Urban Simulation Report

Student number: 23083053

1 Part 1: London's Underground Resilience

The first part of this report aims to address the resilience of the London Underground using network analysis.

1.1 Topological Network

1.1.1 Centrality Measures

Centrality measures are characteristics of nodes showing their *importance* in various aspects. In this section, we will identify nodes on the network with the highest centrality in the following measurements: degree centrality, closeness centrality, and betweenness centrality. The 3 central measures that I will cover in this report are: degree centrality, betweenness centrality, and closeness centrality. We will consider a network of n nodes, and the number of links between nodes i and j will be denoted as A_{ij}.

Degree centrality is the number of links that are connected to each node. Considering the underground as an undirected graph, the degree centrality k_i for node i is calculated as

$$k_i = \sum_j A_{ij} \tag{1}$$

In the context of the underground network, the degree corresponds to the number of lines that serve each station counting 1 for each direction. A high degree centrality indicates there are many lines that serve the station, thus identifies importance of the station as a transit hub that allows for transfer between multiple lines.

The stations with the highest degree centrality are shown in Table 1. These stations match the characteristics of being a large transfer station.

Table 1: Stations with the highest degree centrality. All stations tied at a degree of 6 are listed.

Rank	Station	Degree
1	Stratford	9
2	Bank and Monument	8
3	King's Cross St. Pancras	7
3	Baker Street	7

Rank	Station	Degree
5	Waterloo	6
5	West Ham	6
5	Canning Town	6
5	Liverpool Street	6
5	Earl's Court	6
5	Green Park	6
5	Oxford Circus	6

Betweenness centrality is defined by the number of shortest paths that run through the node (or link). The betweenness centrality x_i can be calculated as

$$x_i = \sum_{st} \frac{n^i_{st}}{g_{st}} \tag{2}$$

where

$$n^i_{st} = \begin{cases} 1 & \text{(if node } i \text{ is on geodesic from } s \text{ to } t) \\ 0 & \text{(otherwise)} \end{cases}$$

and g_{st} is the total number of geodesic paths from s to t. To normalised, the raw value should be divided by all possible combinations of shortest paths between the other $n-1$ nodes: $\frac{(n-1)(n-2)}{2}$. High betweenness centrality on the underground shows there are many passengers travelling through the station during their journeys, which are stations en route of connecting the suburban areas with the city centre, or stations on short-distance routes. When this station becomes inaccessible, a large amount of people will be affected.

The stations with the highest betweenness centrality are shown in Table 2.

Table 2: Stations with highest betweenness centrality.

Rank	Station	Betweenness Centrality
1	Bank and Monument	17,602
2	King's Cross St. Pancras	16,780
3	Stratford	14,548
4	Baker Street	13,200
5	Oxford Circus	12,584
6	Euston	12,401
7	Earl's Court	11,454
8	Shadwell	11,127
9	Waterloo	10,407
10	South Kensington	10,304

Closeness centrality is the inverse of the main geodesic distance l_i of one node to all the other $n-1$ nodes. Given the geodesic distance between nodes i and j as d_{ij}, the closeness centrality C_i is calculated as

$$C_i = \frac{1}{l_i} = \frac{n-1}{\sum_j d_{ij}} \tag{3}$$

A high closeness centrality in a rail network indicates the station is within a short distance from all the other stations, located at the physical centre of the network. Provided that the network spreads out radially from the city centre, the stations with highest closeness centrality are assumed to be located within the city centre, as illustrated by the high-ranked stations in London shown in Table 3. These stations are located at the West End area - which can be assumed as the central area of London.

Table 3: Stations with the highest closeness centrality.

Rank	Station	Closeness Centrality $(\times 10^{-5})$
1	Holborn	7.926
2	King's Cross St. Pancras	7.914
3	Tottenham Court Road	7.891
4	Oxford Circus	7.883
5	Leicester Square	7.837
6	Picadilly Circus	7.834
7	Charing Cross	7.833
8	Chancery Lane	7.825
9	Covent Garden	7.810
10	Embankment	7.802

1.1.2 Impact Measures

For impact measurement, we considered the delta centrality, a concept introduced by Latora and Marchiori (2007) that compares the *performance* of the whole network when a node is removed from the network in question. For a performance measure P of the network G, the delta centrality C_i^{Δ} of node i is defined as

$$C_i^{\Delta} = \frac{\Delta P}{P} = \frac{P(G) - P(G')}{P(G)}$$

where G' denotes the network where node i is removed. This can be applied to any performance measure P - we have considered the measures summarised in Table 4.

Table 4: Performance measures considered for delta centrality. The impact of node removal are measured through the change in these performance values.

Performance Measure P	Explanation
Size of Largest Connected Component	The size of the largest connected component shows the connectivity of the network. A smaller number indicates stations are disattached from the main network, leading to less connectivity.
Network Efficiency	Efficiency measure proposed by Latora and Marchiori (2001), calculated as $E(G) = \frac{1}{n(n-1)} \sum_{i \neq j} \frac{1}{d_{ij}}$. A high value indicates a low average shortest path length between nodes, and a low value represents nodes are further apart or disconnected. Unlike the average shortest path length, this measure can be calculated for disconnected networks. Note the `nx.global_efficiency()` function cannot incorporate edge length, a custom function was defined to calculate this value.

These measures can be applied to any transport network, especially the average shortest path length is directly addressing the puropose of transport systems of connecting different areas. The number of cycles also impact the resilience of transport networks, although may not be applicable for road networks that inherently have a large number of cycles. An applicable example of these measures outside the London underground is the destruction of the subway network in New York City after the September 11 attacks, which resulted in the closure of multiple stations and line segments (Wyatt, 2002; Paaswell, 2012).

