PYTHON

A. SIMON

CONTENTS

- Where to find Python?
  - Log in here `https://jupyter.math.bme.hu:8888/` with your username and password for leibniz. When clicking on `New`, chose `Python3`.
  - You can log in `tarski.math.bme.hu` the same way you log in to leibniz (if you are on leibniz, you just need to write `ssh -Y tarski` in a terminal) and you will find yourself in your home directory. There you can start

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spyder3 or ipython3 in a terminal. (There is spyder and ipython on leibniz, too, but those are based on Python2.)
    - On your own windows machine you probably want this: http://wiki.math.bme.hu/view/AnacondaInstall With this, you'll get a graphical development environment called spyder that people seem to like. (There's spyder on leibniz, too, but, like ipython, it's Python2 based.)
- Reading material:
    - http://math.bme.hu/~asimon/info2/python.pdf (the newest version of the lecture notes (this document))
    - The second part of last semester's Sage lecture notes may also be useful:
        * http://math.bme.hu/~asimon/info1/sageen.pdf (English version)
        * http://math.bme.hu/~asimon/info1/sage.pdf (Hungarian version)
    - http://wiki.math.bme.hu/view/Informatika2-2021 (last year's lecture notes & more – in Hungarian.)
- Exercises for the lab sessions are here:
    - http://math.bme.hu/~asimon/info2/pythex.pdf (English version)
    - http://wiki.math.bme.hu/view/Informatika2-2022/Cs%C3%BCtGyak01 (Hungarian version for Thursday's lab)
    - http://math.bme.hu/~asimon/info2/pytgyak.pdf (Hungarian version for Wednesday's lab).

1. Warmup[1]

Last semester we've learned quite a few things about Python because it is the underlying programming language of Sage. In this first section we collect some basic features of the language that didn't come up back then (or just wasn't emphasized enough).

(1) Every object has a type, and not only can we ask what it is, but also if an object is of a certain type:

```
Python 3.9.10 (main, Jan 17 2022, 00:00:00)
[GCC 11.2.1 20210728 (Red Hat 11.2.1-1)] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> type(1)
<class 'int'>
>>> isinstance(1,int)
True
>>> isinstance(1,list)
False
```

(2) Some "simple" types are listed in Table 1. Complex literals can be written as a+bj, where a and b are int or float literals.

```
>>> (1+1j)**2
2j
```

j in itself is just a variable name:

```
>>> j**2 == -1
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
```

[1]See Appendix A first if these sound too advanced!

```
NameError: name 'j' is not defined
>>> 1j**2 == -1
True
```

But if we want complex numbers, we probably want Sage, too.

A warning: dividing an `int` with another results in a `float`, even if the second `int` divides the first:

```
>>> 2/2, type(2/2)
(1.0, <class 'float'>)
```

And since the precision of `float`s are limited, this can lead to some unexpected results:

```
>>> 18530201888518410 / 2
9265100944259204.0
```

There are many ways to work around such problems. If you know that the result should be an `int`, you can use `//` instead of `/`. If you don't, then there is `divmod` which returns the pair `(x//y, x%y)`, or write
`x//y if x%y == 0 else x/y`
Of course, if you need to use this idiom a lot, you're better off writing a function:

```
>>> #safe int division
>>> def sid(x,y):
...     return x//y if x%y == 0 else x/y
...
>>> sid(1,2), sid(4,2)
(0.5, 2)
```

But this still doesn't solve the underlying problem, which is the lack of a rational number type in Python. (The module `fractions` ameliorates it.) That is why we can't expect $(x/y) * y = x$ to hold for all $y \neq 0$:

```
>>> y = 10**6 - 1; (1/y) * y, (1/y) * y == 1
(0.9999999999999999, False)
```

(3) A not so simple type is `str`ing. We've briefly met it last semester, and will learn more about it in Section 4; for the time being it's enough to recall that a string literal is whatever is written between quotation marks of various kinds, most importantly `'` and `"`. Another important "complex" type is `list`, but what we have learned about lists will be enough for us for a while, assuming we haven't forgotten about the `.append()` method, which we have only used once but is very important: `l.append(o)` appends `o` to the end of the list `l`.

```
>>> l = [1, 2, 3] ; l.append(4) ; l
[1, 2, 3, 4]
```

This is often used for accumulating objects in a list.

Type	Description
`int`	integer
`float`	floating point number
`complex`	complex number
`bool`	boolean (`True` and `False`)
`NoneType`	`None` (null value)

TABLE 1. Some simple types

(4) The condition in an `if` (or while) statement of expression (in the case of `if`) is usually a `bool`ean, but can be of other types, too, as shown in the following examples:

```
>>> if 0: print('Yes')
...
>>> if 10: print('Yes')
...
Yes
>>> if []: print('Yes')
...
>>> if [1]: print('Yes')
...
Yes
>>> if "": print('Yes')
...
>>> if "1": print('Yes')
...
Yes
>>> if None: print('Yes')
...
```

(5) `None` (of type `NoneType`) is what a function with no `return` statement returns. But it has other uses, too.

(6) We know everything there is to know about the `if` statement, except that it can optionally have one or more `elif` clauses:

```
if cond_1:
   do_this_1
   ...
elif cond_2:
   do_this_2
   ...
else:
   do_this_3
   ...
do_this_when_done
```

does the same as, but is more compact than

```
if cond_1:
   do_this_1
   ...
else:
   if cond_2:
       do_this_2
       ...
   else:
       do_this_3
       ...
   do_this_when_done
```

For example:

```
>>> for i in [-5,5,15,25]:
...     if i<0:
...         print("negative")
...     elif i<=10:
...         print("small")
...     elif i<=20:
...         print("medium")
...     else:
...         print("big")
...
negative
small
medium
big
```

Exercise 1.1. Do the same without `elif`!

Don't forget about the `if` *expression* (or ternary `if`)!

```
>>> 'Yes' if False else 'No'
'No'
```

(7) See Table 2 for an (incomplete) list of binary operators. One noteworthy change from Sage is that `^` no longer means exponentiation.

For binary relations `R` and `S`, `x R y S z` means `x R y and y S z`; for example

```
>>> 3 <=4 > 2 < 10
True
```

This lets us get away with fewer `and`s.

(8) Another change compared to what we've learned last semester[2] is that some functions, such as `range()`, `filter()`, `map()`, no longer return lists, they return *iterables* (not in the strict technical sense of the word, but almost – see § 7.6 for some details). But what they return can be converted to lists with the `list()` function. For example,

[2]This is not a difference between Sage and pure Python, but is due to the fact that the version of Sage we used was built on an older version of Python.

Operators and relations	**Description**
`or`	boolean or
`and`	boolean and
`not`	boolean not
`in`, `not in`	membership
`is`, `is not`	identity test
`<`, `<=`, `>`, `>=`, `==`, `!=`	comparison
`+`, `-`	addition, subtraction
`*`, `/`, `//`, `%`	multiplication, division, integer division, remainder
`**`	exponentiation

TABLE 2. Binary operators and relations

```
>>> r = range(10)
>>> r, list(r)
(range(0, 10), [0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
>>> a = filter(lambda n: n%2==1,r)
>>> a, list(a)
(<filter object at 0x7fa52c070eb0>, [1, 3, 5, 7, 9])
>>> list(map(lambda n: n**2,r))
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81]
```

The last examples show that `filter()` and `map()` not just return, but also accept iterables.

(9) We learned about `for` loops that let us iterate over lists and other iterables. Here's a quick reminder:

```
for variable in iterable:
   do_this
   ...
   do_that
else:
   do_this_too
   ...
   do_that_too
```

and the body (`do_this ... do_that`) will be executed with *variable* bound to successive values of the *iterable*. The body may contain `continue`, which skips the rest of the current round and jumps back to the head of the loop, and `break`, which breaks out of the whole loop, altogether. If this happens, the body of the optional `else` clause is skipped.

It's quite common that we want to iterate over something but also keep track of where we are. To help with this, Python provides `enumerate()`, which can be used like this:

```
>>> list(range(10,15))
[10, 11, 12, 13, 14]
>>> for index,number in enumerate(range(10,15)):
...     print(index, number)
...
0 10
1 11
2 12
3 13
4 14
```

This is much more concise than what we would have to write otherwise:

```
index = 0
for number in range(10,15):
    print(index, number)
    index += 1
```

With an optional argument, `enumerate()` can start counting from integers other than 0:

```
>>> for index,number in enumerate(range(10,15), -10):
...     print(number - index)
...
```

```
20
20
20
20
20
```

It may seem strange that we have *two* loop variables, `index` and `number`. We'll learn later (see page 20!) how this works, but the essence is that at each round `enumerate()` returns a pair, whose first member is assigned to the first, and whose second member is assigned to the second variable.

(10) We've learned how to define functions:

```
def  name(parameter1,parameter2,...):
    do_this
    ...
    do_that
    return  a_value
```

(The `return` statement is optional, and need not be at the end of the body of the function.) I just want to add three things for now:

- Whatever you do, do it with a function!
- A function can call itself: if that's the case, it's called a *recursive* function. (Some problems are solved most naturally with recursive function. For performance reason, this doesn't mean that they *should* be solved by one.)
- Function definitions can be nested. Sometimes this is crucial (there will be an example of this later, on page 47), but sometimes it's simply convenient. The inner function is hidden from the outside world (it is *local*, just like local variables), and it can use the local variables of the surrounding function. Here's an example which happens to be recursive, too:

```
def fib(n):
    def rec():
        return fib(n-1) + fib(n-2)
    if n <= 1:
        return n
    else:
        return rec()
```

  In this overcomplicated function which returns the nth Fibonacci number `rec()` is only callable from `fib()`, and it uses the variable `n` that is local to `fib()`.

Exercise 1.2. Write a function that computes the factorial of a natural number! Do a version without recursion, too.

(11) We'll learn about modules later, for now it's enough to know that whenever the keyword `import` appears, it means that some extra functionality will be provided for the rest of the program. In each case it will be clear what. For example:

```
>>> import math
>>> math.sqrt(2), math.pi
(1.4142135623730951, 3.141592653589793)
```

2. I/O

One could go quite far with the few things we learned last semester about Python for Sage. But one area where we'd soon feel constrained is the ability to provide data for our programs and to display their output in a usable form. This, especially the lack of a data source, is often fine for Sage programs, but not for non-mathematically focused Python programs.

2.1. User input and simple output.

Asking for keyboard input from the user is the simplest way to get a small amount of external data with which to work. For a more substantial amount, it's reading from a text file, which will be covered in the next subsection[3].

The `input()` function returns whatever the user types until she presses RETURN, as a string. Here's an example:

```
x = int(input())
print (2*x)
```

This will wait until you enter a number, and will then print its double. The call to `int()` is there to convert the string representation (say `"9"`) of the input to an integer (`9`, in this case).

To make it more usable, we should call `input()` with the optional argument `prompt`, which will be displayed to the user. Try this version:

```
x = int(input("Enter an integer: "))
print (2*x)
```

With this, the user will see that there is something to be done.

Exercise 2.1. Write a program that asks for numbers, one after the other, and if the user presses `RETURN` without entering a number first, it returns the average of the numbers. So an interaction with your program should look something like this:

```
Enter a number: 1
Enter a number: 4
Enter a number: 12
Enter a number: 3
Enter a number:
The average is 5.0
```

For more complex inputs you may have to preprocess the string in other ways to make it usable for your program. For example, if we want a list of integers, we do this:

```
l = input("Please input a list of integers separated by spaces: ").split()
l = [int(i) for i in l]
```

What the `.split()` method does here is to return the list of the words (substrings separated by whitespace) in the string. For example:

```
Python 3.9.10 (main, Jan 17 2022, 00:00:00)
[GCC 11.2.1 20210728 (Red Hat 11.2.1-1)] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> "12  23 42     135  ".split()
['12', '23', '42', '135']
```

[3]other methods include reading from a network socket or a database

Returning to our doubling example: it's OK to return the double of the integer the user typed in, but it's better to tell her what the answer is an answer to, as in

```
The double of the number 21 is 42.
```

This will almost do that:

```
x = int(input("Enter an integer: "))
print ("The double of the number", x, "is", 2*x, ".")
```

That's because `print()` accepts any number of arguments, and prints them, by default, separated by a space. For this reason the output will not exactly be what we wanted, but this:

```
The double of the number 21 is 42 .
```

There are other (arguably better) ways of circumventing this problem (see for example the discussion of f-strings later), but a simple one is to use the `sep` keyword argument to `print()`. This determines what gets printed between the various arguments, and is a space by default.

```
x = int(input("Enter an integer: "))
print ("The double of the number ", x, " is ", 2*x, ".", sep='')
```

The value of `sep` can be any string. There is another useful keyword argument of `print()`, `end`, newline (`'\n'`) by default, which determines what is printed after all the arguments are. For example:

```
>>> for i in range(5): print(i,end='|')
...
0|1|2|3|4|
```

An ugly but simple workaround for getting rid of the last `sep` is to delete it after the loop with a `print('\b')`. `'\b'` is the backspace *backslash escape sequence*, just as `'\n'` is newline and `'\t'` is tabulator. It's best to avoid these, except possibly `'\n'`.

Remark 2.1. For our problem above,

```
>>> print('|'.join([str(i) for i in range(5)]))
0|1|2|3|4
```

is a better solution. This works because if `s` is a string and `l` is a list of strings, then `s.join(l)` is the concatenation of the strings in the list separated by `s`. For example,

```
>>> "<>".join(["this", "looks", "strange"])
'this<>looks<>strange'
```

Exercise 2.2. Write a program that asks for numbers separated by spaces, and prints their average. So an interaction with your program should look something like this:

```
>>> Enter some numbers separated by spaces: 1 4 13 2
The average is  5.0
>>>
```

2.2. **Reading and writing files.** Suppose that

```
$ cat data.txt
one
two
three
very long
four
five
```

We can get the contents of `data.txt` from Python this way:

```
>>> with open('data.txt') as file:
...     for line in file:
...         print(line.rstrip())
...
one
two
three
very long
four
five
```

`with` opens a new block (the semicolon at the end of the line and the indentation of the next line gives a clue); since it is followed by

```
  open('data.txt') as file
```

what is does is open the file named `data.txt` in the current directory for reading, and assigns an iterable of the lines of the file to the variable `file`. The `.rstrip()` is there only to strip whitespace and newline from the end of each line. Try it without `.rstrip()` to see the difference!

To make us feel that our program actually does something, we may want to prepend each line with its line number in the output.

```
>>> with open('data.txt') as file:
...     for i, line in enumerate(file,1):
...         print(str(i)+": "+line.rstrip())
...
1: one
2: two
3: three
4: very long
5: four
6: five
```

(We've met `enumerate()` on page 6.)

There is a more primitive way to open a file (for reading or writing), namely with the function `open`; so our first program above could've been written like this:

```
file = open('data.txt')
for line in file:
    print(line.strip())
file.close()
```

but the `with` construction guarantees that our file will be closed (there's no need to invoke the method `close`) once control leaves its body. This is particularly important when writing files, because closing a file opened for writing ensures that all data sent to it is actually written to it. As an example of writing to a file, let's write to `out.txt` the lines of `data.txt` in reverse order:

```
>>> lines = []

>>> with open('data.txt') as file:
...     for line in file:
...         lines.append(line.rstrip())
```

```
...
>>> with open ('out.txt','wt') as out:
...     for line in lines[::-1]:
...         print(line,file=out)
...
>>> #we check the result by escaping back to the shell
>>> import os
>>> print(os.popen("cat out.txt").read())
five
four
very long
three
two
one
```

Here we opened the the file `out.txt` for writing in text mode; that's what `wt` means in the second argument to `open()`. Some important other possibilities are `rt` (read in text mode — the default), `rb` (read in binary mode) and `wb` (write in binary mode).

Another novelty here is that `print()` has a keyword argument `file`, which can be an open (for writing) file; in that case `print()` writes there instead of the standard output.

