1) Dual expression computation Ridge Regression

Soln

Using the slack variable $\xi_i = y_i - w^T x_i$

First we note that for minimization

$$\Rightarrow \frac{1}{2}\|w\|^2 + C \sum_{i=1}^{n} \xi_i$$ which is subject

to $y_i (w^T x_i) \geq \xi_i \quad \forall_i$

$\Rightarrow$ In this case we only have 2 opt variables w, ξ_i

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$$\min_{w,\xi} \frac{1}{2}\|w\|^2 + C \sum_{i=1}^{n} \xi_i$$

Subject to

$$1 - \xi - y_i (w^T x_i) \leq 0 \quad \forall_i$$

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Subject to

$$1 - \xi - y_i(w^T x_i) \leq 0 \quad \forall_i$$

$$-\xi_i \leq 0 \quad \forall_i$$

The Langrangian of the above becomes

$$L(w, \xi, \mu, \lambda) = \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n} \xi_i + \sum_{i=1}^{n} \mu_i(1-\xi$$

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$$- \sum_{i=1}^{n} \lambda_i \xi_i$$

We then take the derivative of L wrt ξ_i

to $y_i(w^T x_i)$

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$$1 - \xi - y_i(w^T x_i) \le 0 \quad \forall_i$$

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$$L(w, \xi, \mu, \lambda) = \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n} \xi_i + \sum_{i=1}^{n} \mu_i (1 - \xi_i - y_i(w^T x_i)) - \sum_{i=1}^{n} \lambda_i \xi_i$$

We then take the derivative of L w.r.t ξ_i to form the dual form, then

$$\min_{w,\xi} \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n} \xi_i$$

Subject to

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$$\frac{\partial L}{\partial \xi_i} = 0 \Rightarrow \mu_i + \lambda_i = C \quad \forall_i$$

=) $1 - \xi_i - y_i(w^T x_i) \le 0 \quad \forall_i$

This implies that

$\min_{w,\xi} \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n}\xi_i$

Subject to

$$1 - \xi_i - y_i(w^T x_i) \le 0 \quad \forall i$$
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The Lagrangian of the above becomes

$$L(w,\xi,\mu,\lambda) = \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n}\xi_i + \sum_{i=1}^{n}\mu_i(1-\xi_i-y_i(w^T x_i)) - \sum_{i=1}^{n}\lambda_i\xi_i$$

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$$\mu_i(1 - \xi_i - y_i(w^T x_i)) = 0 \; ; \; \lambda_i \xi_i = 0 \quad \forall i$$

$\Rightarrow$ Thus for the complementary slackness condition we have

$$1 - y_i(w^T x_i) = 0$$

$\Rightarrow$ Finding the value for all μ_i the dual function takes the form

$$Q(\mu,\lambda) = \inf_{w,\xi} L(w,\xi,\mu,\lambda)$$

but from the Lagrangian we had that the term

$$\sum_i (C - \mu_i - \lambda_i)\xi_i$$

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=> Finally taking the infimum w.r.t ξ_i and imposing $C - \mu_i - \lambda_i$ $\forall_i$ yields the dual problem

$$\max_{\lambda} q(\lambda) = \sum_{i=1}^{n} \lambda_i - \frac{1}{2} \sum_{i,j=1}^{n} \lambda_i \lambda_j y_i y_j x_i^T x_j$$

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$$\frac{1}{2} \|w\|^2 + C \sum_{i=1}^{n} \xi_i$$

Subject to

$$y_i (w^T x_i) \geq 1 - \xi_i \quad \text{and} \quad C = N \to \infty$$

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$$\frac{1}{2}\|w\|^2 + C \sum_{i=1}^{n} \xi_i$$

Subject to

$$y_i (w^T x_i) \geq 1 - \xi_i \qquad \text{as } C = N \to \infty$$

) Dual ~~expression~~ C

2) Dual expression computation for SVM

soln

using the slack variable

$$\xi_i = y_i - w^T x_i - b$$

For the minimization $\frac{1}{2} w^T w + C \sum_{i=1}^{n} \xi_i$

Subject to

$$y_i (w^T x_i + b) \geq 1 - \xi_i \quad , \forall i$$

with $\xi_i \geq 0 \quad \forall i$

Opt variables in this case w, b, ξ_i

2) Dual expression computation for SVM

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Opt variables in the case w, b, ξ_i