1.1.3 Node Removal

The removal of nodes with high centrality and the impact on the performance of the network are considered. For each centrality measure, two strategies are used to remove 10 nodes from the network: a **non-sequential** strategy removes nodes in order that appears in the rank tables (Table 1; Table 2; Table 3), while a **sequential** strategy removes the node with the highest centrality when recalculated after the previous node removal step. The stations removed for each centrality measure are shown in Table 5.

The change in performance measures caused by the node removal is as illustrated in Figure 1, and the final network is mapped in Figure 2

- Discussions

Based on the two performance measures, we can conclude that the strategy more effective to study resilience is the sequential removal. This strategy evaluates the network after the previous removal, thus able to identify the most important node at the point of removal. Since the characteristics of the network change after removing one node, the node with the next highest centrality measure may not be the most important node after removal of one node. One obvious example can be observed in the non-sequential removal based on highest closeness centrality measures, where the stations located in the centre of London are removed altogether; the impact

Table 5: Stations removed for impact assessment for each centrality measure and strategy. For each scenario, the stations were removed one at a time from the top of the table to the bottom.

(a) Degree Centrality

Non-sequential	Sequential
Stratford	Stratford
Bank and Monument	Bank and Monument
King's Cross St. Pancras	Baker Street
Baker Street	King's Cross St. Pancras
Waterloo	Oxford Circus
West Ham	Earl's Court
Canning Town	Canning Town
Liverpool Street	Waterloo
Earl's Court	Turnham Green
Green Park	Green Park

(b) Betweenness Centrality

Non-sequential	Sequential
Bank and Monument	Bank and Monument
King's Cross St. Pancras	King's Cross St. Pancras
Stratford	Canada Water
Baker Street	West Hampstead
Oxford Circus	Earl's Court
Euston	Oxford Circus
Earl's Court	Shepherd's Bush
Shadwell	Bakes Street
Waterloo	Acton Town
South Kensington	Stratford

(c) Closeness Centrality

Non-sequential	Sequential
Holborn	Holborn
King's Cross St. Pancras	King's Cross St. Pancras
Tottenham Court Road	Embankment
Oxford Circus	Waterloo
Leicester Square	London Bridge
Picadilly Circus	West Hampstead
Charing Cross	Clapham Junction
Chancery Lane	Mile End
Covent Garden	Stratford
Embankment	Notting Hill Gate

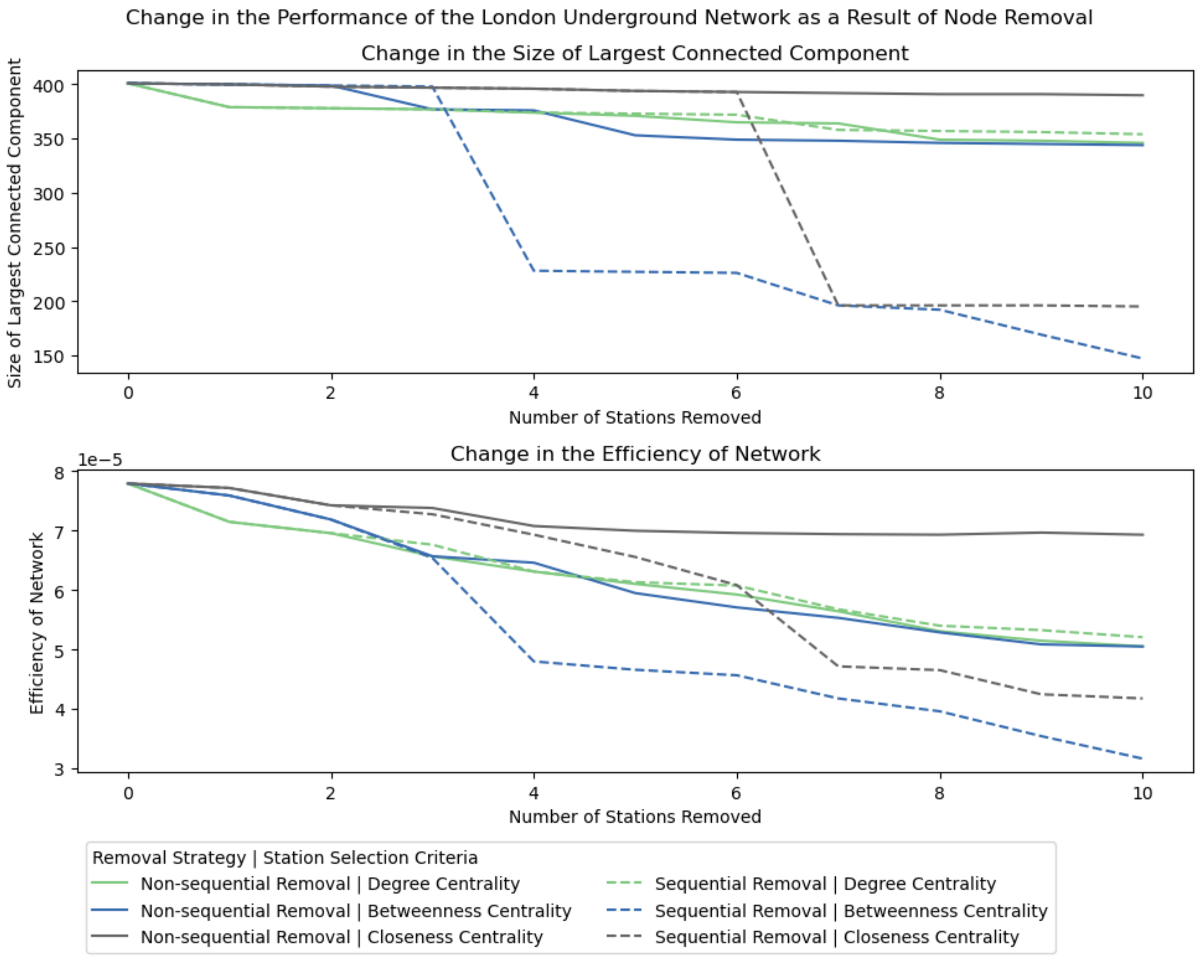

Figure 1: The change in the performance measures as a result of node removal on the London Underground Network.

