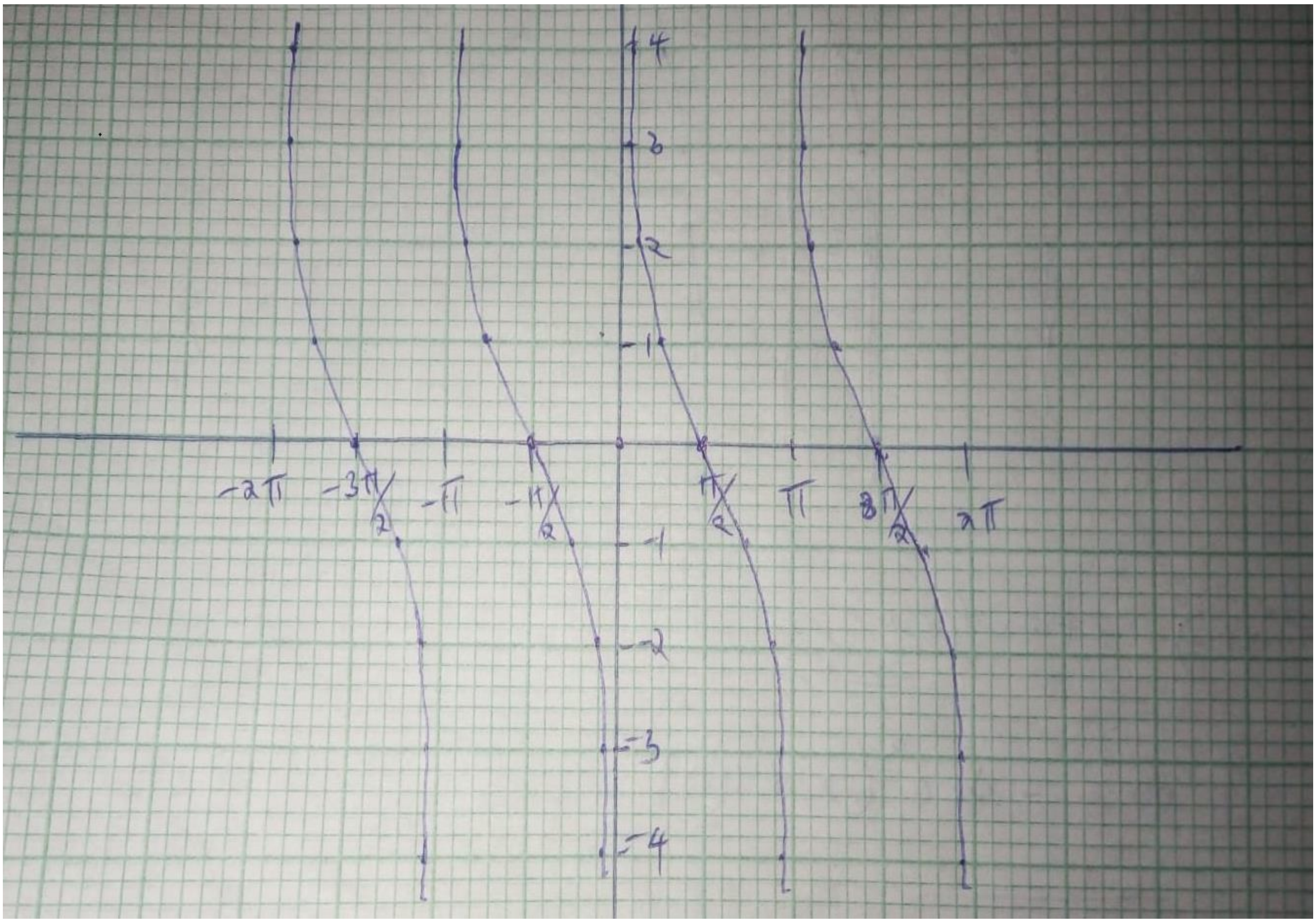
b. Determine $f'(x)$. [4 marks]

Solution

$$f'(x) = -\sec^2\left(x + \pi/2\right)$$

c. Use your knowledge of the tangent function to graph $f(x)$. [5 marks]

Solution



2. A radioactive substance decays so that after t years, the amount remaining, expressed as a percent of the original amount, is $A(t) = 100(1.6)^{-t}$.
a. Determine the function, A', which represents the rate of decay of the substance.

Solution

$$A(t) = 100(1.6)^{-t}$$
$$=> A'(t) = 100 * -1.6^{-t} ln(1.6)$$
$$=> A'(t) = -100 * 1.6^{-t} ln(1.6)$$

[2 marks]
b. What is the half-life for this substance?[2 marks]
c. What is the rate of decay when half the substance has decayed?[3 marks]

(b)

Solution

$$=> \frac{50}{100} = \frac{100}{100}(1.6)^{-t}$$
$$=> 0.5 = 1.6^{-t}$$
$$=> -tln(1.6) = ln(0.5)$$
$$=> -t = \frac{ln(0.5)}{ln(1.6)} = -1.4748$$
$$=> t = 1.4748\ years$$

©

Solution

$$=> A'(t) = -100 * (1.6)^{-t} ln(1.6)\ at,\ t = 1.4748$$
$$=> -100 * (1.6)^{-1.4748} ln(1.6) = 23.5\ substance\ yearly$$

Thinking and Inquiry

1. Determine the derivative $\frac{dy}{dx}$ for $y = \tan^2(e^x)$. [4 marks]

Solution

$$=> 2tan(e^x) * (e^x) * sec^2(e^x)$$
$$=> 2e^x tan(e^x).\ sec^2(e^x)$$

2. The velocity of a car is given by $v(t) = 60(1-(0.7)^t)$, where v is measured in m/s and t is measured in s.

a. Determine the acceleration function. [4 marks]

Solution

$$=> v'(t) = 0 - 60 * 0.7^t * ln(0.7)$$
$$=> a = v'(t) = -60 * 0.7^t * ln(0.7)$$

b. Determine the acceleration at t = 2s. [2 marks]

