- $COV(X,Y) = E[(X-E(X))(Y-E(Y))] = E(XY) - E(X)E(Y)$

(COVARIANCE, NOTE THAT $COV(X,X) = VAR(X)$)

- CORRELATION : $CORR(X,Y) = \dfrac{COV(X,Y)}{D(X) \cdot D(Y)}$
    - IT TAKES VALUES IN $[-1,1]$
    - ITS SIGN HAS IMPORTANT MEANING
    - IF X AND Y ARE INDEPENDENT THEN $CORR(X,Y) = 0$
    - THE REVERSE STATEMENT IS TYPICALLY NOT TRUE

      COUNTEREXAMPLE : $X \sim N(0,1)$, $Y = X^2$. THEN X AND Y ARE OBVIOUSLY NOT INDEPENDENT, HOWEVER,

      $COV(X,Y) = E(X \cdot Y) - 0 \cdot E(Y) = E(X^3) = \int_{-\infty}^{\infty} x^3 \varphi(x)\, dx \overset{\text{ODD FUNCTION}}{=} 0$
- WE WILL SEE THAT " THE CORRELATION MEASURES THE STRENGTH OF THE LINEAR RELATIONSHIP BETWEEN THE TWO VARIABLES

SIMULATION OF A TWO DIMENSION CONTINOUS R.V. (X,Y):

(1) WE SIMULATE X VIA THE FORMULA $F_1^{-1}(RAND())$

(2) LET $F_{2|1}^{-1}(y|x)$ DENOTES THE INVERSE WITH RESPECT TO y OF THE CONDITIONAL DISTRIBUTION FUNCTION $F_{2|1}(y|x)$. WE SIMULATE Y USING THE ALREADY SIMULATED X AND $F_{2|1}^{-1}(y|x)$ VIA THE FORMULA $F_{2|1}^{-1}(RAND() | X)$

- THE PAIR (X,Y) HAS JOINT NORMAL DISTRIBUTION WITH PARAMETERS $\mu_1, \mu_2, \sigma_1, \sigma_2, r$ IF THEIR TWO DIMENSIONAL DENSITY EQUALS ... (SEE PART IV OF THE NOTES WRITTEN BY A. VETIER)
- THE PARAMETERS HAVE IMPORTANT MEANING:

  $E(X)=\mu_1$, $D(X)=\sigma_1$, $E(Y)=\mu_2$, $D(Y)=\sigma_2$, $\mathrm{CORR}(X,Y)=r$
- $X \sim N(\mu_1, \sigma_1)$, $Y \sim N(\mu_2, \sigma_2)$
- $Y \mid X=x \sim N\left(\mu_2 + r \cdot \frac{\sigma_2}{\sigma_1}(x-\mu_1), \sigma_2 \cdot \sqrt{1-r^2}\right)$
- $X \mid Y=y \sim N\left(\mu_1 + r \cdot \frac{\sigma_1}{\sigma_2}(y-\mu_2), \sigma_1 \cdot \sqrt{1-r^2}\right)$

REMARK: IF THE DISTRIBUTION OF (X,Y) IS JOINT NORMAL AND $r=0$ THEN X AND Y ARE INDEPENDENT.