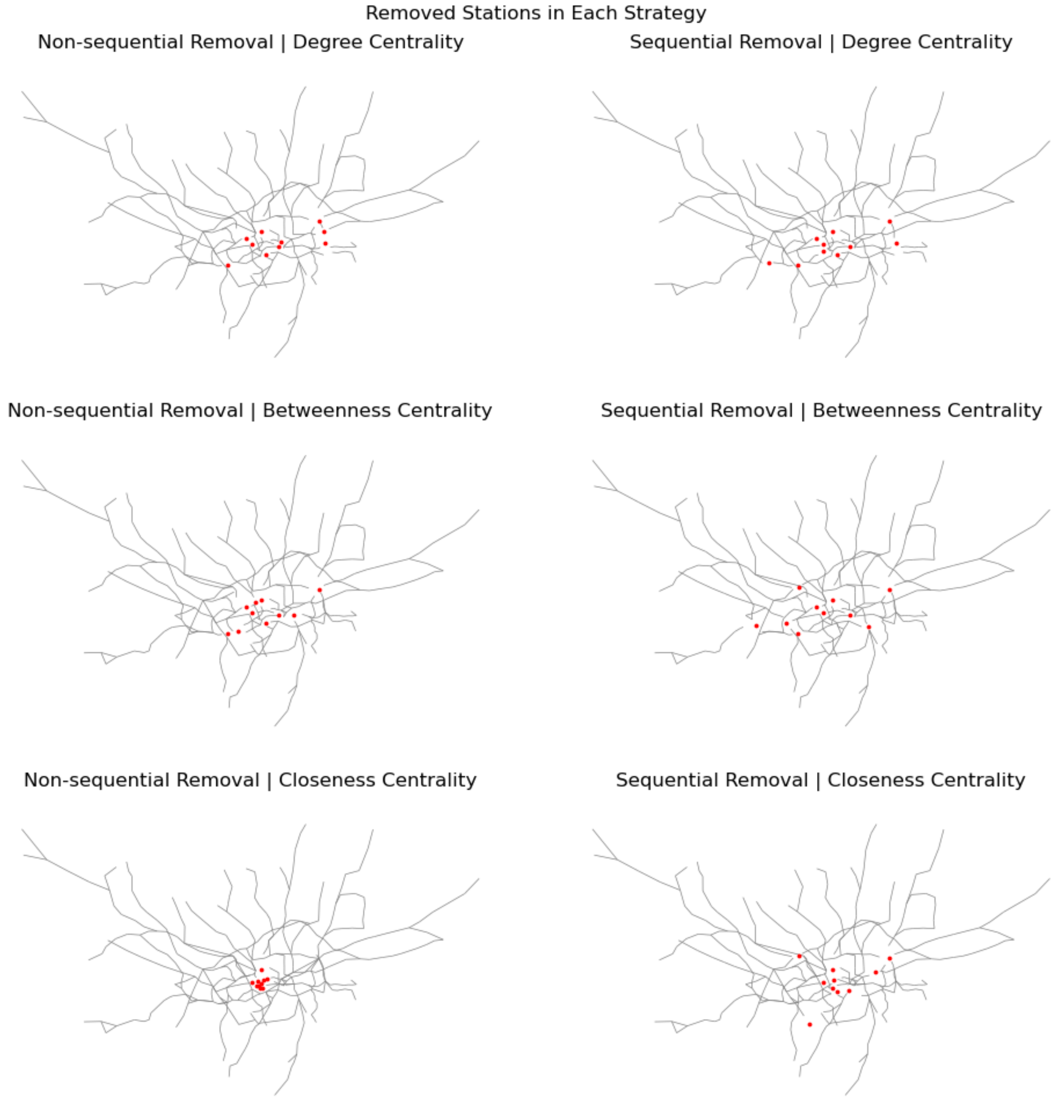

Figure 2: The map of the London Underground Network after each node removal strategy.

to the network after the removal of the first few nodes have significantly lowered the importance to the network in this area. By assessing the importance of node at each step, the sequential removal is capable of identifying the hidden nodes that gain importance when the most obvious nodes have been removed, which may need improvements if we were to prepare for emergency situations.

Comparing between the centrality measures, the betweenness centrality reflects the importance of stations the most. When removing multiple stations based on highest centrality measures, following the betweenness centrality lead to the largest decrease in both of the performance measures. Since a high betweenness centrality indicates high number of flows through the node, removing this node will force the flows to divert using different routes, requiring a higher cost to reach the destination. Given that connecting two points is the fundamental function of a transport network, the betweenness centrality directly addresses the importance regarding this aspect.

It is worth noting that the degree centrality has identified the first station to remove that impacts the performance the most, and has outperformed the closeness centrality until the 6th station. Further investigation is required to identify whether the stations at the centre of the network (high closeness centrality) or terminal stations with many lines operating (high degree centrality) is more critical to the performance of the whole network.

For the comparison of the impact measure, the efficiency appears to be a better evaluation of the performance of the network. The size of the largest connected component does not take into account how well-connected the nodes are within the largest component, therefore unable to assess the impact of station removal unless the removal disconnects a portion of the network. In an underground network where multiple lines intersect at multiple points, removal of a few stations may not always disconnect stations from networks (especially in the central part of the network) but may impact the efficiency. On the other hand, efficiency measures the actual distance between origin and destination, thus the impact on the actual flow within the network is more accurately reported.

