Volatility, Correlation and Tails for Systemic Risk Measurement

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Abstract

This paper proposes an empirical methodology to measure systemic risk. We associate the systemic risk of a financial institution with its contribution to the deterioration of the system capitalization that would be experienced in a crisis. In order to measure this empirically we introduce the SRISK index, the expected capital shortage of a firm conditional on a substantial market decline. The index is a function of the degree of Leverage, Size and Marginal Expected Shortfall (MES) of a firm. MES is the expected loss an equity investor in a financial firm would experience if the market declined substantially. To estimate MES, we introduce a dynamic model for the market and firm returns. The specification is characterized by time varying volatility and correlation as well a nonlinear tail dependence. The empirical application to a set of top U.S. financial firms between 2000 and 2010 is used to illustrate the usefulness of the methodology. Results show that SRISK provides useful ranking of systemically risky firms at various stages of the financial crisis: one year and a half before the Lehman bankruptcy, nine companies out of the SRISK top ten turned out to be troubled institutions. The aggregate SRISK of the financial system provides early warning signals of distress in the real economy: a one standard deviation shock in aggregate SRISK predicts a twentyfive basis points drop in Industrial Production over the next month.

Keywords: Systemic Risk, Volatility, Correlations, Tails, Forecasting.

JEL classification: C22, C23, C53.

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Systemic risk analysis of top U.S. financial firms based on the results of this paper are updated weekly on the Vlab (http://vlab.stern.nyu.edu/).
Acknowledgments are at the back of the paper. All mistakes are ours.

1 Introduction

The Great Recession of 2007-2009 has motivated market participants, academics and regulators to better understand systemic risk. A useful definition of systemic risk by Federal Reserve Governor Daniel Tarullo, is

> *"Financial institutions are systemically important if the failure of the firm to meet its obligations to creditors and customers would have significant adverse consequences for the financial system and the broader economy."*

In this definition, it is the failure of a systemically important firm to meet obligations that is the cause of systemic distress as well as negative externalities to the rest of the economy. Thus, measures of systemic risk are associated with firm bankruptcies or near bankruptcies that are inevitable consequences of a decline in equity valuations for highly levered firms.

This idea is further developed in the theoretical analysis of Acharya *et al.* (2010). A financial firm is unable to function when the value of its equity falls to a sufficiently small fraction of its outstanding liabilities. In good times, such a firm is likely be acquired, may be able to raise new capital or may face an orderly bankruptcy. If this capital shortage occurs just when the financial sector is already financially constrained, then the government faces the question of whether to rescue the firm with taxpayer money as other avenues are no longer available. Such a capital shortage is damaging to the real economy as the failure of this firm will have repercussions throughout the financial and real sectors. Consequently a firm is systemically risky if it is likely to face a large capital shortfall just when the financial sector itself is under distress.

In this paper we propose novel Systemic Risk indices (which we name SRISK for short) to measure the systemic risk contribution of a financial institution as well as the aggregate systemic risk of the whole financial system. The SRISK index of an individual firm is determined by the expected capital shortage a financial firm would experience in case of a systemic event, here defined as a substantial market decline over a given time horizon. The shortage depends on the firm's degree of leverage, its size and its equity loss conditional on a market decline, which is also known as Marginal Expected Shortfall (MES). The companies with the highest SRISK are the companies that contribute the most to the financial sector undercapitalization in a crisis and are therefore the most systemically risky firms. The sum of the SRISKs of the whole financial system represents the potential capital shortage that the government may be pressured to recapitalize in case of a crisis. Conceptually this calculation is similar to the stress tests that are regularly applied to financial firms; however, here it is done with only publicly available information and is quick and inexpensive.

The computation of the SRISK indices requires information on the equity, debt and MES of each firm. While equity and debt can be readily measured, MES needs to be estimated from return data using appropriate econometric methods. To this extent, we propose a conditionally heteroskedastic bivariate model for market and firm equity returns. The specification decomposes the behaviour of returns into time varying volatility, correlation and tails. A multi step modeling approach based on GARCH and DCC (Engle (2002b), Engle (2009)) is used to fit volatility and correlations. Inference on the tails is based on flexible methods that allow for potential nonlinear dependence without making specific distributional assumptions. The model is used to construct one and multi–period ahead MES forecasts, which we name, respectively, short and long term MES.

We employ this methodology to study systemic risk in the U.S. financial system between July 2000 and July 2010. We construct a panel containing the top 94 U.S. financial firms. Companies are grouped in 4 categories: Depositories, Insurance, Broker-Dealers and a residual category labelled Others, which also contains non depositary institutions and real estate related firms. First, we focus on short term MES and understanding the dynamics of risk and interdependence during the crisis. Using our proposed specification, we study in–sample dynamics of short term MES and then carry out an out–of–sample evaluation exercise where our approach is compared with a number of benchmarks. The list of competitors is made up of a rolling static bivariate model, a rolling static factor model, a Dynamic Conditional Beta model (Engle (2012)) and a Dynamic Rotated Gumbel copula model. Second, we examine the SRISK results, constructed using the long term MES predictions of our specification. We begin by reporting the time series dynamics and cross sectional rankings of SRISK. As a sanity check, we compare SRISK rankings with the ones provided by other variables and show how sensitive our results are to the choice of the SRISK index parameters. We then engage in two out–of–sample evaluation exercises. First, we assess whether individual firm SRISK is able to predict actual Fed capital injections during the crisis. The second evaluation exercise consists of carrying out Granger Causality analysis to test if changes in aggregate SRISK predict future adverse movements in real variables like industrial production and unemployment. Finally, in analogy to the analysis in Engle and Rangel (2008) and Bekaert *et al.* (2013), we investigate the determinants of individual and aggregate SRISK.

The MES results show that time varying firm volatility and correlation with the market capture most of the dependence: MES is high for firms that are volatile and are not diversified with respect to the market. A dynamic conditional beta model essentially captures most of the dependence between firm and market returns. The time series analysis reveals that the level of MES during the financial crisis is extreme by historical standards. However, the MES industry group rankings have been stable over time, with Broker-Dealers and the Other sectors being the most exposed ones. In the forecasting application our model performs well relative to the benchmarks. The SRISK results show that our methodology delivers useful rankings of systemically risky firms at various stages of the financial crisis. For instance, one year and a half before the Lehman bankruptcy, nine companies out of the SRISK top ten turned out to be troubled institutions. Results also document the deterioration of the financial system capitalization starting from July 2007 and reveal that as of July 2010 the financial system does not appear to be fully recovered. Ex–ante SRISK is highly correlated with actual Fed capital injections in the crisis. The Granger causality analysis shows that changes in SRISK predict drops in Industrial Production even after controlling for the past history of Industrial Production and Unemployment. The analysis of the determinants of aggregate

SRISK conveys that market volatility is its main determinant, explaining approximately half of the variation of aggregate SRISK. The analysis of the determinants of individual SRISK shows that overvalued companies with low profitability have higher SRISK.

The main contribution of this work is the proposal of the SRISK index and its analysis in the context of the 2007-2009 financial crisis. Our approach builds upon Acharya *et al.* (2010). In their contribution, the authors introduce an economic model that formally links systemic risk and capital shortages. They also propose a *static structural form* approach to estimate a systemic risk index associated with capital shortages called SES. The main shortcoming of SES however, is that it cannot be used for ex-ante systemic risk measurement: their approach requires data from the actual financial crisis for estimation. Thus, it is unclear whether it would have been possible to compute SES before 2007-2009 and it is also not straightforward to use the evidence from 2007-2009 to monitor the system for a future crisis. The contribution of this paper is to propose an alternative *dynamic reduced form* estimation strategy for capital shortages. As we detail in the empirical section, the approaches provide significantly different results.

Our work relates to a growing number of contributions on the analysis of the financial crisis and the measurement of systemic risk. Among others, Brunnermeier and Oehmke (2012) Bisias *et al.* (2012) provide recent surveys in the area. Our contribution relates mostly with the strand of the literature that relies on market data and, specifically, stock prices to infer investors' expectations and the distribution of returns. The CoVaR of Adrian and Brunnermeier (2009) is one of the early proposals in the literature that has received notable attention. CoVaR relates systemic risk to tail spillover effects from individual firms to the whole market. Manganelli *et al.* (2010) develop dynamic multivariate quantile time series models in the same spirit. Billio *et al.* (2012) introduce a methodology to construct networks of spillover effects among financial institutions. Hautsch *et al.* (2010) propose a methodology to construct networks which focuses on tail dependence. Barigozzi and Brownlees (2012) introduce network estimation techniques for generic covariance stationary processess. Contributions that relate to this area of research also include Allen *et al.* (2010), which assesses the predictive power of aggregate systemic risk measures in predicting future downturns and Brownlees (2011), which proposes a model to relate financial firm dynamics to their characteristics. From the point of view of the econometric methodology, this work builds upon the literature on volatility and correlation modeling using, respectively, GARCH and DCC models. A detailed glossary of the ARCH universe can be found in Bollerslev (2008). The DCC approach for correlations has been introduced by Engle (2002a) and recently surveyed in Engle (2009). Contributions in this area include Engle and Sheppard (2001), Aielli (2006) and Engle *et al.* (2009). Dynamic models for Value–at–Risk and Expected Shortfall have been developed in Engle and Manganelli (2004) and Taylor (2008). Finally, the MES concept has been around in the actuarial literature for quite same time under the name of conditional tail expectations (Tasche (2000)).

The rest of the paper is organized as follows. Section 2 introduces the definition of the SRISK indices used in this work. Section 3 describes the econometric methodology used to estimate MES. Section 4 describes the panel of top U.S. financial firms analysed in this work. Section 5 contains the results of the MES analysis while Section 6 SRISK results. Concluding remarks follow in Section 7. The methodology developed in this paper is used to analyse top U.S. financials on a weekly basis in the systemic risk section of the Vlab (`http://vlab.stern.nyu.edu/`).

2 Measuring Systemic Risk

The key insight of the theoretical model of Acharya *et al.* (2010) is that capital shortages of individual firms impose external costs on the real economy when they occur during a period of distress for the whole system. When the economy is in a downturn, the bankruptcy of a firm cannot be absorbed by a stronger competitor, and obligations will spread throughout both the financial and real economy as the natural functions of the financial sector will be curtailed.

When the system is undercapitalized, it will no longer supply credit for ordinary everyday business and the economy will grind to a halt. Thus, a shortage of capital is dangerous for the firm and for its bondholders, but it is dangerous for the whole economy if it occurs just when the rest of the sector is undercapitalized. Note that typically the objective function of the firm does not take into account the negative externality costs that it generates in a crisis. Hence, firms can take excessive leverage in the absence of regulatory intervention. When the volatility of returns is low for instance, risk is low and the optimal leverage is high. Building upon this intuition, our objective is to develop an empirical methodology to measure the aggregate capital shortage generated by a pool of financial institutions in a crisis, as well as the contributions of individual companies.

Various strategies can be devised to measure capital shortages. In this work we follow an approach which is market based in spirit. We merge balance sheet data with stock prices in the attempt to provide a measure of capital shortage that reflects investors' expectations. Consider a panel of financial institutions indexed by $i = 1, ..., I$ observed at times $t = 1, ..., T$. For each firm, D_{it} and W_{it} denote respectively the book value of its debt and the market value of its equity. We also assume that prudential management would restrict each institution to maintain equity as a fraction k of its assets. In this setting we can define the capital buffer of a financial institution at time t as

$$\mathsf{CB}_{\mathsf{it}} = W_{it} - k(D_{it} + W_{it}).$$

The capital buffer represents the working capital of the firm; when the buffer is positive, the firm is going to function properly. On the other hand, when this quantity is negative, the firm experiences a capital shortage which is going to impair the company and, if it occurs when the rest of the economy is in distress, generate negative externalities to the rest of the economy. Thus, we are interested in computing the expected capital shortage in case of market distress. We refer to distress as a systemic event and in this work we define it as a drop of the market index below a threshold C over a given time horizon h. Let $R_{m\,t:t+h}$ denote the simple market return between period t and $t+h$. The systemic event is denoted by $\{R_{m\,t:t+h} < C\}$. The expected capital shortage is then

$$\begin{aligned}
\mathsf{CS}_{i\,t+h|t} &= -\mathsf{E}_t(\mathsf{CB}_{\mathsf{i\,t+h}}|R_{m\,t+h:t} < C) \\
&= -k\mathsf{E}_t(D_{i\,t+h}|R_{m\,t+h:t} < C) + (1-k)\ \mathsf{E}_t(W_{i\,t+k}|R_{m\,t+h:t} < C).
\end{aligned}$$

In order to compute this expectation we further assume that in the case of a systemic event debt cannot be renegotiated, implying that $\mathsf{E}_t(D_{i\,t+h}|R_{m\,t+h:t} < C) = D_{i\,t}$. Using this assumption it follows that

$$\begin{aligned}
\mathsf{CS}_{i\,t+h|t} &= -kD_{i\,t} + (1-k)\ W_{i\,t}\ \mathsf{E}_t(R_{i\,t+h:t}|R_{m\,t+h:t} < C) \\
&= -kD_{i\,t} + (1-k)\ W_{i\,t}\ \mathsf{MES}_{i\,t+h|t}(C)
\end{aligned}$$

where $\mathsf{MES}_{i\,t+h|t}(C) = \mathsf{E}_t(R_{i\,t+h:t}|R_{m\,t+h:t} < C)$ is the tail expectation of the firm equity returns conditional on the systemic event. We define the systemic risk index of institution i as

$$\mathsf{SRISK}_{i\,t} = \max(0, \mathsf{CS}_{i\,t}),$$

and its percentage version as

$$\mathsf{SRISK\%}_{i\,t} = \mathsf{SRISK}_{i\,t} / \sum_{i=1}^{I} \mathsf{SRISK}_{i\,t}.$$

The total amount of systemic risk in the economy is

$$\mathsf{SRISK}_t = \sum_{i=1}^{I} \mathsf{SRISK}_{i\,t}.$$

Aggregate SRISK_t can be thought of as the total amount of capital that the government would have to provide to bailout the financial system in the case of a crisis, and $\mathsf{SRISK}\%_{i\,t}$ is the percentage of the bailout money that would be needed by institution i.

