$\boxed{\text{EXERCISE}}$

LET X_1 AND X_2 BE i.i.d RV-s WITH COMMON DENSITY FUNCTION EQUAL TO

$$f(x) = 2x, \quad 0 < x < 1 \quad (0 \text{ OTHERWISE})$$

$$P\left(\frac{1}{4} < \frac{X_1 + X_2}{2} < \frac{3}{4}\right) = ?$$

$0<x<1$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \int_{0}^{t} 2x\,dx = x^2, \quad 0<x<1$$

$$x^2 = y$$

$$x = \sqrt{y} = F^{-1}(y)$$

RND: UNIFORMLY DISTRIBUTED

0 ——— 1

2·RND IS NOT GOOD, IT IS UNIFORMLY DISTR. ON $[0,2]$

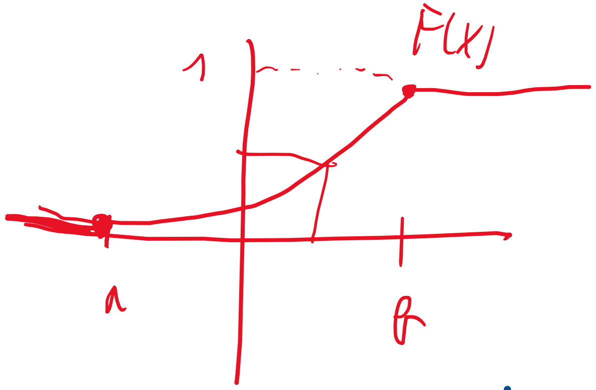
$$X_1 \overset{d}{=} \sqrt{RND}$$

PROPOSITION

Let $F(x)$ be a nice continuous dist. function ($\exists\, a, b$ such that $F(a)=0$, $F(b)=1$ and $F(x)$ is strictly increasing on $[a, b]$)

a can be $-\infty$, b can be ∞

Then $F^{-1}(x)$ is well defined on $(0, 1)$ and the dist function of $F^{-1}(RND)$ equals $F(x)$.



$$g(x) = P(F^{-1}(RND) < x) =$$

$$= P(RND < F(x)) = F(x)$$

$\boxed{\text{STRONG LAW OF LARGE NUMBERS}}$

LET $X_1, X_2, \dots, X_n, \dots$ BE i.i.d RVs WITH FINITE EXPECTED VALUE m

$$\frac{X_1 + \dots + X_n}{n} = \bar{X}_n \xrightarrow{n \to \infty} E(X) = m$$

WITH PROB 1

CENTRAL LIMIT THEOREM (CLT)

DIST FN OF N(0,1) DIST

LET $X_1, X_2, \ldots, X_n, \ldots$ BE i.i.D RV-S WITH FINITE EXPECTATION m AND VARIANCE σ^2

$$P\left(\frac{X_1+X_2+\ldots+X_n-n\cdot m}{\sqrt{n}\cdot\sigma}<x\right)\xrightarrow{n\to\infty}\Phi(x),\ \forall x$$

$$\frac{X_1+\ldots+X_n-n\cdot m}{\sqrt{n}\cdot\sigma}\underset{n\text{ LARGE}}{\overset{d}{\approx}}N(0,1)\underset{n\text{ IS LARGE}}{\overset{d}{\approx}}\frac{\sqrt{n}}{\sigma}\left(\bar{X}_n-m\right)$$