1.2 Flows: Weighted Network

In this section, a weighted network with the flows assigned to the links between stations will be considered. Based on the Transport for London OD data (Transport for London, 2021), the morning peak flows have been assigned assuming that all flows use the shortest path, considering the length of each edge. Hereinafter, w_{ij} is the flow on the link between adjacent stations i and j (sum of both directions).

1.2.1 Centrality Measures

We will consider whether the 3 centrality measures we have identified in the previous section need adjustments to be applied in the context of the weighted network.

The **degree centrality** as the number of connected stations continues to be a valid figure. Theoretically, a flow-weighted version of degree centrality Equation 1 can be defined as

$$s_i = \sum_j A_{ij} w_{ij} \tag{4}$$

which is a measure defined as the strength s_i (Lee *et al.*, 2008). This quantifies the amount of flow that passes through each node - which is highly related to the betweenness centrality in the weighted context. The stations with the higest weighted degree centrality is shown in Table 6.

Table 6: Stations with the highest weighted degree centrality.

Rank	Station	Weighted Degree $(\times 10^5)$
1	Bank and Monument	5.188
2	Oxford Circus	4.889
3	King's Cross St. Pancras	4.671
4	Baker Street	3.358
5	Waterloo	3.344
6	Euston	3.164
7	Green Park	3.143
8	Victoria	2.947
9	Liverpool Street	2.770
10	Embankment	2.539

6 out of 10 stations in this table also appear in Table 1, while the terminals of Euston, Victoria, and Liverpool Street only appear in Table 6, which is a result of the high number of flows in these stations.

The **betweenness centrality** considering flows, given T_{ij} as the number of flows from node i to j, can be proposed as follows:

$$x_i' = \sum_{st} \frac{n_{st}^i}{g_{st}} T_{st} \tag{5}$$

Recalling the strength Equation 4 was the sum of all flows using the links that connect to the node, and Equation 5 only considers the flows that passes through (not beginning nor terminating), these two centrality measures fulfill the following relationship

$$s_i = x_i' + (O_i + D_i - T_{ii}) \tag{6}$$

where O_i, D_i is the number of flows originating and terminating at node i, respectively. By using Equation 6, the stations with the highest values are calculated as shown in Table 7.

Table 7: Stations with the highest weighted betweenness centrality.

Rank	Station	Weighted Betweenness Centrality $(\times 10^5)$
1	Oxford Circus	4.415
2	Bank and Monument	4.104

Rank	Station	Weighted Betweenness Centrality $(\times 10^5)$
3	King's Cross St. Pancras	4.052
4	Baker Street	3.131
5	Green Park	2.852
6	Euston	2.816
7	Waterloo	2.433
8	Embankment	2.393
9	Charing Cross	2.277
10	Victoria	2.236

6 out of 10 stations are in the ranking for the unweighted betweenness centrality Table 2, and 4 stations (Green Park, Embankment, Charing Cross, and Victoria) are only in this table. The topological betweenness centrality, despite its simplicity, is a good estimate for the actual flows that occur within the network. It is notable that because of the similarity between the weighted degree centrality, 9 out of 10 stations in Table 7 appear in Table 6 as well.

When considering the **closeness centrality** with regards to the number of flows, simply considering T_{ij} in Equation 3 as $C'_i = \frac{\sum_j T_{ij}}{\sum_j T_{ij} d_{ij}}$ will return the average distance travelled by the user of the station, which does not correspond to the principle of closeness centrality. (This figure may be larger in the city centre where people are willing to travel from far away, or at the periphery of the network where passengers need to travel long distances in general.) Instead, by defining the social distance of links as the inverse of the flows, and u_{ij} as the shortest social distance between nodes i and j, we will be able to calculate the closeness centrality considering the weights. Since u_{ij} diverges to infinity when no flows between stations, we will use the harmonic closeness centrality, calculated as:

$$C''_i = \frac{1}{n-1} \sum_j \frac{1}{u_{ij}}$$

The 10 stations with the highest weighted closeness centrality is as shown in Table 8.

Table 8: Stations with the highest weighted harmonic closeness centrality.

Rank	Station	Harmonic Weighted Closeness Centrality
1	Bank and Monument	68.816
2	Liverpool Street	66.250
3	Stratford	65.282
4	Waterloo	64.690
5	Green Park	63.928
6	King's Cross St. Pancras	63.876
7	Moorgate	62.571
8	Oxford Circus	61.793
9	Westminster	61.636
10	Baker Street	61.169

Comparing this with Table 3, the weighted results are not geographically concentrated in the west end, but have more focus on terminal stations with high originating or terminating journeys, or stations connected by high-flowing edges. King's Cross St. Pancras and Oxford Circus are the only stations that appear in both Table 3 and Table 8.

1.2.2 Impact Measures

We will consider the need for adjusting the impact measures in Table 4 when considering the passenger flows.

The size of the largest connected components is a valid measure in a weighted network, since it does not consider the flows between nodes. If the weight of the edges are to be incorporated to quantify the *connected-ness* of the nodes, a similar performance measure would be the ratio of journeys that can be completed within a connected component. Denoting this measure as $J(G)$, this could be calculated as:

$$J(G) = \frac{\sum_{i \neq j} T_{ij}\delta_{ij}}{T} \quad \left(\delta_{ij} = \begin{cases} 1 \text{ if } i \text{ and } j \text{ are in same connected component} \\ 0 \text{ otherwise} \end{cases} \right)$$

For the network efficiency, each of the distances should be weighted according to the number of passengers who travel between the two nodes. The efficiency should also account for the flows between points. The equation in Table 4 should be adjusted using T_{ij} as:

$$E(G) = \frac{1}{n(n-1)} \sum_{i \neq j} \frac{T_{ij}}{d_{ij}}$$

1.2.3 Node Removal

Using $J(G)$ and $E(G)$ as performance measures, we will remove 3 nodes with the highest unweighted and weighted betweenness centrality as shown in Table 9.

