CHILD DEVELOPMENT PERSPECTIVES

An Introduction to the Approximate Number System

Darko Odic[1] and Ariel Starr[2]

[1]University of British Columbia and [2]University of California, Berkeley

ABSTRACT—***What are young children's first intuitions about numbers and what role do these play in their later understanding of mathematics? Traditionally, number has been viewed as a culturally derived breakthrough occurring relatively recently in human history that requires years of education to master. Contrary to this view, research in cognitive development indicates that our minds come equipped with a rich and flexible sense of number—the approximate number system (ANS). Recently, several major challenges have been mounted to the existence of the ANS and its value as a domain-specific system for representing number. In this article, we review five questions related to the ANS (what, who, why, where, and how) to argue that the ANS is defined by key behavioral and neural signatures, operates independently from nonnumeric dimensions such as time and space, and is used for a variety of functions (including formal mathematics) throughout life. We identify research questions that help elucidate the nature of the ANS and the role it plays in shaping children's earliest understanding of the world around them.***

KEYWORDS—*approximate number system; number; mathematics; cognitive development*

Numbers dominate children's lives, from knowing how old they are to deciding who has more toys. What intuitions, if any, do young children have about numbers, and what role do these intuitions play as children develop and learn about the world? In this article, we argue that well before they enter school or even hear a single number word, children have an intuitive, abstract, and flexible sense of number—an approximate number system (ANS). We synthesize current research by exploring five questions related to the ANS: What is the ANS? Who has access to the system? Why does it exist? Where in the brain is it localized? And how can it be used for formal mathematics? In each section, we highlight our understanding of the ANS as a phylogenetically ancient system used actively throughout life. We also identify challenges and questions that researchers are investigating in neuroscience as well as in cognitive, developmental, comparative, and computational psychology. And we argue that the ANS is a specialized, domain-specific system for representing number.

WHAT IS THE ANS?

Consider coming home from a concert and being asked to estimate how many people attended. Each of us could easily, albeit approximately, answer this question, as well as several related ones: Was it the largest concert you attended? Could another 50 people have fit inside the venue? Were there more men or women? Our estimates of this kind are necessarily imprecise, but they are possible: When presented with a visual or auditory stimulus, we automatically and efficiently extract the approximate number of items in a scene (1, 2). In turn and in coordination with other cognitive systems (as shown in Figure 1), representations of the ANS can be divided into subgroups (3), manipulated arithmetically (4), compared (5), and—with the help of language—estimated with number words like *fifty-three* (6).

The ANS is not the only way our minds represent number: Small collections of objects can be enumerated precisely, but with a firm capacity limit (1), objects can be counted precisely (6), and negative and imaginary numbers are understood conceptually without any apparent use of the ANS (2), and so forth. But when the ANS is used, it manifests itself through two behavioral signatures (see Figure 2). First, as the number of presented

Darko Odic, University of British Columbia; Ariel Starr, University of California, Berkeley.

Financial support for the research reported in this article was provided by the National Institute of Health (NIH F32HD085736) to Ariel Starr and a Natural Sciences and Engineering Research Council Discovery Grant to Darko Odic.

Correspondence concerning this article should be addressed to Darko Odic, Department of Psychology, University of British Columbia, 2136 West Mall, Vancouver, British Columbia V6T 1Z4; e-mail: darko.odic@psych.ubc.ca.