SRISK has different determinants. An increase in the size of the firm, keeping the leverage ratio constant, increases systemic risk in that it makes the capital shortage potentially larger. An increase in debt also has a positive impact on systemic risk in that it will shrink the capital buffer of the firm. Finally, a high downside exposure of the firm to systematic shocks in the economy contributes positively to an increase in systemic risk. A number of tuning parameters need to be set to compute the index: the value of k as well as the horizon h and magnitude C of the systemic event $\{R_{t:t+h} < C\}$. In the empirical section we show that while the overall level of Systemic Risk is clearly determined by the choice of these parameters, the time series profile and cross sectional ranking are relatively stable to the choice of such parameters.

A couple of remarks on our SRISK index are in order. Conceptually, this calculation is similar to the stress tests that are regularly applied to financial firms. However, here it is done with only publicly available information making the index widely applicable and relatively inexpensive to implement. Admittedly, our index is making a number of simplifying assumptions in order to deliver a measure that can be easily computed in practice. Among the possible extensions of the baseline model, one could think of using different values of k for different types of institutions and/or types of assets. Another limitation of the measure is that it does not employ off–balance sheet information and to this extent it might not appropriately capture the true asset structure of a firm.

An important characteristic of our measurement approach is that it merges together accounting and market information to estimate the capital buffer of a firm. The capital buffer could be measured using solely the accounting value of assets and liabilities. On the other hand, the market value of the equity of the firm provides a market estimate of the future value of the firm. This may differ from the accounting value because the assets or liabilities are evaluated differently from the accounting figures and also because the market value is forward looking and may take into account factors that have yet to occur. To be more precise, consider a firm with nominal liabilities that come due at a time T in the future when the firm will be liquidated. This firm has assets with uncertain value at time T and investors in the equity of this firm must evaluate the distribution of firm value. The risk faced by the equity holders is the risk that the assets will not cover liabilities. It is in this context that Merton equated the value of the equity of the firm to a call option on the firm value with a strike of the face value of the liabilities and a maturity of T. If the firm value falls below the liabilities at T, equity holders receive nothing and debt holders retain the balance. In reality, liabilities are stochastic as well as assets. Bankruptcy will occur whenever firm values are low enough that economic incentives drive the firm to default thereby incurring bankruptcy costs which typically exhaust remaining equity value as well as some portion of debt value. These costs can be viewed as state dependent liabilities. Furthermore, other risks may also be on the liability side such as short term rollover financing which will have uncertain costs and with some probability will become impossible to continue. In other cases, liabilities might include demand deposits which have implicit guarantees but could be withdrawn quickly, or stochastic claims such as for insurance companies. In standard finance theory, the market capitalization of the firm is the discounted present value of the net earnings of the firm where the discount rate reflects the risk. This corresponds to a forward looking measure of the net worth of the firm incorporating the distribution of future assets, liabilities and bankruptcy costs.

We would like to stress that the measurement approach proposed in this section to compute expected capital shortages is quite general and could be applied to other types of firms. Interpreting capital shortages in a crisis as a systemic risk measure requires some caution however. Such extension hinges on the existence of a linkage between individual shortages and economy wide negative externalities. An example of another industry where this might be the indeed the case is the auto industry in the U.S. A bankruptcy of the automotive sector would have triggered massive layoffs and would have added a significant negative impact on all the business partners of the main companies involved. Our methodology could have been used to monitor the level of undercapitalization of the sector. Similar arguments could be used in other industries as well. That said, we believe that capital shortages should be a primary concern in the financial sector since in this industry the level of leverage is typically much higher than in the other sectors.

It is important to explicitly point out our contribution with respect to Acharya *et al.* (2010). In their work, the authors also propose a measure of systemic risk called Systemic Expected Shortfall (SES), the expected capital shortage of an individual firm conditional on a substantial reduction of the capitalization of the system. In order to bring their methodology to the data, they propose a *static structural* estimation approach for SES. They begin by showing that, under a number of assumptions, SES can be expressed as a linear combination of MES and Leverage, where the parameters of such a combination are not known in general. They then regress MES and Leverage on the equity return of each financial institution during the crisis and use the fitted values of this regression to compute the so called fitted realized SES. We believe that this approach has a number of shortcomings. The main concerns are that before the financial crisis one could have not run such a regression and it is not clear under which conditions regression estimates based on the 2007-2009 financial crisis would offer suitable guidance in a future crisis. Overall, a *static structural* estimation approach raises a number of challenges and concerns that limit the usefulness of the basic intuition of their model. To this extent, in this work we follow the alternative route that we have outlined in this section to measure systemic risk. We directly attempt to measure the expected capital shortages using a *dynamic reduced* form approach, starting from the standard definition of capital buffers which is common in the literature. As we detail in the empirical section of the paper, the two measures are substantially different in practice and deliver substantially different rankings for systemically risky firms.

3 Econometric Approach

The computation of the systemic risk indices requires data on the debt, equity and MES of each firm. While debt and equity information is readily available, the estimation of the MES requires the introduction of appropriate time series techniques. In this work we are interested in estimating the capital shortfall over a potentially long time period (say, a quarter or six months) and we need time series methods able to deliver estimates of MES over long horizons. Several strategies can be devised to tackle this problem. In this work we propose to specify a bivariate dynamic time series model for the daily firm and market returns. Once the model is estimated, the process can be extrapolated to produce the MES predictions of interest.

Let $r_{i\,t}$ and $r_{m\,t}$ denote respectively the ith firm's and the market log return on day t. Our approach starts from a description of the bivariate process of the firm and market returns:

$$\begin{aligned} r_{m\,t} &= \sigma_{m\,t}\,\epsilon_{m\,t} \\ r_{i\,t} &= \sigma_{i\,t}\,\rho_{i\,t}\epsilon_{m\,t} + \sigma_{i\,t}\sqrt{1-\rho_{i\,t}^2}\xi_{i\,t} \\ (\epsilon_{m\,t}, \xi_{i\,t}) &\sim F \end{aligned} \tag{1}$$

where $\sigma_{m\,t}$ is the conditional standard deviation of the market return, $\sigma_{i\,t}$ is the conditional standard deviation of the firm return, $\rho_{i\,t}$ is the conditional market/firm correlation and $(\epsilon_{m\,t}, \xi_{i\,t})$ are the shocks that drive the system. The shocks $(\epsilon_{m\,t}, \xi_{i\,t})$ are independent and identically distributed over time and have zero mean, unit variance and zero covariance. However they are not assumed to be independent of each other. Indeed, there are important reasons to believe that extreme values of these disturbances could occur at the same time for systemically risky firms. When the market disturbances are in the tail, the firm disturbances may be even further in the tail if there is serious risk of default. The firm return equation can alternatively be represented as

$$r_{it} = \beta_{it} r_{mt} + \sigma_{it}\sqrt{1-\rho_{it}^2}\xi_{it}$$

where $\beta_{it} = \rho_{it}\frac{\sigma_{it}}{\sigma_{mt}}$. Thus, our specification can be though of as a factor model that allows both for time varying systematic risk and tail dependence between market and idiosyncratic firm shocks.

The stochastic specification is completed by a description of the two conditional standard deviations and the conditional correlation. These will be discussed in the next section but are familiar models of TARCH and DCC. On the other hand, the distribution F is left unspecified and we will adopt a flexible nonparametric approach for inference.

The one period ahead MES can be straightforwardly expressed as a function of volatility, correlation and tail expectations of the standardised innovations distribution

$$\begin{aligned} \mathsf{MES}^1_{it-1}(C) &= \mathsf{E}_{t-1}(r_{it}|r_{mt} < C) \\ &= \sigma_{it}\mathsf{E}_{t-1}(\rho_{it}\epsilon_{mt} + \sqrt{1-\rho_{it}^2}\xi_{it}|\epsilon_{mt} < C/\sigma_{mt}) \\ &= \sigma_{it}\rho_{it}\mathsf{E}_{t-1}(\epsilon_{mt}|\epsilon_{mt} < C/\sigma_{mt}) + \\ &\quad \sigma_{it}\sqrt{1-\rho_{it}^2}\mathsf{E}_{t-1}(\xi_{it}|\epsilon_{mt} < C/\sigma_{mt}), \end{aligned} \tag{2}$$

and the conditional probability of a systemic event is

$$\mathsf{PoS}^1_t(C) = P_{t-1}(r_{mt} < C) = P(\epsilon_{mt} < C/\sigma_{mt}). \tag{3}$$

Some comments on the formulas in Equations (2) and (3) are in order under the assumption that the dependence between the firm and the market is positive. Firstly MES is an increasing function of a firm's own volatility. Depending on whether correlation is high or low, the MES formula gives more weight to, respectively, either the tail expectation of the standardised market residual or the tail expectation of standardised idiosyncratic firm residual. The second term in Equation (2) arises because of the nonlinear dependence assumption between $\epsilon_{m\,t}$ and $\xi_{i\,t}$ and it would otherwise be zero if the dependence was captured entirely by correlation. Second, MES relates to the systematic risk "beta" of the CAPM. If the data are generated by a one factor model then MES is equal to systematic risk multiplied by the Expected Shortfall of the market. Our approach is more flexible in that it allows for time varying moments and focuses on downside exposure. It is also important to stress the implication of the conditioning systemic event C. Typically, VaR and ES are expressed in conditional terms, that is the conditioning event is a quantile from the conditional return distribution. On the other hand, in this work the conditioning event is unconditional. Thus, while in the conventional approach the probability of observing the conditioning event is constant, in our framework such probability is time varying: The higher the volatility the higher the probability of observing a loss above a given threshold. Equation (2) has an approximation error due to the fact that we are using log returns rather than arithmetic returns. In this work we do not add an adjustment term to the formula. This has been worked out in Caporin and De Magistris (2011). In what follows we refer to the one-period ahead MES as the "short term" MES.

The multi–period ahead MES cannot be obtained in closed form and a simulation procedure is used to construct the forecasts. The procedure is designed as follows. In order to produce the h-period MES starting from day t, on day $t-1$ we simulate S return paths of length h

$$\left\{ \begin{array}{c} r^s_{m\,t+\tau-1} \\ r^s_{i\,t+\tau-1} \end{array} \right\}_{\tau=1}^{h} \qquad s = 1, ..., S.$$

We obtain the paths by first drawing pseudo innovations from the innovation distribution F, that is

$$(\epsilon^s_{m\,t+\tau-1}, \xi^s_{m\,t+\tau-1})_{\tau=1}^h \sim F,$$

and by then recolouring them through the DCC and GARCH filters using the current levels of volatility and correlation as starting conditions. The multi-period MES is calculated using the Monte Carlo average of the simulated paths,

$$\mathsf{MES}^h_{i\,t-1}(C) = \frac{\sum_{s=1}^S R^s_{i\,t:t+h-1} I\{R^s_{m\,t:t+h-1} < C\}}{\sum_{s=1}^S I\{R^s_{m\,t:t+h-1} < C\}}, \tag{4}$$

where $R^s_{i\,t:t+h-1}$ denotes the s^{th} simulated cumulative return of firm i from period t to period $t+h-1$, i.e.

$$R^s_{i\,t:t+h-1} = \exp\left\{\sum_{\tau=1}^{h} r^s_{i\,t+\tau-1}\right\} - 1, \tag{5}$$

and analogously for the market return $R^s_{m\,t:t+h-1}$. Analogously, the multi–period probability of a crisis is given by

$$\mathsf{PoS}^h_t(C) = P_t(R_{m\,t:t+h-1} < C) = \frac{1}{S}\sum_{s=1}^{S} I\{R^s_{m\,t:t+h-1} < C\}.$$

This approach is close in spirit to what risk management practitioners call "scenario analysis", that is extrapolating the risk implied by the model conditionally on a stream of adverse outcomes. In what follows we will consider a 6-months period MES and we will refer to this prediction as "long term" MES. A useful feature of these long term forecasts conditionally on the systemic event, is that they partly counter the pro–cyclical nature of short term risk assessment. Dynamic models tend to produce volatilities and correlation forecast that reflect current market conditions. On the other hand, averaging over long sequences of events in which the market has a substantial fall allows us to evaluate the model implied downside exposure of a firm while giving less importance to current market conditions.

Different strategies can be employed to obtain estimates of MES at the horizons of interest. In this work we rely on a multi stage modeling approach which is inspired by the DCC (Engle (2002a), Engle (2009)). In the first step we model volatilities using GARCH models to obtain conditional volatility and standardised residuals. We then resort to a DCC specification to obtain conditional correlation and the standardised idiosyncratic firm residual. Finally, inference on the model innovations is based on the GARCH/DCC residuals. The appealing features of such a modelling paradigm are simplicity and flexibility. Estimation of a fully bivariate conditionally heteroskedastic model with nonlinear residual dependence for a large panel of assets can be quite challenging, especially when the time series are not too long. On the other hand, our approach is much easier to implement and it allows for considerable flexibility in the specifications, by changing the different types of volatility (splines, high frequency based, etc.) and correlation (standard DCC, factor DCC, asymmetric, breaks in correlation, etc.) models.

In this work we make the following specification choices:

Volatility. In the wide universe of GARCH specifications, we pick the TARCH specification to model volatility (Rabemananjara and Zakoïan (1993), Glosten *et al.* (1993)). The evolution of the conditional variance dynamics in this model class is given by

$$\begin{aligned} \sigma^2_{m\,t} &= \omega_{m\,G} + \alpha_{m\,G}\, r^2_{m\,t-1} + \gamma_{m\,G}\, r^2_{m\,t-1} I^-_{m\,t-1} + \beta_{m\,G}\, \sigma^2_{m\,t-1} \\ \sigma^2_{i\,t} &= \omega_{i\,G} + \alpha_{i\,G}\, r^2_{i\,t-1} + \gamma_{i\,G}\, r^2_{i\,t-1} I^-_{i\,t-1} + \beta_{i\,G}\, \sigma^2_{i\,t-1} \end{aligned}$$

with $I^-_{i\,t} = 1$ if $\{r_{i\,t} < 0\}$ and $I^-_{m\,t} = 1$ if $\{r_{m\,t} < 0\}$. Market volatility dynamics are stationary if $(\alpha_{m\,G} + \gamma_{m\,G}/2 + \beta_{m\,G})$ is smaller than one under the assumption that the probability of a negative shock is 1/2 (and the analog condition is required for firm volatility dynamics). The main highlight of this specification is its ability to capture the so called leverage effect, that is the tendency of volatility to increase more with negative news rather than positive news. This model is also successful from a forecasting standpoint and it turns out to be quite difficult to beat. We estimate the model using QML which guarantees consistent estimates of the model parameters as long as the conditional variance equation is correctly specified.