Table 9: Stations removed using the highest unweighted and weighted betweenness centrality. Excerpt from Table 2 and Table 7.

Rank	Unweighted	Weighted
1	Bank and Monument	Oxford Circus
2	King's Cross St. Pancras	Bank and Monument
3	Stratford	King's Cross St. Pancras

The change in the performance measures $J(G), E(G)$ for each of the removal strategies are shown in Figure 3. Comparing these results, removing stations based on the unweighted betweenness centrality measure has the most impact on the network. The weighted efficiency of the network has dropped significantly, indicating journeys are taking longer or impossible to complete with the nodes-removed network. Thus, the stations that have the most impact on the performance of the network are: Bank and Monument, King's Cross St. Pancras, and Stratford.

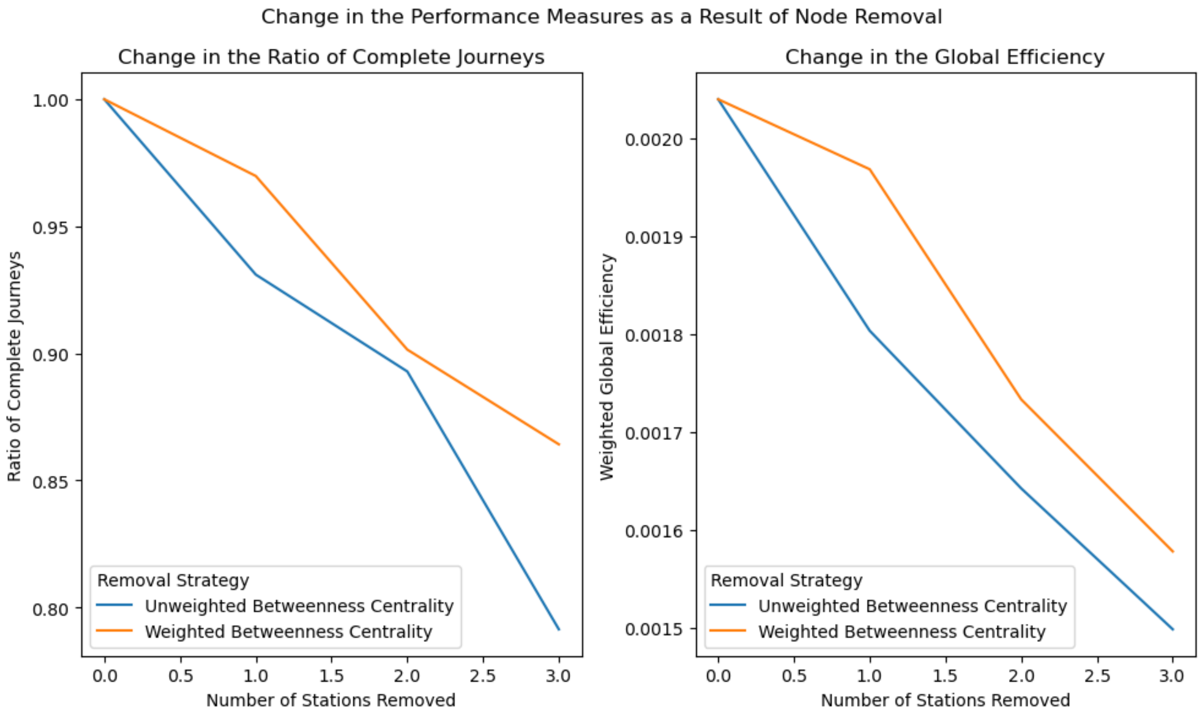

Figure 3: Change in the performance measures as a result of node removal according to the weighted and unweighted betweenness centrality.

The weighted strategy takes into account the flows of passengers, but did not outperform the unweighted strategy. One possible explanation is that the first station removed by the weighted strategy, Oxford Circus, had alternatives that could operate without distorting the efficiency of the network, while Stratford, the third choice for the unweighted strategy immediately disconnects branches from the largest component.

2 Part 2: Spatial Interaction Models

In the second part, we will analyse the OD matrix of the London Underground. The variables will be denoted as follows:

- T_{ij}: estimated flow from station i to j
- T: total flow within a network, thus $T = \sum_{i,j} T_{ij}$
- d_{ij}: distance between stations i and j - the cost function is a function of d_{ij} hence denoted as $f(d_{ij})$
- O_i, D_j: attractors (often population) at the origin and destination

2.1 Models and Calibration

2.1.1 Family of Spatial Interaction Models

The family of spatial interaction models are as follows:

The **unconstrained model** only constrains the model with matching the total flows with the observed value, written as

$$T_{ij} = KO_i^{\alpha} D_j^{\gamma} f(c_{ij})$$

where K is the constant

$$K = \frac{T}{\sum_i \sum_j O_i^{\alpha} D_j^{\gamma} f(d_{ij})}$$

so that the total number of journeys are constrained.

The **singly constrained model** constrains the total number of observations for each component at the origin or the destination. The origin constrained model fixes the total number of journeys at the origin, as:

$$T_{ij} = A_i O_i^{\alpha} D_j^{\gamma} f(d_{ij}) \tag{7}$$

The parameter A_i is determined so that the total at the origin is constrained, that is:

$$\sum_j T_{ij} = O_i \therefore A_i = \frac{1}{\sum_j D_j f(d_{ij})}$$

Similarly, the **destination constrained model** constrains the total at the destination.

$$T_{ij} = O_i^{\alpha} B_j D_j^{\gamma} f(c_{ij}) \quad \text{where} \quad \left(B_j = \frac{1}{\sum_i O_i f(c_{ij})} \right)$$

The **doubly constrained model** constrains both the total at the origin and destination.

$$T_{ij} = A_i O_i^{\alpha} B_j D_j^{\gamma} \,\text{exp}(-\beta d_{ij}) \tag{8}$$

2.1.2 Calibration of Parameters

The dataset used for analysis have the following data for every origin-destination pair for the London Underground Stations.