A shorter way to achieve the same result is this:

```
>>> with open('data.txt') as file:
...     with open ('out.txt','wt') as out:
...         for line in reversed(list(file)):
...             print(line.rstrip(),file=out)
...
```

This works because `list()` creates a list from an iterable, which `reversed()` can reverse. We could have written `list(file)[::-1]` instead of `reversed(list(file))`.

2.3. **Standalone programs.** Suppose someone finds one of our programs in this section so useful, that she wants to use it, too. What can we do to make it usable for her?

The first thing of course is that we need to define a function and make the filename an argument of it.

```
def cat(fn):
    with open(fn) as file:
        for line in file:
            print(line.rstrip())
```

In theory, we can send the user the file that contains this function definition. But in practice, we can't expect the users of our program to start a Python interpreter, load our program and invoke the function (`cat` in this case) that is its main entry point. We need to be able to deliver an executable file, or at least one that can be started with

```
python mycat
```

or perhaps

```
python mycat data.txt
```

from a terminal. (If we can deliver an executable, then the user can omit `python` from the above commands. But the way to do this delivery depends on the operating system.)

If `mycat.py` contains this:

```
 1 import sys
 2 
 3 def cat(fn):
 4     with open(fn) as file:
 5         for line in file:
 6             print(line.rstrip())
 7 
 8 def main():
 9     if len(sys.argv) == 2:
10         cat(sys.argv[1])
11     else:
12         raise SystemExit('Usage: '+ sys.argv[0] + ' [ filename ]')
13 
14 if __name__ == '__main__':
15     main()
```

then

```
[simon@localhost tmp]$ python mycat.py data.txt
one
two
three
very long
four
five
[simon@localhost tmp]$ python mycat.py
Usage: mycat.py [ filename ]
[simon@localhost tmp]$
```

and if we include `#!/usr/bin/python` as the first line of mycat.py, then it can be invoked as `./mycat data.txt` on Linux. (`./` is not needed if `mycat.py` is in a directory that is a member of `$PATH` environment variable — but this has nothing to do with Python.)

The details:

- The role of the last two lines is to arrange that the `main()` function will be called if the program is run in one of the two ways above. The reason is that in this case the built-in variable `__name__` has the value `'__main__'`. (If it's `import`ed[4] in an other file with `import mycat`, its value is `mycat`.)
- `sys.argv` in lines 9, 10 and 12 is a list that contains the words of the invocation (except for `python`): so with

  ```
  [simon@localhost tmp]$ python mycat.py data.txt
  ```

  we get `sys.argv[0]==mycat.py` and `sys.argv[1]==data.txt`. That's how we can access the command line arguments.
- Line 12 raises an exception (signals that "something is wrong") and prints our message explaining the cause:

  ```
  [simon@localhost tmp]$ python mycat.py
  Usage: mycat.py [ filename ]
  [simon@localhost tmp]$ echo $?
  1
  ```

[4]See Section 6

In this case it looks as if we could have just printed the message with `print()`. But the fact that the result of `echo $?` is not 0 shows that our program told the shell that it couldn't successfully terminate, which is potentially very useful [5] We'll learn a little more about exceptions in Section 4.2.1.

Exercise 2.3. If you haven't done it yet, write a function `is_prime()` that returns `True` if its only argument is a prime, and `False` otherwise. Turn this into a standalone program `is_prime.py`! For example

```
[simon@localhost tmp]$ python is_prime.py 13
```

should print `True`, and if called by the wrong number of arguments, it should print a message explaining the correct invocation.

As at the beginning of this section, you will probably need the function `int()`, because the members of `sys.argv` are strings.

3. Debugging

Programming is debugging. It doesn't happen very often that a function or method, not to mention a whole program does what it needs to do the first time it's run. There are two cases: either it throws an exception and we end up with a more or less unintelligible stack trace (often this is the better outcome), or it runs with no errors and produces the wrong result (in this case we're lucky if we realize that the result is wrong). The first kind of problem is better because at least we know where to start looking for the error.

In either case, we have a few options for investigating.

(1) Judicious use of `print()` calls. For example, to "trace" what is going on when we call the function `factorial()`:

```
Python 3.9.10 (main, Jan 17 2022, 00:00:00)
[GCC 11.2.1 20210728 (Red Hat 11.2.1-1)] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> def factorial(n): return n if n <= 1 else n * factorial(n-1)
...
```

we may want to put in an extra `print()` to show what arguments it is called with:

```
>>> def factorial(n):
...     print('n:',n)
...     return n if n <= 1 else n * factorial(n-1)
```

[5] `$ python backup.py && rm -r .` runs `python backup.py` and then, if it terminated normally, `rm -r .`. So it's very important that our programs signal to the shell their exit status. Here's a less dangerous example involving our `mycat.py`:

```
[simon@localhost tmp]$ python mycat.py data.txt && echo "mycat terminated successfully"
one
two
three
very long
four
five
mycat terminated successfully
[simon@localhost tmp]$ python mycat.py && echo "mycat terminated successfully"
Usage: mycat.py [ filename ]
[simon@localhost tmp]$
```

```
...
>>> factorial(4)
n: 4
n: 3
n: 2
n: 1
24
```

Depending on what we want to understand about our function, this may or may not help. If it doesn't, we can try to get a more complete picture:

```
>>> def factorial(n):
...     res = n if n <= 1 else n * factorial(n-1)
...     print('in: ',n,'out: ',res)
...     return res
...
>>> factorial(4)
in:  1 out:  1
in:  2 out:  2
in:  3 out:  6
in:  4 out:  24
24
```

The good thing about this method is that it doesn't need any extra tools, and is very easy to use. The bad is that we have to remove all those `print()`s afterwards, and that there is no interactivity: if, by looking at the value of one variable, we find we also need to know about another, we have to change the function and start all over again. Nevertheless, there is anecdotal evidence showing that this is the most popular debugging method used by Python programmers.

(2) Tracing your function. If you enter this:

```
def trace(f):
    depth = 0
    def wrapper(*args,**kwargs):
        nonlocal depth
        depth += 1
        print(f"{depth:>{2*depth}}: {f.__name__}:", *args, kwargs or "")
        res = f(*args,**kwargs)
        print(f"{depth:>{2*depth}}: {f.__name__} returned: {res}")
        depth -= 1
        return res
    return wrapper
```

(the details are not important), or save it in a file named `trace.py` in your working directory and import it with

```
from trace import trace
```

then any function, whose definition is preceeded by `@trace` will be traced, as in the following example:

```
>>> @trace
... def fact(n):
...     return 1 if n<=1 else n*fact(n-1)
...
```

```
>>> fact(5)
 1: fact: 5
   2: fact: 4
     3: fact: 3
       4: fact: 2
         5: fact: 1
         5: fact returned: 1
       4: fact returned: 2
     3: fact returned: 6
   2: fact returned: 24
 1: fact returned: 120
120
```

For untracing, just redefine the function without the preceeding `@trace`:

```
>>> def fact(n):
...     return 1 if n<=1 else n*fact(n-1)
...
>>> fact(5)
120
```

The good thing about tracing is that there's no need to change the program. On the other hand, it only shows how functions are called and what they return.

(3) Using a debugger. This is the most versatile method, but you need to learn to use a separate software. Or more, because there are many. The standard one, `pdb` (*p*ython *d*ebugger) is always present, but it's not very user friendly. You can try it like this:

```
>>> def fact(n):
...     breakpoint()
...     return 1 if n<=1 else n*fact(n-1)
...
>>> fact(4)
> <stdin>(3)fact()
(Pdb)
```

This is `pdb`'s prompt; you can ask for `help`, or type `q` to quit.

IPython has a version of `pdb` that has the same commands, but is more user friendly. For example, it is easier to enter (there's no need to change the function)[6]:

```
Python 3.9.10 (main, Jan 17 2022, 00:00:00)
Type 'copyright', 'credits' or 'license' for more information
IPython 7.20.0 -- An enhanced Interactive Python. Type '?' for help.

In [1]: def fact(n):
   ...:     if n<=1:
   ...:         return 1
   ...:     else:
   ...:         return n*fact(n-1)
   ...:

In [2]: %debug fact(5)
```

[6]It is also possible to enter the debugger when our program throws an exception, by entering `%debug`.

```
NOTE: Enter 'c' at the ipdb>  prompt to continue execution.
> <string>(1)<module>()

ipdb> s
--Call--
```

We don't want to continue execution, because that would just run the function to completion, but want to "step in" the function, and that's what the `s` command does.

```
> <ipython-input-1-2334ab7e9b53>(1)fact()
----> 1 def fact(n):
      2     if n<=1:
      3         return 1
      4     else:
      5         return n*fact(n-1)
```

Here we can see another aspect of IPython being more user friendly than `pdb`: it shows a bit of context. (This is why I use a more verbose version of `fact()`.)

Now I use `n`, which means "execute the *n*ext statement". In this context, `s` would do the same.

```
ipdb> n
> <ipython-input-1-2334ab7e9b53>(2)fact()
      1 def fact(n):
----> 2     if n<=1:
      3         return 1
      4     else:
      5         return n*fact(n-1)

ipdb> n
> <ipython-input-1-2334ab7e9b53>(5)fact()
      2     if n<=1:
      3         return 1
      4     else:
----> 5         return n*fact(n-1)
      6
```

At this point, `s` is the good choice, because `n` would just execute line 5 and then return immediately. (Which is what we want when our code calls another function that we don't want to debug.)

```
ipdb> s
--Call--
> <ipython-input-1-2334ab7e9b53>(1)fact()
----> 1 def fact(n):
      2     if n<=1:
      3         return 1
      4     else:
      5         return n*fact(n-1)

ipdb> n
> <ipython-input-1-2334ab7e9b53>(2)fact()
      1 def fact(n):
```

```
----> 2     if n<=1:
      3         return 1
      4     else:
      5         return n*fact(n-1)

ipdb> !n
4
```

`!n` shows the value of the variable `n`. (The value of more than one variable can be queried by writing their names after the `!` separated by commas. See also the `display` command!) So we're in the second call into `fact()`. Arguments of the function can also be queried by the command `args`.

```
ipdb> help

Documented commands (type help <topic>):
========================================
EOF    cl         disable  interact  next    psource  rv           undisplay
a      clear      display  j         p       q        s            unt
alias  commands   down     jump      pdef    quit     skip_hidden  until
args   condition  enable   l         pdoc    r        source       up
b      cont       exit     list      pfile   restart  step         w
break  continue   h        ll        pinfo   return   tbreak       whatis
bt     d          help     longlist  pinfo2  retval   u            where
c      debug      ignore   n         pp      run      unalias

ipdb> c

In [3]:
```

Pythontutor is also a kind of debugger; its strong point is that it shows what's happening with our variables, in a beautiful, graphical way.

4. Containers

4.1. `list`s, `tuple`s and `str`ings.

We've encountered plenty of lists already, but there's a lot that can be done with them that we haven't covered yet. A subset of these are applicable to tuples and strings, too. This is not surprising, considering that both tuples and strings are a bit like lists, in that all of them are mappings from a proper initial segment of the natural numbers to all Python objects (in the case of lists and tuples) and characters (in the case of strings). That is, they are all *sequences*[7]. The main difference between lists and tuples is that tuples are immutable, meaning that their members cannot be changed:

```
Python 3.9.12 (main, Mar 25 2022, 00:00:00)
[GCC 11.2.1 20220127 (Red Hat 11.2.1-9)] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> l = list(range(0,10,2)) ; l  # a new way of creating a list
[0, 2, 4, 6, 8]

>>> t = tuple(l) ; t  # one way to create a tuple
```

[7]as are `range`s, which are the return values of the function `range()`

```
(0, 2, 4, 6, 8)

>>> l[3]
6

>>> l[3] = 5 ; l[3]
5

>>> t[3]
6

>>> t[3] = 5
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: 'tuple' object does not support item assignment
>>> s = 'abcdef ghijk' ; s
'abcdef ghijk'

>>> s[3]
'd'

>>> s[3] = 'q'
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: 'str' object does not support item assignment
```

But other than this, everything that we have learned last semester about lists (mostly various methods of indexing them), applies to tuples and strings, too. So do the following, which are new:

```
>>> l + l, t + t
([0, 2, 4, 5, 8, 0, 2, 4, 5, 8], (0, 2, 4, 6, 8, 0, 2, 4, 6, 8))
>>> l*2, t*2
([0, 2, 4, 5, 8, 0, 2, 4, 5, 8], (0, 2, 4, 6, 8, 0, 2, 4, 6, 8))
>>> s + s, s*3
('abcdef ghijkabcdef ghijk', 'abcdef ghijkabcdef ghijkabcdef ghijk')
```

Exercise 4.1.★ Why do you think the following works? Doesn't this contradict the fact that tuples are immutable?

```
>>> tup = ([0,1],[2]) ; tup[0][1] = 'a' ; tup
([0, 'a'], [2])
```

Hint: try it in Pythontutor!

Here are the list of methods applicable to lists, and to tuples (courtesy of IPython's `TAB`-completion[8]):

```
l.
  append()   count()   insert()   reverse()
```

[8]An alternative way of obtaining a list of them is `dir(list)`, or `dir(l)`, where `l` is a list. So for example `dir([])` works, too. If you want them together with their documentation, enter `help(list)` (or `help(l)` if `l` is a list) at the interpreter's prompt. But with either of these techniques (which of course work for other types, too), ignore the methods whose name starts with an underscore (_) character. We will see later why.

`s[i]`	Element `i` of `s`
`s[i:j]`	A slice of `s`
`s[i:j:stride]`	An extended slice of `s`
`len(s)`	Length of `s`
`min(s)`	Minimum value in `s`
`max(s)`	Maximum value in `s`
`sum(s [,initial])`	Sum of items in `s` (not applicable to strings – use `.join()`)
`all(s)`	`True` iff all items in `s` are `True`
`any(s)`	`True` iff there is an item in `s` that is `True`
`x in s`	`True` iff `x` is a member of `s`

TABLE 3. Operations and functions on sequences

`s[i] = v`	Item assignment
`s[i:j] = v`	Slice assignment
`s[i:j:stride] = v`	Extended slice assignment
`del s[i]`	Item deletion
`del s[i:j]`	Slice deletion
`del s[i:j:stride]`	Extended slice deletion

TABLE 4. Operations applicable to lists

```
  clear()   extend()  pop()     sort()
  copy()    index()   remove()
```

```
t.
  count() index()
```

Some of the list methods are understandably missing from tuples: for example, `l.sort()` sorts the list `l` *in place*, so, unless `l` is already sorted, it must do “item assignment” (to use the terminology of the error message above). [9] The same holds for `.reverse()`:

```
>>> l.reverse() ; l
[8, 5, 4, 2, 0]
>>> l.sort(); l
[0, 2, 4, 5, 8]
```

which doesn’t mean we can’t easily reverse a tuple or a string:

```
>>> t[::-1]
(8, 6, 4, 2, 0)
>>> s[::-1]
'kjihg fedcba'
```

but, unlike `reverse()`, this of course doesn’t change the tuple or string itself.[10] `l.append(obj)` appends `obj` to the end of the list `l`, and `l.extend(it)` extends `l` with the members of

[9] There is a `sorted()` *function*, applicable to both lists, tuples and strings (and in fact, any iterable, and, in particular, any sequences). But whatever the type of its argument, it returns a list. `.reverse()` also has a function counterpart, `reversed()`, but it returns an iterable (technically, an *iterator*) that is not a list.

