1) Vector perpendicular to sum of $\vec{v}+\vec{w}$

Soln

we have.

$\vec{v} = (1,1)$ and $\vec{w} = (2,2)$

$\Rightarrow \vec{v} = 1i + 1j$ and $\vec{w} = 2i + 2j$

$\Rightarrow Z \Rightarrow$

$$\Rightarrow \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 2 & 2 & 0 \end{vmatrix}$$

$$\Rightarrow i(0) + j(0) + 0\vec{k}$$

$\Rightarrow$ This is equal to

$$Z = 3i + 3j$$

$$\Rightarrow \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

Vector perpendicular is given by

$$\Rightarrow \frac{3i}{\sqrt{18}} + \frac{3j}{\sqrt{18}} = \frac{3i}{3\sqrt{2}} + \frac{3j}{3\sqrt{2}}$$

$$= \frac{i}{\sqrt{2}} + \frac{j}{\sqrt{2}}$$

2) directional derivative

$f(x,y) = \frac{x}{x+y}$ at $P(1,2)$ in direction $Q(-1,5)$

Soln.

$DD = \nabla f \cdot \hat{a}$

$\Rightarrow$

$f_x(x,y) = \frac{(x+y) - x}{(x+y)^2} = \frac{y}{(x+y)^2}$

$f_y(x,y) = \frac{0 - x}{(x+y)^2} = \frac{-x}{(x+y)^2}$

$$\left(\frac{y}{(x+y)^2}\right)i + \left(\frac{-x}{(x+y)^2}\right)j + 0k \Bigg|_{(1,2)}$$

$$= \frac{2}{9}i - \frac{1}{9}j + 0k$$

$$\hat{a} = \frac{-i + 5j}{\sqrt{1+25}} = \frac{-i + 5j}{\sqrt{26}}$$

$$\Rightarrow \left(\frac{2}{9}i - \frac{1}{9}j\right)\left(\frac{-i + 5j}{\sqrt{26}}\right) = \frac{-2}{9\sqrt{26}} - \frac{5}{9\sqrt{26}}$$