Correlations. We model time varying correlations using the DCC approach (Engle (2002a), Engle (2009)). Let P_{it} denote the time varying correlation matrix of the market and firm return, that is, using matrix notation,

$$\mathsf{Var}_{t-1}\begin{pmatrix} r_{it} \\ r_{mt} \end{pmatrix} = D_{it}\, P_{it}\, D_{it} = \begin{bmatrix} \sigma_{it} & 0 \\ 0 & \sigma_{mt} \end{bmatrix} \begin{bmatrix} 1 & \rho_{it} \\ \rho_{it} & 1 \end{bmatrix} \begin{bmatrix} \sigma_{it} & 0 \\ 0 & \sigma_{mt} \end{bmatrix}.$$

Rather than directly modeling the P_{it} matrix, the DCC framework models the so–called pseudo correlation matrix Q_{it}, a positive definite matrix which is then mapped in a correlation matrix through the transformation

$$P_{it} = \mathsf{diag}(Q_{it})^{-1/2}\, Q_{it}\, \mathsf{diag}(Q_{it})^{-1/2},$$

where the $\mathsf{diag}(A)$ matrix operator denotes a matrix with the same elements of the A matrix on the diagonal and zero otherwise. We formulate the pseudo correlation matrix Q_t dynamics using the DCC formulation proposed in Aielli (2006).

The basic (scalar) symmetric DCC specification is defined as

$$Q_{it} = (1 - \alpha_C - \beta_C)S_i + \alpha_C\, \epsilon^*_{it-1}\epsilon^{*\prime}_{it-1} + \beta_C\, Q_{it-1}, \tag{6}$$

where S_i is an intercept matrix and ϵ^*_{it} contains the rescaled standardised (or degarched) returns, that is $\epsilon^*_{it} = Q^*_{it-1}\epsilon_{it}$ with $Q^*_{it} = \mathsf{diag}(Q_{it})^{1/2}$. The pseudo conditional correlation matrix Q_{it} is thus an exponentially weighted moving average of past outer products of the rescaled standardised returns. Necessary and sufficient conditions for Q_{it} to be positive definite are $\alpha_C > 0$, $\beta_C > 0$, $\alpha_C + \beta_C < 1$ and the positive definiteness of the S_i matrix. The rescaling device ensures that $\{\epsilon^*_{it}, Q_{it}\}$ is a MGARCH process (Ding and Engle (2001)) and, under the assumption of stationarity of the model ($\alpha_C + \beta_C < 1$), this implies that S_i is the unconditional covariance matrix of ϵ^*_{it}

$$S_i = \mathsf{E}(\epsilon^*_{it}\epsilon^{*\prime}_{it}).$$

This property is useful for highly dimensional DCC estimation in that it justifies the use of the unconditional covariance matrix of the ϵ^*_{it} as a correlation targeting (Mezrich and Engle (1996)) estimator for S_i, that is

$$\hat{S}_i = \frac{1}{n}\sum \epsilon^*_{it}\epsilon^{*\prime}_{it},$$

which drastically reduces the number of parameters that need to be optimized to estimate the model.

Tail Expectations for Short Term MES. To compute short term MES in Equation (2) we need to estimate the tail expectations

$$\mathsf{E}(\epsilon_{mt}|\epsilon_{mt} < \kappa) \qquad \text{and} \qquad \mathsf{E}(\xi_{it}|\epsilon_{mt} < \kappa).$$

These expectation can be estimated for a particular value of the variances $(\sigma^2_{mt}, \sigma^2_{it})$ and conditional correlation ρ_t by simply looking at the average of the two residuals in all cases which satisfy the condition $\epsilon_{mt} < \kappa$. However, when $-\kappa$ is large, this estimator will be unstable as there are only a small number of observations. A nonparametric kernel estimation approach can be used to improve the efficiency of these simple estimators. Let

$$K_h(t) = \int_{\infty}^{t/h} k(u)du,$$

where $k(u)$ is a kernel function and h is a positive bandwidth. Then

$$\widehat{\mathsf{E}}_h(\epsilon_{m\,t}|\epsilon_{m\,t} < \kappa) = \frac{\sum_{i=1}^{n} \epsilon_{m\,t} K_h(\epsilon_{m\,t} - \kappa)}{(n\hat{p}_h)}, \tag{7}$$

and

$$\widehat{\mathsf{E}}_h(\xi_{i\,t}|\epsilon_{m\,t} < \kappa) = \frac{\sum_{i=1}^{n} \xi_{i\,t} K_h(\epsilon_{m\,t} - \kappa)}{(n\hat{p}_h)}, \tag{8}$$

where

$$\hat{p}_h = \frac{\sum_{i=1}^{n} K_h(\epsilon_{m\,t} - \kappa)}{n}.$$

An advantage of the nonparametric estimators defined in Equations (7) and (8) is that they are smooth functions of the cutoff point κ which, in turn, deliver smooth estimates of short term MES as a function of κ. The properties of these type of nonparametric tail expectation estimators are discussed in Scaillet (2005).

Simulations for Long Term MES. To compute the long term MES of Equation (4), we need to draw samples from the innovation distribution F. The sampling strategy we adopt is to sample with replacement from the empirical cumulative density function of the estimated residuals $\widehat{F}$. A number of algorithmic shortcuts can be implemented to substantially reduce the computational burden of the long term MES computation in large panels. The strategy we adopt is to simulate S market paths and check which paths meet the systemic event condition. For each of these paths, we store the sequence of id's of the selected draws. Then, the simulation of the individual firm trajectories consists of constructing for each selected path the sequence of individual firm shocks which correspond to those of the market. This speeds up the simulations in that it avoids having to simulate and select paths for each firm/market return pair in the panel.

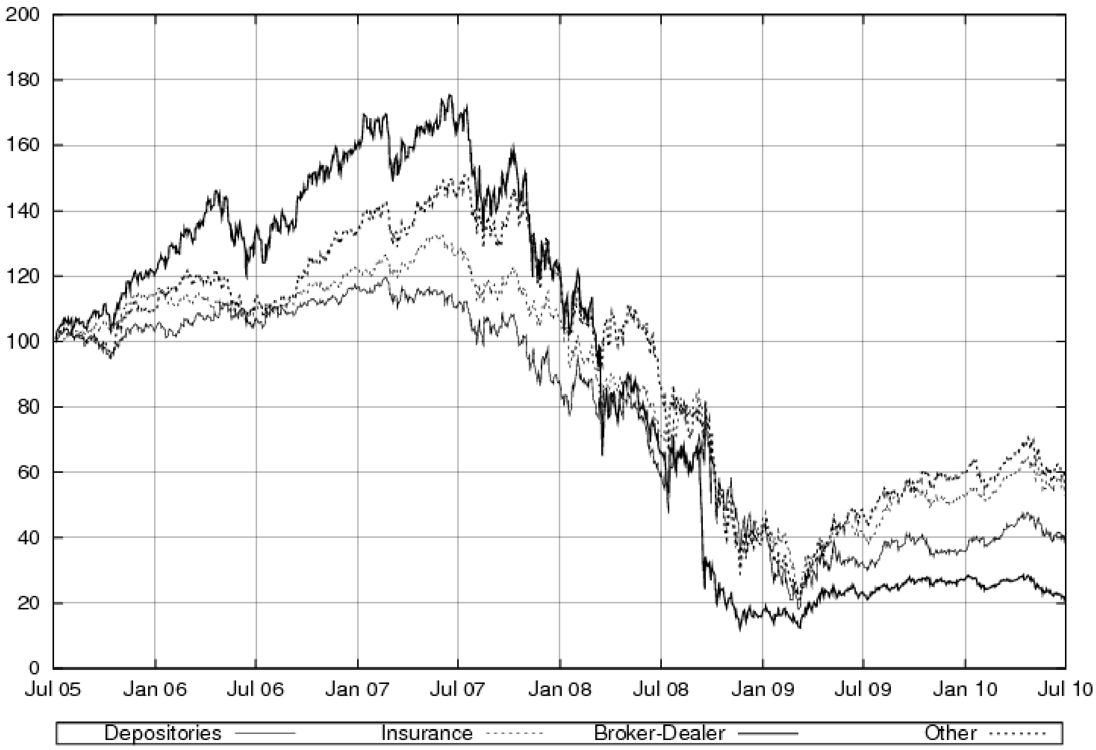

Figure 1: Cumulative average return by industry group. The figure reports the cumulative return for the Depositories, Insurance and Broker-Dealers and Others groups between July 2005 and June 2010.

4 Data

We study the same panel of institutions studied in Acharya *et al.* (2010) between July 3, 2000 and June 30, 2010. The panel contains all U.S. financial firms with a market capitalization greater than 5 bln USD as of the end of June 2007, and it is unbalanced in that not all companies have been trading continuously during the sample period. We extract daily returns and market capitalization from CRSP and the quarterly book value of equity (ceqq) from COMPUSTAT. SIC codes are used to divide firms into 4 groups: Depositories (such as Bank of America or JP Morgan Chase), Broker-Dealers (Goldman Sachs or Lehman Brothers), Insurance (AIG) and Others (non depositary institutions, real estate, like Freddie and Fannie). We make one exception to this rule, Goldman Sachs (GS) should have been classified within the Others group, but instead we put it with Brokers-Dealers. We also use the daily CRSP market

Depositories (29)		Insurance (32)		Broker-Dealers (10)		Others (23)	
BAC	Bank of America	ABK	Ambac Financial Group	AGE	A.G. Edwards	ACAS	American Capital
BBT	BB&T	AET	Aetna	BSC	Bear Stearns	AMP	Ameriprise Financial
BK	Bank of New York Mellon	AFL	Aflac	ETFC	E-Trade Financial	AMTD	TD Ameritrade
C	Citigroup	AIG	American International Group	GS	Goldman Sachs	AXP	American Express
CBH	Commerce Bancorp	AIZ	Assurant	LEH	Lehman Brothers	BEN	Franklin Resources
CMA	Comerica inc	ALL	Allstate Corp	MER	Merrill Lynch	BLK	Blackrock
HBAN	Huntington Bancshares	AOC	Aon Corp	MS	Morgan Stanley	BOT	CBOT Holdings
HCBK	Hudson City Bancorp	WRB	W.R. Berkley Corp	NMX	Nymex Holdings	CBG	C.B. Richard Ellis Group
JPM	JP Morgan Chase	BRK	Berkshire Hathaway	SCHW	Schwab Charles	CBSS	Compass Bancshares
KEY	Keycorp	CB	Chubb Corp	TROW	T. Rowe Price	CIT	CIT Group
MI	Marshall & Ilsley	CFC	Countrywide Financial			CME	CME Group
MTB	M & T Bank Corp	CI	CIGNA Corp			COF	Capital One Financial
NCC	National City Corp	CINF	Cincinnati Financial Corp			EV	Eaton Vance
NTRS	Northern Trust	CNA	CNA Financial corp			FITB	Fifth Third Bancorp
NYB	New York Community Bancorp	CVH	Coventry Health Care			FNM	Fannie Mae
PBCT	Peoples United Financial	FNF	Fidelity National Financial			FRE	Freddie Mac
PNC	PNC Financial Services	GNW	Genworth Financial			HRB	H&R Block
RF	Regions Financial	HIG	Hartford Financial Group			ICE	Intercontinental Exchange
SNV	Synovus Financial	HNT	Health Net			JNS	Janus Capital
SOV	Sovereign Bancorp	HUM	Humana			LM	Legg Mason
STI	Suntrust Banks	LNC	Lincoln National			NYX	NYSE Euronext
STT	State Street	MBI	MBIA			SEIC	SEI Investments Company
UB	Unionbancal Corp	MET	Metlife			SLM	SLM Corp
USB	US Bancorp	MMC	Marsh & McLennan				
WB	Wachovia	PFG	Principal Financial Group				
WFC	Wells Fargo & Co	PGR	Progressive				
WM	Washington Mutual	PRU	Prudential Financial				
WU	Western Union	SAF	Safeco				
ZION	Zion	TMK	Torchmark				
		TRV	Travelers				
		UNH	Unitedhealth Group				
		UNM	Unum Group				

Table 1: Tickers, company names, industry groups.

		Avg. Ret.	Vol.	Corr.	Beta	QLVG
Pre-Crisis (2005-07 to 2007-07)						
Depos.	Q_1	0.01	14.85	0.53	0.86	4.92
	$Median$	0.03	15.95	0.60	0.93	5.90
	Q_3	0.08	17.80	0.66	1.00	6.86
Insur.	Q_1	0.06	15.56	0.36	0.76	2.45
	$Median$	0.12	18.10	0.50	0.83	3.08
	Q_3	0.17	24.88	0.57	1.00	6.83
Bro.-Deal.	Q_1	0.21	20.81	0.62	1.39	1.67
	$Median$	0.23	23.43	0.69	1.55	8.55
	Q_3	0.30	27.84	0.74	1.67	13.05
Other	Q_1	0.00	19.66	0.42	0.96	1.14
	$Median$	0.17	24.20	0.49	1.21	1.51
	Q_3	0.31	31.13	0.56	1.49	5.26
Crisis (2007-07 to 2009-07)						
Depos.	Q_1	-0.98	67.08	0.60	1.37	7.20
	$Median$	-0.37	92.44	0.67	1.78	13.02
	Q_3	-0.18	112.31	0.71	2.09	18.22
Insur.	Q_1	-0.58	55.57	0.56	0.99	2.40
	$Median$	-0.37	68.86	0.65	1.33	4.19
	Q_3	-0.24	105.06	0.73	2.11	19.60
Bro.-Deal.	Q_1	-1.39	65.32	0.52	1.47	1.46
	$Median$	-0.41	86.48	0.76	1.87	20.76
	Q_3	-0.11	136.69	0.77	2.33	39.10
Other	Q_1	-0.70	64.89	0.61	1.45	1.73
	$Median$	-0.45	78.32	0.72	1.76	2.29
	Q_3	-0.24	113.40	0.80	2.06	11.48
Post-Crisis (2009-07 to 2010-07)						
Depos.	Q_1	-0.10	32.33	0.58	1.17	7.31
	$Median$	0.24	37.18	0.64	1.38	10.84
	Q_3	0.38	52.62	0.69	1.62	15.93
Insur.	Q_1	0.12	25.05	0.48	0.84	2.45
	$Median$	0.22	36.45	0.70	1.14	4.48
	Q_3	0.33	43.50	0.80	1.82	17.44
Bro.-Deal.	Q_1	-0.21	33.00	0.61	1.12	3.36
	$Median$	-0.14	34.14	0.66	1.40	10.41
	Q_3	-0.07	42.10	0.76	1.56	18.44
Other	Q_1	-0.11	32.48	0.64	1.15	1.73
	$Median$	0.04	37.96	0.71	1.43	2.24
	Q_3	0.40	47.08	0.79	1.76	7.16

Table 2: Descriptive statistics. The table reports the average return, annualized volatility, correlation, beta and quasi leverage for the firms in the panel. The table reports the 1st quartile, median and 3rd quartile of the statistics across each financial industry sub group. Descriptive statistics are computed over different sample periods: Pre-Crisis (July 2005 to June 2007), Crisis (July 2007 to June 2009) and Post-Crisis (July 2007 to June 2010).

value weighted index return as the market index return. The full list of tickers and company names organized by industry groups is reported in Table 1.

