Hints for the solution of the 3rd assignment

It is not necessery to follow the hints in your solution, but the following ideas might help:

A. Bertrand paradox

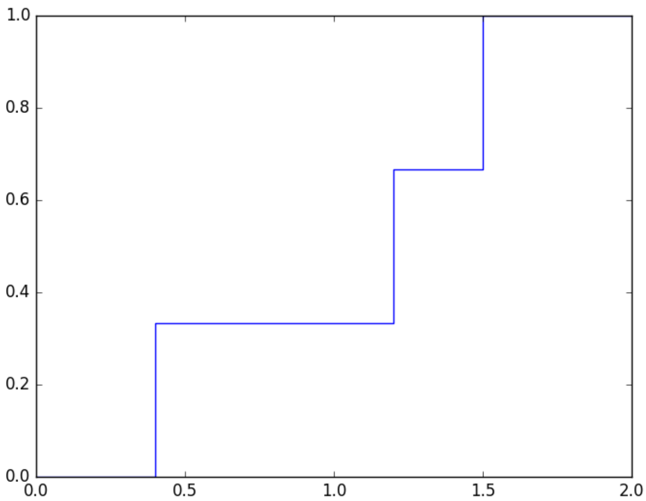
- First model: choose a random number r from the interval $[-1, 1]$, and calculate the length of the chord perpendicular to the horizontal line at $(r, 0)$ (you may use the functions `acos` and `sin` from the modul `math` modul).
- Second model: let the first point be $(1, 0)$, and the second be a random point from the line of the circle, that is the point $(\cos\alpha, \sin\alpha)$, where α is a random number from the interval $[0, 2\pi)$.
- Third model: choose an arbitrary pont from the square $[-1, 1] \times [-1, 1]$ (that is choose two random numbers from the interval $[-1, 1]$, and make a pair from them), and check if it is inside the circle. If not, choose an other one, and so on. The length of the chord can be calculated similarly to the first model.

B. Buffon's needle problem

- Note that if $L < 1$, then the probability of line-crossing is $2L/\pi$, so the relative frequency of the line-crossings approximates this value.
- Simulation can be simplified because of symmetry reasons in several ways. For example, the angle between the needle and the lines can be considered a uniformly distributed variable on the interval $[0, \pi/2]$. Similarly, the distance of its center from the nearest line is uniformly distributed on $[0, 0.5]$. One may calculate by supposing that the "left" end point of the needle is uniformly distributed on $[0, 1]$, and so on.
- If you like challenges may try to calculate the probabilities for a particular L (e.g. $L = 2$ or $L = 5$) comparing the results with the experimental values. This isn't a compulsory part of this exercise!

Plotting the experimental cumulative distribution function

The *experimental cumulative distribution function* at x will say that how much is the relative frequency of the $\{X < x\}$ event. This is a step function, which approximates the *cumulative distribution function* – defined by $F_X(x) = \mathbb{P}(X < x)$. For example, if you make $N = 3$ experiments, and the values of X are 1.5, 0.4 and 1.2 respectively, then the graph of the experimental cumulative distribution function looks like this (the graph made by the `step` function of the `matplotlib.pyplot` library; do not worry about the vertical lines in the graph, and consider the function at these places continuous from the left):



When drawing the experimental cumulative distribution function with over the interval $[0, A]$,

- After simulating N experiments, sort the numbers with the `sort()` method. Write a 0 at the beginning of the list and a 1 at the end. This list gives the points on the horizontal axis. Above these points the step function "jumps". At each point the value of the function is increasing by $1/N$. The value of the random variable X positive with probability 1 in both problems, so the list of function values is $y = [0, 0, 1/N, 2/N, 3/N, ..., 1]$.
- For the plotting use the function `matplotlib.pyplot.step(x,y)`, where x and y are the two previously constructed lists.
- After completing this program, do experiments with different values for N.