- population of origin
- jobs at the destination
- distance between origin and destination
- flow from the origin to the destination

Using this dataset, we will first use the doubly constrained model to estimate the best cost function and parameters by comparing with the observed flow. Then, the origin-constrained model that will be used for modelling the scenarios are used to calibrate the γ parameter. For the cost function, constraining both the total journeys for both the origin and destination will enable the full utilisation of observed data, enabling the most accurate calibration. The γ parameter does not appear in the doubly constrained model, and is required to calibrate in the origin-constrained model.

* Calibration of the cost function

The doubly constrained model after a logarithm transformation of Equation 8 into a Poisson expression can be written as:

$$\ln(T_{ij}) = \ln A_i + \ln O_i^{\alpha} + \ln B_j + \ln D_j^{\gamma} + \ln f(d_{ij})$$

We will compare the negative exponential and inverse power relationships as the cost function $f(d_{ij})$.

$$f(d_{ij}) = \begin{cases} \exp(-\beta d_{ij}) \text{ (Negative Exponential)} \\ d_{ij}^{-\beta} \text{ (Inverse Power)} \end{cases}$$

For each cost function, we have run the Poisson Regression to calculate the optimal β.

Table 10: Comparison of results for the doubly constrained model using inverse power and negative exponential cost functions.

Cost Function	Parameter β	R^2 value
Negative exponential $f(d_{ij}) = \exp(-\beta d_{ij})$	$\beta = 1.543 \times 10^{-4}$	$R^2 = 0.4979$
Inverse Power $f(d_{ij}) = d_{ij}^{-\beta}$	$\beta = 9.096 \times 10^{-1}$	$R^2 = 0.4077$

From these results, we have seen the negative exponential model (Equation 9) for the cost function has a better fit to the observed flows.

$$f(d_{ij}) = \exp(-\beta d_{ij}) \quad (\beta = -1.543 \times 10^{-4}) \tag{9}$$

* Calibration of γ

Since the origin-constrained model was used for the analysis of the scenarios, the gamma variable introduced in Equation 7 must be calibrated before applying to the new scenarios. γ with the highest R-squared value for estimating the original flow was used, resulting as follows:

$$\gamma = 7.556 \times 10^{-1} \quad (R^2 = 0.4680) \tag{10}$$

2.2 Scenarios

The scenarios we have considered are summarised in Table 11.

Table 11: The scenarios explored in this report

Scenario	Explanation
Scenario A	Jobs at Canary Wharf decrease by 50 %
Scenario B	Increase in cost of transport - considering 2 parameters

Since scenario A involves the change in the characteristics of the destination, the origin constrained model is used for the analysis to preserve the number of commuters starting their journeys in each area.

2.2.1 Scenario A

We have first decreased the number of jobs at Canary Wharf by 50%, from the original 58,772 to 29,386. Using the origin-constrained model, the procedure was as follows:

1. Reduce the number of jobs at Canary Wharf (transform D_j into D'_j)
2. Recalibrate the adjusted A'_i parameter with the new distribution of using the relationship $A'_i = \frac{1}{\sum_j D'^{\gamma}_j f(d_{ij})}$
3. Calculate the new flows using the relationship $T'_{ij} = A'_i O_i D'^{\gamma}_j$ exp$(-\beta d_{ij})$

Using the origin constrained model, we have observed how the destination of commuters changed in reaction to the decrease in jobs in Canary Wharf.

A significant drop in the number of journeys terminating at Canary Wharf was observed, from 47,690 in the original simulation to 29,496 (61.9 % of original flow) in scenario A. As observable from Figure 4, the decrease in the flows to Canary Wharf occured evenly among all origins, and been redistributed into other destinations.