DOI: 10.1111/cdep.12288

Child Development Perspectives

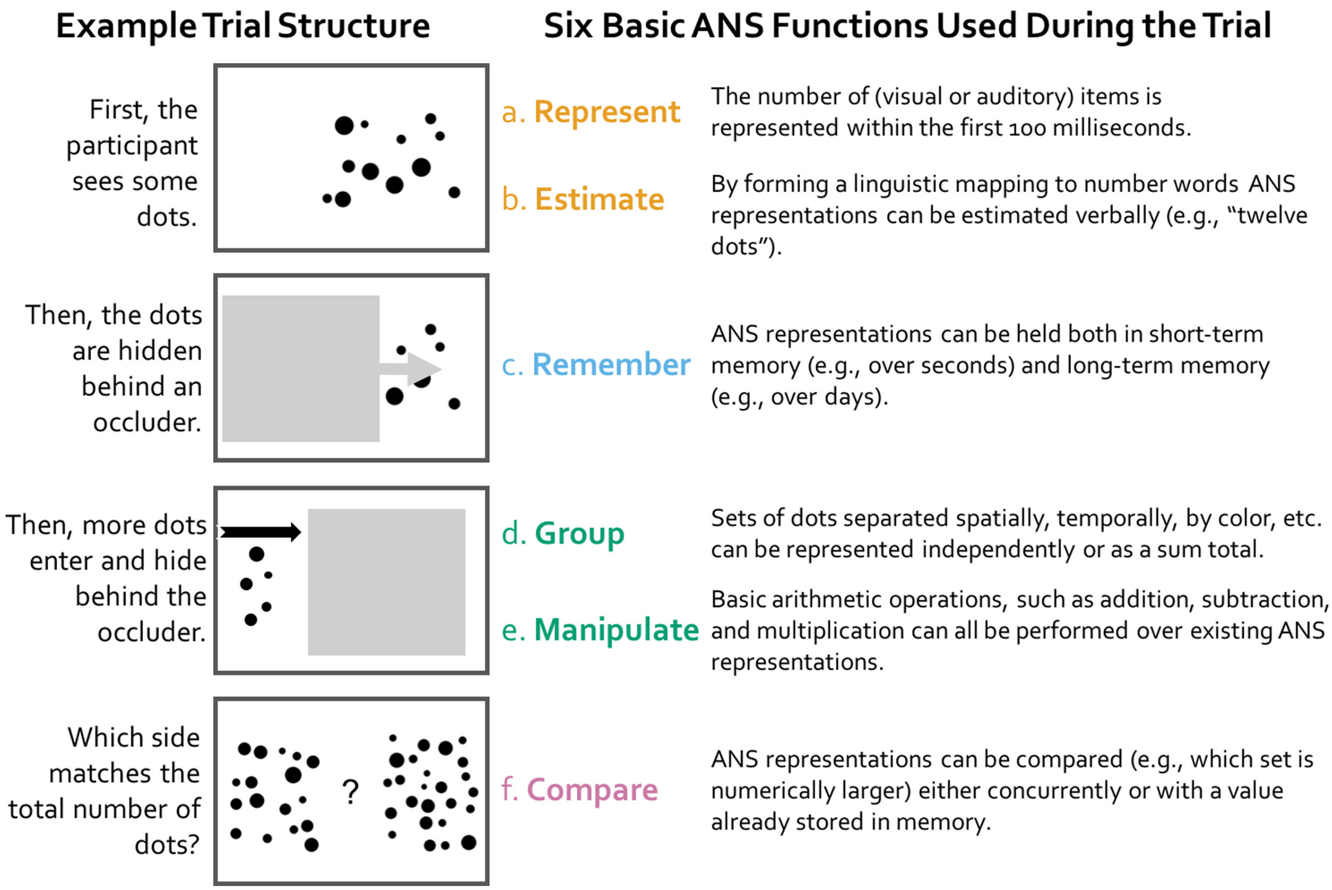

Figure 1. The six basic functions of the approximate number system (ANS), explored within a simple approximate addition task.
Note. Some functions (e.g., estimate, remember, compare) require coordination between the ANS and broader cognitive systems for language, memory, attention, and so forth. [Color figure can be viewed at wileyonlinelibrary.com]