⇒ Note that for all poible deviations for change in $w^T x$ and b we have.

$$w^T x + b = 1$$
$$w^T x + b = 0$$
$$w^T x + b = -1$$

⇒ We then consider $w^T x + b = 0$, and let the

Subject to

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Note $w^T w =$

Subject to

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⇒ We then consider $w^T x + b = 0$, and let the optimization problem be given as

$$\min_{w, b, \xi_i} \frac{1}{2} w^T w + C \sum_{i=1}^{n} \xi_i \qquad \text{note } w^T w = \|w\|^2$$

Subject to

$$1 - \xi_i - y_i (w^T x_i + b) \leq 0 \quad \forall_i$$
$$-\xi_i \leq 0 \quad \forall_i$$

⇒ Thus the Lagrangian becomes

$$L(w, b, \xi, \mu, \lambda) = \frac{1}{2} w^T w + C \sum_{i=1}^{n} \xi_i + \cdots +$$

$$+ \sum_{i=1}^{n} \mu_i (1 - \xi_i - y_i (w^T x_i + b)) - \sum_{i=1}^{n} \lambda_i \xi_i$$

$\Rightarrow$ Taking derivatives of L w.r.t b and ξ respectively we have

$$\nabla_w L = 0 \Rightarrow w^* = \sum_{i=1}^{n} \mu_i x y_i x_i$$

$$\Rightarrow \frac{\partial L}{\partial b} = 0 \Rightarrow \sum \mu_i x y_i = 0$$

$$\Rightarrow \frac{\partial L}{\partial \xi_i} = 0 \Rightarrow \mu_i x \lambda_i = C \quad \forall_i$$

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$$\Rightarrow 1 - \xi_i - y_i (w^T x_i + b) \leq 0 \; ; \; \xi_i \geq 0 \quad \forall_i$$

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$1 - y_i (w^T x_i + b) = 0 \quad —①$

$b = y_i - x_j^T w$, j such that $0 < \mu_j < C \quad —②$

$\Rightarrow$ Thus finding value for all μ_i, the dual functi

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$$Q(\mu, \lambda) = \inf_{w, b, \xi} L(w, b, \xi, \mu, \lambda)$$

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$\Rightarrow \mu_i(1 - \xi_i - y_i(w^T x_i + b)) = 0 \quad ; \quad \lambda_i \xi_i = 0 \quad , \forall_i$

$\Rightarrow$ For the complementary slackness condition, we have that

$$1 - y_i(w^T x_i + b) = 0 \quad \text{---(1)}$$

$$b = y_i - x_j^T w \quad , j \text{ such that } 0 < \mu_j < C \quad \text{---(2)}$$

$\Rightarrow$ Thus finding value for all μ_i, the dual function becomes

$$q(\mu, \lambda) = \inf_{w, b, \xi} L(w, b, \xi, \mu, \lambda)$$

but from lagrangian we have the term

$$\sum_i (C - \mu_i - \lambda_i)\xi_i$$

$\Rightarrow$ Now taking the infimum w.r.t ξ_i and imposing $C = \mu_i + \lambda_i \quad \forall_i$, the dual problem becomes

$$\max_{\mu} q(\mu) = \sum_{i=1}^{n} \mu_i - \frac{1}{2} \sum_{i,j=1}^{n} \mu_i \mu_j y_i y_j x_j^T x_j$$

Subject to

$$0 \leq \mu_i \leq C \quad i = 1, \ldots n \qquad \sum_{i=1}^{n} y_i \mu_i = 0$$

And for the primal problem we have

$$\text{minimize } \frac{1}{2} w^T w + C \sum_i \xi_i$$

=) Now taking the infimum w.r.t ξ_i and imposing $C = \alpha_i + \lambda_i$ $\forall_i$, the dual problem becomes;

$$\max_{\alpha} \; q(\alpha) = \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i,j=1}^{n} \alpha_i \alpha_j y_i y_j x_j^T x_j$$