Figure 1 gives visual insights on the boom and bust of the financial sector. The figure shows the cumulative average return by industry group from July 2005 to July 2010. Between July 2005 and June 2007 all financial groups had steep growth, which is particularly strong for the Broker-Dealer group. Starting from July 2007, the financial firms fell dramatically,

		Volatility				Correlation			Tails
		vol	α_V	γ_V	β_V	cor	α_C	β_C	TD
Depos.	Q_1	32.21	0.02	0.08	0.87	0.55	0.03	0.89	0.23
	$Median$	36.92	0.04	0.10	0.91	0.62	0.04	0.93	0.27
	Q_3	40.14	0.07	0.12	0.92	0.66	0.05	0.96	0.38
Insur.	Q_1	29.93	0.02	0.09	0.86	0.44	0.01	0.91	0.11
	$Median$	37.64	0.04	0.11	0.90	0.52	0.03	0.95	0.29
	Q_3	44.12	0.05	0.13	0.92	0.58	0.05	0.98	0.39
Bro.–Deal.	Q_1	37.86	0.00	0.09	0.89	0.65	0.01	0.92	-0.03
	$Median$	41.55	0.01	0.12	0.93	0.69	0.03	0.95	0.28
	Q_3	45.99	0.03	0.22	0.95	0.73	0.04	0.97	0.36
Other	Q_1	36.03	0.02	0.07	0.87	0.47	0.01	0.92	0.11
	$Median$	42.57	0.04	0.09	0.90	0.58	0.02	0.96	0.19
	Q_3	49.84	0.05	0.11	0.92	0.65	0.04	0.98	0.27

Table 3: In–sample estimation results. The table reports the TARCH estimates, the DCC estimates and Excess residual tail dependence index. Instead of the intercept of the volatility and correlation equations, the table reports the unconditional volatility and correlation implied by the parameter estimates. The table reports the 1st quartile, median and 3rd quartile of the estimates across each financial industry sub group.

with the biggest winners transforming into the biggest losers. Financials hit the bottom in March 2010 and started a slow recovery that was interrupted by the European crisis of May 2010. Table 2 reports descriptive statistics of the set of tickers organized by industry group over three subsamples (the pre-crisis, crisis and post-crisis). For each statistic the table reports the median, 1st and 3rd quartiles across each group.

5 MES Analysis

In this section we use the methodology introduced in Section 3 to study the panel. We focus on MES and the analysis of the lower tail dependence dynamics between firm and market returns. We first show in–sample results of our specification and then engage in an out–of–sample prediction exercise where we compare our approach against a set of alternative prediction models.

5.1 In–Sample MES Analysis

Volatilities, correlations and tail expectations as well as short term MES are estimated for each firm over the whole sample period. Here we report summary information on the parameter estimates and fitted series.

The dynamics of the firms in the panel do not have a strong degree of heterogeneity. In Table 3 we show selected quantiles of the parameter estimates of the TARCH (left), DCC (center) and excess residual tail dependence (right) for each group. TARCH parameters do not fluctuate much, with the exception of the intercept that is on average higher for Broker-Dealers and Others. The range of the asymmetric coefficient γ reaches more extreme values for Broker-Dealers, signaling higher sensitivy to large volatility increases in the case of a drop of the stock. Overall, the point estimates are in line with the typical TARCH estimates, with slightly higher α's and γ's together with lower β's implying a higher level of unconditional kurtosis. Turning to the DCC, parameters are again close to the typical set of estimates and, intercept aside, parameters are similar across groups. Broker-Dealers have the highest level of unconditional correlation, followed by Others, Insurance and Depositories. The tail dependence index reports the excess percentage of residuals observed in the 1% lower tail of the bivariate distribution of $(\epsilon_{m\,t}, \xi_{i\,t})$ under the assumption of independence. Interestingly, the excess percentage is almost always positive and quite sizeable (the median excess varies between 19% and 29% across groups), signaling the fact that independence of $(\epsilon_{m\,t}, \xi_{i\,t})$ might be an unrealistic assumption. However, in only a few cases is such excess dependence significant. Inference in the tails typically relies on a few observations and it is often statistically challenging to draw precise conclusions.

The industry group time series averages of volatility, correlation and short term MES provide interesting insights on the dynamics of risk before, during and after the crisis.

Figure 2 displays average volatility by industry group between 2005 and 2010. The plots are dominated by the explosion in variability at the peak of the crisis, which is so prominent

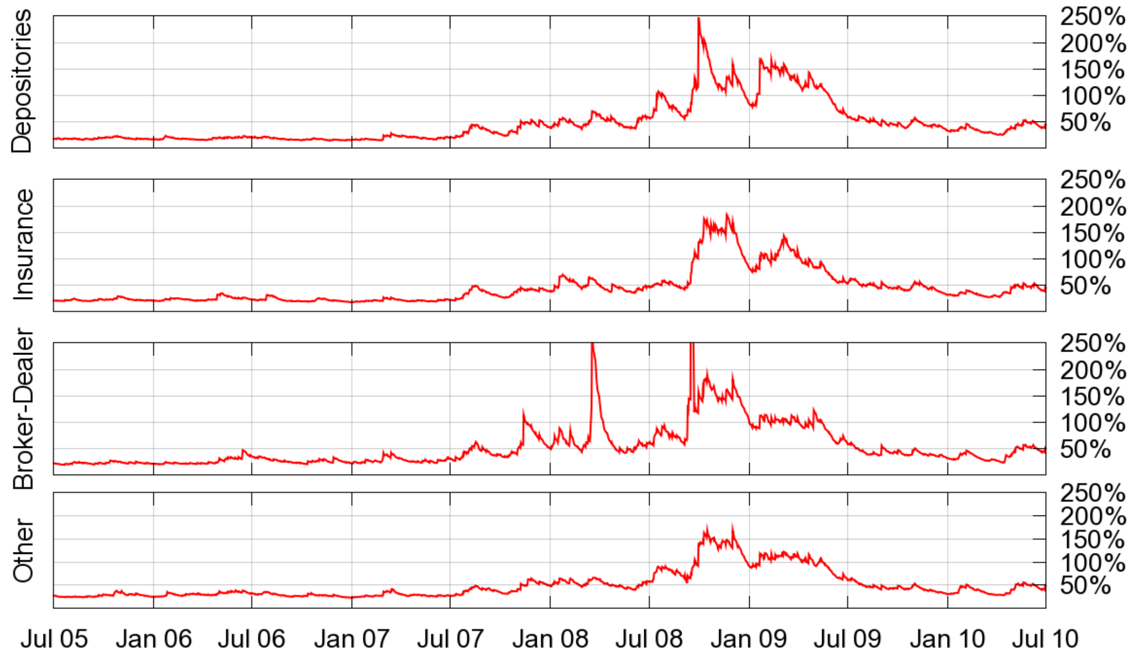

Figure 2: Volatilities. The plots display the average in-sample volatility between July 2005 and June 2010 for each financial industry group.

that it visually dominates all other periods of distress in the sample. The Depositories and Broker–Dealer series have extreme volatility spikes corresponding to well known events in the financial crisis time line: Depositories jump over 200% with the bankruptcy of Washington Mutual (September 2008) and Broker-Dealers go beyond 200% with the acquisition of Bears Sterns (March 2008) and the bankruptcy of Lehman (September 2008). Overall, all groups exhibit a similar time series trend which is also similar to that of market volatility. The pre crisis period is characterised by extremely low levels of variability. Starting from July 2007, volatility gradually increases as the financial crisis unwraps and in October 2008, after the Lehman bankruptcy, it peaks to the highest levels measured since the 1987 crash. It then starts to slowly decay and by March 2009, when the market recovery begins, its level is roughly the same as in the beginning of 2007. All groups progressively cool down until May 2010, when news concerning the European crisis produce a new, yet moderate, surge in volatility. While the overall volatility trend is similar across groups the main difference

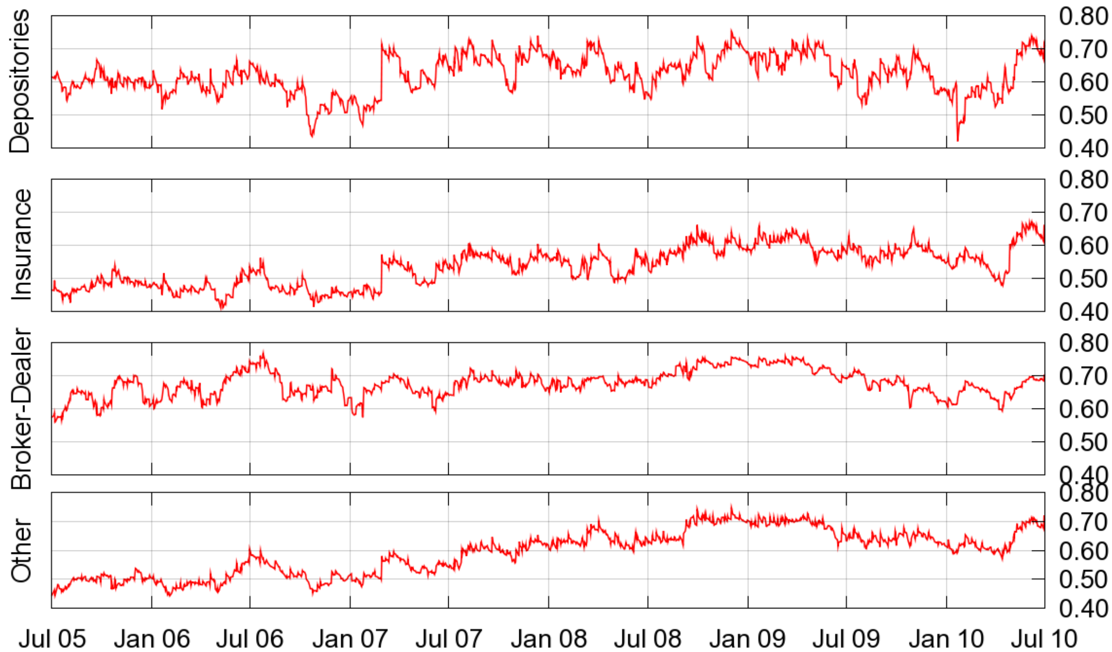

Figure 3: Correlations. The plots display the average in-sample correlation with the market index between July 2005 and June 2010 for each financial industry group.

between the series is their average level. Broker-Dealers together with Other are the most volatile ones, followed by Depositories and Insurance.

Figure 3 shows average correlation by industry group. Once again, the time series pattern is similar although there are some differences between Broker-Dealers and the other three groups. Interestingly, while volatilities before the crisis are low, correlations are moderately high by historical standards, ranging between 0.40 and 0.70. The so–called Chinese correction of February 29 further shifts correlations upwards, with Depositories having the largest increase. As the financial crisis unwinds correlations continue increasing, with Depositories and Broker-Dealers exceeding 0.75 in December 2008. In the post crisis period, correlations slowly begin to decrease until May 2010, when there is a steep increase caused by the European crisis which parallels the surge of volatility. As for volatility, the average levels of correlation differ across groups. Broker-Dealers have the highest correlation, Insurance the lowest one and Depositories together with the Other group lie in the middle.

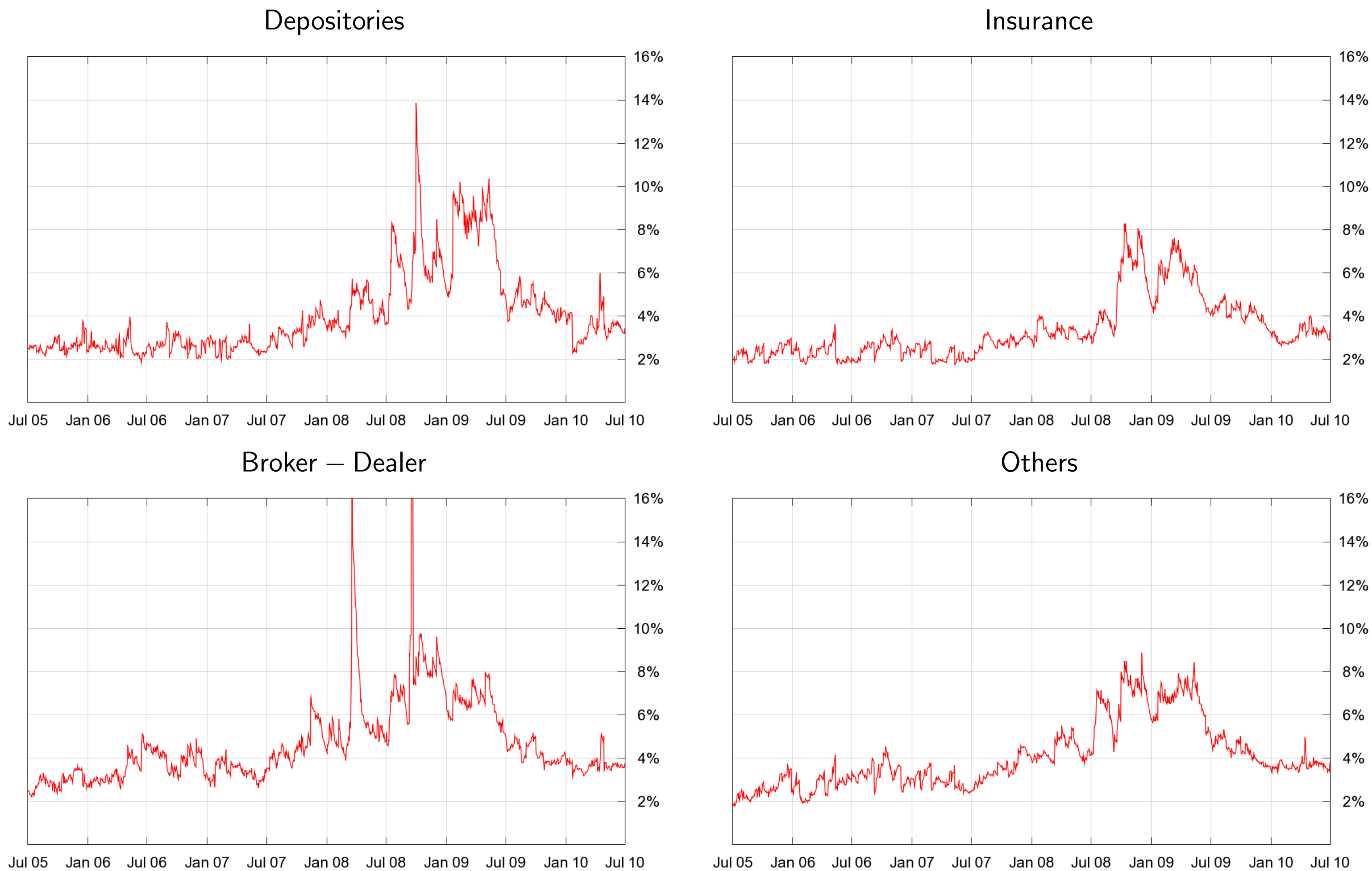

Figure 4: MES. The plots display the average in-sample one step ahead MES between July 2005 and June 2010 for each financial industry group.