[10] We can write `t = t[::-1]`, thereby changing the value of `t` to a tuple that has the same members as `t`’s original value, but in reverse order — but this is nevertheless a *new* tuple under an old name (i.e., assigned to the same variable that held the original tuple). Try `print(id(l)) ; l.reverse() ; print(id(l))` and `print(id(t)) ; t = t[::-1] ; print(id(t))` to see the difference!

the iterable (list, tuple, string, ...) `it`. This example should make clear the difference between the two:

```
>>> l
[0, 2, 4, 5, 8]
>>> l.append(['a','b','c']) ;l
[0, 2, 4, 5, 8, ['a', 'b', 'c']]
>>> l.extend(['a','b','c']) ;l
[0, 2, 4, 5, 8, ['a', 'b', 'c'], 'a', 'b', 'c']
```

`.extend()` differs from `+` (concatenation) in two respects: first, `l.extend(it)` modifies `l`, it doesn't create a new list, unlike concatenation. (That is why neither tuples, nor strings have this method.) And second, in `l.extend(it)`, `it` can be any iterable, not just a list, while the arguments of `+` must be of the same type.

```
>>> l = l[:5] ; l.extend('def') ; l
[0, 2, 4, 5, 8, 'd', 'e', 'f']
```

but

```
>>> l + 'def'
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: can only concatenate list (not "str") to list
```

Finally, the `index()` method returns the index of the first occurrence of its argument in the list, tuple or string (and throws a `ValueError` if it's not a member of the sequence), and `count()` returns the number of occurrences of its argument in the sequence.

Although it is perfectly fine to access members of a tuple by indexing it, as in, say, `t[2]`, it's more common to access them by *variable unpacking*:

```
>>> t
(0, 2, 4, 6, 8)
>>> a, _, c, _, e = t
>>> c, a
(4, 0)
```

(This works, but is used less with other kinds of sequences, too.) The underscore signals that we're not interested in (that is, don't want to assign to a variable) the corresponding value.

Variable unpacking features in a very common pattern: when traversing some iterable which consists of tuples. We've seen an example of this with `enumerate()` in Section 2.2. Here's an other: suppose we have three lists, the first containg names of goods, the second containing the corresponding unit prices, and the third the amounts stocked, and our task is to produce a list with (good, total value) pairs.

```
>>> goods = ['ball', 'table', 'racket', 'net']

>>> amounts = [570, 3, 12, 17]

>>> uprices = [0.13, 2000, 185, 23]

>>> [(good, a*up) for good, a, up in zip(goods, amounts, uprices)]
[('ball', 74.10000000000001), ('table', 6000), ('racket', 2220), ('net', 391)]
```

What's new here is `zip()`, which, according to the documentation:

> returns an iterable of n-length tuples, where n is the number of iterables passed as positional arguments to `zip()`. The i-th element in every tuple comes from the i-th iterable argument to `zip()`. This continues until the shortest argument is exhausted.

Very useful if we want to iterate parallel over more than one iterable.

So, in the example

```
>>> list(zip(goods, amounts, uprices))
[('ball', 570, 0.13), ('table', 3, 2000), ('racket', 12, 185), ('net', 17, 23)]
```

Tuples can be written as literals, the same way as lists, but enclosed in parentheses instead of brackets:

```
>>> type((1,2,3))
<class 'tuple'>
```

In fact, it's the comma that is important, not the parentheses:

```
>>> x = 1,2,3 #we've been using this all the time
>>> x
(1, 2, 3)
>>> type(x)
<class 'tuple'>
```

There is a quirk though: for a tuple of length one, we need to signal to Python that it is a tuple, by writing a comma after its only member (since any expression can be surrounded by parentheses, they don't help here):

```
>>> (1) == 1
True
>>> type((1)), type((1,))
(<class 'int'>, <class 'tuple'>)
```

This is an ugly corner case which one should be aware of but probably never going to encounter.

Finally, a last word about variable unpacking: what if we don't know in advance the length of the tuple on the right hand side of an assignment? We should be able to indicate that some variable is there to receive all the members that don't go to other variables. And "all the members" can only mean "some kind of collection of all the members". This can be done with a $*$ preceding the name of the variable, and the "kind of collection" is always a list:

```
>>> u,_,v,*w = list(range(10))
>>> u,w
(0, [3, 4, 5, 6, 7, 8, 9])

>>> _,_,v,*w,u = tuple(range(10))
>>> u,w
(9, [3, 4, 5, 6, 7, 8])

>>> u,_,v,*w = "what goes where"
>>> u,w
('w', ['t', ' ', 'g', 'o', 'e', 's', ' ', 'w', 'h', 'e', 'r', 'e'])
```

We will see something similar when we learn about variable number of arguments in §5.

4.1.1. *More on strings.* String literals can be written in four different ways:

```
>>> 'ab' == "ab" == """ab""" == '''ab'''
True
```

Each have their uses. For example,

```
>>> print("No I don't") #no need to escape '
No I don't
>>> print('"Yes," he said') #no need to escape "
"Yes," he said
>>> print("This is
  File "<stdin>", line 1
    print("This is
                  ^
SyntaxError: EOL while scanning string literal
>>> too long to fit in one line")
  File "<stdin>", line 1
    too long to fit in one line")
        ^
SyntaxError: invalid syntax

>>> print("This is \
... too long to fit in one line")
This is too long to fit in one line

>>> print("""This is
... too long to fit in one line""")  #handy for multiline strings
This is
too long to fit in one line
>>> print("This is \ntoo long to fit in one line") #but not strictly necessary
This is
too long to fit in one line
```

Methods applicable to strings:

```
s.
  capitalize()   encode()       format()       isalpha()
  casefold()     endswith()     format_map()   isascii()
  center()       expandtabs()   index()        isdecimal()      >
  count()        find()         isalnum()      isdigit()

  isidentifier() isspace()      ljust()        partition()
  islower()      istitle()      lower()        removeprefix()
< isnumeric()    isupper()      lstrip()       removesuffix()   >
  isprintable()  join()         maketrans()    replace()

  rfind()        rsplit()       startswith()   translate()
  rindex()       rstrip()       strip()        upper()
< rjust()        split()        swapcase()     zfill()
  rpartition()   splitlines()   title()
```

We have already met `.join()` in section 2.1 and `.rstrip()` in Section 2.2.

Some of the above methods are particularly useful. For example, `.replace()` replaces (by default, all) occurrences of a substring with another.

```
>>> s = "you think you can do it"
>>> s.replace("you","we")
'we think we can do it'
>>> s.replace("you","we",1)
'we think you can do it'
```

For more complex replacements, *regular expressions* are used.

Another useful method is `.split()`. With no arguments, it splits the string into a list of words:

```
>>> s.split()
['you', 'think', 'you', 'can', 'do', 'it']
```

But with an argument, which is a string, it considers that string at the boundary of “words”:

```
>>> s.split("ou")
['y', ' think y', ' can do it']
>>> [w.strip() for w in s.split("ou")]
['y', 'think y', 'can do it']
```

The method `.strip()` that was used here is the symmetric version of `.rstrip()`: with no argument, it strips the whitespace from each end of the string it is called on.

We have already met the `str` function, which usually returns a string representation of an object.

```
>>> 2**3+1
9
>>> str(2**3)+str(1)
'81'
```

Conversely, the function `int()` can be used to turn a string representation of an integer into an `int`, and the corresponding function for floating point numbers is `float()`[11].

```
>>> int("2")**int("3")+int("1")
9
>>> float("1.4142135623730951")**2
2.0000000000000004
```

If `int()` or or `float()` cannot parse its string argument into an `int` or a `float`, it will throw a `ValueError`.

```
>>> int("nine")
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
ValueError: invalid literal for int() with base 10: 'nine'
```

We’ll see in Section 4.2.1 how we can deal with this situation. But how we *should* is a harder question and the answer depends on the context. (For example: should we silently return 0? Or 1? Or 42? Or return some number but warn the user? Or give her a chance to specify another number? Or just let her face the error?)

Being able to move between an object, a number, for example, and its string representation is important, among other things because text (file or message) is a very popular medium of communication. For example, every spreadsheet program (MS Excel,

[11]Both `int` and `float()` have other uses, too.

Openoffice Calc) can export and import sheets (tables) in text format (we'll meet this format, CSV, below). And the main protocol that the web uses (the Hypertext Transfer Protocol (HTTP)), or the popular JSON data interchange format, is text based. And since we often want to send/receive not just genuine text, such as names or newspaper articles, but also numbers, it's important to have functions that can recover a number from its text representation.[12]

f-strings. A piece of information one wants to display is almost always part constant, part variable. For example, even though the result a long computation may be 42, it's never a good idea to print just a number. Printing something like the following

```
The answer is: 42
Please enter your question:
```

is much more useful. This is especially true when the result consists of more numbers (and/or other types of data). For example, suppose we have a little "database" of goods, their amounts an unit prices.

```
>>> db = [
...     ('ball', 570, 0.13),
...     ('table', 3, 2000),
...     ('racket', 12, 185),
...     ('net', 17, 23)
... ]
```

If we want to present it, or something derived from it, as we did earlier:

```
>>> [(good, a*up) for good, a, up in db]
[('ball', 74.10000000000001), ('table', 6000), ('racket', 2220), ('net', 391)]
```

labeling the various items displayed is a mimimum requirement, unless we are the only user of our program. At the very least, we want something like this:

```
Name: ball
Total price: 74.10000000000001
Name: table
Total price: 6000
Name: racket
Total price: 2220
Name: net
Total price: 391
```

Every line here consist of two parts: a constant string, for example `Name: ` and a variable (string representation of a) number, such as 6000. We've encountered this problem already, and solved this by converting everything to string (with the function `str()`) and concatenating the results with +. But f-strings (*formatted string literals*) are much better suited to this task: they are strings with "holes" (the official name is *replacement fields*) in them, which are Python expressions enclosed in braces. What's inside these holes get evaluated at the time of printing. For example:

```
>>> f'1+1 = {1+1}'
'1+1 = 2'
>>> f'The first record of our database is {db[0]}'
"The first record of our database is ('ball', 570, 0.13)"
```

[12]Of course, a newspaper article may also contain numbers, but it's safe to treat them as text, because we usually don't need to *use* them as numbers, e.g. square them.

The `f` signifies that the string that follows is not to be printed blindly, because Python code may be found in it between braces. With this, we can present our database in the desired form:

```
>>> for good, a, up in db:
...     print(f'Name: {good}\nTotal price: {a*up}')
...
Name: ball
Total price: 74.10000000000001
Name: table
Total price: 6000
Name: racket
Total price: 2220
Name: net
Total price: 391
```

There's much more to f-strings. Among other things, we have more control over how the data within braces is presented. For example, if we're bothered by the lots of decimals, we can write `{a*up:.2f}` in place of `{a*up}` (the part after the colon is called a *format specifier*) and this will ensure that the number will be written as a `float` with exactly 2 decimal places.

```
>>> for good, a, up in db:
...     print(f'Name: {good}\nTotal price: {a*up:.2f}')
...
Name: ball
Total price: 74.10
Name: table
Total price: 6000.00
Name: racket
Total price: 2220.00
Name: net
Total price: 391.00
```

We can also declare the width of a replacement field, which is useful for presenting data in tabular form[13]:

```
>>> def doit():
...     print(f'{"Name":20}Total price')
...     for good, a, up in db:
...         print(f'{good:20}{a*up:10.2f}')
...
>>> doit()
Name                Total price
ball                     74.10
table                  6000.00
racket                 2220.00
net                     391.00
```

What's new here is that `"Name":20` and `good:20` ensure that `"Name"` and `good` are printed in a column of width 20 characters, and because of `a*up:10.2f`, `a*up` is printed in a column of width 10 characters, right aligned, because it is a numeric field. We

[13]There's no reason to define a function for this; I did it here for LaTeXnical reasons.

could've forced it to be left aligned with `a*up:<10.2f` and centered with `a*up:^10.2f`. Here are some more examples:

```
>>> ans = 42
>>> print(f'|{ans:7d}|'); print('|1234567|') # 'd' stands for 'decimal'
|     42|
|1234567|
>>> print(f'|{ans:<7d}|'); print('|1234567|')
|42     |
|1234567|
>>> print(f'|{ans:^7d}|'); print('|1234567|')
|  42   |
|1234567|
>>> print(f'|{ans:07d}|'); print('|1234567|') #padding by '0'
|0000042|
|1234567|
>>> print(f'|{ans:7b}|'); print('|1234567|') # 'b' stands for 'binary'
| 101010|
|1234567|
>>> print(f'|{ans:7.2f}|'); print('|1234567|')
|  42.00|
|1234567|
>>> print(f'|{ans:07.2f}|'); print('|1234567|')
|0042.00|
|1234567|
```

The official documentation of f-strings can be found here.

4.2. `dict`s.

Like lists and tuples, `dict`ionaries hold a collection of objects, but unlike them, these objects are not indexed by natural numbers, but by keys (strings, numbers, tuples, ...), just like in real world dictionaries. So in the above example with goods and their values, it would make more sense to put the result of our computation in a dictionary, and not in a list of tuples, because then the total value of balls could be accessed simply by `total_values['ball']`. So we'll redo that example in a minute with a dictionary (and dictionary comprehension) in place of list and list comprehension.

But let's first see the basics of `dict`s.

```
>>> d = dict()
>>> type(d)
<class 'dict'>
>>> d
{}
>>> d['one']=1 ; d['two']=2; d
{'one': 1, 'two': 2}
>>> d['two']
2
>>> d.get('two')
2
>>> d['three']
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
```

```
KeyError: 'three'
>>> 'two' in d, 'three' in d
(True, False)
>>> None == d.get('three')
True
>>> d.get('three',"can't find it")
"can't find it"
>>> d.get('two',"can't find it")
2
```

As you can see, the method `.get()` has a second, optional argument, `None` by default, which is returned in case the key (its first argument) is not present in the `dict`.

Other methods applicable to `dict`s, again, courtesy of IPython's `Tab`-completion:

```
d.
 clear()      get()        pop()        update()
 copy()       items()      popitem()    values()
 fromkeys()   keys()       setdefault()
```

Just as for lists and tuples, `dict`s can be written as literals, and in exactly the same way they're printed. So `d` above could've been defined with `d = {'one': 1, 'two': 2}`.

Now we can come back to our inventory example.

```
>>> db
[('ball', 570, 0.13), ('table', 3, 2000), ('racket', 12, 185), ('net', 17, 23)]

>>> total_values = {good: a*up for good, a, up in db}
>>> total_values
{'ball': 74.10000000000001, 'table': 6000, 'racket': 2220, 'net': 391}
>>> total_values['ball']
74.10000000000001
```

So dictionary comprehension is very much like list comprehension, with parentheses replaced by braces, and it collects "key:value" pairs and not just any objects.

4.2.1. *Exceptions.* Programs sometimes run into situations they can't deal with. For example:

```
>>> 1/0
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
ZeroDivisionError: division by zero
```

or

```
>>> 1 / int('two')
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
ValueError: invalid literal for int() with base 10: 'two'
```

It's unlikely that we write `1/0` directly, but perhaps `0` or `'two'` came from user input that wasn't carefully checked. Such an error need not lead to the termination of our programs. We can use the `try: ... except: ...` construction to give us a chance to continue.

```
>>> try:
...     print(1/0)
... except:
...     print("Something is wrong: I assume you wanted 42")
...
Something is wrong: I assume you wanted 42
```

or

```
>>> try:
...     print(1 / int('two'))
... except:
...     print("Something is wrong: I assume you wanted 42")
...
Something is wrong: I assume you wanted 42
```

This doesn't solve the problem, just hides it. But we can always give the user a second chance:

```
def divide():
    divisor = int(input("Enter the divisor: "))
    return 1 / int(divisor)

try:
    print(divide())
except:
    print("This didn't work, sorry. Let's try again!")
    print(divide())
```

This is better, but the program (and hence the user) doesn't know what kind of problem it (she) is facing. If it did, perhaps it would take the appropriate measure. What's wrong with the input? Because `except:` catches everything, we can't even be sure that the problem has something to do with the input. Maybe what happened was that the computer ran out of memory, resulting in a `MemoryError`.