Subject to

$$0 \le \alpha_i \le C \quad i = 1, \dots n \qquad \sum_{i=1}^{n} y_i \alpha_i = 0$$

and for the primal problem we have

$$\text{Minimize}_p \; \frac{1}{2} w^T w + C \sum_{i=1}^{n} \xi_i$$

Subject to

$$y_i (w^T x_i + b) \ge 1 - \xi_i \quad \forall_i$$

as $\epsilon = N \to \infty$

How to obtain

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(c) How to obtain prediction for ~~x~~

Soln

=) We let K be a $n \times n$ matrix with $K_{ij} = k(x_i, x_j)$ such that K is positive semi-definite $\forall n$ and all data sets $\{x_1, \dots x_n\}$

and for the primal problem we have

minimize $\frac{1}{2} w^T w + C \sum_{i=1}^{n} \xi_i$

Subject to

$y_i (w^T x_i + b) \geq 1 - \xi_i \quad \forall_i$

as $\epsilon = N \to \infty$

3 (a) How to obtain prediction for x_t

Soln

⇒ We let k be a $n \times n$ matrix with $k_{ij} = k(x_i, x_j)$ such that k is positive semi-definite $\forall n$ and all data sets $\{x_1, \ldots x_n\}$

⇒ Thus, given any n, and feature vectors $x_1, \ldots x_n$ we will have that for all scalars $c_1, \ldots c_n$

$$\sum_{i,j=1}^{n} c_i c_j k(x_i, x_j) \geq 0 \quad \longleftarrow (1)$$

minimize $\frac{1}{2} w^T w + C \sum_{i=1}^{n} \xi_i$

Subject to

$y_i (w^T x_i + b) \geq 1 - \xi_i \quad \forall_i$

as $\epsilon = N \to \infty$

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$$\sum_{i,j=1}^{n} c_i c_j k(x_i, x_j) \geq 0 \quad \longleftarrow (1)$$

⇒ Now for the kernel function given below

$$k(x_i, x_j) = \exp\left(-\|x_i - x_j\|^2\right) / 2\sigma^2$$

Subject to $y_i (w^T x_i + b) \geq 1 - \xi_i$ $\forall_i$ as $C = N \to \infty$

3 (a) How to obtain prediction for x_t

Soln

⇒ We let K be a $n \times n$ matrix with $k_{ij} = k(x_i, x_j)$ such that K is positive semi-definite $\forall n$ and all data sets $\{x_1 \dots x_n\}$

⇒ Thus, given any n, and feature vectors $x_1 \dots x_n$ we will have that for all scalars $c_1 \dots c_n$

$$\sum_{i,j=1}^{n} c_i c_j k(x_i, x_j) = 0 \quad \longleftarrow (1)$$

⇒ Now for the kernel function given below

$$k(x_i, x_j) = \exp\left(-\|x_i - x_j\|^2\right) / 2\sigma^2$$

any data values x_i and x_j feed forward into the function (1) will yield the result of the predicted x_t values

⇒ This can be generally classified by the ~~function~~ function $f(x)$ for x_t prediction as

$$f(x) = \sum_{i=1}^{n} \alpha_i y_i k(x_i, x_j) + b$$

$$= \sum_{i=1}^{n} \alpha_i y_i e^{\frac{\|x_i - x_j\|^2}{2\sigma^2}} + b$$

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4) Maximum likelihood Normal distribution

=) Compute MAP to the MLE estimation (MAP MLE) as $N \to \infty$

Soln

Given $x_1, x_2, x_3, \ldots x_n$ for estimation we know that the p.d.f for normal distribution is generally

$$= \sum_{i=1}^{n} \alpha_i y_i e^{\frac{\|x_i - x_j\|^2}{2\sigma^2}} + b$$

4) Maximum likelihood Normal distribution

=) Compute MAP to the MLE estimation (MAP MLE) as $N \to \infty$

Soln

Given $x_1, x_2, x_3, \ldots x_n$ for estimation we know that the p.d.f for normal distribution is generally given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

) If we apply the equation to n - number of observation we have.

$$\frac{1}{(x_i - \mu)^2}$$

4) Maximum likelihood Normal distribution
=) Compute MAP to the MLE estimation
(MAPLE) as $N \to \infty$