Solution

$$=> acceleration\ at, t = 2s$$
$$=> v'(t) = 0 - 60 * 0.7^t * ln(0.7)$$
$$=> a = v'(t) = -60 * 0.7^2 * ln(0.7) = 10.49m/s^2$$

c. What is the initial velocity and what does this mean physically? [4 marks]

Solution

$$=> Initial\ velocity\ will\ be\ at, t = 0$$
$$=> v(t) = 60(1 - 0.7^t) = 60(1 - 0.7^0) = 0m/s$$
$$=> This\ means\ the\ car\ is\ stationary(not\ moving)$$

d. Determine the time at which the acceleration is 3 m/s^2. [3 marks]

Solution

$$=> 3 = -60(0.7^t)ln(0.7)$$
$$=> 0.7^t = \frac{3}{-60*ln(0.7)} = 0.14018$$
$$=> tln(0.7) = ln(0,14018)$$
$$=> t = 5.5093$$

3. Determine the minimum value of the function $f(x) = e^x - 8$. [3 marks]

Solution

$$=> f'(x) = e^x - 0 = e^x$$
$$=> critical\ points\ will\ be\ found\ as\ x\ approaches\ -\infty$$
$$=> f''(x) = e^x|_{x\mapsto -\infty} = 0$$
$$=> This\ function\ has\ a\ sadlle\ point\ but\ no\ minimum\ value$$

Communication

1 a) List two applications in which the exponential function is used. [2 marks]

b) What is the most important aspect of the function $f(x) = e^x$? [2 marks]

c) Show that the inverse of $f(x) = e^x$ is the function $g(x) = \ln x$. [2 marks]

(a)

Solution

(i) Used in bacterial and human growth population predictions

(ii) Used in calculating compound interests

(b)

Solution

This function is vital in representing large numbers in a simple form such as reproduction of human cells which is approximated to grow exponentially

©

Solution

$$=> let\ f(x) = e^x = p$$
$$=> ln(xln(e)) = ln(f(x)) = ln(p)$$
$$=> ln(e^x) = xln(e) = x$$
$$=> Thus\ e^x\ is\ inverse\ of\ ln(x)$$

2 a) Explain the difference between the exponential function e^x and the general exponential function b^x. [2 marks]

Solution

The difference only comes in for the values,in that, b^x is the general form for any exponential growth where b can take in different values depending on target while in , e^x the e value is already known and that its value can not be altered as in , b^x

b) Explain the difference between the function 2^x and the function $(\frac{1}{2})^x$. [2 marks]

Solution

With, 2^x it means that the growth is happening with a constant factor of 2 whilest in, $(\frac{1}{2})^x$,implies that the growth rate is happening by a factor of , $\frac{1}{2}$

c) Explain the difference between the function $(\frac{1}{2})^x$ and the function 2^{-x}. [2 marks]

Solution

$$=> 2^x = \frac{1}{2^x} = (\frac{1}{2})^x \text{ , } which\ is\ the\ same\ as, (\frac{1}{2})^x$$

3a) Explain how to derive the derivative of the function $f(x) = \csc x$ two different ways. (Hint: They are similar but use different rules.) [4 marks]

Solution

(i) One way would be to differentiate the function csc(x) directly to obtain -cot(x)csc(x) by the use of the first principle of derivatives that is with the definition of limits

(ii) Second would be to use chain rule to differentiate the function using certain trigonometric properties and identities

b) Derive the derivative using one of the two ways. [4 marks]

Solution

$$=> using\ chain\ rule \frac{\mathrm{d}(csc(x))}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}(\frac{1}{sin(x)})$$

$$=> using\ quotient\ rule$$

$$=> \frac{sin(x)*0-(1)*cos(x)}{sin^2(x)} = \frac{-cos(x)}{sn^2(x)} = -\frac{cos(x)}{sin(x)} * \frac{1}{sin(x)} = -csc(x)cot(x)$$

Indicate whether the statement is true or false. [2 marks each]

_____ 1. Equal vectors have the same direction and have the same magnitude

True

_____ 2. Opposite vectors have different magnitudes and directions

False

_____ 3. $(m+n)\vec{a} = m\vec{a} + n\vec{a}$ conveys the associative property of scalars.

False

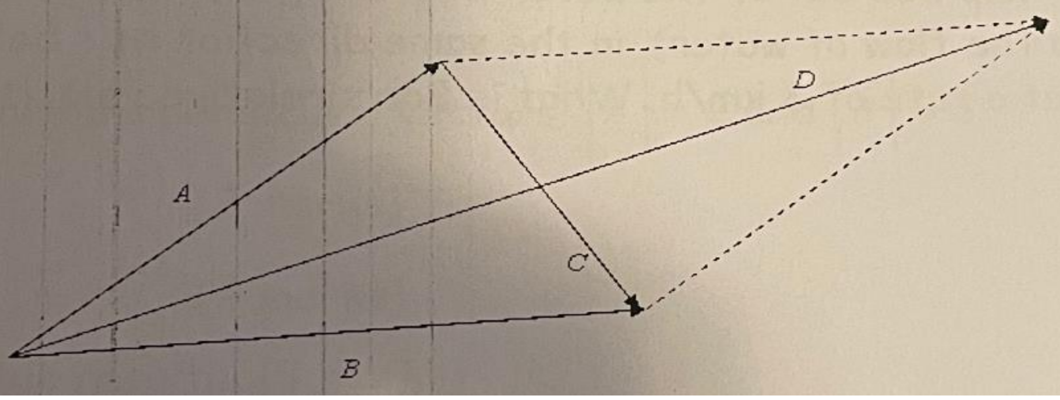
_____ 4. To determine the vector $\overrightarrow{AB}$, we subtract A from B.