$$= \frac{1}{9\sqrt{26}}(-2 - 5) = \frac{-7}{9\sqrt{26}}$$

3) Determine critical points of

$f(x,y) = x^2 + 2y^2 - x^2y$ and classify them

Soln,

$f_x(x,y) = 2x - 2xy = 0$

$f_y(x,y) = 4y - x^2 = 0$

$\Rightarrow 2x(1-y) = 0$ and

$4y - x^2 = 0$

$\Rightarrow 2x = 0$ and $(1-y) = 0$

$\Rightarrow x = 0$ and $1 - y = 0 \Rightarrow y = 1$

at $x = 0$ | we have

$4y - 0 = 0 = y = 0$

at $y = 1$, we have $4 = x^2 = x = 2$

Critical points

$(0,1)$, $(0,0)$, $(2,1)$

$f_{xx}(x,y) = 2 - 2y$, $f_{yy}(x,y) = 4$

$f_{xy}(x,y) = -2x$

at $(0,0) = \left| f_{xx} \right|_{(0,0)} = 2 > 0$ $f_{yy} = 4 > 0$

$\Delta > 0$ Thus $(0,0)$ is a relative minimum point ~~saddle point~~

(ii) $(0,1)$

$$f_{xx} = 0 \geq 0 \qquad f_{yy} = 4 > 0$$

$\Delta = 0$ ~~relative maximum point~~

Thus is a saddle point

(iii) $(2,1)$ $\quad f_{xx} = 0 \geq 0 \quad f_{yy} = 4 > 0$

$$\Delta = -4 < 0$$

Thus $0 - -4 = 4 > 0$

Thus is relative minimum point

A Company produces products A and B as follows

$$x^2 + y^2 = 100 \text{ curve}$$

Soln

We have that let

$$f(x,y) = x^2 + y^2 - 100$$

$$f_x(x,y) = 2x - 0 = 0 \quad \text{. and}$$

$$f_y(x,y) = 2y - 0 = 0 \Rightarrow x = 0, \; y = 0$$

Thus at $x = 0$ and $y = 0$, we have the maximum revenue output

Reverse the integration order

$$\int_{y^{\frac{1}{3}}}^{2}\int_{0}^{8} e^{x^4}\, dy\, dx$$

Soln

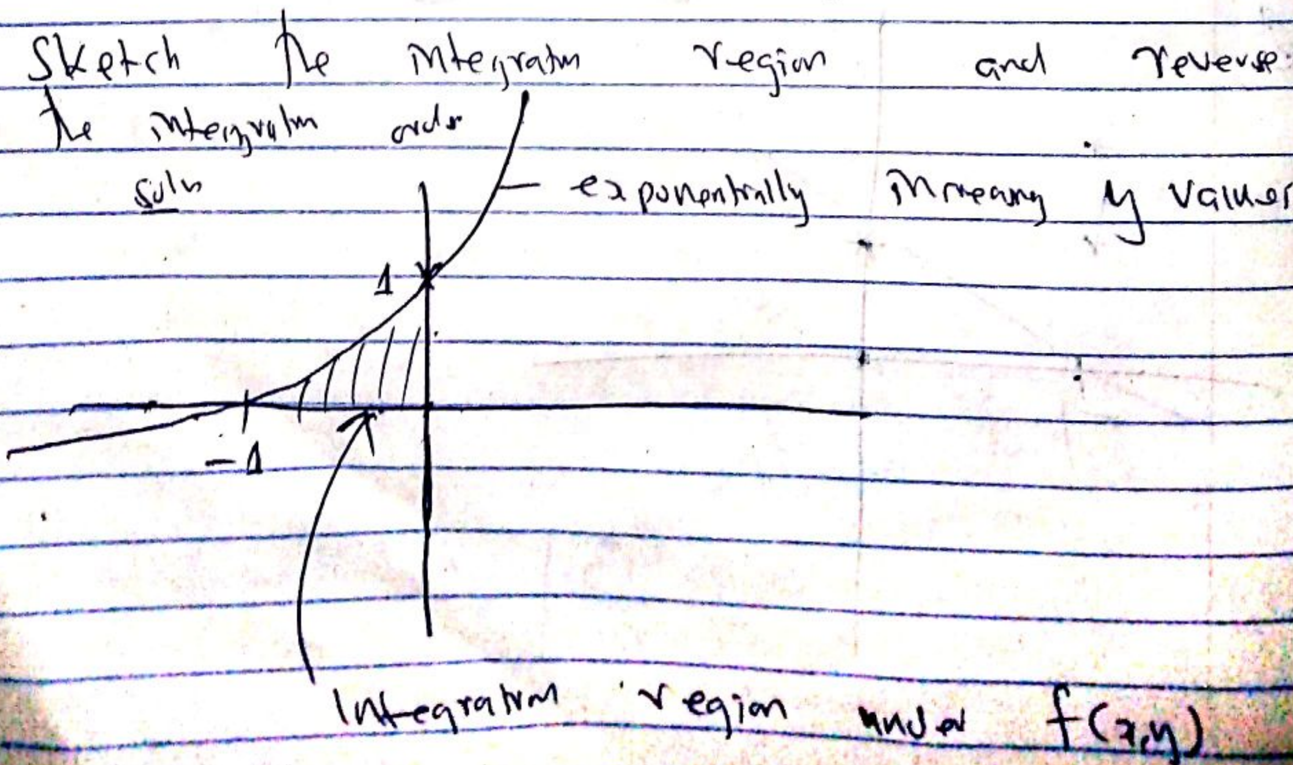
$$\Rightarrow \int_{y^{\frac{1}{3}}}^{2} e^{x^4} y \Big|_{0}^{8} dx = \int_{y^{\frac{1}{3}}}^{2} 8 e^{x^4} dx.$$

$$\Rightarrow \frac{8e^{x^4}}{4x^3}\Bigg|_{y^{\frac{1}{3}}}^{2} = \frac{2e^{x^4}}{x^3}\Bigg|_{y^{\frac{1}{3}}}^{2}$$

$$= \frac{2e^{16}}{8} - \frac{2e^{y^{\frac{4}{3}}}}{y} = \frac{e^{16}}{4} - \frac{2e^{y^{\frac{4}{3}}}}{y}$$

Sketch the integration region and reverse the integration order

Soln



For reverse integram order we have

$$\int_{-1}^{e^2}\int_{0}^{1} f(x,y)\, dx \cdot dy$$

4) Determine if sequence converges or not

$$b_n = \frac{\ln \ln (n+2)}{(n+2)}$$

Soln

We show that given $\varepsilon > 0 \ \exists \ N(\varepsilon) \in -\exists$ such that

$$|b_n - b| < \varepsilon \quad \forall n > N(\varepsilon)$$

$$\Rightarrow \left| \frac{\ln(\ln(n+2))}{n+2} - 0 \right| \Rightarrow \left| \frac{\ln \ln (n+2)}{n+2} \right| < \varepsilon$$

$$\Rightarrow \frac{1}{n+2} < \varepsilon \Rightarrow n+2 < \frac{1}{\varepsilon}$$

$$= n > \frac{1}{\varepsilon} - 2$$

$$\Rightarrow N(\varepsilon) = \frac{1}{\varepsilon} - 2$$

Thus the sequence is convergent

$$\lim_{n \to \infty} \frac{\ln \ln (n+2)}{n+2} = \frac{\ln \ln (n+2)}{\infty + 2} = \frac{\ln \ln (n+2)}{\infty}$$

$$= 0$$

Thus the sequence is convergent

$1 + \frac{4}{5} + \left(\frac{4}{5}\right)^2 + \dots$

Determine if Series is is conversent or not and find its sum.

$$\sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^{n-1}$$

Soln

$$= \sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^{n} \cdot \left(\frac{4}{5}\right)^{-1} = \left(\frac{4}{5}\right)^{-1} \sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^{n}$$

$$\Rightarrow r = \frac{4}{5} \qquad |r| < 1$$

This is a geometric series which is Convergent

$\Rightarrow$ Sum is given by

$$\frac{a}{1-r} = \frac{1}{1-\frac{4}{5}}$$

$$= \underline{\underline{5}}$$

6) Consider the function $f(x) = \cos x$ find taylors series for $a = \pi$

Soln

$f(x) = \cos x \qquad f(\pi) = -1 = \cos 180 = -1$

$f'(x) = -\sin x \qquad f'(\pi) = -\sin \pi = 0$

$f''(x) = -\cos x \qquad f''(\pi) = 1$

$f'''(x) = \sin x \qquad f'''(\pi) = 0$

$f^{iv}(x) = \cos x \qquad f^{(iv)}(\pi) = -1$

Thus the taylor series expansion is given by

$$f(x) = f(\pi) + f'(\pi)(x-\pi) + \frac{f''(\pi)(x-\pi)^2}{2!} + \frac{f'''(\pi)(x-\pi)^3}{3!}$$

$$+ \frac{f^{(iv)}(\pi)(x-\pi)^4}{4!} + \cdots$$

$$\Rightarrow -1 + 0 + \frac{1(x-\pi)^2}{2!} + 0 - \frac{1(x-\pi)^4}{4!} + \cdots$$

$$\Rightarrow -1 + \frac{(x-\pi)^2}{2} - \frac{1(x-\pi)^4}{24} + \cdots$$

$$\Rightarrow -1 + \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24} + \cdots$$