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Given $x_1, x_2, x_3, \ldots x_n$ for estimation we know that the p.d.f for normal distribution is generally given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

=) If we apply the equation to n-number of observation we have

$$f(x, \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2\sigma^2}(x_i-\mu)^2}$$

$$L(\mu, x_i, \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x_i-\mu)^3}{2\sigma^2}}$$

=) Compute MAP to the MLE estimation
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Given $x_1, x_2, x_3, \ldots x_n$ for estimation we know that the p.d.f for normal distribution is generally given by

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$$L(\mu, x_i, \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x_i-\mu)^3}{2\sigma^2}}$$

=) Simplifying this we obtain

$$\Rightarrow \frac{1}{\sigma\sqrt{2\pi}} \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot \cdots \cdot \frac{1}{\sigma\sqrt{2\pi}} = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n$$

$$= e^{-\frac{1}{2\sigma^2}(x_1-\mu)^2} \cdot e^{-\frac{1}{2\sigma^2}(x_2-\mu)^2} \cdot \cdots \cdot e^{-\frac{1}{2\sigma^2}(x_i-\mu)^2}$$

$$= e^{-\frac{1}{2\sigma^2}\cdot\sum_{i=1}^{n}(x_i-\mu)^2} = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n \cdot e^{-\frac{1}{2\sigma^2}(x_i-\mu)^2}$$

Taking the log of the above function we have

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$$= e^{-\frac{1}{2\sigma^2}\cdot\sum_{i=1}^{n}(x_i-\mu)^2} = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n \cdot e^{-\frac{1}{2\sigma^2}(x_i-\mu)^2}$$

=) Taking the log of the above function we have

$$\log\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n + \log\left(e^{-\frac{1}{2\sigma^2}\sum(x_i-\mu)^2}\right)$$

$$\Rightarrow \frac{1}{\sigma\sqrt{2\pi}} \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot \cdots \cdot \frac{1}{\sigma\sqrt{2\pi}} = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n$$

$$= e^{-\frac{1}{2\sigma^2}(x_1-\mu)^2} \cdot e^{-\frac{1}{2\sigma^2}(x_2-\mu)^2} \cdot \cdots \cdot e^{-\frac{1}{2\sigma^2}(x_i-\mu)^2}$$

$$= e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2} = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n \cdot e^{-\frac{1}{2\sigma^2}(x_i-\mu)^2}$$

$\Rightarrow$ Taking the log of the above function we have

$$\log\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n + \log\left(e^{-\frac{1}{2\sigma^2}\sum(x_i-\mu)^2}\right)$$

$$\Rightarrow n\log\left(\frac{1}{\sigma\sqrt{2\pi}}\right) + -\frac{1}{2\sigma^2}\sum(x_i-\mu)^2 = n\log(1) - n\log(\sigma(2\pi$$

$$- \frac{1}{2\sigma^2}\sum(x_i-\mu)^2$$

$$= e^{-\frac{1}{2\sigma^2}(x_1-\mu)^2} \cdot e^{-\frac{1}{2\sigma^2}(x_2-\mu)^2} \cdot \cdots \cdot e^{-\frac{1}{2\sigma^2}(x_i-\mu)^2}$$

$$= e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2} = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n \cdot e^{-\frac{1}{2\sigma^2}(x_i-\mu)^2}$$

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$= e^{\frac{1}{2\sigma^2} \cdot \sum_{i=1}^{n} (x_i - \mu)^2} = \left(\frac{1}{\sigma\sqrt{2\pi}}\right) \cdot e$

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$$\Rightarrow \mu = \frac{1}{n}\sum_{i=1}^{n} x_i \Rightarrow \text{estimation}$$

5) Decision Trees

a) Discussion.