True

_____ 5. Velocity is a vector, it requires a magnitude and direction. Speed is a scalar; it just has a magnitude.

True

6. What does vector $\overrightarrow{D}$ represent? [3 marks]



Solution

$$=> \vec{D} \text{ represents the length of the diagonal}$$

$$=> |\vec{A} + \vec{B}| = |\vec{B} + \vec{A}|$$

7. $\vec{x} = 2\vec{i} + 3\vec{j}$ and $\vec{y} = -2\vec{i} - \vec{j}$. Determine $\frac{1}{2}(\vec{x} - \vec{y})$. [4 marks]

Solution

$$=> \vec{x} - \vec{y} = (2i + 3j) - (-2i - j)$$
$$=> 2i + 2i + 3j + j = 4i + 4j$$
$$=> \tfrac{1}{2}(4i + 4j) = 2i + 2j$$

Application

1. Bob can swim at a rate of 5 km/h. He is in a river that is flowing at a rate of 9 km/h.

a. If Bob swims upstream, what is his relative velocity to the ground? [3 marks]

Solution

9km/h-5km/h=4km/h

b. If Bob swims downstream, what is his relative velocity to the ground? [2 marks]

Solution

9km/h+5km/h=14km/h

c. Someone decides to help Bob out of the water with a rope. Bob swims across the river (perpendicular to the flow of water) in the same direction he's being pulled. The rescuer is pulling at a rate of 3 km/h. What is Bob's relative velocity to the ground? [4 marks]

Solution

$$=> 9^{+}3^{2} = \sqrt{9^{+}3^{2}} = \sqrt{90} = 9.487 km/h$$

2. A rectangle is formed in R^2 by the vectors $\overrightarrow{OA}$ = (1, 2) and $\overrightarrow{OB}$ = (-6, 3).

a. Determine its perimeter. [3marks]

Solution

$$=> |\overrightarrow{OB}| = \sqrt{(-6)^2 + (3)^2} = 45$$
$$=> |\overrightarrow{OA}| = \sqrt{1^2 + 2^2} = 5$$
$$=> 45 + 45 + 5 + 5 = 100$$

b. Determine its area. [2marks]

Solution

Area=45*5=225 sq. units

c. Determine the length of its diagonals. [4 marks]

Solution

$$=> \sqrt{45^2 + 5^2} = \sqrt{2050} = 45.28$$

Thinking and Inquiry

1. $\overrightarrow{A}$ and $\overrightarrow{B}$ are perpendicular vectors. $\left|\overrightarrow{A}\right| = 2$ and $\left|\overrightarrow{B}\right| = 3$.

a. Calculate $\left|\overrightarrow{A} + \overrightarrow{B}\right|$. [2 marks]

Solution

$$=> |\overrightarrow{A} + \overrightarrow{B}| = |\overrightarrow{A}| + |\overrightarrow{B}| = 2 + 3 = 5$$

b. Calculate $\left|\overrightarrow{A} - \overrightarrow{B}\right|$. [2 marks]

Solution

$$=> |\overrightarrow{A} - \overrightarrow{B}| = |\overrightarrow{A}| - |\overrightarrow{B}| = 2 - 3 = 1$$

c. Explain your results. [3 marks]

Solution

Both vectors A and B has a positive resultant vector of 5 units whilst their difference results into a negative vector of -1 units implying that their positional vector coordinates will be much influenced by the directions of both vectors

2. A parallelogram is formed in $\mathbb{R}^2$ and has vertices $A = (1, 2)$, $B = (3, 7)$, $C = (4, 4)$ and $D = (7, 11)$. Determine the vectors that form this parallelogram, with their tails starting at A. Then determine the perimeter of the parallelogram. [9 marks]

Solution

(i) Determine the vectors

$$=> \overrightarrow{AB} = (3,7) - (1,2) = (2,5) = 2i + 5j$$

$$=> \overrightarrow{AC} = (4,4) - (1,2) = (3,2) = 3i + 2j$$

$$=> \overrightarrow{AD} = (7,11) - (1,2) = (6,9) = 6i + 9j$$

$$=> \overrightarrow{OA} = (1,2) - (0,0) = (1,2) = i + 2j$$

(ii) Perimeter

$$=> |\overrightarrow{AB}| = (3,7) - (1,2) = (2,5) = 2i + 5j = \sqrt{2^2 + 5^2} = \sqrt{29}$$

$$=> |\overrightarrow{BC}| = (4,4) - (3,7) = \sqrt{1^2 + (-3)^2} = \sqrt{10}$$

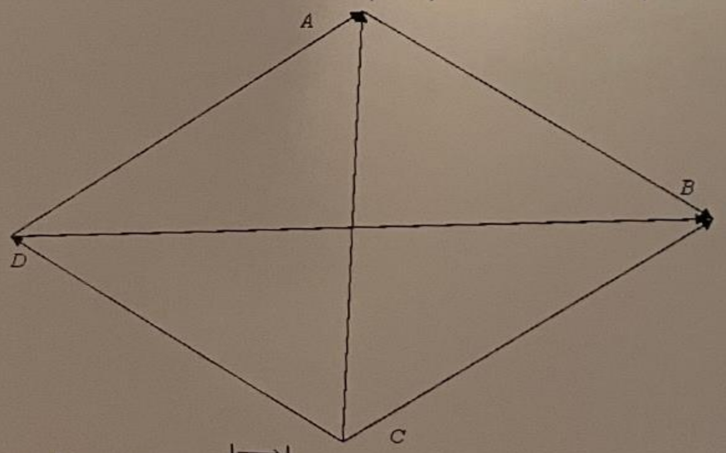
$$=> |\overrightarrow{DC}| = (7,11) - (4,4) = \sqrt{3^2 + 7^2} = \sqrt{58}$$

$$=> |\overrightarrow{DA}| = (1,2) - (7,11) = \sqrt{(-6)^2 + (-9)^2} = \sqrt{117}$$

$$=> \sqrt{29} + \sqrt{10} + \sqrt{58} + \sqrt{117} = 26.98$$

Communication

1 In the parallelogram, $|\overrightarrow{DA}| = 5$ and $|\overrightarrow{CA}| = 10$.



a. Determine $|\overrightarrow{CB}|$ and explain your reasoning. [3 marks]

Solution

$|\overrightarrow{CB}|$, is equal to 5 since in a pallelogram opposite sides are congruent i.e DA=CB, implying that their magnitude will be equal i.e, $|DA| = |CB|$

b. Name the two opposite vectors of $\overrightarrow{CB}$.[3 marks]