But if we look at how Python reported the errors above, before we caught it with `try: ... except: ...`, we see the type of the error (printed in red), and that is not just a name, but an object on which we can discriminate by writing it after `except`.

```
try:
    print(divide())
except ValueError:
    print("I want a NUMBER, not some junk!")
    print(divide())
except ZeroDivisionError:
    print("Can't divide by 0. Perhaps later, in version 2.0.")
    print(divide())
```

or even

```
while True:
    try:
        print(divide())
        break
    except ValueError:
        print("I want a NUMBER, not some junk!")
    except ZeroDivisionError:
        print("Can't divide by 0. Perhaps later, in version 2.0.")
```

to give the user as many chances as possible. Now an interaction with the user may look like this:

```
Enter the divisor: zero
I want a NUMBER, not some junk!
Enter the divisor: 0
Can't divide by 0. Perhaps later, in version 2.0.
Enter the divisor: 5
0.2
```

This is good, and solves the "the error may have a completely different origin" problem, too. For if a third kind of exception occurs, our exception handlers will not catch it, and we won't ask the user to reinput the number, which is good. But we can make this a little more elegant by saying goodbye before bailing out. Here is how:

```python
while True:
    try:
        print(divide())
        break
    except ValueError:
        print("I want a NUMBER, not some junk!")
    except ZeroDivisionError:
        print("Can't divide by 0. Perhaps later, in version 2.0.")
    except:
        print("\nI don't know what happened and I can't deal with it. Sorry!")
        raise
```

```
Enter the divisor: two
I want a NUMBER, not some junk!
Enter the divisor:      #here I pressed Ctrl-d which means End-Of-File
I don't know what happened and I can't deal with it.
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File "/tmp/mc.py", line 67, in <module>
  File "/tmp/mc.py", line 62, in divide
EOFError
>>>
```

Here the final `except` clause is activated if the exception is not of type `ValueError` or `ZeroDivisionError`; and `raise` raises the same exception that got us here. This is good practice because we don't want to hide information from the user (who, though not in this case, but in general, may be another part of our program).

For a different kind of example: suppose we want to throw a dice many times and record the distribution of the various faces. We can put this data in a dictionary where the keys are the numbers $1, \ldots, 6$ and the values are the corresponding number of occurrences. If we don't start with a dictionary which has all the six keys in place already, we need to do something special (put the new key with value 1 in the dictionary) when a face comes up for the first time.

```python
>>> import random

>>> num_count = {} #empty dictionary
>>> for num in [random.randint(1,6) for _ in range(100)]:
...     if num in num_count:
...         num_count[num]+=1
...     else:
...         num_count[num]=1
...
>>> num_count
{6: 17, 2: 23, 1: 16, 4: 13, 3: 20, 5: 11}
>>> sum(map(lambda k: num_count[k], num_count))
100
```

Alternatively, we can omit checking every time whether the face is in the dictionary already, and instead, only put it in there if its absence leads to an error.

```
>>> num_count = {}

>>> for num in [random.randint(1,6) for _ in range(100)]:
...     try:
...         num_count[num]+=1
...     except KeyError:
...         num_count[num]=1
...

>>> num_count
{2: 19, 4: 15, 5: 18, 6: 20, 1: 15, 3: 13}
```

This has nothing to do with handling exceptions, but if we want to see the results ordered by the faces, we can do this, using the `.keys()` method of `dict`s, which return all the keys (as an iterable, which needs to be converted into a list before being sorted):

```
>>> [(key, num_count[key]) for key in sorted(list(num_count.keys()))]
[(1, 15), (2, 19), (3, 13), (4, 15), (5, 18), (6, 20)]
```

Or we can use the `.items()` method of `dict`s, which return all key-value pairs, again, as an iterable that we need to convert to a list if we want to sort it:

```
>>> sorted(list(num_count.items()),key=lambda kv: kv[0])
[(1, 15), (2, 19), (3, 13), (4, 15), (5, 18), (6, 20)]
```

The `key` keyword argument (this has nothing to do with keys in a `dict`) to `sorted()` lets us supply a function of one argument, which given a member of the list to be sorted, returns a value on which sorting should be based. In out case, it's the face in the (face, number of occurrences) pair.

This example has shown how to handle a specific error. But in real life, the best way to solve a problem like this is not using a dictionary at all, but a `Counter`.

```
>>> from collections import Counter

>>> num_count = Counter()

>>> for num in [random.randint(1,6) for _ in range(100)]:
...     num_count[num]+=1
...
>>> num_count
Counter({2: 22, 4: 19, 6: 18, 1: 15, 3: 14, 5: 12})
```

Remark 4.1. There are some fine points about exceptions that are good to be aware of.

- More than one kind of exception can be dealt with in the same `except` clause: we need to write them as a tuple. For example:
  ```
  except (ValueError, ZeroDivisionError):
  ```
- The `try: ... except: ...` block may have an `else:` clause, too. It will get executed if the `try:` succeeded.

  Exercise 4.2. What's the point of `else:`? Why not just write
  ```
  try:
      do_something
      do_this_if_all_went_well
  except:
      do_something_else
  ```
  instead of
  ```
  try:
      do_something
  except:
      do_something_else
  else:
      do_this_if_all_went_well
  ```
- There may also be a `finally:` clause, which will get executed no matter what, and in particular, whether an exception occurred in the `try` clause or not.
  ```
  while True:
      try:
          print(divide())
          break
      except ZeroDivisionError:
          print("Can't divide by 0. Perhaps later, in version 2.0.")
      finally:
          print("Finally!")
      print("At last")
  ```
  Here, if the input is correct, `"At last"` will not be printed (since control leaves `while` because of the `break`), but `"Finally!"` will, because of the "no matter what" rule.

When writing bigger programs it's important to be able to define and raise various exceptions. But there is one way (apart from *re*raising, which we have done before) to raise an exception that can be useful even in the simplest functions. It's done with the `assert` statement. Its first argument must be an expression that evaluates to a boolean (or something that can be cast to a boolean, see Section 1, item 4). When this evaluates to `False`, an `AssertionError` exception is raised, and if the second, optional argument to `assert` is present, it is printed.

This is a great way to ensure that things are as they ought to be. For example, if we write a function `day_to_date()`, that, given a number, returns the corresponding date (month, day pair), it makes sense to check first that the argument is between 1 and 365:

```
MONTHS = [31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31]

def day_to_date(day):
```

```
    assert 1 <= day <= 365, f"{day} is not a day of a year"
    days = [sum(MONTHS[:i]) for i in range(1+len(MONTHS))]
    for index in range(len(days)):
        if day <= days[index]:
            return (index,day-days[index-1])
>>> day_to_date(74)
(3, 15)
>>> day_to_date(-74)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File "<stdin>", line 2, in day_to_date
AssertionError: -74 is not a day of a year
```

`assert` is also useful in debugging. For example, if something doesn't work quite right, we can put various `assert`s in our function to make sure that things are as they should be.

Exercise 4.3. Write a function `date_to_day()` that is the inverse of `day_to_date()`. That is, given a month and a day in that month, it should return the serial number (?) of that day in the year. For example:

```
>>> date_to_day(3,15)     #31+28+15
74
>>> day_to_date(date_to_day(3,15))
(3, 15)
```

4.3. `set`**s.** This is the last and least important kind of container. Its methods are:

```
add()        difference_update()   isdisjoint() remove()
clear()      discard()             issubset()   symmetric_difference()
copy()       intersection()        issuperset() symmetric_difference_update()
difference() intersection_update() pop()        union()
                                                update()
```

The syntax for literal sets is writing the set's elements separated by commas between braces:

```
>>> type({1,2,3})
<class 'set'>
```

The `set()` function, given an iterable as argument, also returns a set:

```
>>> set(range(5))
{0, 1, 2, 3, 4}
```

This makes it easy to remove duplicates from a list:

```
>>> list(set([1,3,2,3,1]))
[1, 2, 3]
```

But the usefulness of sets is limited by the fact that not every object can be put in a set:

```
>>> set([1,[2,3]])   #doesn't work
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: unhashable type: 'list'
```

The error message means that Python cannot encode lists in such a way that identical lists get the same code. (Just think about what would/should happen if a list in a set is changed. Should the code change, too?) And that would make it problematic to check whether a list is *in* the set or not. For the same reason, a `set` also cannot be put in a `set`, and neither can it be a key in in a `dict`.

The names of most of the important methods speak for themselves. For example:

```
>>> s1 = {1,2,3}; s2 = {2,3,4} ; s1.intersection(s2)
{2, 3}
>>> s1.difference(s2)
{1}
>>> s1
{1, 2, 3}
```

But some don't:

```
>>> s1.difference_update(s2) ; s1
{1}
>>> s1.update(s2) ; s1   # why not union_update()?
{1, 2, 3, 4}
>>> s1.discard(5) ; s1
{1, 2, 3, 4}
>>> s1.discard(2) ; s1
{1, 3, 4}

>>> try:
...     s1.remove(1) ; s1
... except KeyError:
...     "Can't remove what's not there!"
...
{3, 4}
>>> try:
...     s1.remove(1) ; s1
... except KeyError:
...     "Can't remove what's not there!"
...
"Can't remove what's not there!"
```

As a data structure, `set` doesn't offer much over `dict` (with values all set to `None`). But the applicable methods listed above may come in handy.

4.4. **More on list comprehension.** We know that if `l` is a list, then

```
result = [expr for i in l]
```

is equivalent to

```
result = []
for i in l:
    result.append(expr)
```

and

```
result = [expr for x in l if c]
```

is equivalent to

```
result = []
for i in l:
    if c:
        result.append(expr)
```

For example,

```
>>> [i**2 for i in range(20) if i%2 == 1]
[1, 9, 25, 49, 81, 121, 169, 225, 289, 361]
```

But more generally,

```
[expr for i1 in l1 if c1
     for i2 in l2 if c2
     ...
     for iN in lN if cN ]
```

is equivalent to

```
result = []
for i1 in l1:
    if c1:
        for i2 in l2:
           if c2:
               ...
               for iN in lN:
                   if cN:
                       result.append(expr)
```

For example,

```
>>> [(i,j) for i in [1,2,3] for j in ['a','b']]
[(1, 'a'), (1, 'b'), (2, 'a'), (2, 'b'), (3, 'a'), (3, 'b')]
```

because it should be read "from left from right" unlike nested list comprehensions:

```
>>> [[(i,j) for i in [1,2,3]] for j in ['a','b']]
[[(1, 'a'), (2, 'a'), (3, 'a')], [(1, 'b'), (2, 'b'), (3, 'b')]]
```

which are read "outside in".

Here's another example that shows that in a list comprehension such as

```
[(i,j) for i in [1,2,3] for j in ['a','b']]
```

the inner loop (here `for j in ...`) the local variable established by the outer loop (`for i in ...`) is available:

```
>>> [j for i in range(3) for j in [range(3),range(3,6),range(6,8)][i]]
[0, 1, 2, 3, 4, 5, 6, 7]
```

It's worth abstracting away the essence of this in a function:

Exercise 4.4. Write a function `concatenate()` that concatenates the list of lists that it is passed. For example:

```
>>> concatenate([list(range(3)),list(range(3,6)),list(range(6,8))])
[0, 1, 2, 3, 4, 5, 6, 7]
```

The original example with a condition:

```
>>> [(i,j) for i in [1,2,3] if i%2 ==1 for j in ['a','b']]
[(1, 'a'), (1, 'b'), (3, 'a'), (3, 'b')]
```

The concatenating example with multiple conditions:

```
>>> [j for i in range(3) if i%2==1
...   for j in [range(3), range(3,10), range(11,15)][i]
...   if j%2==0]
[4, 6, 8]
```

Example 4.1. Suppose that `l` is a list of lists of numbers that are all smaller than `len(l)`. The idea is that `l[i]` is the list of neighbours of `i`. Here is a function `n2(i,l)` that returns the list of all neighbours of neighbours of `i`:

```
>>> def n2(i,l):
...     assert i < len(l)
...     return [k for j in l[i] for k in l[j]]
...
```

and then for example

```
>>> n2(2,[[1,2,3],[1],[0],[2]])
[1, 2, 3]
```

5. Functions

We have been using and defining functions since last semester. We know that a function definition looks like this:

```
def fname(par_1,par_2,...):
    statement_1
    statement_2
    ...
```

When a function defined this way is called with `fname(arg_1,arg_2,...)`, what happens is that the arguments `arg_1`, `arg_2`,... get evaluated (so for example, if `arg_1` is another function call, that function is called before `fname`), and then the statements in the body of the definition get evaluated, with `par_1` set to the value of `arg_1`, `par_2` set to the value of `arg_2`, etc. `par_1`, ... are the *parameters* of the function. In the body, these are local variables, so if there is a variable of the same name in the program, it is "shadowed" by the parameter. Its value cannot be seen or changed by the function. Assignment to a variable in the body makes that variable local, too. Here's the example from last semester that shows this.

```
Python 3.9.12 (main, Mar 25 2022, 00:00:00)
[GCC 11.2.1 20220127 (Red Hat 11.2.1-9)] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> a = 1 ; b = 2
>>> def some_fun(a):
...     b = a
...     print(f'In the body of the function, a={a} and b={b} after the assignment')
...
>>> some_fun(42)
In the body of the function, a=42 and b=42 after the assignment
>>> a,b
(1, 2)
```

The global value of `b` is available in the body but only if we don't create a local variable with the same name:

```
>>> a = 1 ; b = 2
>>> def some_fun(a):
...     print(f'In the body of the function a={a} and b={b}')
...
>>> some_fun(42)
In the body of the function a=42 and b=2
>>> a,b
(1, 2)
```

but

```
>>> a = 1 ; b = 2
>>> def some_fun(a):
...     print(f'In the body of the function, b={b} before the assignment')
...     b = a
...     print(f'In the body of the function, a={a} and b={b} after the assignment')
...
>>> some_fun(42)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File "<stdin>", line 2, in some_fun
UnboundLocalError: local variable 'b' referenced before assignment
```

doesn't work because even if it happens later, Python knows that we have created a local variable `b` (but used it before having assigned a value to it). The fact that we never actually get to touch `b` doesn't change the fact that it's a local variable:

```
>>> a = 1 ; b = 2
>>> def some_fun(a):
...     print(f'In the body of the function, b={b} before the assignment')
...     if False:
...         b = a
...         print(f'In the body of the function, a={a} and b={b} after the assignment')
...
>>> some_fun(42)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File "<stdin>", line 2, in some_fun
UnboundLocalError: local variable 'b' referenced before assignment
```

The statement `b = a`, even if it is guaranteed not to be executed, is the givaway.

If, for some reason, we *did* want to change the value of the global variable `b` in the function, we could do it like this:

```
>>> a = 1 ; b = 2
>>> def some_fun(a):
...     global b
...     b = a
...     print(f'In the body of the function a={a} and b={b} after the assignment')
...
>>> some_fun(42)
In the body of the function a=42 and b=42 after the assignment
```

```
>>> a,b
(1, 42)
```

but we shouldn't.