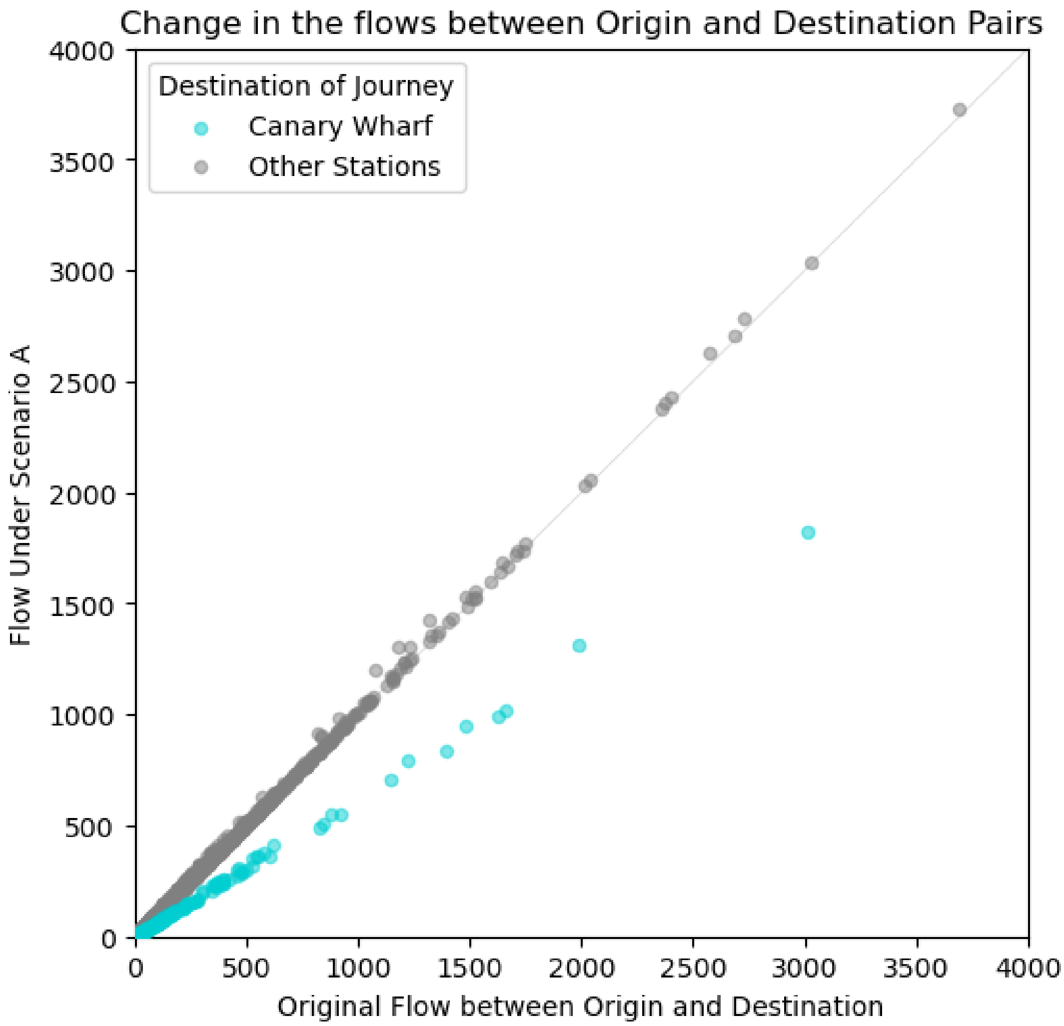

Figure 4: Comparison of the flows of OD pairs the original simulation and the flows under scenario A.

2.2.2 Scenario B

2 scenarios for the change in the cost were considered. Given the original parameter as β, the parameters β_1, β_2 for the two scenarios B1 and B2 will be modified as shown in Equation 11 and Figure 5.

$$\begin{cases} \beta_1 = 2\beta \\ \beta_2 = 10\beta \end{cases} \qquad (\beta = -1.543 \times 10^{-4}) \tag{11}$$

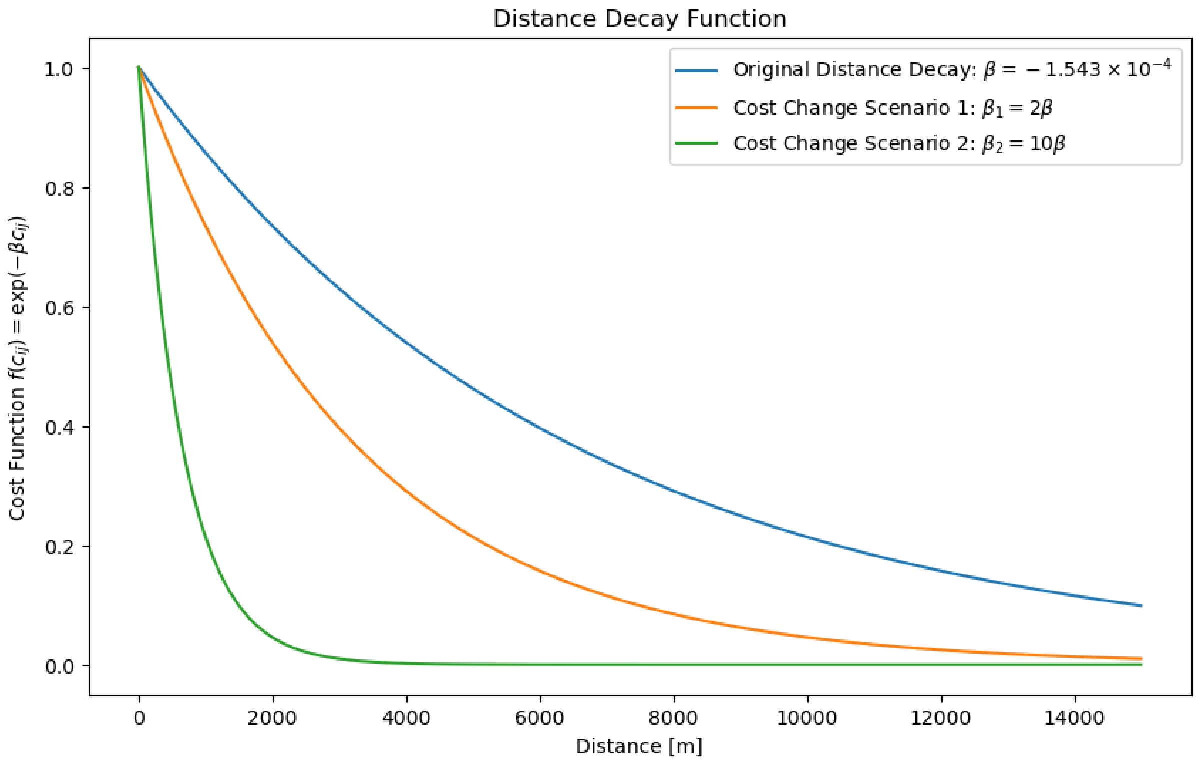

Figure 5: Distance decay function for the original scenario and the two modified scenarios. This indicates the cost for travelling a fixed distance will be equivalent to that of travelling twice (B1) or 10 times (B2) the distance in the original model.

The flows between origin-destination pairs for each scenario, plotted by distance of journeys, are shown in Figure 6. As the β increases, the longer distance journeys are disencouraged, and the distribution of the distance of journeys become negatively skewed.

2.2.3 Discussion

The measurements summarised in Table 12 are considered to examine the changes that occured as a result of each scenario.