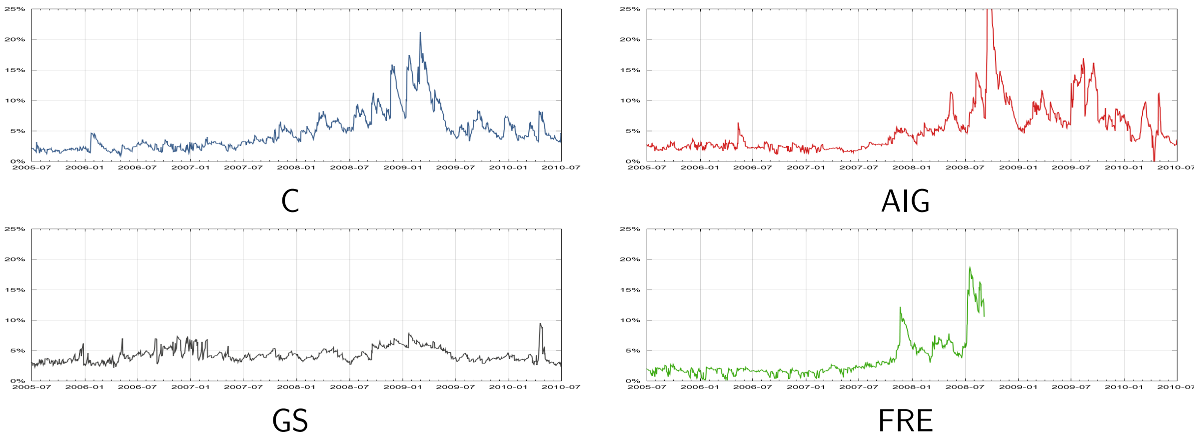

Figure 5: MES. The plots display the in-sample one step ahead MES between July 2005 and June 2010 for Citigroup (C), AIG (AIG), Goldman Sachs (GS) and Freddie (FRE).

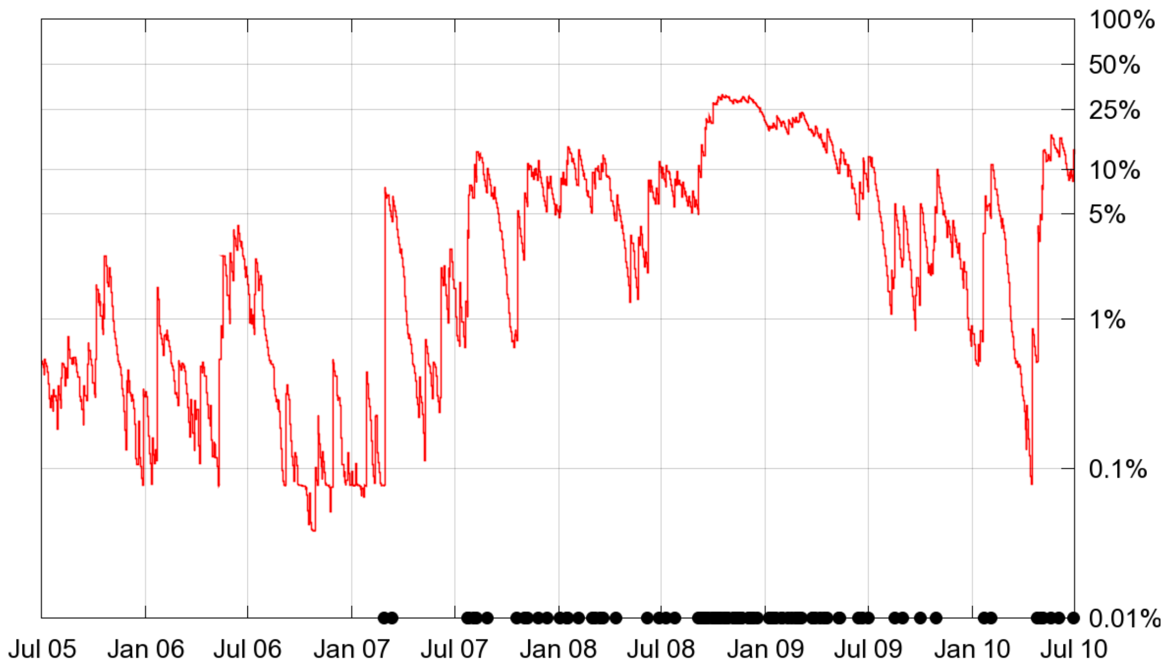

Figure 6: Probability of a 2% market drop. The plot displays the in–sample 1-step-ahead probability of a daily 2% loss in the market between July 2005 and June 2010.

Figure 4 reports the average short term MES by industry group. The short term MES is computed by setting the value of the threshold C equal to -2%, which corresponds approximately to the 5% quantile of the empirical unconditional market return distribution in the whole sample. In the pre crisis period, the levels of MES appear to be roughly similar. The series start to increase from July 2007, reaching their peaks in October 2008. Depositories reach a MES of roughly 7% and go beyond 10% at the end of September 2008, corresponding to the bankruptcy of Washington Mutual. Broker-Dealers on the other hand have the biggest increase in mid March 2008 and mid September 2008 with the acquisition of Bears Sterns and the liquidation of Lehman. MES starts to drop only after March 2009 and by January 2010 is back to its pre crisis levels. The European crisis does increase the level of MES but its impact is moderate. Once more, the main difference across the series lies in their average levels. Broker-Dealers are the most exposed group followed by the Others, Depositories and Insurance. We report individual MES plots for selected companies in Figure 5. We select one company from each group: Citigroup for Depositories, AIG for Insurance, Goldman-Sachs for Broker-Dealers and Freddi Mac for Others. The time series profiles of the companies are close to their group averages with the exception of Goldman-Sachs whose unconditional level of MES is higher than the average but does not fluctuate much during the crisis.

We also report in Figure 6 the time varying probability of observing the systemic event $\{r_{mt} < C\}$. The rug plot displays the actual days on which a systemic event is observed. The number of event days when the market returns are below -2% in this period is 76, most of which are concentrated between the end of 2008 and the beginning of 2009. The probability of a systemic event is a function of the market volatility and, as it can be readily noticed, this series has essentially the same time series profile. We also report that the probability of larger lossess over longer horizons have a similar profile but different level. This is the consequence of the fact that volatility is typically strongly persisent. Thus the probability of observing a drop over a given horizon has a time series profile which is tightly related by the current level of volatility.

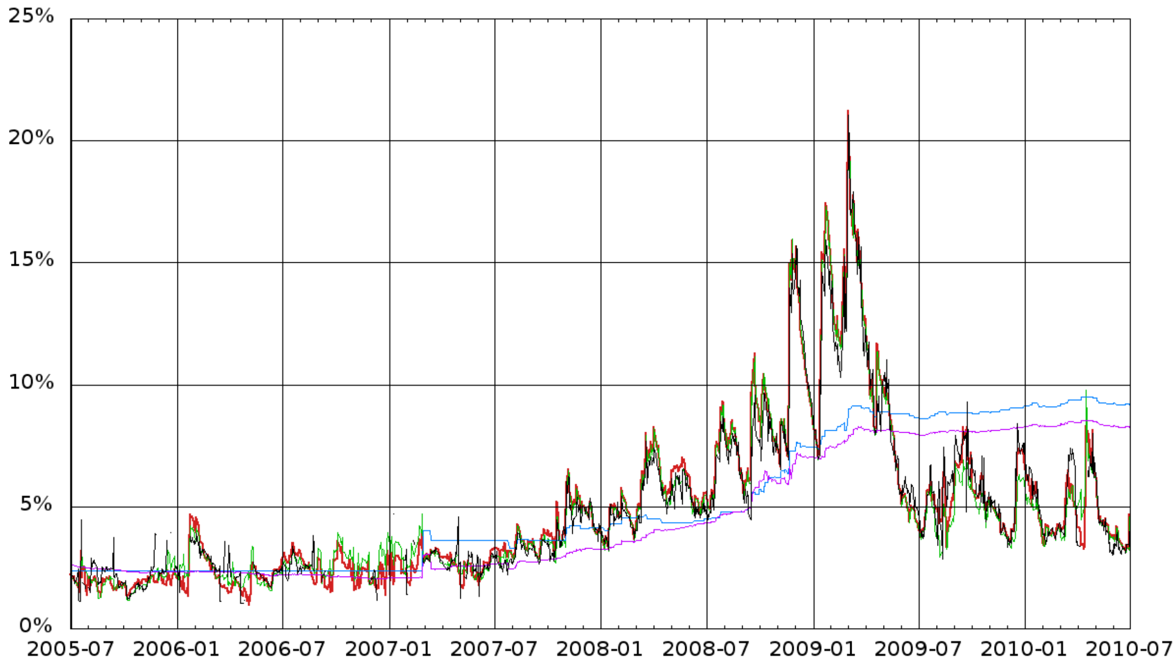

Figure 7: MES forecasts. The plot displays the one period MES forecasts for Citigroup between July 2005 and June 2010. The forecasting methods are the specification proposed in Section 3 (black line), Dynamic Conditional Beta Model (green line), Dynamic Copula Model (red line), Rolling Static Bivariate Model (purple line) and Rolling Static Factor Model (blue line).

5.2 Out–of–Sample MES Analysis

In this section we carry out an out–of–sample predictive exercise to evaluate our proposed modelling approach. We compare the model presented in Section 3 with two static and two dynamic alternatives: a (Rolling) Static Bivariate model, a (Rolling) Static Factor model, a Dynamic Beta model (based on GARCH-DCC, see also Engle (2012)) and a Dynamic Copula model (based on the Rotated Gumbel Copula, see also Patton (2004), Patton (2006)). Models are evaluated on the basis of their ability to adequately capture tail dependence and to produce accurate short term MES predictions.

The (Rolling) Static Bivariate model is the one proposed in Acharya *et al.* (2010) to estimate MES. A simple approach that can be used to carry out inference on the firm and

		UC Rej	DQ Rej	RMSE
VCT	Depos.	13.8	3.4	1.68
	Insurance	34.4	9.4	1.34
	Bro. Deal.	0	10	1.22
	Other	39.1	8.7	2.17
Dynamic Beta	Depos.	31	0	1.67
	Insurance	34.4	12.5	1.51
	Bro. Deal.	10	10	1.22
	Other	39.1	8.7	1.41
Dynamic Copula	Depos.	27.6	0	1.94
	Insurance	53.1	9.4	2.04
	Bro. Deal.	0	10	1.38
	Other	30.4	4.3	1.73
Static Bivariate	Depos.	100	100	2.25
	Insurance	100	100	2.04
	Bro. Deal.	100	70	4.93
	Other	95.7	91.3	2.55
Static Factor	Depos.	96.6	55.2	2.18
	Insurance	87.5	87.5	2.40
	Bro. Deal.	70	30	5.78
	Other	82.6	47.8	2.37

Table 4: Out–of–Sample Performance. The table reports out–of–sample performance metrics of our proposed modelling approach (VCT) time varying beta model (Dynamic Conditional Beta), time varying copula model (Dynamic Coupla), static rolling bivariate model (Static Bivariate) and static factor model (Static Factor). For each specification and industry group, the table reports the percentage of rejections at the 5% significance levele of the UC test, DQ test and the average Relative Mean Square Error of the MES predictions.