items grows, estimates become more variable: The variability in estimating 100 items is scalar, that is, about twice as large as the variability in estimating 50 (7). Second, when deciding which of two sets is larger numerically, performance is ratio-dependent: Distinguishing 50 dots from 40 dots is significantly harder than distinguishing 50 dots from 25 dots (1). Individual and developmental differences in the ANS are most often captured as Weber fractions (w), which correspond roughly to the smallest, and therefore most difficult, ratio that can be discriminated reliably, but correspond more realistically to the underlying noise in the tuning curves that neurophysiologically instantiate the ANS (8, 9). Together, these signatures suggest that the ANS represents numbers as noisy Gaussian curves along an ordered mental number line. Other number representations, such as counting, have distinct signatures (e.g., binomial variability; 7), allowing researchers to identify distinct number representations by monitoring for scalar variability and ratio dependence.

Individual and developmental differences in ANS performance are usually tested by briefly flashing sets of dots (or playing a sequential series of tones) quickly enough that they cannot be counted. Participants are then asked to compare that stimulus to another set of dots presented simultaneously or concurrently, or to a particular number word. Infants are usually tested through habituation to a particular number of dots, or by preferential looking to one of two streams of dots, with one constant and the other varying in number. In more complicated paradigms, sets of dots can be hidden and then manipulated (e.g., by subtracting objects or adding new ones; Figure 1). Each individual's ANS acuity can then be measured by examining the most difficult ratio that can be discriminated successfully (see 5 for details on models that accomplish this).

WHO HAS ACCESS TO THE ANS? WHEN DOES IT EMERGE?

If the ANS is one of many forms of number representations, what sets it apart? Dozens of studies have highlighted that the ANS acts as the first route by which we understand numbers, and exists across cultures, ages, and species of animals. For example, newborns spontaneously match the number of tones they hear to the number of objects in front of them (10).

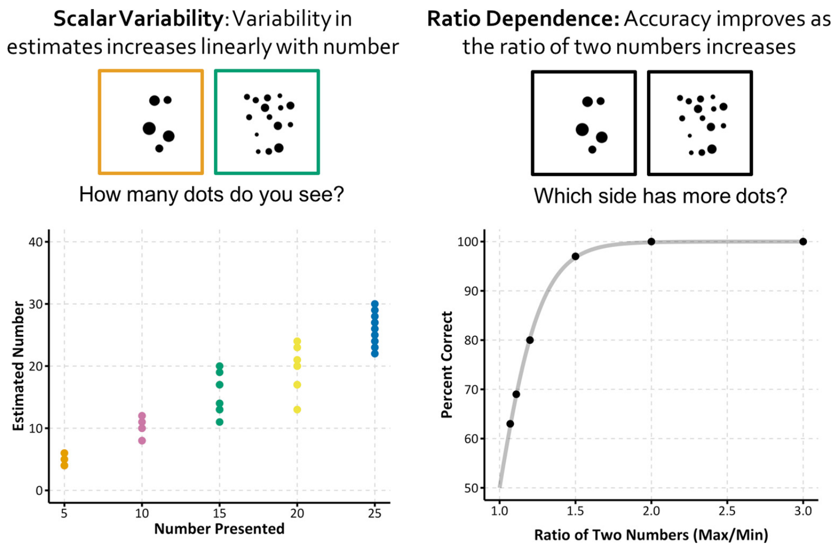

Figure 2. The two signatures of the approximate number system. Left: scalar variability, the linearly increasing variability as the number of items presented rises. Right: ratio dependency, the increasing accuracy and decreasing reaction time as the ratio of two numbers grows. [Color figure can be viewed at wileyonlinelibrary.com]

Members of cultures without any number words show ratio dependency when discriminating number (11). Newly hatched baby chicks, without training, show ratio dependency when foraging for food among collections of objects (for a review of the ANS's behavioral and neural signatures across species, see 12). Therefore, the ANS apparently has a long phylogenetic history.

The ANS develops even if typical sensory input is disrupted. For example, congenitally deaf individuals growing up using Nicaraguan Sign Language reason about number using the ANS (13). Congenitally blind individuals also demonstrate ratio dependence when discriminating sequences of tones based on number, and further exhibit compensatory activity in early visual cortices in ways that appear specific to number (14).