Solution

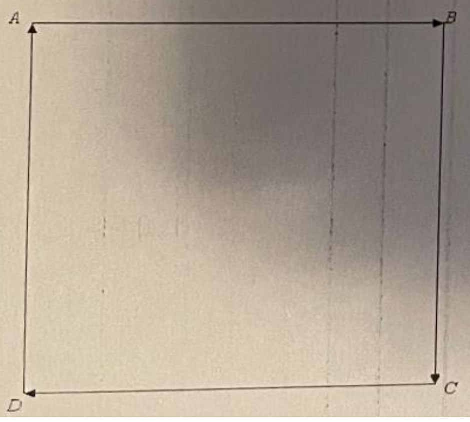
$=> \overrightarrow{DA}\ and\ \overrightarrow{AD}$

c. Determine $|\overrightarrow{AC}|$ and explain your reasoning. [3 marks]

Solution

Since |CA|=10, it implies that |AC| is also the magnitude of vectors AC and CA will be always equal regardless of the direction

2. The diagonals of the square $ABCD$ meet at X. Determine two ways to write $\overrightarrow{BX}$ using $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$. Explain. [2 marks]



Solution

$$=> \overrightarrow{BX} = -\overrightarrow{AB} + \tfrac{1}{2}\overrightarrow{AC}$$
$$=> -\overrightarrow{AB} + \tfrac{1}{2}\overrightarrow{AB} + \tfrac{1}{2}\overrightarrow{BC}$$
$$=> \tfrac{1}{2}\overrightarrow{BC} - \tfrac{1}{2}\overrightarrow{AB}$$

$$=> \overrightarrow{BX} = -\overrightarrow{BC} + \overrightarrow{CX}$$
$$=> \overrightarrow{BC} - \tfrac{1}{2}\overrightarrow{AB} - \tfrac{1}{2}\overrightarrow{BC}$$
$$=> \tfrac{1}{2}\overrightarrow{BC} - \tfrac{1}{2}\overrightarrow{AB}$$

To find the 2 ways we need to first calculate the diagonal AC equivalence, in terms of the given vectors

Then we split the diagonal to obtain any vector to the point X, Lastly we take the two possible ways which will result to BX as shown above

3. Name two pairs of vectors that could span R^2. Show how the vector (3, 2) could be written as a linear combination of your spanning set. [3 marks]

Solution

$=> \{(1,2),(2,1)\}$

$=> \textit{we have that } c1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$=> c1 + 2c2 = 3$

$=> 2c1 + c2 = 2$

$=> c1 = \frac{1}{3}, c2 = \frac{4}{3}$

$=> (3,2) = \frac{1}{3}(1,2) + \frac{4}{3}(2,1)$

_____ 1. A force of magnitude 5 N acts on an object. A second force of magnitude 7 N acts at 50° to the other force. What is the magnitude of the resultant of these two forces?

a. 8.60 N
b. 10.91 N
c. 11.60 N
d. 12.00 N

b

_____ 2. Two forces of 3N and 8N act on an object at an angle of 30□ to each other. What is the dot product of these force vectors?

a. 4.24
b. 20.78
c. 12.00
d. 24.00

b

_____ 3. Suppose that $\vec{a} \cdot \left(\vec{b} - 3\vec{c}\right) = 0$ and $\vec{a} \cdot \vec{c} = 2$. What is $\vec{a} \cdot \vec{b}$ equal to?

a. 6
b. 2
c. □6
d. □2

a

_____ 4. Suppose $\vec{x} = 3$, $\vec{y} = 5$, and the angle between $\vec{x}$ and $\vec{y}$ is 40□. What is the dot product of the vectors $\vec{x}$ and $\vec{y}$?

a. 2.97
b. 9.64
c. 19.58
d. 11.49

d

_____ 5. Given that the vectors $\vec{a} = (6, -2, 2s)$ and $\vec{b} = (-1, s+1, 2)$ are perpendicular, what is the value of s?

a. -4
b. -1
c. 0
d. 4

d

_____ 6. If ABC is a triangle with vertices $A(2, 2)$, $B(3, 0)$, and $C(4, 6)$, then wh
is the scalar projection of $\overrightarrow{AB}$ on $\overrightarrow{AC}$?

a. $-\frac{3}{5}$

b. $-\frac{3}{\sqrt{5}}$

c. $-\frac{6}{\sqrt{5}}$

d. $\frac{6}{\sqrt{10}}$

b

_____ 7. If ABC is a triangle with vertices $A(1, 1, □1)$, $B(1, 0, 1)$, and $C(1 + a, 0, 2)$
and $\overrightarrow{AB} \times \overrightarrow{AC} = (□1, 2, 1)$, then what is the value of a?

a. 1

b. 0

c. -2

d. -1

a

_____ 8. Determine which line is perpendicular to the line $2x - 3y + 17 = 0$.

a. $\vec{r} = (2, -3) + s(3, -2)$, $s \in \mathbf{R}$

b. $\vec{r} = (1, 2) + s(3, 2)$, $s \in \mathbf{R}$

c. $\vec{r} = (1, 7) + s(2, -3)$, $s \in \mathbf{R}$

d. $\vec{r} = s(3, 2)$, $s \in \mathbf{R}$

b

_____ 9. Determine which line is perpendicular to the line $2x - 3y + 17 = 0$.

a. $\vec{r} = (2, -3) + s(3, -2)$, $s \in \mathbf{R}$

b. $\vec{r} = (1, 2) + s(3, 2)$, $s \in \mathbf{R}$

c. $\vec{r} = (1, 7) + s(2, -3)$, $s \in \mathbf{R}$

d. $\vec{r} = s(3, 2)$, $s \in \mathbf{R}$

b

_____ 10. Which of the following is not a plane?

a. $\vec{r} = (1, 3, 4) + s(2, -1, 2) + t(1, 1, 1)$, $s, t \in \mathbf{R}$

b. $\vec{r} = (2, 4, 2) + s(1, -2, 3) + t(3, 2, 2)$, $s, t \in \mathbf{R}$

c. $\vec{r} = (3, 2, 3) + s(4, -4, 2) + t(-2, 2, -1)$, $s, t \in \mathbf{R}$

d. $\vec{r} = (-2, 1, 4) + s(2, 2, -1) + t(2, 2, 1)$, $s, t \in \mathbf{R}$

a

_____ 11. A plane is defined by the equation $3x - 2z = 4y + 1$. Which of the following is the normal vector of this plane?