market returns is to assume that they are generated by a bivariate distribution

$$\begin{bmatrix} r_{t\,i} \\ r_{t\,m} \end{bmatrix} \sim F,$$

where F is unspecified. Inference on the bivariate distribution F can be carried out by non-parametric methods. A limitation of the approach is that it does not take into account the fact that the distribution of returns changes over time. Thus, a rolling window estimation scheme is used to produce adaptive estimates. In this framework, Acharya *et al.* (2010) suggest to estimate MES as

$$\mathsf{MES}^{1\,\mathsf{his}}_{i\,t}(C) \equiv \frac{\sum_{\tau=t-W}^{t-1} r_{i\,\tau} I(r_{m\,\tau} < C)}{\sum_{\tau=t-W}^{t-1} I(r_{m\,\tau} < C)}, \tag{9}$$

that is the average of firm returns on event days over a given window of the most recent observations (we use $W = 2$ years as in Acharya *et al.* (2010)). The approach is inspired by the risk management practice where rolling averages are often used to obtain estimates of ES or VaR. The (Rolling) Static Factor model assumes that firms are generated by a one factor model

$$r_{i\,t} \;\; = \;\; \beta_i \, r_{m\,t} + \xi_{i\,t} \quad \xi_{i\,t} \sim F_i$$

where the market return $r_{m\,t}$ and the idiosyncratic innovation $\xi_{i\,t}$ are assumed to be independent draws from two unspecified distributions. Once again, the model is estimated using a rolling window approach in order to capture time varying features of the data. The corresponding short term MES predictor in this setting is

$$\mathsf{MES}^{1\,\mathsf{sf}}_{i\,t}(C) \equiv \hat{\beta}_{i\,t} \, \widehat{\mathsf{ES}}_{m\,t}(C), \tag{10}$$

where $\hat{\beta}_{i\,t}$ is the rolling least square estimator of the factor loading using a window of size W and $\widehat{ES}_{m\,t}(C)$ is the rolling estimator of the Expected Shortfall of the market, that is

$$\widehat{\mathsf{ES}}_{m\,t}(C) \equiv \frac{\sum_{\tau=t-W}^{t-1} r_{m\,\tau} I(r_{m\,\tau} < C)}{\sum_{\tau=t-W}^{t-1} I(r_{m\,\tau} < C)},$$

where we take $W = 2$ years. The Dynamic Conditional Beta and Dynamic Copula models are variants of the specification described in Equation (2). The Dynamic Conidtional Beta model is defined as

$$\begin{aligned} r_{mt} &= \sigma_{mt}\epsilon_{mt} \quad \epsilon_{mt} \sim F_m \\ r_{it} &= \sigma_{it}\rho_{it}\epsilon_{mt} + \sigma_{it}\sqrt{1-\rho_{it}^2}\xi_{it} \quad \xi_{it} \sim F_i \end{aligned}$$

with ϵ_{mt} and ξ_{it} are independent. We name this model Dynamic Conditional Beta model since the relation between firm and market returns can be rewritten as

$$r_{it} = \beta_{it} r_{mt} + \sigma_{it}\sqrt{1-\rho_{it}^2}\xi_{it}$$

where $\beta_{it} = \rho_{it}\frac{\sigma_{it}}{\sigma_{mt}}$. In this setting, the one-step ahead MES is given by the formula of Equation (2) without the correction for the tail dependence between idiosyncratic and market returns, that is

$$\mathsf{MES}^{1\,\mathsf{db}}_{it-1}(C) = \sigma_{it}\rho_{it}\mathsf{E}_{t-1}(\epsilon_{mt}|\epsilon_{mt} < C/\sigma_{mt}) = \beta_{it}\mathsf{E}_{t-1}(r_{mt}|r_{mt} < C).$$

The time varying volatility and correlations are modelled using GARCH and DCC models and the tail expectation is estimated using the nonparametric tail estimators previously introduced. Time varying dependence between market and firm returns can also be modelled using dependence measures other then correlation, like measures based on copulas. To this end, the we consider the Dynamic Rotated Gumbel model (cf. Patton (2004), Patton (2006)). We choose this particular specification since Patton (2006) documents that empirically this model fits the data well. Let F_{mt} and F_{it} denote the conditional marginal cumulative distributions of market and firm returns respectively. We can then define the following variables

$$u_{mt} = F_{mt}(r_{m,t}, \theta_m) \quad \text{and} \quad u_{it} = F_{it}(r_{i,t}, \theta_i),$$

which we call uniform margins. The conditional copula function is defined as the conditional cumulative distribution function of these uniform margins

$$C_t(u_{m\,t}, u_{i\,t}) = P_t(U_{m\,t} \leq u_{m\,t}, U_{i\,t} \leq u_{i\,t}),$$

and it characterizes the dependence structure between the market and firm returns. Several types of copulas can be introduced and several modelling devices can be formulated to have time varying dynamics. In what follows, we use the techniques proposed by Patton (2006). We use a rotated Gumbel copula, which uses one parameter to determine the degree of dependence in the lower tail. Specifically, the distribution is defined as

$$C(u_{m\,t}, u_{i\,t}|\delta_t) = u_{m\,t} + u_{i\,t} - 1 + \exp\{-((-\log(1-u_{m\,t}))^{\delta_t} + (-\log(1-u_{i\,t}))^{\delta_t})^{1/\delta_t}\}$$

where $\delta_t \in [1, \infty)$. The parameter δ_t which determines the degree of dependence evolves according to the following autoregressive equation

$$\delta_t = 1 + \left(\omega + \alpha \frac{1}{10}\sum_{\tau=1}^{10} |u_{m\,t} - u_{i\,t}| + \beta\delta_{t-1}\right)^2.$$

Note that this formulation ensures that δ_t is always greater than one so that the copula distribution is well defined. The model is fitted by estimating the marginal models first and then by maximising the copula likelihood using the fitted uniform margins. The specification we employ for the marginals is the previously introduced TARCH model. The rotated Gumbel model does not deliver closed form expressions for MES and we use a simulation based method to compute the tail expectations of interest.

The out–of–sample predictive ability of our proposed model and the four alternative specifications is evaluated using two adequacy tests for lower tail dependence and a loss function that measures the precision of the MES predictions. The adequacy tests are constructed as follows. The predictive distribution implied by each model is used to define regions containing the most extreme negative predicted realizations of the returns process. The regions are chosen as to have a nominal coverage rate of 1%. We then define a series of "hits", that is a series of binary indicators equal to one when the market and firm return pair falls inside the region. If the predictive distribution implied by the forecasting model is correct, then the sequence of hits behaves as an iid Bernoulli sequence with $p = 0.01$. This can be used to construct adequacy tests against a number of different alternatives. Here we use two tests proposed in Christoffersen (1998) and Engle and Manganelli (2004): the unconditional coverage test (labelled as UC) and the dynamic quantile test (labelled as DQ). The null hypothesis of the unconditional coverage test is that the percentage of hits is 1%, which is tested against the alternative of a different proportion of violations. The null hypothesis of the dynamic quantile test (in the version used in this work) is that the series of hits has no dependence, and is tested against the alternative that past hits up to one week prior change the conditional probability of observing a new hit. The precision of MES predictions is evaluated by computing the relative mean square error of the forecasts on event days, that is

$$RMSE_i = \frac{1}{|E_C|} \sum_{t \in E_C} \left(\frac{-r_{i\,t} - \mathsf{MES}_{i\,t}(C)}{\mathsf{MES}_{i\,t}(C)} \right)^2,$$

where E_c is the set of systemic event dates t. We opt for a Relative MSE rather then absolute in that the series of losses exhibits strong heteroskedasticity due to time varying volatility. Standardizing by the MES predictions allows us to make a more fair comparison in that it avoids realized losses on volatile periods to essentially determine the value of this performance measure.

The forecasting exercise is implemented as follows. Starting from July 2005 we compute one-period ahead MES predictions conditionally on a 2% market drop. The parameter estimates needed to produce the forecasts are estimated once a week on the last weekday using all data available starting from the beginning of the sample. The number of event days when the market returns are below -2% in this period is 76, most of which are concentrated between the end of 2008 and the beginning of 2009.

In order to give insights on the differences among the forecasting methods, Figure 7 displays the plot of the short term MES forecasts for Citigroup. The time series profile of the forecasts of the dynamic models is rather similar and forecasts are hard to distinguish. The static forecasting approaches also produce similar forecast, the main difference being the average level of predictions, which is slightly higher for the static factor model. The picture makes a compelling case for the use of dynamic models for forecasting in a risk management context. The static approaches react to the crisis with substantial delay. Similarly, after the crisis, they decline at a rate which appears to be too slow: one year and a half after the Lehman events, predictions are still close to the levels of January 2009. Inspection of the corresponding plots for the other companies in the sample reveals that these patterns are common across tickers.

Table 4 reports summary statistics of the short term MES forecasts for the five methods. For each financial group, the table reports the percentage of 5% violations of the two adequacy tests as well as the group averages of the MES RMSE loss. Inspection of the adequacy test results shows that the static forecasting approach delivers a substantially poor fit of the tails: there is strong evidence for almost all tickers that the proportion of hits is higher than the nominal and that the hit sequence exhibits serial dependence. The dynamic models have a proportion of hits that is close to the nominal one and perform reasonably well in terms of removing dependence in the series of hits. Looking at the RMSE group loss, the static models never perform better than any of the dynamic specifications, and the RMSE of the dynamic models are approximately 50% lower than those obtained with the alternative approaches.

Overall, these results show that using a dynamic model is crucial to capture dependence adequately and produce accurate MES predictions. Admittedly, we do not attempt to carry out a detailed forecasting comparison exercise across these dynamic models and other possible candidates; such an analysis would be beyond the scope of the present paper. The point we wish to make, however, is that using dynamic specifications is important for accurate real time measurement and that the typical models used in the financial econometric toolbox perform satisfactorily when compared to more sophisticated models like dynamic copula models. Our results suggest that the simplest time series specification for the dynamics, the Dynamic Conditional Beta model, captures most of the dependence and that the role of nonlinear tail dependence is only marginally relevant. In the rest of the analysis we will stick to our proposed approach, but we acknowledge that results based on the Dynamic Conditional Beta are substantially similar.

6 SRISK Analysis

In this section we investigate the SRISK index proposed in Section 2. Using balance sheet data and long term MES predictions, we compute firm specific and system wide SRISK indices for our panel of financial companies. We begin with the study of the time series and cross sectional characteristics of SRISK in the sample. We then focus on evaluating the ability of the indices to provide early warning signals that can be useful for regulation and macro-prudential policy. Finally, we investigate the determinants of SRISK. It is important to stress that throughout this section the SRISK index is always computed in a purely forward looking manner and that it has no look ahead bias.

6.1 Time Series Profile and Cross Sectional Rankings of SRISK

We compute SRISK as follows. At the end of each month, starting from July 2005 until July 2010, we estimate our specification for each firm in the panel that has been trading for at least two years. For each of these tickers we estimate the volatility and correlation models introduced in Section 3 using all information available from the beginning of sample (July 2010). The simulation based procedure previously described is applied to compute long term MES for each firm: Starting from the end of the estimation sample, we simulate the bivariate firm/market returns system six months ahead and then use the simulated paths to compute the expected return of the firm conditional on a market dropped beyond -40%. We

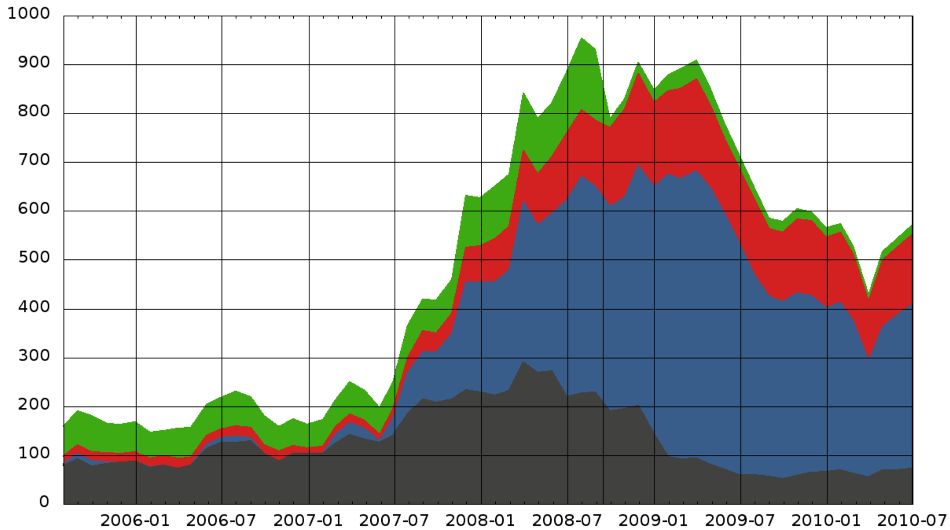

Figure 8: Aggregate SRISK. The plot displays the aggregate SRISK of the top U.S. financial institutions broken down by financial subindustry group between July 2005 and June 2010. The sub industry groups are ordered top to bottom: Others, Insurance, Depositories and Broker-Dealers

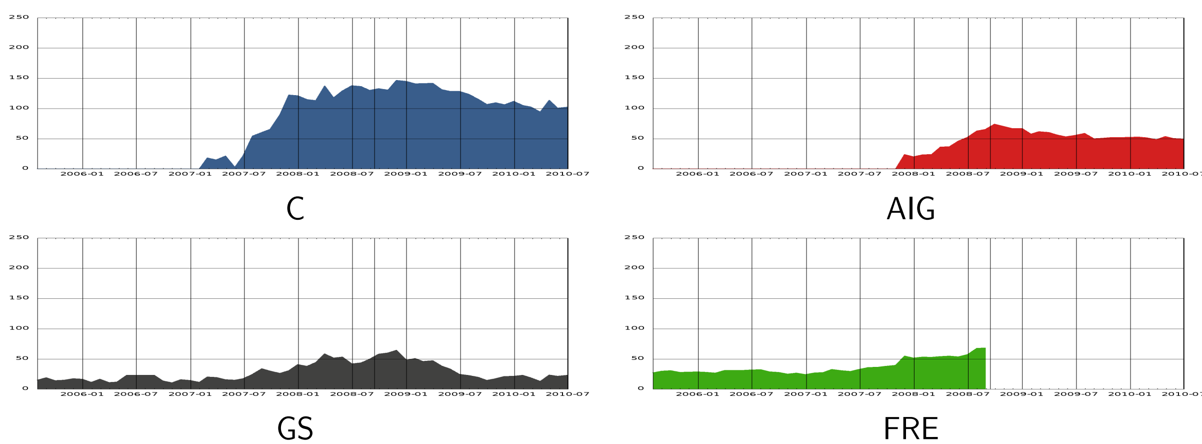

Figure 9: Individual SRISK. The plots display the SRISK between July 2005 and June 2010 for Citigroup (C), AIG (AIG), Goldman Sachs (GS) and Freddie (FRE).

chose a -40% decline over six months to approximately match the magnitude of the market correction in the crisis. Using the book value of debt and the market value of equity we then compute the expected capital shortfall and, finally, the SRISK index. The book debt data come from the COMPUSTAT database and are updated each quarter. The SRISK results we show are based on a prudential ratio k equal to 8%. We later investigate the consequences of alternative choices of these values. Our methodology allows for computation of SRISK on a daily basis as well. However, we chose to update the series on a monthly basis only to reduce the computational burden of the analysis.

