

$$Z = X_1 + X_2$$

$$\begin{aligned}
 h(z) &= \int_{-\infty}^{\infty} f_1(x) \cdot f_2(z-x) dx = \int_0^1 2x \cdot 4z(z-x) dx = \\
 &= \begin{cases} \int_0^z 2x \cdot 4(z-x) dx = \int_0^z 4xz - 4x^2 dx = 2z^3 - \frac{4}{3}z^3 = \frac{2z^3}{3} & 0 < z < 1 \\ \int_{z-1}^1 4xz - 4x^2 dx = 2z \left(1 - (z-1)^2 \right) - \frac{4}{3} \left(1 - (z-1)^3 \right) = & 1 < z < 2 \end{cases} \\
 &= -\frac{2}{3}z^3 + 4z - \frac{8}{3}
 \end{aligned}$$

$$\begin{aligned}
 P\left(\frac{1}{4} < \frac{X_1 + X_2}{2} < \frac{3}{4}\right) &= P\left(\frac{1}{2} < X_1 + X_2 < \frac{3}{2}\right) = \\
 &= \int_{\frac{1}{2}}^1 \frac{2z^3}{3} dz + \int_1^{\frac{3}{2}} \left(-\frac{2}{3}z^3 + 4z - \frac{8}{3} \right) dz = \frac{31}{48} \approx 0.645833
 \end{aligned}$$