Remark 5.1. There's a way to change (usually inadvertently) the value of a global variable in a different way, as the following example shows. Suppose we want to define a function that sorts a list of numbers using the "bubble sort" algorithm. The idea of this algorithm is that whenever we find a pair of numbers in the wrong order, we reverse it.

```
>>> def bubble(l):    #bad
...     for i in range(len(l)):
...         for j in range(len(l[i+1:])):
...             jj = i+1+j
...             if l[jj]<l[i]:
...                 l[jj], l[i] = l[i], l[jj]
...     return l
...
>>> import random
>>> l = [random.randint(0,100) for _ in range(10)]
>>> l
[18, 12, 64, 77, 42, 33, 14, 12, 55, 38]
>>> bubble(l)
[12, 12, 14, 18, 33, 38, 42, 55, 64, 77]
```

The return value looks good, but there is a problem:

```
>>> l
[12, 12, 14, 18, 33, 38, 42, 55, 64, 77]
```

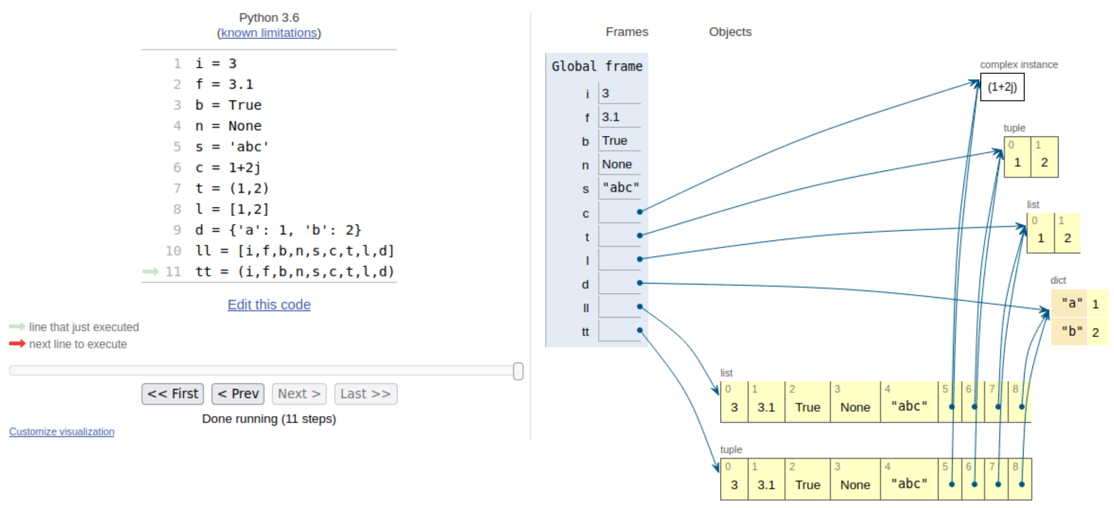
The function was not supposed to change its argument.[14] Here's the problem in a simpler context:

```
>>> def side_effect(a):
...     a=42
...
>>> b = 0; side_effect(b); b
0
>>> #so far, so good
>>> def side_effect(a):
...     a[0]=42
...
>>> b = [0]; side_effect(b); b
[42]
```

What's happened is not that after the call to the function a new object was assigned to `b` – that can't have happened, because there was no `global b` declaration in the body of the function. It's the old object itself, the one that is the value of `b`, that has changed. (It's worth watching on Pythontutor what's going on here.) This is because for such complex values as lists (and all other containers, except for strings), when they are assigned to a variable, what the variable contains is not the list itself, but a reference

[14]When the purpose of a function or method is to have a side effect, such as changing its arguments, it's customary in Python for it to not return a value.

to it (in all likelihood its address in memory). And a list, being a mutable object, can



```
i = 3
f = 3.1
b = True
n = None
s = 'abc'
c = 1+2j
t = (1,2)
l = [1,2]
d = {'a': 1, 'b': 2}
ll = [i,f,b,n,s,c,t,l,d]
tt = (i,f,b,n,s,c,t,l,d)
```

FIGURE 1. Memory (as shown by Pythontutor)

change without its address having changed. This is what happens to the value of b in the example. The way to avoid this problem (and this usually, though not always, *is* a problem) is to make a copy of the complex object and mutate the copy:

```
>>> def side_effect(a):
...     a = a[:] #could use a.copy() instead
...     a[0]=42
...
>>> b = [0]; side_effect(b); b
[0]
```

Here the variable a that is assigned to in the second line is a new local variable. (We could have given it any other name, but it's a good practice to use the name of the corresponding parameter.) And what is assigned to it is a brand new list.

But this isn't always enough:

```
>>> def side_effect(a):
...     a = a[:]
...     a[0][0]=42
...
>>> b = [[0]]; side_effect(b); b
[[42]]
```

The problem is that a[:] and a.copy() creates a *shallow copy* of the list. A shallow copy of a list is a new list, but if the original contained a list, then what it really contained is a reference to it, and that reference will be copied into the new list. So the same problem will crop up, only at another level. What we need here is a *deep copy*, which, instead of copying a reference (in any level) creates a new list (which is again a deep copy of the list the reference referred to) and write *that* into the newly created list. Something like this:

```
def deep_copy(l):
    return [deep_copy(i) for i in l] if isinstance(l, list) else l
```

would solve the problem as long as the value we pass into `side_effect` doesn't contain other kinds of complex values buried deep inside.

```
>>> def side_effect(a):
...     a = deep_copy(a)
...     a[0][0]=42
...
>>> b = [[0]]; side_effect(b); b
[[0]]
```

But the real solution is using the `deepcopy()` function from the `copy` module:

```
>>> import copy
>>> def side_effect(a):
...     a = copy.deepcopy(a)
...     a[0][0]=42
...
>>> b = [[0]]; side_effect(b); b
[[0]]
```

This takes care not just of lists but other complex values, too. For our bubble sort function, we don't need a deep copy, a shallow one will do.

```
>>> def bubble(l):
...     l = l[:]
...     for i in range(len(l)):
...         for j in range(len(l[i+1:])):
...             jj = i+1+j
...             if l[jj]<l[i]:
...                 l[jj], l[i] = l[i], l[jj]
...     return l
...
>>> l = [random.randint(0,100) for _ in range(10)]
>>> l
[5, 41, 75, 28, 5, 36, 42, 29, 11, 62]
>>> bubble(l)
[5, 5, 11, 28, 29, 36, 41, 42, 62, 75]
>>> l
[5, 41, 75, 28, 5, 36, 42, 29, 11, 62]
```

A final word on the problem of deep vs. shallow copy: it can come up in other situations, not involving function calls, too, as the following example shows:

```
>>> b = [[0]] ; c = b ; c[0][0] = 42; b
[[42]]
```

And the solution is always the same:

```
>>> b = [[0]] ; c = copy.deepcopy(b) ; c[0][0] = 42; b
[[0]]
```

Exercise 5.1. Consider the following piece of code:

```
def side_effect_or_not (l):
    l = [2*i for i in l]
    return l

mylist = [1,2,3] ; side_effect_or_not(mylist)
```

What will be the value of `mylist` after running it, and why?

Keyword and optional arguments. In a call

```
>>> f(1,2)
x=1, y=2
```

to a function defined by

```
def f(x,y):
    print(f'x={x}, y={y}')
```

Python knows `1` should be assigned to `x` and `2` to `y` because of their respective positions. That's why these are sometimes called *positional parameters*. But we can be more explicit about what values to assign to which parameter with keyword arguments:

```
>>> f(y=2,x=1)
x=1, y=2
```

We can change the definition of the function so that the caller is forced to use some arguments as keyword arguments. For example,

```
>>> def f(x,*,y):
...     print(f'x={x}, y={y}')
...
>>> f(1,2) #wrong
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: f() takes 1 positional argument but 2 were given
>>> f(1) #wrong
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: f() missing 1 required keyword-only argument: 'y'
>>> f(1,y=2) #finally...
x=1, y=2
```

It is also possible to provide default values for parameters, like this:

```
>>> def f(x,y=3):
...     print(f'x={x}, y={y}')
...
>>> f(1,2)
x=1, y=2
>>> f(1)
x=1, y=3
```

But we can't put non-default positional arguments after a default argument:

```
>>> def f(x,y=3,z):   #wrong
  File "<stdin>", line 1
    def f(x,y=3,z):   #wrong
                ^
SyntaxError: non-default argument follows default argument
```

This is only logical, for if the function definition started with `def f(x,y=3,z):` and we called `f` with two arguments (and we should be able to, because one of the three parameters has a default value), the second one should be assigned to z, which is very confusing. If you want to mix default and non-default parameters, use keyword arguments:

```
>>> def f(x,*,y=3,z):
...     print(f'x={x}, y={y}, z={z}')
...
>>> f(1,z=2)
x=1, y=3, z=2
```

It is also possible to define functions with variable number of arguments. Of course it doesn't make sense to write "variable number of parameters" in the definition. So we write just one, and mark it with an asterisk.[15] This signals to Python that this parameter should receive all the remaining (positional) arguments as a tuple. ("Remaining", because some might have been assigned to preceeding normal, positional parameters.) Suppose for example that we want to compute the average of an unknown number of numbers. Here's how we can do this:

```
>>> def avg(*nums):
...     return(sum(nums)/len(nums))
...
>>> avg(2,3)
2.5
>>> avg(2,3,5,9,6)
5.0
```

It wouldn't make sense to have two such variadic parameters (which one would get which argument?), but positional parameters can preceed one. For example, suppose that sometimes we want to compute the geometric mean, not the arithmetic one. Then a first argument can receive the type of mean we want computed, and the rest of the arguments are the numbers themselves.

```
>>> from functools import reduce
>>> def avg(type, *nums):
...     return \
...         reduce(lambda x, y: x*y,nums)**(1/len(nums)) \
...         if type == 'g' \
...            else sum(nums)/len(nums)
...
>>> avg('whatever',2,3,5,9,6)
5.0
>>> avg('g',2,3,5,9,6)
4.384327654865777
```

The first argument went into the parameter `type`, and the rest into `nums`. This is fine for illustrating where arguments go if there are positional parameters preceeding a variadic one, but it's not a very æstetic user interface. Since there is a sensible default here (we'd probably want arithmetic mean most of the time), `type` should be made into a default parameter. But we can't put the variadic parameter after a default one, so let's do it the other way round!

```
>>> def avg(*nums, type='a'):
...     return sum(nums)/len(nums) if type == 'a' \
```

[15]This resembles the variable unpacking in §4, but is more restricted.

```
...         else reduce(lambda x, y: x*y,nums)**(1/len(nums))
...
>>> avg(2,3,5,9,6)
5.0
>>> avg(2,3,5,9,6,type='g')
4.384327654865777
```

In this case Python knows where it should stop collecting arguments in `nums`, because it recognizes the keyword argument from the equality symbol. And it is a "mandatory" keyword argument, because it is after a variadic one. (The `*` above, which was used to force the subsequent parameters to be keyword parameters, can be thought of as a "dummy variadic parameter" accepting exactly zero arguments, whose only reason to exists is to force the subsequent parameters to be keyword parameters.)

There's a kind of inverse to variadic arguments: if `a` is a list, `f(*a)` calls `f` with the members of the lists as arguments. For example,

```
>>> (lambda x,y: x+y)(*[1,2])
3
```

Example 5.1. This is an example that uses both variadic arguments and this "inverse". It's a simplified version of the `map()` function. Its first argument is a function of any number of arguments, and the rest of its arguments are that many lists. The result is the list of the result of applying the function to successive elements of the lists. For example,

```
>>> mymap(lambda x,y,z: (y,z,x), [1,2,3],[4,5,6],['a','b','c'])
[(4, 'a', 1), (5, 'b', 2), (6, 'c', 3)]
```

Here's the definition:

```
def mymap(fn, *lists):
    return [fn(*i) for i in  zip(*lists)]
```

We need `*lists`, because we don't know in advance the arity of `fn` and hence the number of lists `mymap()` will be called with. In the body of the function `lists`'s value is a tuple of lists. We feed these lists as separate arguments to `zip()` (which, fortunately, is also a function that accepts any number of arguments), which returns a list (actually, an iterable, but that doesn't matter and shouldn't concern us now) of tuples: the first (which will be the value of `i` in the first iteration) contains the first members of all the lists that were the arguments of `mymap()`, the second tuple contains their second members, etc. And, in each iteration, `fn` will be called with the members of `i`, so on the first iteration, the first members of the lists, on the second the second members of the lists, etc. And the result is the list of the results `fn` returns.

Documentation. Since the body of a function is a series of statements and expressions which get evaluated in the order they are written, it doesn't make a difference in the behaviour of a function if a literal object (such as `42` or `"nice wheather, eh?"`) is included in this series. Now if that literal object happens to be a string, and it's inserted as the very first statement, then it's called a *documentation string*, and is stored in the `__doc__` attribute of the function.

```
>>> def fun():
...     """
...     This function does nothing, but does it well.
...     Usage: fun()
...     """
...     pass
...
>>> print(fun.__doc__)

    This function does nothing, but does it well.
    Usage: fun()
```

The docstring is retrievable by the various IDEs. For example, by `fun?` in IPython. But under the hood, it almost surely uses the `__doc__` attribute.

Documenting the functions we write this way is very good practice.

Anonymous functions. We've seen and used `lambda`s before, but a short overview of what they are and why they are useful doesn't hurt.

First of all, `lambda`s are not indispensable. (Very few constructs are.) They construct functions whose bodies consist of one expression only, which will be the return value of the constructed function. So, wherever

```
lambda v1,...,vn: expr
```

appears in our program, we can always write `f` instead, supposing we have also written

```
def f(v1,...,vn):
    return expr
```

before, and that `f` is not used otherwise as the name of a function.

And this shows two reasons why `lambda`s are useful. First, when we need a function only once, it's not just an overkill to give it a name, but potentially dangerous, too: we need to make sure that there is no function of the same name elsewhere in the program. The other reason is that we see immediately what our `lambda` does, there's no need to look up the definition of a function defined elsewhere. And the definition *must* often be elsewhere, because, unlike a `lambda`, a `def` is a statement, not an expression, so cannot be written where an expression is expected, such as in a list.

⋮

Exercise 5.2. Write a function computing the factorial of positive integers using `reduce()` from `functools`.

Higher order functions. Functions that take functions as arguments or return functions are called higher order functions.

One use of higher order function is avoiding code duplication. For example, suppose we need to do various operations on lists of numbers. We could write functions for each of these:

```
>>> def inc_list(l):
...     return [x+1 for x in l]
...
>>> def prod_list(l):
...     return [2*x for x in l]
...
>>> inc_list(list(range(10)))
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
>>> prod_list(list(range(10)))
[0, 2, 4, 6, 8, 10, 12, 14, 16, 18]
```

but these two functions are practically the same; the only difference is what they are doing with the members of the list. So it make sense to turn *that* into an extra parameter:

```
>>> def process_list(fun,l):
...     return [fun(x) for x in l]
...
>>> process_list(lambda x: x+1, list(range(10)))
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
>>> process_list(lambda x: 2*x, list(range(10)))
[0, 2, 4, 6, 8, 10, 12, 14, 16, 18]
```

Actually, `process_list()` is a primitive version of the function `map()` that we've encountered briefly.

Exercise 5.3. Define a function `apply()` of two arguments, which returns the result of applying its first argument, which should be a one-argument function, to its second.

We have seen functions taking functions as arguments already: `map`, `filter` and `reduce`. Beware that unlike last semester, where Sage/Python returned lists for the first two of these (the reason being that the Sage version we used was built on an older version of Python) it now returns an *iterable* (something over which one can iterate). But that can be converted to a list if that's what we need:

```
>>> list(map(lambda x: x**2,range(10)))
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81]
```

Here's an example of a function that, besides having a function as an argument, returns a function:

```
>>> def compose(f1,f2):
...     return lambda x: (f1(f2(x)))
...
>>> compose(lambda x : x+1,lambda x: 2*x)(4)
9
```

Exercise 5.4. Write a function `self_compose` of two arguments such that

```
self_compose(fun, n)
```

where `fun` is a function of one argument and `n` is a natural number, returns `fun` composed with itself `n` times. In particular, `self_compose(fun, 1)` should return `fun` itself, `self_compose(fun, 2)` the composition of `fun` with itself, etc. What should `self_compose(fun, 0)` return? (If you don't know, just assume that the second argument is always positive.)

6. Modules

Some lesser used parts of Python, and all "third party" provided functionalities are not loaded by default. They are collected in *modules* that can be `import`ed in different ways.

(1) `import math` This imports everything in the module `math`; you can access them by prefixing their name by `math.` For example, `math.sqrt()`.
(2) `import math as pd` The same as before, but using `pd` as an *alias*. This just means that the function `sqrt()` of the module `math` is now accessible as `pd.sqrt()`.