Figure 8 displays the aggregate SRISK in the panel broken down by financial industry group. The aggregate SRISK is defined as the sum of all the positive capital shortfalls in the system. In the pre-crisis period the total shortfall is estimated to be close to 200 bln USD. Most of the capital shortage comes from the Broker-Dealers and Others groups. This is due to the fact that in these groups there are several institutions with high levels of Leverage and, as seen in the previous section, also high MES. The main contributors in the Other sectors are Freddie and Fannie. Aggregate SRISK rapidly escalates starting from 2007, and especially after July 2007 when the effects of the subprime crisis become more apparent. The increase of SRISK also corresponds to a change in its composition: the relevance of Depositories and Insurance becomes progressively larger. In January 2008 the level of SRISK has roughly tripled with respect to the previous year. The series peaks around the Lehman bankruptcy at roughly 1000 bln USD. The contribution of the Others becomes quite small shortly after these events as soon as Freddie and Fannie are place under conservatorship (if Goldman-Sachs would have been kept in this group, the SRISK of this sector would have been much larger). After its peak, the recovery is rather slow and it becomes more sustained only after March 2009, with the beginning of the market rally. As of July 2010 the system still looks far away from a full recovery.

Figure 9 displays the individual SRISK series for four institutions in the panel (cf Figure 5 with the corresponding MES series). In the pre-crisis period Goldman-Sachs and Freddie have large capital shortfalls while Citigroup and AIG are appropriately capitalized. Starting from January 2007 capital shortages start to rise quickly and Citigroup and AIG have a sharp increase. The increase is particularly severe for Citigroup, which by January 2008 exceeds 100 bln USD. Overall, the time trend of the SRISK series is rather uniform across firms and the groups to which they belong.

Table 5 reports the rankings of the most systemically risky U.S financial institutions. On eight pre selected dates we report the top ten most risky institutions according to the SRISK% measure. At the beginning of 2007, when the total capital shortfall of the system is modest, the list contains mostly highly levered Broker-Dealers firms together with Freddie and Fannie. As the crisis unwinds, large commercial banks, like Citigroup, Bank of America and JP Morgan, start rising up in the top ten. The change in the composition of the top ten is consistent with the evidence of Figures 8 and 9. It is important to stress that as of March 2007, approximately 1 year and a half before the Lehman bankruptcy, nine firms out of the SRISK top ten are institutions that in different ways have been severely hit by the crisis and that produced negative externalities to the economy.

Detailed results for each company in the panel as well as historical rankings can be viewed on-line at vlab (`http://vlab.stern.nyu.edu`), where results are updated on a weekly basis.

6.1.1 Choice of the SRISK parameters and Alternative Ranking Comparison

In this section we carry out a number of sanity checks to assess how sensitive SRISK is to changing its parameters and how close SRISK rankings are to those provided by other indices.

We use Citigroup to show different SRISK profiles depending on the choice of the systemic event threshold C and the prudential ratio k (we keep the MES horizon fixed to h equal to six months in all cases). Figure 10 shows SRISK computed using $k = 8\%$ and $C = -40\%$ (our preferred choice), $k = 8\%$ and $C = -30\%$ and $k = 12\%$ and $C = -40\%$. It is straightforward to see from Equation 1 that increasing k or $-C$ increases the SRISK index: the stricter the capital requirements and/or the more extreme the threshold of the systemic event, the larger the capital shortage. Figure 10 quantifies the size of these increments for Citigroup. It is important to stress that the overall time series profile of SRISK is unaffected

2007-03-30			2007-06-29			2007-12-31			2008-02-29		
1	MS	22.3%	1	MS	19.23%	1	C	19.6%	1	C	19.3%
2	FNM	13.4%	2	FRE	13.22%	2	MS	9.9%	2	MS	9.8%
3	FRE	13.0%	3	MER	13.22%	3	MER	9.2%	3	MER	9.6%
4	MER	9.6%	4	FNM	8.95%	4	FRE	8.6%	4	FRE	8.2%
5	LEH	9.4%	5	C	8.83%	5	FNM	6.7%	5	FNM	6.8%
6	BSC	7.7%	6	LEH	8.73%	6	GS	6.5%	6	GS	6.5%
7	GS	7.7%	7	BSC	8.68%	7	JPM	6.1%	7	JPM	6.5%
8	C	6.0%	8	GS	7.07%	8	LEH	5.9%	8	LEH	5.9%
9	MET	3.5%	9	JPM	4.63%	9	BSC	3.9%	9	BSC	4.0%
10	JPM	3.4%	10	MET	3.951%	10	WM	3.1%	10	AIG	3.2%
2008-06-30			2008-08-29			2009-01-30			2010-06-30		
1	C	15.6%	1	C	14.0%	1	C	16.0%	1	BAC	18.5%
2	BAC	9.0%	2	JPM	9.6%	2	JPM	15.6%	2	C	17.9%
3	JPM	7.8%	3	BAC	8.7%	3	BAC	14.1%	3	JPM	14.3%
4	MS	7.5%	4	FRE	7.3%	4	WFC	8.6%	4	AIG	7.7%
5	MER	7.1%	5	AIG	7.0%	5	AIG	6.6%	5	WFC	7.1%
6	FRE	6.5%	6	MS	7.0%	6	GS	5.8%	6	MS	4.4%
7	FNM	6.1%	7	FNM	6.9%	7	MS	4.8%	7	MET	3.7%
8	AIG	5.9%	8	MER	6.9%	8	MET	3.3%	8	GS	3.4%
9	LEH	4.9%	9	GS	5.3%	9	PRU	3.1%	9	PRU	3.0%
10	WB	4.9%	10	WB	5.0%	10	HIG	2.4%	10	HIG	2.8%

Table 5: SRIKS rankings. The table displays the SRISK rankings of the institutions in the panel on a number of selected dates. For each date the table reports the firms in the top ten ordered by rank and the SRISK%, the proportion of the total capital shortage due to an individual firm.

Date	$C = -30\%$ $k = 8\%$	$C = -40\%$ $k = 12\%$
2007-03-30	0.8	1.0
2007-06-29	0.9	1.0
2007-12-31	0.9	0.9
2008-02-29	1.0	0.8
2008-06-30	1.0	0.9
2008-08-29	0.9	1.0
2009-01-30	1.0	1.0
2009-06-30	1.0	1.0

Table 6: SRISK sensitivity. The table reports the proportion of common companies in the SRISK top ten produced on a number of selected dates using $k = 8\%$ and $C = -40\%$ (our preferred choice) versus $k = 8\%$ and $C = -30\%$ (left column) and $k = 12\%$ and $C = -40\%$ (right column).

Date	MES	Size	Leverage	SES
2007-03-30	0.30	0.08	-0.01	-0.00
2007-06-29	0.29	0.17	-0.10	-0.11
2007-12-31	0.29	0.18	0.00	-0.01
2008-02-29	0.33	0.15	0.03	0.05
2008-06-30	0.22	0.19	0.02	0.03
2008-08-29	0.32	0.26	0.10	0.05
2009-01-30	0.40	0.24	0.45	0.44
2009-06-30	0.48	0.28	0.26	0.30

Table 7: SRISK Ranking Comparison. The table reports the rank correlation between the SRISK rankings and the ones produced using MES, Size, Leverage and SES (Acharya *et al.* (2010))

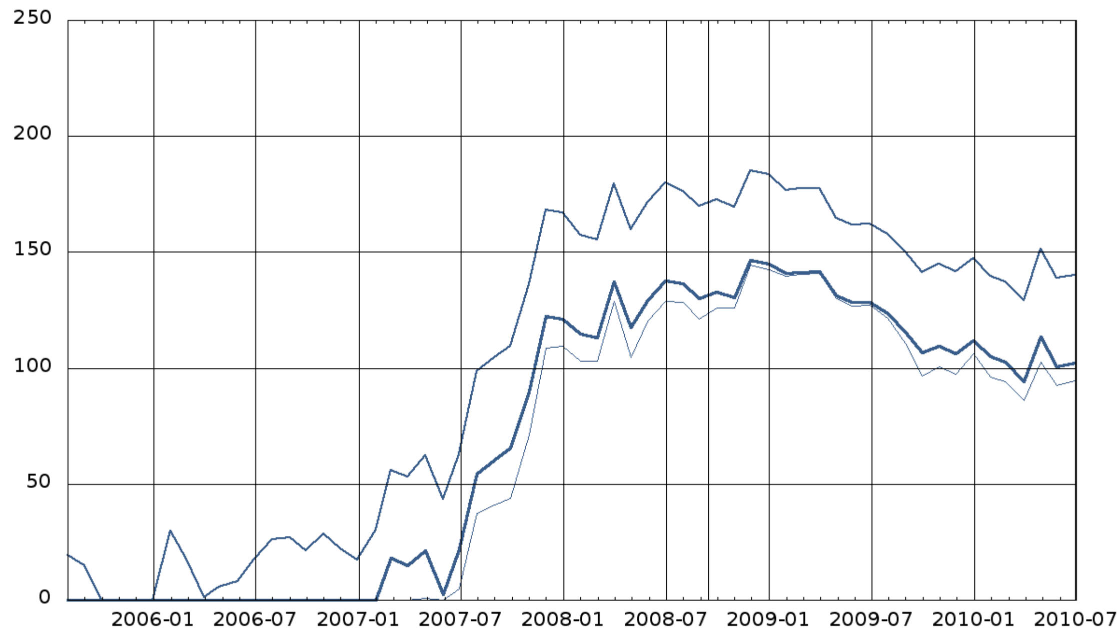

Figure 10: SRISK sensitivity. The plot displays the SRISK index for different values of the SRISK parameters k and C for Citigroup. The SRISK series in the middle is computed using $k = 8\%$ and $C = -40\%$ (our preferred choice). The SRISK series on top is computed using $k = 12\%$ and $C = -40\%$ and the one on the bottom with $k = 8\%$ and $C = -30\%$.

by the choice of k and C and that typically the increments across series in the panel are of comparable magnitude. Table 6 shows the sensitivity of the SRSIK top ten to alternative choices of these parameters. The table reports the percentage of companies in the SRISK top ten that are common to our preferred choice of k and C and the two alternatives. The percentage is never smaller than 80% and it is often 100%. Detailed inspection of the results shows that the companies at the very low end of the rankings are quite sensitive to changes to k and $-C$. However, the top positions are relatively stable for reasonable choices of the parameters. On the companion website, we allow users to select their preferred choice of k and see how this affects rankings.

Another important exercise to assess the value of the SRISK methodology is to understand how close SRISK rankings are to those produced by other variables. Table 7 reports the rank correlation of the SRISK rankings and the ones provided by MES, Size (Market Cap),

	(1)	(2)
const	8.76***	7.80***
Insurance FE	-1.72**	-2.02***
Broker-Dealer FE	2.20**	1.26
Other FE	-2.87**	-2.55**
SRISK		0.18**
R^2	0.39	0.48

Table 8: Fed Injections Regression. The table reports the regression results of the max Fed capital injection after March 2008 received by each firm in the panel accessing the Fed programs. The table reports parameter estimates as well as the R^2 index. Asterisks denote significance at the standard confidence levels.

Leverage and SES (Acharya *et al.* (2010)). Rank correlation is generally quite low. The variable that provides the highest rank correlation is MES, but the rank correlation is never higher than 0.48. This, however, is due to the fact that the SRISK index is also a function of MES. Interestingly, the rank correlation with the SES is substantially smaller and is even negative on a number of dates, reinforcing our claim that SRISK and SES provide empirically different approaches to measure systemic risk.

6.2 Evaluating SRISK

A challenge of empirical research on systemic risk measurement is the design of appropriate evaluation methodologies. In this work, as well as in other research, systemic risk indices stem from particular notions and definitions of systemic risk, which makes the task of assessment and comparison more challenging. In this section we present two evaluation strategies. We first relate firm specific SRISK, which is a measure of capital shortage in periods of distress, to actual capital injections carried out by the Fed during the crisis. The second exercise consists of assessing whether aggregate SRISK can provide early warning signals of distress in the real economy.

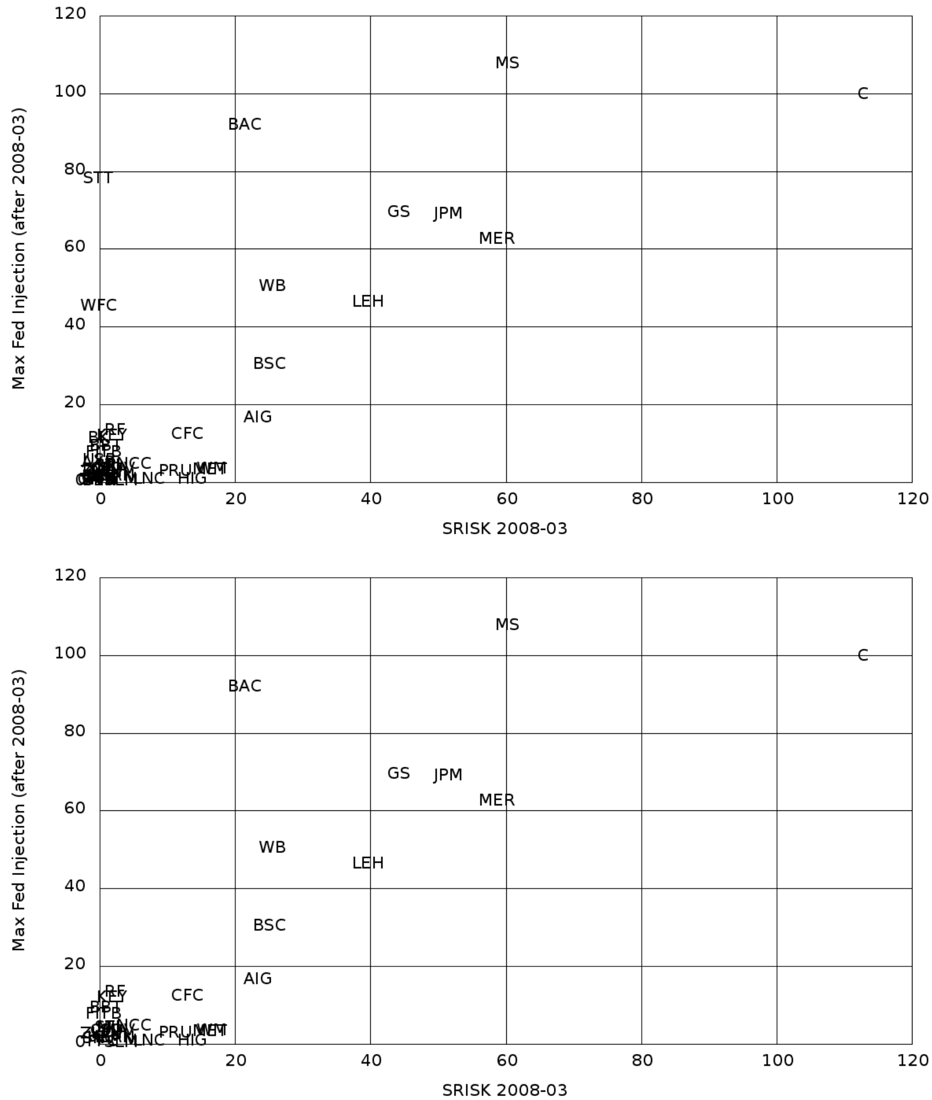

Figure 11: Fed Injections Scatter. The plot shows the scatter plot of Fed capital injections versus SRISK. For each firm in the our panel that has also been accepted in the Fed capital programs, the scatter shows the plot of the max capital injection after March 2008 versus SRISK as of March 2008. The top panel reports both companies with positive and zero SRISK while the bottom one reports those with positive SRISK only.