In this problem we gonna be discussing our decision tree information gain based on Entropy which is used to determine or Calculate the purity of a substance. In other words we going to use Information gain to help in getting and understanding its role in building decision tree by getting the total average for entropy based on each split in the below example

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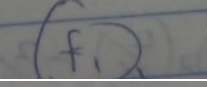

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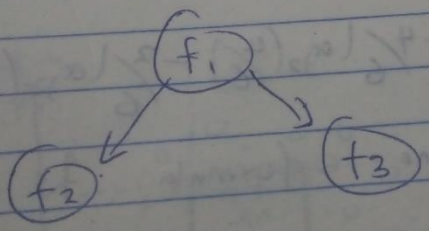

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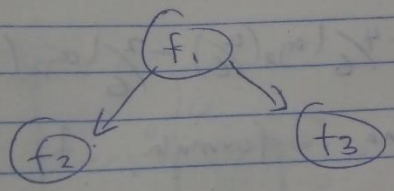
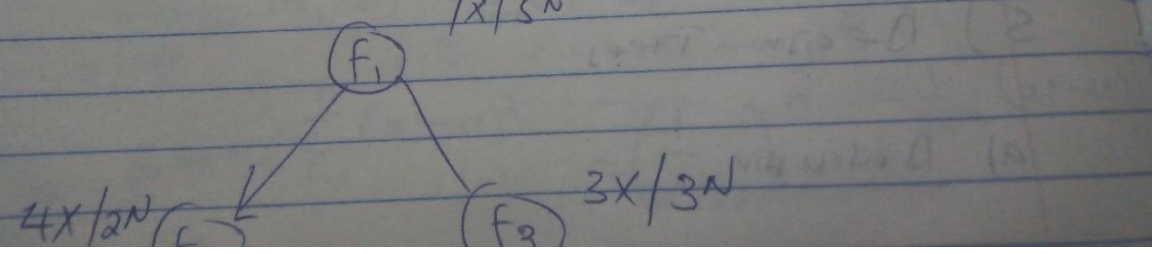


Note that Gain function is given by

$$Gain(S, A) = H(S) - \sum_{v \in Val} \frac{S_v}{S} H(S_v)$$

where S_v is the subset after splitting and
S is the total sub set

=> Now, Lets say we have to split (f_1) and we are splitting it based on (f_2) and (f_3) as shown below



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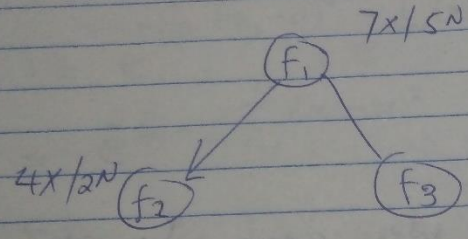

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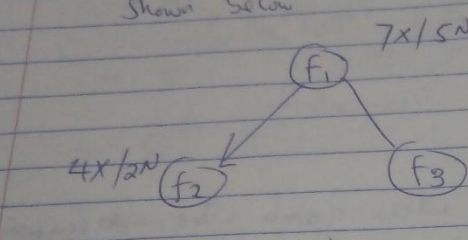

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$$H(S) = -P_+ \log_2 P_+ - P_- \log_2 P_-$$

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0 -> For complete purity

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$$Gain(S, f_1) = H(S) - \frac{6}{12} H(f_2) - \frac{6}{12} H(f_3)$$

$$= 0.87 - \frac{6}{12} \times 0.73 - \frac{6}{12} \times 1 = 0.005$$

=) So basically if the above split gives the highest information gain value then the above structure will actually be used to split the particular dataset itself inorder to construct the decision tree

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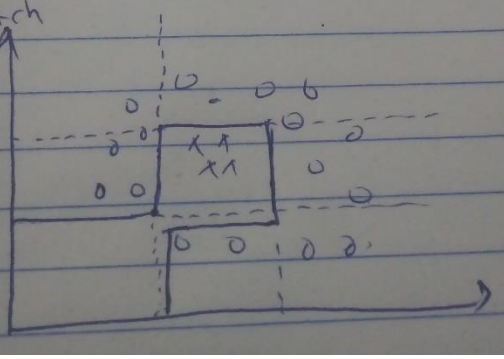

- Understanding how decision tree works is that decision boundaries will always be perpendicular to X and Y axis as for that case we made the above estimation

⇒ So, basically the above sketch of the model can be said to be overfitted and this happens when we increase the depth of the tree
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⇒ Thus, we see that if no limit on the depth of the algorithm, then this will be prone to overfitting implying that we going to have a complex decision boundary accomodating

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