(3) `from math import sqrt` This will not import the whole of `math`, just `sqrt`, but make it accessible without qualification, that is, by simply writing `sqrt`. You can import a list of functions (and classes, etc.) by listing them separated by commas. For example,
`from math import sqrt, isqrt`

To learn the details of a module (what it's good for, what functions, classes, variables, etc. it provides), enter

```
>>> help("math")
```

at the command prompt (or `math?` in IPython). And if you're only interested in one function, say, `isqrt()`, of the module, enter

```
Python 3.9.12 (main, Mar 25 2022, 00:00:00)
[GCC 11.2.1 20220127 (Red Hat 11.2.1-9)] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> help("math.isqrt")
Help on built-in function isqrt in math:

math.isqrt = isqrt(n, /)
    Return the integer part of the square root of the input.
```

(Or `math.isqrt?` in IPython.)

Of course, we can write and use our own modules, too. To create a module named `foo`, we need to create a file named `foo.py` and write the definitions of the functions, classes and whatever else we want to have in the module. If there's a problem importing it, check and perhaps change the list contained in the variable `path` in the `sys` module.

7. Classes

7.1. **Local state.** Earlier we've said that a function manipulating a global variable was not a good idea. One reason is that a global variable can be seen and be changed by every part of the program, including those which (should) have nothing to do with it. For example, suppose that we have write that part of a program for a bank which deals with customer's accounts. For simplicity, assume that we're only interested in the balance of an account, not for example its history (past transactions, etc.), and that all we need to do is to provide a way to report the balance and to change it.

One solution is to set up a dictionary named `accounts` where the keys are the customer's names and the values are their respective balances. If there is a new customer, we just need to add him/her to this dictionary. For reporting we do lookup, and for change, lookup and update.

```
Python 3.9.12 (main, Mar 25 2022, 00:00:00)
[GCC 11.2.1 20220127 (Red Hat 11.2.1-9)] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> accounts = {}

>>> def make_account(name,balance):
...     accounts[name]=balance
...
>>> def deposit(name,amount):
...     accounts[name] += amount
...
```

```
>>> def withdraw(name,amount):
...     accounts[name] -= amount
...
>>> def report(name):
...     print(f'{name} has ${accounts[name]} on his/her account.')
...
>>> make_account('Alice',100); make_account('Bob',200);
>>> report('Alice'); report('Bob')
Alice has $100 on his/her account.
Bob has $200 on his/her account.
>>> deposit('Alice',50); withdraw('Bob',20); report('Alice'); report('Bob')
Alice has $150 on his/her account.
Bob has $180 on his/her account.
```

This seems to work well enough. (As long as we ignore the problem of having different customers with the same name. But alleviating this is not a programming task, that is, it's not solved by checking in `make_account` if there is already a customer with `name`, but by not using names as keys in the first place.) And there could be other modules of the program which use `accounts`, say, for creating annual reports, or sending letters to customers based on their balances. (Of course, we would kindly ask these programmers not to change `accounts`.) But what if one day the bank decides that balances should be stored in cents (or bitcoins) instead of dollars? How can we make sure that all parts of a million line software that deals with accounts is updated accordingly? How can we be sure that there hasn't remained any that is not?

One solution is to not let others mess with `accounts` other than through the means we provide (`deposit`, `withdraw` and `report` as things stand now). (They can ask us to give them different means.) If this protocol is observed, then when a change like the above happens, all there is to do is for us to update our part.

But if noone else is supposed to touch `accounts` directly, why should they even see it? It should be private to `deposit`, `withdraw` and `report` (and possibly other functions that we will write later). This is where `class`es come into the picture, because their instances are exactly what is needed here: a bunch of data ("attributes") and functions ("methods") operating on them. We make an extra step here and dispense with `accounts`, our central repository of accounts, altogether.

```
class Account:
    def __init__(self,name,balance):
        self._balance = balance
        self._name = name

    def deposit(self,amount):
        self._balance += amount

    def withdraw(self,amount):
        self._balance -= amount

    def report(self):
        print(f'{self._name} has ${self._balance} on his/her account.')
>>> a = Account('Alice', 100) ; b = Account('Bob',200)
>>> a.report() ; b.report()
```

```
Alice has $100 on his/her account.
Bob has $200 on his/her account.
>>> a.deposit(50) ; b.withdraw(20); a.report() ; b.report()
Alice has $150 on his/her account.
Bob has $180 on his/her account.
```

There's a lot that is new here.

(1) The definition of a class resembles the definition of a function: it's a block, the first line of which begins with a keyword (`class` in this case) that is followed by the name of the class and then a colon. Between the name of the class and the colon there may be a comma separated list of the names of other classes between parentheses. We will see examples of this later.
(2) In the body, the `def`s look exactly like function definitions, but these define *methods*, not functions.
(3) The first argument (its name doesn't matter, but it's customary to call is `self`) of a method will be bound to the instance of the class on which the method is invoked. That is, if `a` is an instance of `Account`, `a.report()` calls `.report()` with `self` bound to `a`.
(4) The "magic" `.__init__()` method is run when an instance of `Account` (from now on: an `Account`) is created by calling `Account()`. The newly created instance will be bound to the first argument of `.__init__()` and the arguments to `Account` to the rest.
(5) `_name` and `_balance` are *attributes* of the class. They hold instance-specific data. In our case, the name and the balance of the account holder. The underscore signals that even though they can, they shouldn't be accessed (read or set) directly.
```
>>> a._name, a._balance
('Alice', 150)
```

Local state by higher order functions⋆. It turns out that the machinery of classes is not strictly necessary for solving the accounts problem. We can give a solution in the same spirit using ordinary functions.

```
def make_account(name, balance):
    balance = balance
    name = name

    def deposit(amount):
        nonlocal balance
        balance += amount

    def withdraw(amount):
        nonlocal balance
        balance -= amount

    def report():
        print(f'{name} has ${balance} on his/her account.')

    return deposit,withdraw,report
```

```
>>> a_deposit, a_withdraw, a_report = make_account('Alice',100)
>>> b_deposit, b_withdraw, b_report = make_account('Bob',200)
>>> a_report() ; b_report()
Alice has $100 on his/her account.
Bob has $200 on his/her account.
>>> a_deposit(50) ; b_withdraw(20) ; a_report() ; b_report()
Alice has $150 on his/her account.
Bob has $180 on his/her account.
```

There is just one novelty here: the `nonlocal balance` declaration. `nonlocal` is similar to `global`, except that while the latter really means that the variable is not local, the former only says that it's not local in `deposit` and `withdraw`. But it *is* local in `make_account()`.

7.2. **Inheritance.** Suppose that some customers of the bank want to keep their money in the currency of the country they earned it in, not in US dollars.

One thing we could do is to create a new class `CurrencyAccount` by copy/paste-ing the definition of `Account` and then changing the definition of `.__init__()` and the `.report()` methods, since we want to specify the kind of currency when we create the account, and the currency should also be reflected when reporting it. But there are two problems with this. First, instances of `CurrencyAccount` would not be instances of `Account`, even though they are accounts, too. And, second, if something needs be changed in all accounts (for example, the way to deal with overdraft) we would have to revise the definition of both `Account` and of `CurrencyAccount`. (This can get really hard if there are hundreds of different kinds of accounts.)

So what we do instead is create `CurrencyAccount` as a specialized `Account`. The official terminology is *subclass* or *derived class*; and `Account` is a *superclass* or *base class* of `CurrencyAccount`. A subclass, such as `CurrencyAccount`, will *inherit* everything from its superclass, except what is *overridden* in its definition, which, in `CurrencyAccount`'s case, are the `.__init__()` and the `.report()` methods.

```
class CurrencyAccount(Account):

    def __init__(self,name,amount,currency):
        super().__init__(name, amount)
        self._currency = currency

    def report(self):
        print(f'{self._name} has \
{self._currency} {self._balance} on his/her account.')
```

Before trying to understand how this works, let's create an account and see if it does.

```
>>> c = CurrencyAccount('Cecil',100,'EUR')
>>> c.report()
Cecil has EUR 100 on his/her account.
>>> c.deposit(10) ; c.report()
Cecil has EUR 110 on his/her account.
>>> isinstance(a, Account), isinstance(a, CurrencyAccount)
(True, False)
>>> isinstance(c, Account), isinstance(c, CurrencyAccount)
```

```
(True, True)
>>> #so Cecil's CurrencyAccount is an Account, too
```

The first line of the definition declares the base class or classes of the new class. (If there are more, they are separated by commas.) The definition of the `.report()` method completely overrides the definition given by the base class. With `.__init__()`, the situation is similar, in that it overrides `Account`'s `.__init__()`. The difference is that it uses it. And the key to achieve this is the function `super()`, which returns a reference to the superclass part of the object; so

```
super().__init__(name, amount)
```

initializes an `Account`. Once that is done, we do the rest, the `CurrencyAccount`-specific part of the work.

⋮

Class variables and methods. Next, suppose that our boss asks us to create a new kind of account, one that is still an `Account`, but works in a slightly different way: if you deposit money, the bank will give you 5% premium. He intends to open such accounts for his friends.

Now we know that this new kind of account, `FriendAccount`, should be a subclass of `Account`, and in this case only the `.deposit()` method should be overridden.

```
class FriendAccount(Account):

    def deposit(self,amount):
        self._balance += amount if amount <=0 else amount * 1.05
```

Let's try it!

```
>>> d = FriendAccount('Diana',100)
>>> d.withdraw(10); d.report()
Diana has $90 on his/her account.
>>> d.deposit(10); d.report()
Diana has $100.5 on his/her account.
```

Now suppose the boss says he wants to change sometimes the "premium" his favourite customers get for every deposit. Each of them gets the same amount, but not necessarily 5%. What we need is an attribute `premium` that is common to all instances of `FriendAccount` (a *class variable*), so that when the boss decides to change it, there's just one place where it needs to be changed.

```
class FriendAccount(Account):

    premium = 1.05

    def deposit(self,amount):
        self._balance += amount * FriendAccount.premium
```

The reason `premium` is a class variable is that it is not set in `.__init__()`.

```
>>> d = FriendAccount('Diana',100)
>>> d.deposit(10) ; d.report()
Diana has $110.5 on his/her account.
>>> FriendAccount.premium=2.0
>>> d.deposit(10) ; d.report()
Diana has $130.5 on his/her account.
```

```
>>> vars(d)
{'_balance': 130.5, '_name': 'Diana'}
```

It's not only attributes, but also methods, that can belong to the class itself (as opposed to its intances). This of course means that they don't have access to instance variables (attributes), simply because there's no instance involved when they are called. Accordingly, their first argument is the class itself, and not some instance. But they can access class variables (as can normal methods), as in the following version of `FriendAccount`.

```
class FriendAccount(Account):

    _premium = 1.05

    @classmethod
    def set_premium(cls,new_premium):
        cls._premium = new_premium

    def deposit(self,amount):
        self._balance += amount * FriendAccount._premium
```

With methods, it's `@classmethod`'s job to indicate that the next method is a class method. A stylistic change introduced here is renaming `premium` to `_premium` which signals (to the reader of the code, not to Python) that it shouldn't be accessed directly. Now that there is a (class) method with which to adjust it, it's not necessary anymore.

```
>>> d = FriendAccount('Diana',100)
>>> FriendAccount.set_premium(2.0)
>>> d.deposit(10)
>>> d.report()
Diana has $120.0 on his/her account.
```

Exercise 7.1. Write a subclass of `Account` that keeps track of the number of its instances. That is, it should have a class variable `no_of_accounts`, initially 0, that is increased every time a new instance of it is created.

We could go a step further and extend `Account` in such a way that it keeps a list of all such accounts.

```
class CollectedAccount(Account):

    all_accounts = []

    def __init__(self,name,balance):
        super().__init__(name,balance)
        CollectedAccount.all_accounts.append(self)
```

Let's see how useful this is:

```
>>> e = CollectedAccount('Emily',1000)
>>> f = CollectedAccount('Freddy',200)
>>> CollectedAccount.all_accounts
[<__console__.CollectedAccount object at 0x7f2624a0bfd0>, <__console__.CollectedAccount objec
```

At first, it doesn't seem to be. But it is, because we can further process this list:

```
>>> for acc in CollectedAccount.all_accounts:
...     acc.report()
...
Emily has $1000 on his/her account.
Freddy has $200 on his/her account.
```

Nevertheless, at least for debugging purposes, it'd be nice if a `CollectedAccount` appeared in a more readable way. This can be arranged by defining a `.__repr__()` method of one argument, `self`.

```
class CollectedAccount(Account):

    all_accounts = []

    def __init__(self,name,balance):
        super().__init__(name,balance)
        CollectedAccount.all_accounts.append(self)

    def __repr__(self):
        return f"{self._name}'s account"
```

And now even without further processing we can see some useful information.

```
>>> e = CollectedAccount('Emily',1000)
>>> f = CollectedAccount('Freddy',200)
>>> CollectedAccount.all_accounts
[Emily's account, Freddy's account]
```

For this reason it's always a good idea to define a `.__repr__()` method for our classes.

7.3. **Composition.** Inheritance is not always the best way to reuse a class. For example, suppose we want a class `Queue` that has two methods, `.queue()`, which puts its argument in the queue, and `.dequeue()`, which retuns and removes from the queue the object that was put in earliest. So a queue is like a queue at the cashier: you join the rear end of the queue, and the first one to have joined it is the one who can pay and leave first. Or a pipe in which you're allowed to put in balls at one end and get them out at the other.

It makes sense to put the items in a list when implementing queues, since we need to keep them in order. But this doesn't mean that `Queue` should be a subclass of `list`. `Queue` is not a `list` with extra capabilities. `Queue` just happens to *have* a `list` as its internal storage.

```
class Queue:

    def __init__(self):
        self.storage = []

    def __repr__(self):
        return str(self.storage)

    def queue(self,value):
        self.storage.append(value)

    def dequeue(self):
        if self.storage == []:
            return None
        value = self.storage[0]
        self.storage = self.storage[1:]
        return value
>>> q = Queue()
>>> q.dequeue()
>>> q.queue(13) ; q.queue(13) ; q.queue(23)
>>> q.dequeue()
13
>>> q.dequeue()
13
>>> q.dequeue()
23
>>> q.dequeue()
```

Exercise 7.2. Modify the definition of `Queue` (or extend it) by adding two methods: `.peek()` and `.is_empty()`. Both accept zero argument; the first return what `dequeue()` would, withot removing it from the queue, the second returns `True` if the queue is empty and `False` otherwise.

Exercise 7.3. Perhaps using your `lookup()` function from Exercise **??** as a starting point, define a class `MyDict` that has similar functionality to Python's built in `dict` type. It should have a method `.put()` so that `md.put(key, val)` stores `val` under the `key` in `md` if `md` is a `MyDict` (and overwrite any possible earlier associations to `key`), and a method `.get()` such that `md.get(key)` returns that value associated to `key` if there is one, `None` otherwise.

`MyDict` should use a list of pairs as its storage.

```
>>> md = MyDict()
>>> md.put(12,'twelve')
>>> md.put('twelve',12)
>>> md.get(13)
>>> print(md.get(13))
None
>>> md.put(13,'thirten')
>>> md.put(3,'three')
>>> md.get(13)
'thirten'
>>> md.put(13,'thirteen')
>>> md.get(13)
'thirteen'
>>> md
[(12, 'twelve'), ('twelve', 12), (13, 'thirten'), (3, 'three'), (13, 'thirteen')]
```

7.4. **Multiple "constructors".** Suppose we want a datatype for computing with matrices. We could easily represent matrices by nested lists: for example, by the list of rows, where a row is represented by the list of its members. So `[[1,0,0],[0,1,0],[0,0,1]]` would represent the 3×3 identity matrix. If `m` is such a matrix, we could get or set the jth member of its ith row by `m[i][j]` and `m[i][j] = v`.

This works, but the lack of abstraction (we need to deal with nested lists instead of matrices) leads to all kinds of difficulties, the most important being that if we ever come up with a better representation, we need to change all our code that deals with matrices. For example, we may find out later that we need to deal with sparse matrices (matrices where almost all elements are the same): representing them by nested lists is a waste of space.

Defining functions that access or change our matrices goes a long way towards ameliorating this problem, for if we only deal with matrices through these accessors, only these will need to be changed. We'll still need to deal with the problem of special (for example, sparse) matrices, though. Having for example a `product_sparse()` function besides the normal `product()` is not very elegant (both do multiplication, after all), so we would probably end up redefining `product()` to make case distinction based on the type of its arguments.