Despite the ubiquity of the ANS, the ANS is more precise in some people than in others. For example, while a typical college student can easily discriminate 25 dots from 22 dots, some can distinguish 25 from 24, whereas others struggle to discriminate 25 from 15 (15). These individual differences emerge early in development and stay relatively stable with age—precision at 6 months predicts precision in preschool (16). As is the case with many cognitive traits, the sources of these individual differences are difficult to pinpoint. Genetic heritability of the ANS is only moderate (17), and although experience with number words and formal education improves ANS acuity (18), many other unexplored factors likely also contribute to these individual differences.

WHY DO WE HAVE AN ANS?

Researchers have debated whether humans need a specialized system such as the ANS to perceive and represent number, or whether we could simply infer numerical information from other dimensions, such as density or area. In typical ANS dot displays (as in Figures 1 and 2), as number varies within the arrays, many other features necessarily covary as well. For example, if the size of the dots remains constant, increasing the number of dots will also increase the total cumulative area covered by the dots, whereas if the area in which the dots are drawn remains constant, the density of the dot array will increase. The natural covariation of these features could allow individuals to select the more numerous array without representing number, either by using these dimensions in place of number or by combining and averaging across many different dimensions to infer number (19–21).

The ANS's potential dependence on other magnitudes is important because it questions the existence of and need for a domain-specific number system, and because it carries implications for cognitive development: For example, if number is inferred by learning its natural covariation with other dimensions, infants and young children without sufficient learning experiences should also lack an ANS. A full discussion of this debate is not possible here. Instead, we argue that, for three reasons, the ANS is best conceptualized as a domain-specific and specialized system for number representations that is not extracted or inferred from nonnumeric dimensions. (For information on the opposing view, see 20, 21.)

First, studies examining the ANS across perceptual modalities —for example, tasks in which participants must combine or compare visual and auditory presentations of number—argue against using nonnumeric cues during number perception. Because vision and audition use drastically different perceptual features to represent their respective inputs (e.g., spatial frequency vs. pitch), studying the ANS across perceptual modalities bypasses shared encoding procedures and instead investigates shared representations between number and other dimensions. Consistent with a separation between the ANS and nonnumerical representations, many studies support amodal ANS representations. For example, neonates spontaneously match visual and auditory presentations of number (10), and children and adults can both compare and add approximate numerosities across visual and auditory modalities with no apparent cost to performance within modalities (22). Perhaps most impressively, visually adapting participants to number (e.g., asking participants to stare at a patch of dots for about 20 s) subsequently decreases their auditory number estimates and vice versa (23). These results suggest that number representations are not a mere byproduct of representations of other nonnumerical features (for an alternative view, see 21). And although this integration of auditory and visual signals likely occurs at midrepresentational rather than early sensory levels (21, 23), the finding that even newborns integrate these signals suggests that early, domain-specific ANS representations automatically bridge numerical information across diverse perceptual features and sensory modalities.

Our second argument involves *congruency effects*, the finding that participants are slower and less accurate when the array with more dots is incongruent with other nonnumerical dimensions (e.g., the array with fewer dots has larger or denser dots than the array with more dots), even when participants are instructed to attend only to number. The interpretation of congruency effects is a major topic of discussion in ANS research. Many researchers have taken such effects as evidence that number perception is dependent on or domain-general with nonnumeric dimensions because congruency effects may stem from participants actually using nonnumeric features, such as density or area, to perceive number.