a. $\vec{n} = (3, -2, 4)$

b. $\vec{n} = (3, 4, -2)$

c. $\vec{n} = (3, 2, 4)$

d. $\vec{n} = (3, -4, -2)$

d

_____ 12. Which three points are on the plane $2x - 7y + 3z - 5 = 0$?

a. $P(1, 0, 1)$, $Q(3, 1, 2)$, and $R(4, 3, 6)$

b. $P(1, 0, 1)$, $Q(2, 2, 3)$, and $R(3, 1, 2)$

c. $P(3, 1, 2)$, $Q(4, 3, 6)$, and $R(5, 0, -2)$

d. $P(4, 3, 6)$, $Q(0, 0, 0)$, and $R(3, 1, 2)$

a

Problem

11. a) An object with a weight of 60 N is suspended by two lengths of rope from the ceiling. The angles that both lengths make with the ceiling are the same. The tension in each length is 40 N. Determine the angle that the lengths of ropes make with the ceiling. [8 marks]

Solution

$$=> F_y = T_1 sin\theta + T_2 sin\theta - 60 = 0$$
$$=> 40 * sin\theta + 40 * sin\theta - 60 = 0$$
$$=> 80 * sin\theta = 60$$
$$=> sin\theta = \frac{60}{80} = 0.75$$
$$=> \theta = 48.59$$

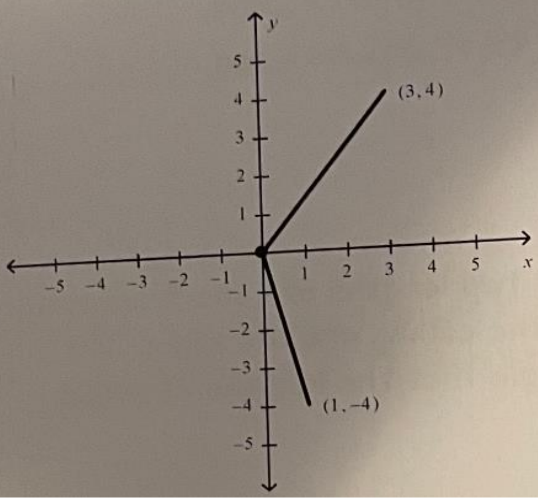
12. Forces of 4 N, 5 N, and 7 N are in equilibrium. Determine the angle between the two smaller vectors. [7 marks]

Solution

For force in equilibrium, the sum of the 2 smaller forces must be equal to the 3rd force

$$=> R^2 = a^2 + b^2 + 2abcos\theta$$
$$=> 7^2 = 4^2 + 5^2 + 2 * 4 * 5cos\theta$$
$$=> cos\theta = \frac{7^2 - 4^2 - 5^2}{2*4*5} = 0.2$$
$$=> \theta = 78.46$$

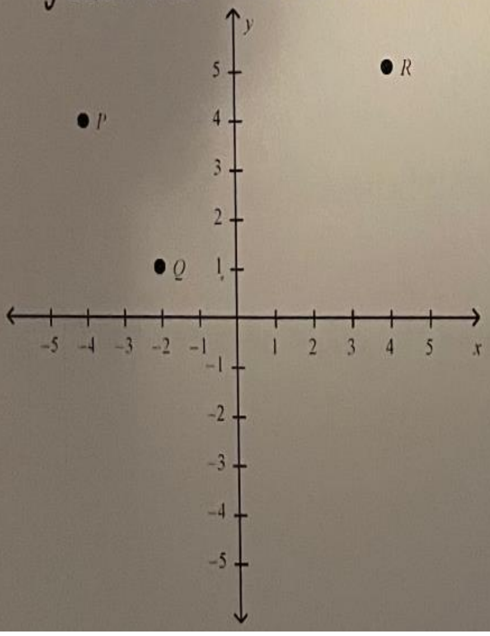
13. Determine the dot product of the two vectors in the diagram. [4 marks]



Solution

$$=> \overrightarrow{OA} = (3,4) - (0,0) = (3,4) = 3i + 4j$$
$$=> \overrightarrow{OB} = (1,-4) - (0,0) = (1,-4) = i - 4j$$
$$=> \overrightarrow{OA}.\overrightarrow{OB} = (3i+4j).(i-4j) = (3*1) + (4*-4) = -13$$

14. The vectors $\vec{a}$ and $\vec{b}$ are unit vectors such that $\vec{a} \cdot \vec{b} = \frac{1}{2}$. Determine the value of j such that the vectors $\vec{a} + \vec{b}$ and $2\vec{a} + j\vec{b}$ are perpendicular. [7 marks]

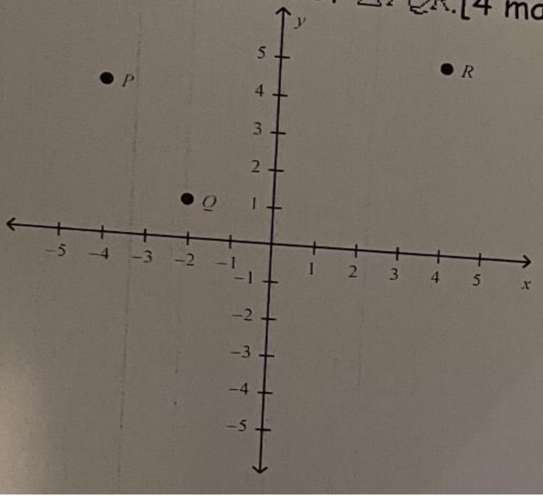


Solution

$$=> (\vec{a} + \vec{b}) \rightarrow \vec{p}$$
$$=> (\overrightarrow{2a} + \overrightarrow{jb}) \rightarrow \vec{q}$$
$$=> For\ perperdicular$$
$$=> \vec{p}\,.\,\vec{q} = 0$$
$$=> (1 * 2) + (1 * j) = 0$$
$$=> 2 + j = 0$$
$$=> j = -2$$