At this point we'd better cave in and define a class.

```
class Mtx():

    def __init__(self, list):
        nc = len(list[0])
        #rows must be of equal length
        assert all([len(l) == nc for l in list[1:]]), 'dimension mismatch'
        self._list = list
        self._no_of_rows = len(list)
        self._no_of_columns = nc

    def no_of_rows(self):
        return self._no_of_rows

    def no_of_cols(self):
        return self._no_of_columns

    def get_row(self,rn):
        return self._list[rn]

    def get_col(self,cn):
        return [l[cn] for l in self._list]

    def get(self,r,c):
        return self._list[r][c]

    def set(self,r,c,value):
        self._list[r][c] = value

    def __repr__(self):
        return f'{self._no_of_rows} times {self._no_of_columns} Mtx: {self._list}'

    def __str__(self):
        s = ""
        for i in self._list:
            s += str(i)+'\n'
        return s
```

We have met the `assert` statement in Section 4.2.1. It helps us produce meaningful error messages, such as the one below.

```
>>> m = Mtx([[1,2,3],[4,5,6],[7,8]]) #should fail
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File "<stdin>", line 5, in __init__
AssertionError: dimension mismatch

>>> m = Mtx([[1,2,3],[4,5,6],[7,8,9]])

>>> m.set(1,1,-5)

>>> m
3 times 3 Mtx: [[1, 2, 3], [4, -5, 6], [7, 8, 9]]

>>> print(m)
[1, 2, 3]
[4, -5, 6]
[7, 8, 9]
```

Now we would like to define matrix addition and multiplication (either as functions[16] or as methods). Let's concentrate on addition first: the conceptually simplest way to implement it would be to start with a brand new matrix, and setting its "coordinates" one by one before returning it. But we have only one way to construct a `Mtx`, and it is foolish to have to build a list of lists of the correct sizes just so that we can rewrite all their members. Not to mention that we'll probably want to do this when defining `product()`, too. It's much better to build this possibility into the class itself. We need a method which will return a new `Mtx` of the correct dimensions. But this is the same kind of method as `.__init__()`: we call it with some arguments and expect it to return a new `Mtx`. The problem is that there can be only one `.__init__()` method. What we can do though is to define a class method, say `.empty()` that will do the work. So we augment our class definition (and while we are at it, we define another new constructor (it's not called that officially, but that is its function), which returns an identity matrix):

```
    # The pythonic way to have multiple "constructors"
    @classmethod
    def empty(cls,no_of_rows,no_of_columns,default = None):
        return cls([no_of_columns*[default] for _ in range(no_of_rows)])

    @classmethod
    def id(cls,no_of_rows_and_columns):
        m = cls.empty(no_of_rows_and_columns,no_of_rows_and_columns,0)
        for i in range(no_of_rows_and_columns): m.set(i,i,1)
        return m
```

Let's see first what these do before we look at how they work.

[16]The problem about having to define separate functions for sparse matrices doesn't come up here, because the implementation details are hidden behind `Mtx`'s public methods.

```
>>> zero = Mtx.empty(3,3,0) ; id = Mtx.id(3)
>>> print(zero); print(id)
[0, 0, 0]
[0, 0, 0]
[0, 0, 0]

[1, 0, 0]
[0, 1, 0]
[0, 0, 1]
```

We've defined the class method `.empty()` in a slightly more general way than planned: there is an optional argument, `default`, which will be used for onitializing the members of the new `Mtx`. This comes in handy when defining `.id()`, and in the definition of `zero`, too, which, by the way, is a candidate for being another constructor. Both new constructors just prepare the right argument for, and passes it to, the real one, `.__init__()`.

Now we can return to what we wanted to do: define methods `.add()` and `.prod()` for adding and multiplying `Mtx`s.

```
    def add(self, other):
        assert (self.no_of_cols() == other.no_of_cols()
                and self.no_of_rows() == other.no_of_rows())
        nr = self.no_of_rows()
        nc = self.no_of_cols()
        m = Mtx.empty(nr,nc)
        for i in range(nr):
            for j in range(nc):
                m.set(i,j,self.get(i,j)+other.get(i,j))
        return m
```

(The only reason the first argument of `assert` is in a pair of parentheses is that otherwise Python doesn't let me break the line.)

Exercise 7.4. Define `.prod()`! Is should check that the dimensions of the argument fit. (And should of course only use the public methods of `Mtx`.)

Let's try these:

```
>>> print(m.add(zero))
[1, 2, 3]
[4, -5, 6]
[7, 8, 9]

>>> print(m.add(Mtx.id(4))) #should fail
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File "<stdin>", line 39, in add
  File "<stdin>", line 43, in __add__
AssertionError
>>> print(m.add(id)) ; print(m.prod(id)) ; print(m.prod(m))
[2, 2, 3]
[4, -4, 6]
[7, 8, 10]
```

```
[1, 2, 3]
[4, -5, 6]
[7, 8, 9]

[30, 16, 42]
[26, 81, 36]
[102, 46, 150]
```

Exercise 7.5.* Define `add()` using list comprehension to create the right argument to the original constructor, with no assignments.

Exercise 7.6. Define the class `MtxSq` of square matrices as a subclass of `Mtx`! (In fact, this, rather than `Mtx`, would be the natural home for the `.id()` class method.) Define the method `.determinant()` and perhaps other methods that only make sense for square matrices.

7.5. **Operator overloading.** If we're going to use our `Mtx` class interactively, we will soon get tired of typing those `add`s and `prod`s all the time. It would be much more convenient and natural to being able just do something like:

```
>>> print(m + id) ; print(m * id) ; print(m * m) ; m += id ; print(m)
[2, 2, 3]
[4, -4, 6]
[7, 8, 10]

[1, 2, 3]
[4, -5, 6]
[7, 8, 9]

[30, 16, 42]
[26, 81, 36]
[102, 46, 150]

[2, 2, 3]
[4, -4, 6]
[7, 8, 10]
```

and get the right result. It turns out that this is very easy to arrange. All we have to do is renaming `add` to `__add__` and `prod` to `__mul__`. When Python sees that the first argument of + has a method named `__add__`, it calls it instead of what it would otherwise try (and fail) to do. This technique is called *operator overloading* and it's what is at work when we "add" strings or "multiply" a list by an integer.

Exercise 7.7. Instead of addig the methods `__add__` and `__mul__` to `Mtx`, define a subclass, say `MtxOver` that has these methods, which can of course call `.add()` and `.prod()` of the superclass. Do you need to override any method of `Mtx`? Experiment!

One can go much further in this direction. Here's one final example. With an extra method named `__getitem__` we can control what is returned when a `Mtx` is indexed with the help of the indexing operator `[]`. For example, if we define `__getitem__` in the definition of `Mtx` as follows:

```
    def __getitem__(self, index):
        assert(isinstance(index,tuple))
        r, c = index
        assert (0 <= r < self._no_of_rows
                and 0 <= c < self._no_of_columns), 'index out of bound'
        return self.get(r,c)
```

then we can get the members of a matrix like this:

```
>>> m = Mtx([[1,2,3],[4,5,6],[7,8,9]]) ; m[1,2]
6
```

Of course it would be easy to arrange for indexing to be 1 instead of 0-based, as in mathematics.

Exercise 7.8. Collect all the pieces of the definition of `Mtx` in this subsection. Play with it, and extend it with new methods if you feel that something is missing. For example, you can overload `__subtract__` to be able to subtract one `Mtx` from another using `-`, and, more interestingly, `__setim__`, which lets you not just get `m[1,2]` but also set it with `m[1,2]=3`.

7.6. **Iteration.** Now that we know about classes, we can finally understand what's going on when we iterate over some "iterable", a list, for example:

```
>>> l = [1,2,3]
>>> for i in l:
...     print(i)
...
1
2
3
```

Behind the scenes, Python obtains an *iterator* for `l`, and uses the iterator's `.__next__()` method to get the next element. When there isn't any more, `.__next__()` raises a `StopIteration` exception.

```
>>> l = [1,2,3]
>>> it = iter(l)
>>> type(it)
<class 'list_iterator'>
>>> it.__next__()
1
>>> it.__next__()
2
>>> it.__next__()
3
>>> it.__next__()
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
StopIteration
```

An iterator is an object that has a `.__next__()` method that behaves as above: it returns successive elements of some stream of data, and raises a `StopIteration` exception when there's no more. And an iterable is an object for which `iter()` returns an iterator.

In the next section we'll learn how to build an iterator ourselves.

8. Generators

8.1. Generator expressions. Generator expressions resemble list comprehensions, both syntactically (the only difference is that they're enclosed in parentheses instead of brackets) and semantically: even though they produce *generators*, not lists, we can iterate over these just as we can iterate over lists.

```
Python 3.9.12 (main, Mar 25 2022, 00:00:00)
[GCC 11.2.1 20220127 (Red Hat 11.2.1-9)] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> gen = (i**2 for i in range(1,6))

>>> for x in gen: print(x)
...
1
4
9
16
25
>>> for x in gen: print(x)
...
```

Two crucial deviations from the corresponding list comprehension are the following:

- Executing the first line above doesn't actually make Python count from 1 to 5 and compute squares. It just creates something that, when iterated over, generates those squares. The point of the generator is its lazy nature. It only computes something if/when it must.
- Once it has generated all of them, it will signal to whatever is iterating over it that it is exhausted. That's why the last line above prints nothing.

We can turn the generator into a list by

```
>>> gen = (i**2 for i in range(1,6))

>>> [i for i in gen]
[1, 4, 9, 16, 25]
```

or simply by `list(gen)` but if we want a list, we should use list comprehension in the first place, and not a generator expression.

What is this laziness good for? Suppose there is a source of data (a file, or a network socket) from which we want to read lines or bytes, do something with them, say, until some condition is met (this can depend on what we've just read, or the time that has elapsed since we started reading, or just the time on the clock). For concreteness, suppose that we read lines from a file `data.txt`:

```
>>> print(os.popen("cat data.txt").read())
one
two
three
very long
four
five
```

and want to print the reverse of each line until we encounter a line longer then 5 characters or arrive at the end of the file.

We could in principle do something like this:

```
>>> with open("data.txt") as f:
...     lines = [line.strip()[::-1] for line in f]
...     for l in lines:
...        if len(l)>5: break
...        print(l)
...
eno
owt
eerht
```

That is, read and collect in a list the reverse of all lines, and then print the members of this list as long as no line that is too long is encountered.

But reading all lines that should be processed, or possibly even more than that, and then starting the processing line by line could use up all the memory we have.

Changing
`lines = [line.strip()[::-1] for line in f]`
to
`lines = (line.strip()[::-1] for line in f)`
solves this problem completely and doesn't require any other adjustments. Instead of the reversed lines, `lines` contain the "recipe" to produce them.

An additional benefit is that once we're done, we could pass `lines` to some some other function that then processes the rest of the lines in some other way.

Of course, all this, including the "don't read what you don't need" part can be done without a generator:

```
>>> with open("data.txt") as f:
...     for line in f:
...         l = line.strip()[::-1]
...         if len(l)>5: break
...         print(l)
...
eno
owt
eerht
```

But the list comprehension version is arguably more elegant, because the reading and the processing part is kept separate, and the generator version inherits this elegance while being as efficient (by not reading what's not needed) as the last snippet.

Because of the lazy nature of generators, they can not only contain (the recipe for) an arbitrarily big list, but even of an "infinite list", such as the "list" of all natural numbers or of all primes. But, unless we already have such an infinite iterable, generator expressions are not enough to produce these. We need to get to a more basic level.

8.2. **Writing generators by hand*.** "Unlike regular functions which on encountering a return statement terminate entirely, generators use a yield statement in which the state of the function is saved from the last call and can be picked up or resumed the next time we call the generator function." (source)

```
>>> def natural_numbers():
...     n = 1
...     while True:
...         print('NaNu('+str(n)+')')
...         yield n
...         n += 1
...

>>> nanus = natural_numbers()

>>> type(nanus)
<class 'generator'>
```

The `print` statement is only here to help us understand what is going on (and when!). Note that even though we've already called `natural_numbers()`, no `NaNu`s have been printed so far.

```
>>> evens = (x for x in nanus if x % 2 == 0)
>>> even_squares = (x ** 2 for x in evens)
```

Still no `NaNu`s! This is a sign of (healthy) laziness. But now we start to iterate over `even_squares`, so, implicitly over `evens` and so `nanus`:

```
>>> for i in even_squares:
...     if i > 100: break
...     print(i)
...
NaNu(1)
NaNu(2)
4
NaNu(3)
NaNu(4)
16
NaNu(5)
NaNu(6)
36
NaNu(7)
NaNu(8)
64
NaNu(9)
NaNu(10)
100
NaNu(11)
NaNu(12)
```

We can see, that behind the scenes, there is an iteration going on through *all* natural numbers, not just the squares or the even numbers. (The reason why 12 is the biggest generated number is that that is the first even number with a square bigger than 100.)

Here's another example, the "list" of primes and their squares, but first we redefine `natural_numbers` to not print anything:

```
>>> def natural_numbers():
...     n = 1
...     while True:
...         yield n
...         n += 1
...
>>> nanus = natural_numbers()
```

Nothing new so far. But now we need a function which decides whether a number is a prime.

```
>>> def is_prime(n):
...     for d in range(2,n):
...         if n%d==0: return False
...         if d**2 > n: return True
...     return n > 1 # n==2
...
```

A quick check that it does what it's supposed to do:

```
>>> [i for i in range(50) if is_prime(i)]
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47]
```

And now we're ready to generate all primes:

```
>>> primes = (n for n in nanus if is_prime(n))

>>> prime_squares = (n**2 for n in primes)
>>> # prime_squares = map(lambda n : n**2,primes) would work, too

>>> for i in primes:
...     if i > 100: break
...     print(i)
...
2
3
5
7
11
13
17
19
23
29
31
37
41
43
47
53
59
61
67
71
73
```

```
79
83
89
97
>>> for i in prime_squares:
...     if i > 20000: break
...     print(i)
...
10609
11449
11881
12769
16129
17161
18769
19321
```

The reason why the last output starts this late (at $10609 = 103^2$) is that we've already consumed the first few `primes`, and with them, their squares, too. Actually, by consuming the first few `primes`, we've consumed the first few `nanus`, too.

It is possible, but not a good idea to ask whether something is in an infinite "list". For if it is not, we get into an infinite loop. And even if it is, it won't be, after we've asked:

```
>>> nanus = natural_numbers()
>>> 42 in nanus
True
>>> #42 in nanus # would never terminate
```

Appendix A. Python from scratch

A.1. **Python as a calculator.** If you start the Python interpreter `python3`, or, preferably, `ipython3` from a terminal, you get a prompt, such as this:

```
>>>
```

or this:

```
In [84]:
```

Here you can type Python commands and the interpreter will execute (or evaluate) them and return the results (if any) of this. Exactly as in Sage, so most of you should have some experience with it. For example:

```
Python 3.9.10 (main, Jan 17 2022, 00:00:00)
[GCC 11.2.1 20210728 (Red Hat 11.2.1-1)] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> 2+3
5
```

or

```
>>> 2**3
8
```

or

```
>>> 2**3 == 8
True
>>> (123**13) % 13 == 123 % 13  #because of Fermat's little theorem
True
```

(Whatever is written after an `#` on a line is ignored by Python; it's a *comment* for the human reader.)