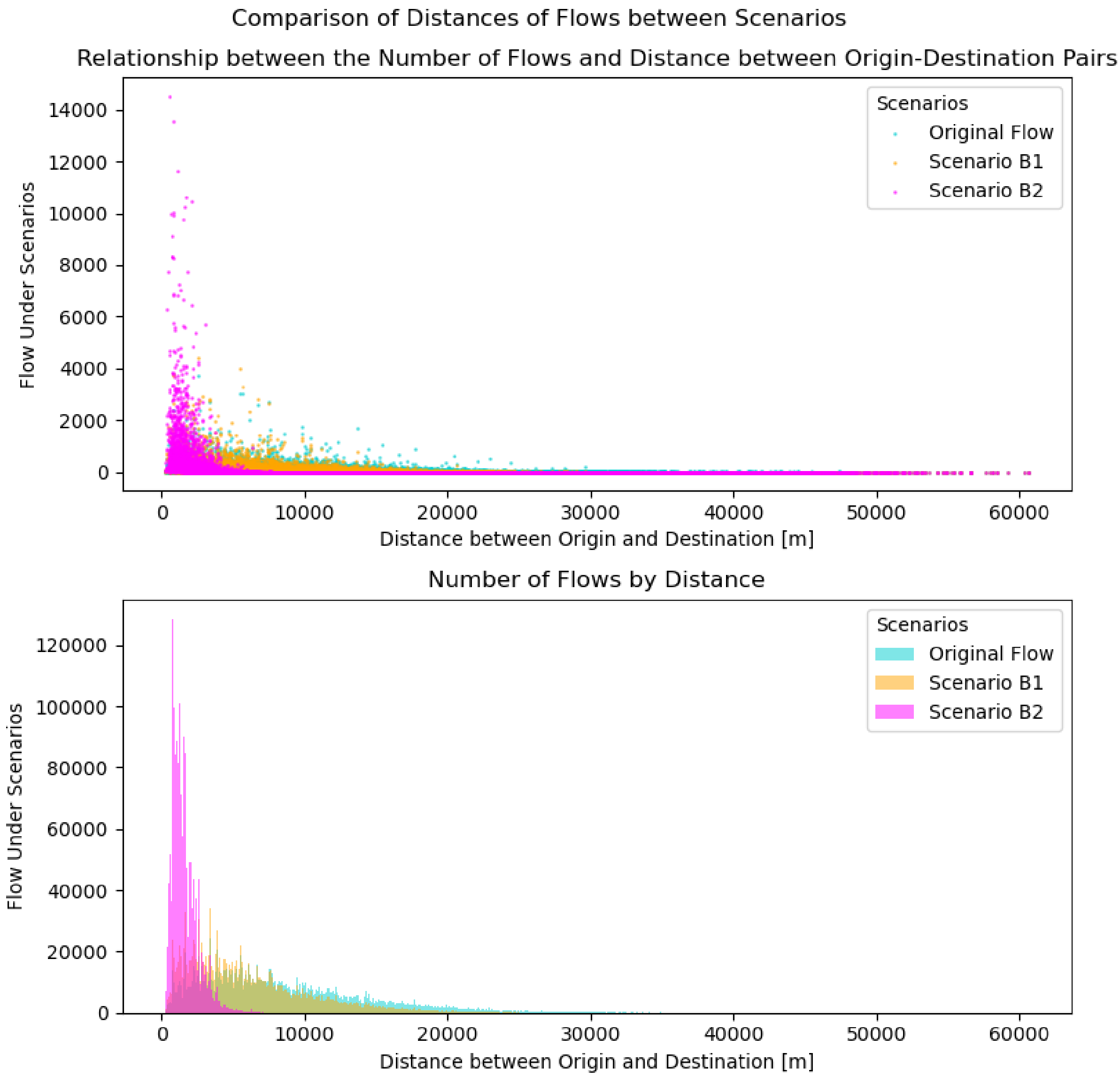

Figure 6: Comparison of the relationship between the distance and the number of flows for scenario B and the original model. Raising the cost function results in a negatively skewed distance distribution.

Table 12: Measurements consider to quantify the change in flows for each scenario. T'_{ij} is the number of journeys from station i to j under the scenario.

Measurement	Explanation
Quantity of change in destination T_{diff}	The number of flows that have different destination compared to the original scenario. A larger number of change in destination indicate larger impact.
Mean distance of journeys $\bar{\delta^X}$	The mean distance of all journeys simulated in each scenario. A larger change in the distance compared to the original simulation indicate larger impact.

For each scenario, values for these measurements are calculated as shown in Table 13. The number of destinations changed and the change in the mean distance both indicate that scenario B2 had the largest impact on the number of flows, and scenario A being the least impactful.

Table 13: Value of measurements for each scenario.

Scenario	T_{diff}	$\bar{\delta}$ [m]
Scenario A	18,193 (1.2 % of total flows)	8,579
Scenario B1	346,503 (22.47 %)	6,030
Scenario B2	1,222,191 (79.25 %)	1,613
Original Simulation	Total flows: 1,542,283	8,583

The intervention to the travel cost (scenario B) directly impacts all OD pairs, while the reduction of jobs in one area (scenario A) only impacts journeys that end at the station affected. This different nature of the interventions dictate the magnitude of change it causes; thus scenario B has a larger impact on the flows on the network as a whole. Recalling the impact of scenario A on Canary Wharf was a 39 % decrease in the total inflows, this figure falls between scenarios B1 and B2 in terms of T_{diff}. Since $\bar{\delta}$ remains unchanged the impact on Canary Wharf is difficult to compare between scenarios A and B1, but we can conclude scenario B2 has the largest impact even if the focus is on Canary Wharf. To conclude, a drastic impact on the cost function has more impact throughout the network compared to a large change in the demand in a particular area. Impact of the actual changes in the underground fares that occured should be investigated to compare with the results of our model.

Word count: x words

GitHub repository (as hyperlink): Urban_Simulation_Report

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