17. a) Determine vector equations for the line containing $\overrightarrow{PQ}$ and the line containing $\overrightarrow{QR}$.[7 marks]

b) Find the area of $\triangle PQR$.[4 marks]



Solution

a)

$$=> vector\ eqn\ containing\ \overrightarrow{PQ}$$
$$=> \overrightarrow{PQ} = \vec{Q} - \vec{P} = (-2, 1) - (-4, 4) = (2, -3)$$
$$=> \tilde{r} = \langle -4, 4 \rangle + \lambda \langle 2, -3 \rangle$$
$$=> vector\ eqn\ containing\ \overrightarrow{QR}$$
$$=> \overrightarrow{QR} = \vec{R} - \vec{Q} = (4, 5) - (-2, 1) = (6, 4)$$
$$=> \tilde{r} = \langle 4, 5 \rangle + \lambda \langle 6, 4 \rangle$$

b)

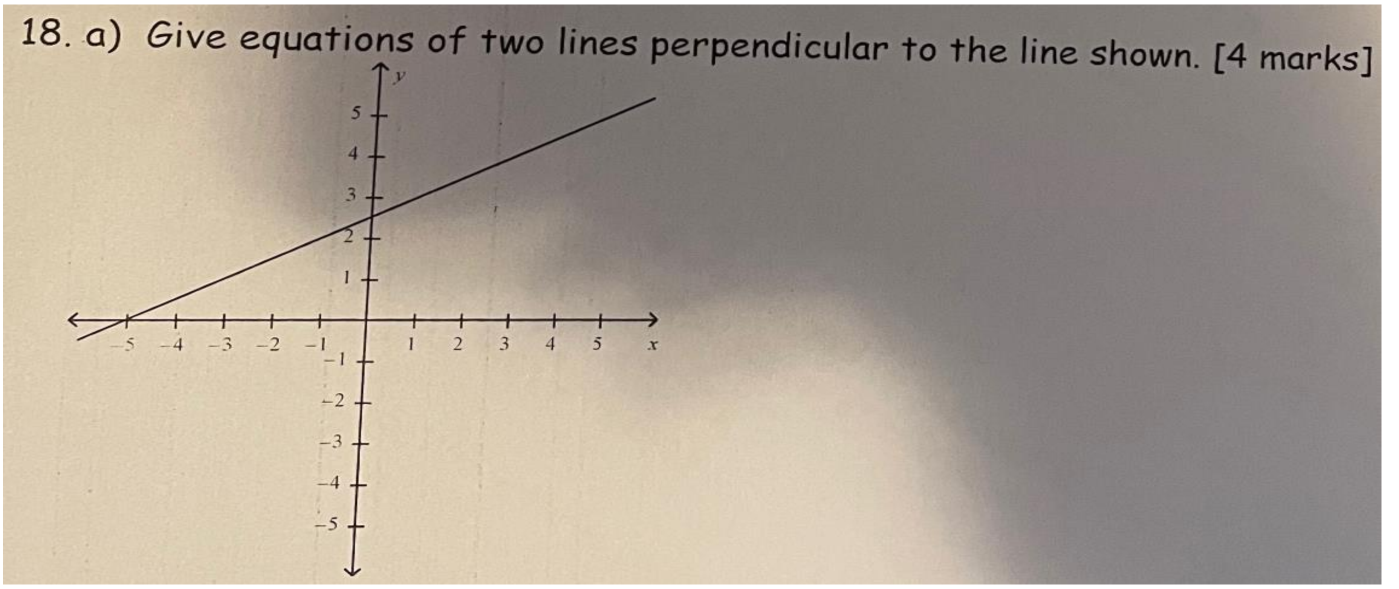
$$=> area\,of\,PQR = \frac{1}{2}|\overrightarrow{PQ} * \overrightarrow{PR}|$$

$$=> \overrightarrow{PQ} = (-2,1) - (-4,4) = (2,-3) = 2i - 3j$$

$$=> \overrightarrow{PR} = (4,5) - (-4,4) = (8,1) = 8i + j$$

$$\overrightarrow{PQ} * \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ 2 & -3 & 0 \\ 8 & 1 & 0 \end{vmatrix} = i(0) - j(0) + k(2+24) = 26k$$

$$\Rightarrow |\overrightarrow{PQ} * \overrightarrow{PR}| = \sqrt{26^2} = 26, \Rightarrow \frac{1}{2} * 26 = 13 \text{ sq. units}$$



Solution

$$=> first\,line$$

$$=> \frac{y-o}{x+5} = -2$$

$$=> y = -2(x+5)$$

$$=> y = -2x - 10$$

$$=> second\,line$$

$$=> \frac{y-2.5}{x-0} = -2$$

$$=> y - 2.5 = -2x$$

$$=> y = -2x + 2.5$$

b) Why are there many possibilities to part a.? [4 marks]

Solution

This is because we already know the gradient for any other line to be perpendicular to the line shown will be equal to -2, implying that we can obtain as many lines as possible at any given coordinate in which we find the slope and equate to -2

19. a) Explain why the lines with equations $2x - 5y + 3 = 0$ and $-4x + 10y - 5 = 1$ are actually the same line. [4 marks]

Solution

They are the same since the first line is just multiplied with a factor of -2 to obtain the second line. Thus, they are actually the same line with a different multiplication factor

b) Write the line from part a. in vector form. [3 marks]

Solution

$$=> at\ x = 0, y = \frac{3}{5}$$
$$=> at\ y = 0, x = \frac{-3}{2}$$
$$=> we\ now\ find\ \overrightarrow{AB} = (\tfrac{-3}{2}, 0) - (0, \tfrac{3}{5}) = (\tfrac{-3}{2}, \tfrac{-3}{5})$$
$$=> in\ vector\ form:$$
$$=> \tilde{r} = (0, \tfrac{3}{5}) + \lambda(\tfrac{-3}{2}, \tfrac{-3}{5})$$