6.2.1 Individual SRISK as a Proxy of Capital Injections

Between 2007 and 2009 the Federal Reserve carried out several programs to inject capital into the financial system, the most famous and largest one being the Troubled Asset Relief Program (TARP). A natural question is whether SRISK could have predicted individual Fed capital injections during the crisis.

The results of this type of exercise give interesting insights into the predictive power of SRISK. However, they also need to be interpreted with caution. Government and firm incentives play a crucial role in the way such programs are carried out, and this is a topic of active research. Bayazitova and Shivdasani (2012) provides a detailed assessment of these Fed programs in the context of the financial crisis. Among other findings, the authors document that there is strong evidence of self selection bias: Only strong institutions applied to the programs. On the other hand, the Fed approval data show that it approved firms that were on average larger, with strong asset quality and posed greater systemic risk. Moreover, not all approved firms tapped into the programs because of capital needs. A number of companies, one the most famous being State Street (ST), have been criticized for tapping into the programs for profit: Firms took advantage of the substantially low interest rates on the Fed loans and reinvested the capital in higher interest investments.

The data used for this exercise is the Bloomberg Loan Crisis Data, a Bloomberg compiled dataset using data from the Fed. Of the 407 banks and companies that accessed the Federal Reserve programs, 40 financial institutions are also included in our our sample. We focus on assessing if SRISK predicts the amount of capital borrowed by these institutions during the crisis.

Figure 11 shows a scatter plot of SRISK as of March 2008 versus the maximum level of borrowings after March 2008 for each firm in the panel that accessed the Fed programs. The points in the scatter can be clustered into three groups: the first group has low or zero SRISK and a low level of borrowings, the second group has a high level of SRISK and high level of borrowings, and the third group is characterized by low or zero SRISK and high level of borrowings. Overall, the scatter conveys that SRISK is able to successfully identify the capital needs of individual firms: Almost all companies with an SRISK higher than 20 bln ended up receiving a significant amount of bailout funds from the emergency programs. There are also companies that are flagged as not risky by SRISK that ended up

	SRISK	INDPRD	URATE
SRISK		0.02	0.15
INDPRD	5.05**		10.79***
URATE	1.59	9.04***	

Table 9: Granger Causality Tests. The table reports the test static of the Granger Causality test. The i, j entry of the table reports the test statistic to assess if series j Granger Causes series i. Asterisks denote significance at the standard confidence levels.

receiving significant funds, like Wells Fargo (WFC) and State Street (ST). However, for the reasons explained before, only Wells Fargo should be rightly consider as a "true negative". Table 8 reports the results of the regression of the (log) max borrowing after March 2008 on group fixed effects, SRISK as of March 2008. Instead of the level of SRISK, we take $\log(1 + \text{SRISK})$ as SRISK can be 0. The regression results show that SRISK significantly explains the cross sectional variation of Fed capital injections and increases the R^2 of the cross sectional regression by 10% with respect to a baseline model containing fixed effects only.

6.2.2 Aggregate SRISK as an Early Warning Signal

A common feature of most systemic risk definitions, including the one implied by Federal Reserve Governor Tarullo, is the fact that a surge in Systemic Risk has negative spillover effects on the real economy. Building upon this notion, in this section we investigate whether SRISK provides early warning signals of distress in the real economy.

To this end, we consider a multivariate system made up of the monthly growth rates of aggregate SRISK, Industrial Production and Unemployment, that is,

$$y_t = \begin{bmatrix} \Delta \log(SRISK_t) \\ \Delta \log(INDPRD_t) \\ \Delta \log(URATE_t) \end{bmatrix},$$

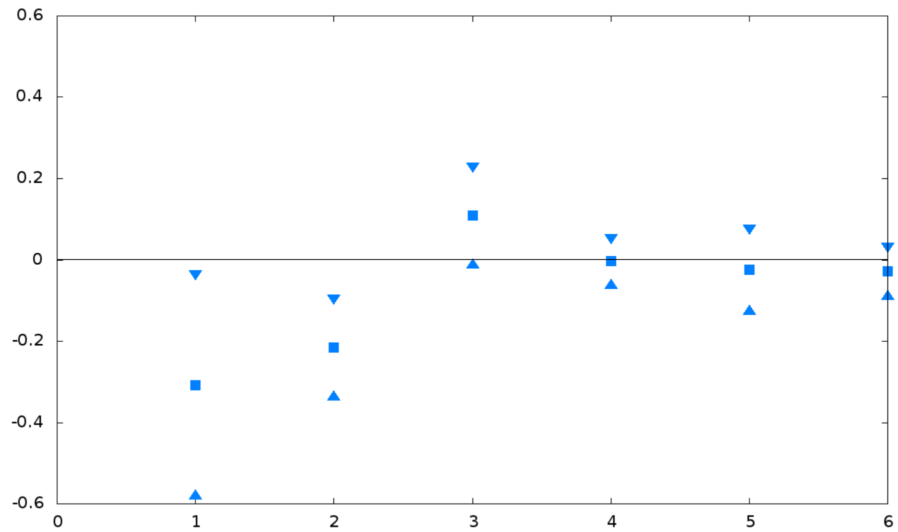

Figure 12: Impulse response function of SRISK on Industrial Production The plot displays the Impulse response function of SRISK changes on Industrial Production from lag one to six. Square denote point the estimates and the triangles 95% confidence intervals.

and we model it as a vector autoregressive (VAR) process

$$y_t = \sum_{l=1}^{p} A_j y_{t-l} + \epsilon_t.$$

In order to analyse the role of each variable in the system we use to Granger causality tests: We rely on a multivariate notion of Granger causality: variable i is said to Granger cause variable j if i contributes to the prediction of j once we condition on the entire past of history of j and *all* other variables in the system.

The model is estimated between 2005 and 2010 (which delivers a sample size of 60 observations) via least squares. The order of the VAR process is one. Increasing the order of the VAR process delivers similar estimates but significance deteriorates quickly as VAR parameters proliferate. We use Industrial Production rather than GDP as a proxy of the growth in the economy as GDP is only released quarterly. Moreover, the time span of the application makes it challenging to use quarterly data. The monthly data on Industrial Production and Unemployment are from the St. Luis Fred database.

We report the outcome of the Granger causality analysis in Table 9. Results show that SRISK is not influenced by the real variables. On the other hand, Industrial Production is Granger caused by SRISK. Figure 12 plots the impulse response function of SRISK on Industrial Production. The graphs confirms that the "sign" of the relationship is the expected one: one standard deviation shock to SRISK decreases Industrial Production by approximately 30 basis points in the following month and 20 basis points in two months. Unemployment is not Granger caused by SRISK but, on the other hand, is Granger caused by Industrial Production. Thus an SRISK shock will have an indirect impact on Unemployment through the Industrial Production channel.

Overall, results convey a picture that is in line with economic intuition. Capital shortages are generated within the financial system and are not predicted by real variables. When capital shortages increase and the economy is in distress, the financial system will stop functioning properly and this will impair industrial production as, for instance, business will have problems accessing credit. Through the industrial production channel, the consequences of capital shortage will then propagate to other real variables such as unemployment.

6.3 SRISK Determinants

In this section we investigate which variables explain the dynamics of individual and aggregate SRISK. We follow the approach used, among others, by Engle and Rangel (2008), Bekaert *et al.* (2013): We regress estimated (individual or aggregate) SRISK and regress it on a set of explanatory variables. The set contains standard variables also used in other studies, like Adrian and Brunnermeier (2009). Both exercises are performed using monthly series of SRISK from July 2005 to July 2010.

	(1)	(2)	(3)	(4)	(5)
const	2.09***	5.85***	2.77***	4.57**	5.55***
Insurance FE	-3.85***	-4.65***	-3.86***	-5.93***	-4.34***
Broker-Dealer FE	4.62**	4.77**	2.69	2.70	4.92**
Other FE	-0.69	0.45	-1.17	-2.68	0.37
mm	1.35***				0.57*
m2b		-2.00***			-1.37***
roe			-47.82***		-25.84***
liq				-3.36	-1.02
R^2	0.10	0.16	0.14	0.07	0.18

Table 10: Determinants of Individual Systemic Risk. The table reports the regression results of Individual SRISK. The table reports parameter estimates as well as the R^2 index. Asterisks denote significance at the standard confidence levels.

6.3.1 Individual SRISK Determinants

We now turn to determinants of individual firms' SRISK. We transform the index as $\log(1 + SRISK)$ to avoid issues with zeros. The set of explanatory variables we consider contains: Market to Book (m2b), Maturity Mismatch (mm), Return on Equity (roe) and the Quick Liquidity ratio (liq). We also include group fixed effect to capture cross-sectional heterogeneity in the panel. Note that we have excluded from these regressions variables that are used to construct the index.

Table 10 reports the regression estimates. The table shows both simple regressions results, where each candidate determinant is considered individually, as well as the multiple regression results where all variables enter the equation simultaneously. The impact and significance of institution characteristics is similar the same across regressions, with the exception of Maturity Mismatch that becomes significant at the 10% level when modelled jointly with the other variables. The impact of Market to Book on systemic risk is negative: Undervalued companies tend to be associated with lower systemic risk. The marginal effect of ROE is also significant and with the expected sign: Firms with higher return on equity have lower systemic risk. Results show that systemic risk is higher for weaker financial institutions. Also, the impact of Maturity Mismatch on individual SRISK is positive: companies

	(1)	(2)	(3)	(4)	(5)	(6)
const	12.04***	12.86***	12.90***	12.91***	12.82***	11.94***
vix	0.47***					0.54***
dtb3		-0.25***				-0.11
dcs			0.07			0.04
rm				-0.16*		0.16
d3houst					-0.17**	0.07
R^2	0.50	0.12	0.01	0.04	0.05	0.52

Table 11: Determinants of Aggregate Systemic Risk. The table reports the regression results of Aggregate SRISK. The table reports parameter estimates as well as the R^2 index. Asterisks denote significance at the standard confidence levels.

with a higher degree of balance sheet maturity mismatch have higher expected shortages in a downturn. Judging from the R^2, Market to Book and ROE are the variables that contribute the most in explaining the cross sectional variation in the panel, and all together the model explain slightly less than 20% of the overall variability in SRISK. In all specification, the Insurance and Broker Dealer fixed effects are significant, implying that *ceteris paribus* the level of systemic risk is lower for Insurance companies and higher for Broker Dealers.

6.3.2 Aggregate SRISK Determinants

First, we attempt to identify drivers of Aggregate SRISK. To this extent, we consider the series of log monthly aggregate SRISK between July 2005 and July 2010. The set of explanatory variables we consider contains:

vix: the end of month value of the Volatility Index;

dtb: the monthly change in the three month term Treasury bill rate;

dcs: the monthly change in the credit spread which is defined as the spread between 10 year BAA rated bonds and the treasury rate with the same maturity;

mkt: the monthly market return on the S&P 500;

dhouse: the monthly percentage change in the number of new privately owned housing units in the U.S.

Table 11 reports the regression estimates. The table shows both simple regressions results, where each candidate determinant is considered individually, as well as the multiple regression results where all variables enter the equation simultaneously. The simple regressions detect that the VIX, the change in the three month treasury bill and the percentage change in housing significantly explain the level of SRISK. Signs are in line with economic intuition: The marginal impact of volatility on SRISK is positive, while the impact of a change in the bill rate or housing is negative. However, the joint regression estimation results show that only volatility remains significant. The results convey that the most relevant determinant of SRISK is indeed market volatility, which explains about half of the total variation in aggregate SRISK. It is important to stress however, that SRISK is not simply a function a volatility. This can be seen by comparing the plot of aggregate SRISK (Figure 8) and volatility (for instance, Figure 2). The most qualitatively striking difference between the two series is that after the peak of the crisis (following the Lehman bankruptcy) volatility reverted back to lower levels rather promptly while SRISK levels remained fairly stable. This is a consequence of fact that our systemic risk index also depends on the assets of the firm: After the crisis even if volatility reverted to pre-crisis levels fairly quickly, the assets of the financial institutions in the panel were still weak and required more time to heal.

7 Conclusions

The 2007-2009 financial crisis highlighted the need for a better understanding of systemic risk. In this paper we propose a systemic risk index called SRISK that measures the expected capital shortfall of a financial institution in a crisis. SRISK is a function of the leverage, size and MES of the firm. While the leverage and size of a firm can be measured using balance sheet data, the measurement of MES requires appropriate time series methodology. To this extent, we develop an econometric model for the bivariate firm and market returns which decomposes the dynamics of the system in terms of time varying volatility, correlation and possibly nonlinearly dependent shocks. We use this methodology to analyse the systemic risk of top U.S. financial firms between 2005 and 2010. The SRISK analysis provides useful insights for monitoring the financial system and, retrospectively, it captures several of the early signs of the crisis. Among other findings, we show that individual firm SRISK produces useful rankings of the most systemically risky firms at various stages of the crisis while aggregate SRISK produces useful early warning signals of distress in the real economy.

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