It's not just integers (`int`s) that we can work with, but various other types of data, too. For example:

```
>>> 3.14 #a floating point number (float)
3.14

>>> "This is a string" #a string
'This is a string'

>>> [1, 2, "Hello", 3.5] #a list (of ints, a string, and a float)
[1, 2, 'Hello', 3.5]

>>> [1, 2, "Hello", 3.5][2] #the 2nd member of the list (indexing starts at 0)
'Hello'
```

We can also store values (that you type in or get as a result) in *variable*s. (The usual terminology is *assigning a value to* a variable.) You can think of a variable as a box with a name into which you put the value. (That name should only contain letters of the alphabet, lower case, in general, and the underscore (_) character.)

```
>>> my_first_var = 2**3
```

From now on, `my_first_var` will contain 8 until we change it (or quit the Python interpreter):

```
>>> my_first_var
8

>>> 2 * my_first_var
16
```

and I can write `my_first_var` whenever I mean 8.

A.2. **Python as a programming language.** Why not write 8 directly? There are quite a few reasons (for example, we don't want to write `[1, 2, "Hello", 3.5]` every time we need it) but the most important by far is that Python is not just a calculator. Typically, I want to do something (say `Action`) with lots of different values, and the way to achieve that is to do what I want to do with the variable, and change it to the different values, instead of doing `Action` with each of the literal values. For example, one such action is `print`ing. If I want to see the square of a few numbers, I can do this:

```
>>> print(1**2)
1

>>> print(2**2)
4

>>> print(3**2)
9

>>> print(4**2)
16
```

but the following is much more practical:

```
>>> number = 1
>>> while (number < 5):
...     print(number ** 2)
...     number = number + 1
...
1
4
9
16
```

Before trying to understand how this worked, let's see why this is much better. The main reason is that we only had write how to do `Action` *once* (on the third line). The rest is just specifying for which values of the variable `number` we want to do it. One benefit is that if we decide to print more squares, we just have to change 4 to a bigger number. Or if we want to print only every third square, we only have to change the last line:

```
>>> number = 1
>>> while (number < 15):
...     print(number ** 2)
...     number = number + 3
...
1
16
```

```
49
100
169
```

This was an example of a *loop*, more specifically a `while` loop. Loops are what make variables not only useful but indispensable. (And loops are one of the two components that turn a calculator into a programming language.)

Think about how this could've been achieved with our individual `print`s! And it's not just a matter of convenience. There are ways for a program to get input from a user (see § 2!). Now what if `15` above was not a constant, but the result of a user input? How could we modify our list of `print`s to print the amount of squares the user wishes us to print?

The syntax of a while loop is this:

```
while condition:
   do_this
   ...
   do_that
```

This executes repeatedly everything in its body (the indented block[17], `do_this`,..., `do_that`) as long as *condition* holds, and then control goes to the part of the program (if any) the follows the body. Python knows where that is, because it is indented at most as far as the `while` keyword itself. For example,

```
number = 1
while (number < 15):
    print(number ** 2)
    number = number + 3
print("Done")
```

```
1
16
49
100
169
Done
```

Remark A.1. There is a huge difference between

```
>>> 42
42
```

and

```
>>> print(42)
42
```

The first returns a value, the second only prints one: `print()` is a built-in function, but one that is called only for its *side-effect*, namely printing its argument(s) on the console, not for its value (which is `None`). Compare this:

```
>>> a = 42
>>> print(a)
42
```

with this:

[17] Those `...`s at the beginning of each line are not part of the definition. They're the prompts of the command line interface (just like >>>) that show that it expects more lines.

```
>>> a = print(42)
42
>>> print(a)
None
```

In other words: printing is for giving information to a user (a.k.a. a human) only.

Back to our looping example. There is another, often more convenient way of looping: the `for` loop. Here's how our first `while` loop above:

```
>>> number = 1
>>> while (number < 5):
...     print(number ** 2)
...     number = number + 1
...
1
4
9
16
```

can be written with a `for` loop:

```
>>> for number in range(1,5):
...     print(number ** 2)
...
1
4
9
16
```

The nice thing about the `for` loop is that it can iterate over not just a range of numbers, but almost anything for which this make sense (these things are called *iterables*): the characters of a string, the lines of a file, records of a database table,…and, perhaps most importantly, the elements of a list. For example, here is one way to compute the product of the elements of a list:

```
>>> #I store the list in a variable, because I want to use it later.
>>> l = [4,2,5,9]
>>> product = 1

>>> for n in l:
...     product = product * n
...
>>> product
360
```

We could've done this with a `while` loop, too:

```
>>> product = 1
>>> index = 0
>>> while index < len(l):
...     product = product * l[index]
...     index = index + 1
...
>>> product
360
```

but it's *much* less elegant.

One technical detail about both kinds of loops: we can jump out of a loop early with the `break` statement, and also go immediately to the next iteration with `continue`. But to be able to show any meaningful examples of these, we need the other construction that turns a calculator into a full-blown programming langue: the *if* statement.

```
if condition:
    do_this_if
    ...
    do_that_if
```

which executes everything in its body (`do_this_if`,...,`do_that_if`) but only if *condition* holds; and the extenden version:

```
if condition:
    do_this_if
    ...
    do_that_if
else:
    do_this_if_not
    ...
    do_that_if_not
```

which executes the body of the `else` clause if the condition doesn't hold. For example:

```
>>> if 2<3:
...     print("OK")
...
OK
>>> if 3<2:
...     print("OK")
...
>>> if 3<2:
...     print("OK")
... else:
...     print("Not OK")
...
Not OK
```

Now that we have the `if` statement, we can illustrate what `break` and `continue` does.

Suppose that we only want to compute the product of the odd elements of a list. This is one way to achieve this:

```
>>> l = [4,2,5,9]
>>> product = 1

>>> for n in l:
...     if n%2 == 0:
...         continue
...     product = product * n
...
```

```
>>> product
45
```

Exercise A.1. Could we change the loop to achieve the same effect without using `continue`?

What if we only want to take the product until we encounter an odd number? Here's where `break` helps:

```
>>> l = [4,2,5,9]
>>> product = 1

>>> for n in l:
...     if n%2 == 1:
...         break
...     product = product * n
...
>>> product
8
```

See the section "Approximating the limit of Leibniz series" in the Sage lecture notes[18] for a slightly more realistic example of the use of `break` both in `for` and `while` loops!

The last important building block that we need is the ability to define (and of course call) functions. This is not *absolutely* necessary, but would be very hard to live without.

What problems are they supposed to solve? A few examples ago we computed the product of the members of a list. If we had to do that for one list, it's more than likely that we'll want to do that again with other lists of numbers. So what we do is "abstract away" the concrete list from that program, and give the whole thing a name (`product` seems like a good choice). Here's the result:

```
>>> def product(l):
...     result = 1
...     for n in l:
...         result = result * n
...     return result
...
```

which can be used (*called*) like this:

```
>>> product([4,2,5,9])
360

>>> product([4,2,5,9,10])
3600
```

The first line of the definition says that the name of the function being defined is `product` and its only parameter (or argument) is `l`. This means that this function has to be called with one argument, which will be assigned to the variable (more specifically, the parameter) `l`, which can be used in the *body* of the function. It is a *local variable*, which means that even if we have a variable of the same name outside the function definition, its values will be restored when the function returns. The same is true for `result` defined on the second line. Here's an example that shows this:

[18]"Leibniz-sor összegének közelítése" in the Hungarian version

```
>>> a = 1
>>> b = 2
>>> def fun(a):
...     b = a
...     return b
...
>>> fun(42)
42
>>> a
1
>>> b
2
```

Why is defining `product()` as a function better than copying our old code that computed the product of a list with the list replaced by a new one every time we need it? Apart form the obvious reason (that code is just a few lines, but what if we're talking about another, which is a few thousand lines?), there is a decisive one: if we find out that there is a bug in our implementation of `product()`, we only need to correct it in one place, the definition of the function.

Here's the general syntax of a function definition:

```
def name(parameter1,parameter2,...):
    do_this
    ...
    do_that
```

There might be one or more

```
    return a_value
```

statements in the body (the indented part) of the function. What it does is make the function to return immediately, and give back (return) the

a_value

to the caller. The function can return even without a `return` statement, but it will then return the value `None` which is as good as returning nothing. We have already seen that `print()` does this. There are lots of other examples where a function doesn't return anything but it's still useful. For example, it might write to a file (or delete all our files), make a phone call, etc. Here's an example of a function that doesn't return anything:

```
>>> def show(arg):
...     print(arg, "is of", type(arg))
...     if isinstance(arg,list) or isinstance(arg,str):
...         print("It is of length", len(arg))
...     else:
...         if isinstance(arg,int):
...             print("It is", "odd." if arg%2 == 1 else "even.")
...         else:
...             print("I can't tell you more about it.")
...
>>> show(['Hell', 'o', 12])
['Hell', 'o', 12] is of <class 'list'>
It is of length 3
>>> show('Hello')
```

```
Hello is of <class 'str'>
It is of length 5
>>> show(3)
3 is of <class 'int'>
It is odd.
>>> show(3.14)
3.14 is of <class 'float'>
I can't tell you more about it.
```

Here we used the `if` *expression* (as opposed to the `if` *statement*), which returns its first argument (the one before the keyword `if`) if its second argument (the one between `if` and `else`) is `True`, and its third argument (the one after `else`) otherwise:

```
>>> 'Yes' if True else 'No'
'Yes'

>>> 'Yes' if False else 'No'
'No'
```

There's a special kind of function, called *lambda*, or anonymous function, for those cases when we need a function only once. This is an atypical example:

```
>>> (lambda x : x ** 2)(3)
9
```

and here are two typical ones:

```
>>> list(map(lambda x : x ** 2, range(10)))
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81]
>>> list(filter(lambda x : x % 2 == 0, range(10)))
[0, 2, 4, 6, 8]

>>> from functools import reduce
>>> reduce(lambda x, y: x*y, range(1,11))
3628800
```

A `lambda`'s body can only have one expression in it, and it returns its value (there's no need for the `return` statement).

A.3. **A few more things we learned last year.**

Slices. We have seen that lists can be indexed with the operator `[]`:

```
>>> numbers = list(range(18))
>>> numbers
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]
>>> numbers[10]
10
```

But there's more to `[]` than that. One can can extract not just individual members, but various slices of a list:

```
>>> numbers[:10]   #the first 10 members
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
>>> numbers[10:15]   #from the 10th up to (but not including) the 15th member
[10, 11, 12, 13, 14]
>>> numbers[15:]   #everything from the 15th member
[15, 16, 17]
```

```
>>> numbers[::2]  #only the ones with even indices
[0, 2, 4, 6, 8, 10, 12, 14, 16]
>>> numbers[1::2]  #only the ones with odd indices
[1, 3, 5, 7, 9, 11, 13, 15, 17]
```

When an index `i` is negative, `len() - i` is used instead (so we count from the end, but this backward indexing starts at 1, not 0).

```
>>> numbers[-10]  #the 10th member counting from the end
8
>>> numbers[-10:]  #the last 10 members
[8, 9, 10, 11, 12, 13, 14, 15, 16, 17]
>>> numbers[-10:-3]  #the last 10 members except for the last 3
[8, 9, 10, 11, 12, 13, 14]
>>> numbers[:-3][-7:]  #the same: forget the last 3 and then take the last 7
[8, 9, 10, 11, 12, 13, 14]
>>> numbers[-10:][:7]  #the same: take the last 10 and then take its first 7
[8, 9, 10, 11, 12, 13, 14]
>>> numbers[::-1]  #all of them, backwards
[17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0]
>>> numbers[::-2]  #every second of them, backwards
[17, 15, 13, 11, 9, 7, 5, 3, 1]
```

All these tricks work for strings, too. For example:

```
>>> s = 'abcdefgh'
>>> s[3]
'd'
>>> s[-3]
'f'
>>> s[5:]
'fgh'
>>> s[5::-1]
'fedcba'
```

But there is one feature of the indexing and slicing operators that don't apply to strings (because strings are *immutable*): they can be used for assignment, too:

```
>>> numbers
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]
>>> numbers[0]=-10
>>> numbers
[-10, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]
>>> numbers[1:4] = [11,22,33]
>>> numbers
[-10, 11, 22, 33, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]
>>> numbers[-1:-4:-1] = [111,222,333]
>>> numbers
[-10, 11, 22, 33, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 333, 222, 111]
```

Exercise A.2. What's the difference between `numbers[1] = [True]` and `numbers[1:2] = [True]`? Is one of these equivalent to `numbers[1] = True`?

List comprehension. Given a list (or in fact any iterable, such as a `range`) `l`, `[f(x) for x in l]` returns a list whose *i*th member is the result of `f` applied to the *i*th member of `l`. So for example

```
>>> [x/2 for x in range(10)]
[0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5]
```

We can filter `l` if we want:

```
>>> [x/2 for x in range(10) if x%2 == 0]
[0.0, 1.0, 2.0, 3.0, 4.0]
```

List comprehensions can of course be nested:

```
>>> [[str(i)+j for j in 'abc'] for i in range(3)]
[['0a', '0b', '0c'], ['1a', '1b', '1c'], ['2a', '2b', '2c']]
```

Here we use the `str()` function, which returns a string representation of its argument, and the fact that the sum of two strings is their concatenation.

Variable unpacking (basic version). Besides lists, there is another data structure, `tuple`, that can hold objects in a sequential order. If you write

```
>>> 2+3, 2*3, 2**3
(5, 6, 8)
```

the result is not three separate object, it's one `tuple`. Here's proof of that:

```
>>> a = 2+3, 2*3, 2**3
>>> a
(5, 6, 8)
>>> type(a)
<class 'tuple'>
```

But it's easy to unpack a tuple into it's components:

```
>>> a, b = 2+3, 2*3
>>> a
5
>>> b
6
```

A useful consequence of this is that instead of writing

```
>>> a = 5
>>> b = 10
```

one can write

```
>>> a, b = 5, 10
```

The two are not completely equivalent, because in the second case, the two assignments happen in parallel. But that means, that we can exchange the values of two variables like this:

```
>>> a, b
(5, 10)
>>> a, b = b, a
>>> a, b
(10, 5)
```

instead of having to use a temporary variable, as in more primitive languages:

```
>>> temp = a ; a = b; b = temp
>>> a, b
(5, 10)
```

To save space, here I used the fact that it is possible to write more than one statement on a line, separated by semicolons. They will be executed sequentially. It's not something one does in a program, but it's occasionally useful when one is using Python interactively.

Methods. We'll frequently talk about *methods* without formally introducing them (until § 7). It's safe to think that they're just like ordinary functions but called in a peculiar manner. Instead of writing `upper('abc')` to get the uppercase version of `'abc'`, we write

```
>>> 'abc'.upper()
'ABC'
```

and say that `.upper()` is a method of strings. The latter means that this call only makes sense if what's before the dot is (or evaluates to) a string. Another example is `.append()`, which is a method of lists. It appends its argument to the list it's called on:

```
>>> l = [1,2,3]
>>> l.append(100)
>>> l
[1, 2, 3, 100]
```

Note that unlike `.upper()`, it doesn't return a value: it changes the object it's called on.

A.4. **Some examples.**

Example. Define a function `repeats()` of one argument, a list of numbers, which returns `True` if two consecutive members of the list are equal, and `False` otherwise.

For example:

```
>>> repeats([])
False
>>> repeats(range(10))
False
>>> repeats([1,2,1,4])
False
>>> repeats([1,2,1,4,4])
True
>>> repeats([1,1,2,1,4])
True
```

```
def repeats(l):
    for i in range(1,len(l)):
        if l[i-1]==l[i]:
            return True
    return False
```

or

```
def repeats(l):
    if l == []: return False   #need to check because of the next line
    last = l[0]
    for i in l[1:]:
        if i==last:
            return True
        else:
            last = i
    return False
```

Example. Write a function `squares()` of two arguments, so that `squares(m,n)` returns the list of squares between *m* and *n*.

```
def squares(m,n):
    return [i**2 for i in range(n) if m <= i**2 <= n]
```

or

```
def squares(m,n):
    res = []
    i = isquare = 0

    while isquare <= n:
        if isquare >= m:
            res.append(isquare)
        i += 1
        isquare = i**2
    return res
```

or

```
import math
def squares(m,n):
    return [i**2 for i in range(math.ceil(math.sqrt(m)),1+math.isqrt(n))]
```