_____ 1. Determine which line is perpendicular to the line $2x - 3y + 17 = 0$.

a. $\vec{r} = (2, -3) + s(3, -2),\ s \in \mathbf{R}$

b. $\vec{r} = (1, 2) + s(3, 2),\ s \in \mathbf{R}$

c. $\vec{r} = (1, 7) + s(2, -3),\ s \in \mathbf{R}$

d. $\vec{r} = s(3, 2),\ s \in \mathbf{R}$

b

_____ 2. Determine which line is perpendicular to the line $2x - 3y + 17 = 0$.

a. $\vec{r} = (2, -3) + s(3, -2),\ s \in \mathbf{R}$

b. $\vec{r} = (1, 2) + s(3, 2),\ s \in \mathbf{R}$

c. $\vec{r} = (1, 7) + s(2, -3),\ s \in \mathbf{R}$

d. $\vec{r} = s(3, 2),\ s \in \mathbf{R}$

b

____ 3. Which of the following is not a plane?

a. $\vec{r} = (1,3,4) + s(2,-1,2) + t(1,1,1)$, $s, t \in \mathbf{R}$

b. $\vec{r} = (2,4,2) + s(1,-2,3) + t(3,2,2)$, $s, t \in \mathbf{R}$

c. $\vec{r} = (3,2,3) + s(4,-4,2) + t(-2,2,-1)$, $s, t \in \mathbf{R}$

d. $\vec{r} = (-2,1,4) + s(2,2,-1) + t(2,2,1)$, $s, t \in \mathbf{R}$

a

____ 4. A plane is defined by the equation $3x - 2z = 4y + 1$. Which of the following is the normal vector of this plane?

a. $\vec{n} = (3,-2,4)$

b. $\vec{n} = (3,4,-2)$

c. $\vec{n} = (3,2,4)$

d. $\vec{n} = (3,-4,-2)$

d

____ 5. Which three points are on the plane $2x - 7y + 3z - 5 = 0$?

a. $P(1,0,1)$, $Q(3,1,2)$, and $R(4,3,6)$

b. $P(1,0,1)$, $Q(2,2,3)$, and $R(3,1,2)$

c. $P(3,1,2)$, $Q(4,3,6)$, and $R(5,0,-2)$

d. $P(4,3,6)$, $Q(0,0,0)$, and $R(3,1,2)$

a

____ 6. Which of these points is not on the plane $3x + 8y - 2z + 10 = 0$?

a. $(-2,-1,-2)$

b. $(2,-2,0)$

c. $(-4,1,3)$

d. $(1,3,-2)$

d

____ 7. If two lines have no points of intersection and the same direction vector, they are:

a. intersecting lines

b. skew lines

c. parallel lines

d. coincident lines

c

b. skew lines

_____ 8. How many solutions are there to the system of equations $2x + 9y = 31$
$-10x + 6y = -2$?

a. 0

b. 1

c. 3

d. Infinity

b

_____ 9. What is the solution for the system of equations $20x - 4y = 8$ and
$5x + 7y = 26$?

a. $x = 1, y = 3$

b. $x = 0, y = -2$

c. $x = \frac{16}{5}, y = \frac{8}{7}$

d. $x = 2, y = 2$

a

_____ 10. What is the point of intersection for the following lines?

$$y = 3x - \frac{3}{2}$$

$$y = \frac{5}{2}x + 3$$

a. $\left(3, \frac{21}{2}\right)$

b. $\left(2, \frac{9}{2}\right)$

c. $\left(9, \frac{51}{2}\right)$

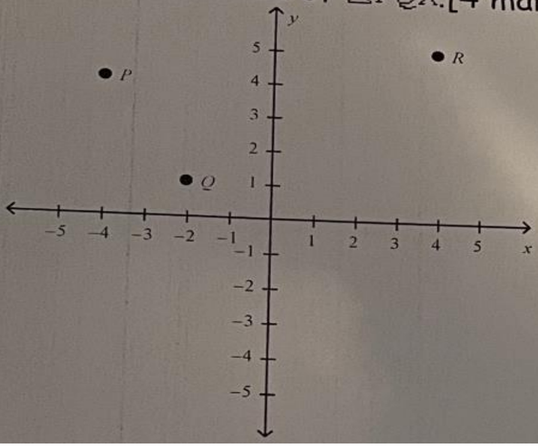
d. none

c

Problem

11. a) Determine vector equations for the line containing $\overrightarrow{PQ}$ and the line containing $\overrightarrow{QR}$.[7 marks]

b) Find the area of $\triangle PQR$.[4 marks]



Solution

a)

$$=> vector\ eqn\ containing\ \overrightarrow{PQ}$$
$$=> \overrightarrow{PQ} = \vec{Q} - \vec{P} = (-2,1) - (-4,4) = (2,-3)$$
$$=> \tilde{r} = \langle -4,4 \rangle + \lambda\langle 2,-3 \rangle$$
$$=> vector\ eqn\ containing\ \overrightarrow{QR}$$
$$=> \overrightarrow{QR} = \vec{R} - \vec{Q} = (4,5) - (-2,1) = (6,4)$$
$$=> \tilde{r} = \langle 4,5 \rangle + \lambda\langle 6,4 \rangle$$

b)

$$=> area\ of\ PQR = \tfrac{1}{2}|\overrightarrow{PQ} * \overrightarrow{PR}|$$
$$=> \overrightarrow{PQ} = (-2,1) - (-4,4) = (2,-3) = 2i - 3j$$
$$=> \overrightarrow{PR} = (4,5) - (-4,4) = (8,1) = 8i + j$$

$$\overrightarrow{PQ} * \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ 2 & -3 & 0 \\ 8 & 1 & 0 \end{vmatrix} = i(0) - j(0) + k(2+24) = 26k$$

