

Given that the daily share prices are denoted by $x_0, x_1, \dots, x_t, x_{t+1}$,

\Rightarrow Then we need to develop a linear model which can be used to train a model based on this data value

\Rightarrow Using share prices on 2 preceding days, then our predicted y value is given by

$$y = \vec{f}(x)$$

\Rightarrow Thus the general function for y is given by

$$y = \beta_0 + \beta_1 x_0 + \beta_2 x_1 + \dots + \beta_t x_t + \beta_{t-1} x_{t-1}$$

where $\beta_0 = y$ -intercept parameter

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1) From the above general equation (1) then by using simple Regression equation

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where $E(y)$ is the expected value of y for given values of $x_0, x_1, \dots, x_t, x_{t-1}$

\Rightarrow Since from the given share prices data, the values are in increasing sequence, then our expected value equals to

$$E(y) = \beta_0 + \beta_1 x$$

\Rightarrow So now for \hat{y} which is the mean value of y for given x value will be given as

$$\hat{y} = b_0 + b_1 x$$

Therefore the linear model for prediction will be given as

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$$e = \beta_0 + \beta_1 x + \varepsilon - (\hat{\beta}_0 + \hat{\beta}_1 x)$$

$$\Rightarrow e = (\beta_0 - \hat{\beta}_0) + (\beta_1 - \hat{\beta}_1)x + \dots + (\beta_t - \hat{\beta}_t)x_t + (\beta_{t+1} - \hat{\beta}_{t+1})x_{t+1} + \varepsilon$$

\Rightarrow Thus the avg min function will be written as

$$\text{avg min } \sum_{i=1}^n (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

\Rightarrow Differentiating the w.r.t β_0 , we have

$$\sum_{i=1}^n (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0$$

$$\Rightarrow \hat{\beta}_0 = \frac{\sum_{i=1}^n y_i}{n} - \hat{\beta}_1 \frac{\sum_{i=1}^n x_i}{n}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \quad \text{where } \bar{y} \text{ and } \bar{x} \text{ are the sample mean scores}$$

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\Rightarrow Again, differentiating the avg min function w.r.t β_1 , we have

$$\sum_{i=1}^n (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)(x_i) = 0$$

$$\Rightarrow \sum_{i=1}^n x_i y_i - \hat{\beta}_0 \sum_{i=1}^n x_i - \hat{\beta}_1 \sum_{i=1}^n x_i^2 = 0$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n x_i y_i - \bar{y} \sum_{i=1}^n x_i}{\sum_{i=1}^n x_i^2 - \bar{x} \sum_{i=1}^n x_i}$$

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⇒ Thus, using $\hat{\beta}_0$ and $\hat{\beta}_1$, our linear model function becomes,

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

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(3) Using Δ_t to predict Δ_{t+1} ,

$$\text{let } y = f(x) + \varepsilon$$

$$\text{where } f(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_t x_t$$

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 $\Rightarrow n \times (p-1)^2$

\Rightarrow From the share prices vals $(x_0, x_1, \dots, x_{t-1})$ we have

$$y = X\beta + \varepsilon \quad \text{with} \quad \hat{y} = X\hat{\beta}$$
 \Rightarrow Finding the estimated \hat{y} for the Δ_{t+1} we have

$$\hat{y} = X\hat{\beta}_0 + x_1 \hat{\beta}_1 + \varepsilon$$

(4) Using Δ_t and x_t to predict Δ_{t+1}
 \Rightarrow From $\hat{y} = f(x) = X\hat{\beta}$
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$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \dots + \hat{\beta}_t x_t + \varepsilon$$
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=> These 2 models take in the independent x variable and then integrate them into making predictions with no bias term, unlike in (3) and (4) where the bias term plays a key factor in detecting the

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=> First, we generate the samples, training datagen, in our case we are using the share price values,

=> We use the random function to perform shuffling of the data so as to obtain the training and testing data

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\Rightarrow Next, we treat the training samples as our independent variables whose real values are known.

\Rightarrow We then feed forward these training samples to our linear model function

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\Rightarrow This will yield an estimate of the predicted y value
 \Rightarrow Since our linear model has been trained perfectly on the share price data values, it can make precise predictions (\hat{y}) which is the unknown function to be estimated by our linear model algorithm