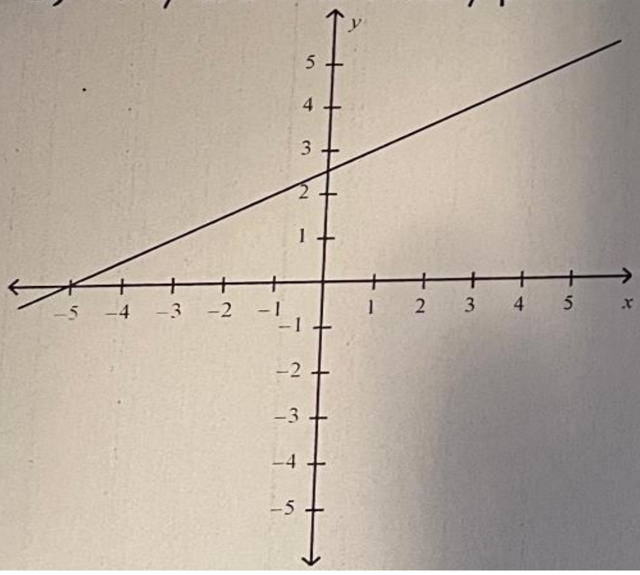
$$\Rightarrow |\overrightarrow{PQ} * \overrightarrow{PR}| = \sqrt{26^2} = 26, \Rightarrow \frac{1}{2} * 26 = 13 \text{ sq. units}$$

12. a) Give equations of two lines perpendicular to the line shown. [4 marks]

Solution

$$=> first\ line$$
$$=> \frac{y-o}{x+5} = -2$$
$$=> y = -2(x+5)$$
$$=> y = -2x - 10$$
$$=> second\ line$$
$$=> \frac{y-2.5}{x-0} = -2$$
$$=> y - 2.5 = -2x$$
$$=> y = -2x + 2.5$$

b) Why are there many possibilities to part a.? [4 marks]



Solution

This is because we already know the gradient for any other line to be perpendicular to the line shown will be equal to -2, implying that we can obtain as many lines as possible at any given coordinate in which we find the slope and equate to -2

13. a) Explain why the lines with equations $2x - 5y + 3 = 0$ and $-4x + 10y - 5 = 1$ are actually the same line. [4 marks]

b) Write the line from part a. in vector form. [3 marks]

Solution

a)

They are the same since the first line is just multiplied with a factor of -2 to obtain the second line. Thus, they are actually the same line with a different multiplication factor

b)

$$=> at\ x = 0, y = \frac{3}{5}$$
$$=> at\ y = 0, x = \frac{-3}{2}$$
$$=> we\ now\ find\ \overrightarrow{AB} = (\frac{-3}{2}, 0) - (0, \frac{3}{5}) = (\frac{-3}{2}, \frac{-3}{5})$$
$$=> in\ vector\ form:$$
$$=> \tilde{r} = (0, \frac{3}{5}) + \lambda(\frac{-3}{2}, \frac{-3}{5})$$

14. What are the slopes of the following lines, and what type of intersection do you expect?
Check your intersection against your expectation. [6 marks]

$4x - y = 8$

$-2x + 3y = 6$

Solution

$$=> from\ , 4x - y = 8\ we\ have, y = 4x - 8$$
$$=> from, -2x + 3y = 6, we\ have, y = \frac{2}{3}x + 2$$
$$=> slopes\ are, 4\ for\ line1\ , and\ \frac{2}{3}\ for\ line2$$

The two lines intersect with each other at an acute angle on the positive first quadrant

Let's check the angle of intersection using the slopes

$$=> \theta = tan^{-1}|\frac{\frac{2}{3}-4}{1+4*\frac{2}{3}}| = |\frac{-10}{11}| = \frac{10}{11}$$
$$=> \theta = 42.27$$

This is correct as per the expectation being an acute angle with the first quadrant and its verified having an angle of 42.27

15. Determine values for k for which the following system has one solution, no solutions, and an infinite number of solutions. [8 marks]

$2k + 4y = 20$

$3x + 6y = 30$

Solution

$$\Rightarrow \begin{pmatrix} 2k & 4 \\ 3 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 20 \\ 30 \end{pmatrix} \longrightarrow \begin{pmatrix} 2k & 4| & 20 \\ 3 & 6| & 30 \end{pmatrix} \longrightarrow \begin{pmatrix} 2k & 4| & 20 \\ 0 & 12-12k| & 20-60k \end{pmatrix}$$

(i) No solution

$$=> 12-12k = 0, and\ , 20-60k \neq 0$$
$$=> k = 1, k \neq \tfrac{1}{3}$$

(ii) One solution

$$=> 12-12k = 0, and\ , 20-60k = 0$$
$$=> k \neq 1, k = \tfrac{1}{3}$$

(iii) Infinite solutions

$$=> (12-12k)y = 20-60k$$
$$=> \forall k \in R,\ but\ not\ , k \neq 1\ and\ k \neq \tfrac{1}{3}$$

16. Two lines with slopes $m_1 = \frac{4}{3}$ and $m_2 = -\frac{7}{2}$ intersect at $(3,4)$. Determine the equations of the two lines and check your answer by solving them. [10 marks]

Solution

$$=> 3 = \frac{c2-c1}{\frac{-7}{2}-\frac{4}{3}} = 3 = -\tfrac{6}{29}c2 + \tfrac{6}{29}c1 - - - (i)$$
$$=> 4 = \frac{\frac{-7}{2}c1 - \frac{4}{3}c2}{-\frac{7}{2}-\frac{4}{3}} = 4 = \tfrac{21}{29}c1 + \tfrac{8}{29}c2 - - - (ii)$$
$$solving\ for\ c1, c2\ simulteneously\ from\ (i), (ii)$$
$$=> 87 = -6c2 + 6c1$$
$$=> 116 = 8c2 + 21c1$$
$$=> c1 = 8, c2 = -6.5$$
$$=> eqn1, y = \tfrac{4}{3}x + 8$$
$$=> eqn2, y = -\tfrac{7}{2}x - 6.5$$
$$=> checking\ intersection\ point$$

$$=> x_0 = \frac{-[(3*13)-(2*-24)]}{(-4*2)-(7*3)} = \frac{-87}{-29} = 3$$
$$=> y_0 = \frac{(-24*7)-(13*-4)}{(-4*2)-(7*3)} = \frac{-116}{-29} = 4$$
$$=> (x_0, y_0) = (3,4)$$

THE END