In contrast, we argue that congruency effects alone cannot provide evidence for or against the separation between the ANS and nonnumerical representations because such effects can stem from many levels of processing. Congruency effects in number tasks may simply be a byproduct of response competition between the ANS and other perceptual representations, such as density or area, rather than a product of the actual use of nonnumeric features during number perception. For example, consider that the classic Stroop Task, in which the irrelevant but automatic reading of a word interferes with the task of naming the word's color, is not evidence for a unitary process underlying color perception and reading, but instead points to competition between two otherwise independent processes. Similarly, the observation that participants select the denser patch of dots does not necessitate that density is used to perceive number, but instead that density and number may compete for the same response. Indeed, neurophysiological evidence suggests that interactions between numerical and nonnumerical representations vary as a function of task demands and can reflect conflict at the level of both stimulus encoding and response selection (24).

Finally, the influence of nonnumerical dimensions on numerical representations becomes clearer in experimental paradigms that eliminate response conflicts. For example, monkeys, preschoolers, and adults from both numerate and innumerate societies spontaneously categorize stimuli based on number rather than size (25). Because participants are not given explicit instructions or time limits, this method enables them to focus on whatever they find most salient while eliminating the potential response conflicts inherent in speeded discrimination tasks. Likewise, modeling techniques that parse the influence of numerical and nonnumerical cues in numerical discrimination tasks indicate that participants base their decisions primarily on number, not other dimensions (26). Electrophysiological evidence also suggests that areas early in the visual processing stream are sensitive to changes in numerosity independent of changes in other visual features, even under passive viewing conditions in which participants do not attend or respond to number (27). Finally, preverbal infants are not more sensitive to changes in nonnumerical features than in number (28), and developmental improvements in ANS acuity occur independent of other dimensions, including area, density, length, and time (5). Together, these findings suggest that the ANS has a critical degree of independence from nonnumerical dimensions.

The debate on the role of nonnumerical dimensions in number perception remains active and productive (19, 20). Although we cannot discuss every study on the ANS's dependence on other magnitudes, any such theory must at least account for three basic phenomena: Crossmodal effects suggest that ANS representations persist even when divorced from a single sensory modality, congruency effects alone cannot pinpoint the locus of interaction between the ANS and nonnumerical dimensions, and finally, spontaneous categorization and developmental work strongly suggests an ontogenetic primacy of numerical representations. We propose that the natural covariation of numerical and nonnumerical features may offer an efficient way to combine redundant information across many perceptual cues to make more optimal decisions about number, especially when numerical ratios are low or other cues are salient. In other words, congruency effects likely demonstrate that the ANS—like other representational systems—is very flexible and efficient, not that it relies entirely on nonnumerical dimensions.

WHERE IN THE BRAIN IS THE ANS LOCALIZED?

Convergent findings across methods, species, and ages demonstrate that the posterior parietal cortex, and the intraparietal sulcus (IPS) in particular, are integral for processing numerical magnitudes (29). Neural activity in these regions exhibits the same ratio-dependent hallmark of ANS representations that are observed behaviorally, and ratio-dependent activity within the IPS is found both when participants perform numerical tasks and when they passively view numerically changing stimuli (30). Ratio dependence can also be seen at the level of single neurons in monkeys trained to perform a numerical match-to-sample task. Single-cell recordings from the IPS in these monkeys reveal populations of neurons that respond maximally to a preferred numerical value, with the firing rate decreasing as the presented value deviates from the preferred value (29).

Sensitivity to numerosity in the posterior parietal cortex emerges early in human development, long before children learn to count or begin formal schooling. For example, fluctuations in blood flow as measured by functional near-infrared spectroscopy indicate that activity in the right parietal cortex of 6-month-olds is modulated by changes in the number of objects within an array but not by changes in shape (31). Likewise, functional magnetic resonance imaging demonstrates that activity in the IPS of 4-year-olds responds to changes in number but not changes in shape (32). In adults, regions in the parietal cortex respond to numerical information regardless of whether it is presented as arrays of dots, Arabic digits, or auditory number words (33). Children as young as 6 also have similarly abstract representations of number within the IPS (34). Taken together, the neuroimaging evidence suggests that the IPS supports amodal, abstract representations of number. Furthermore, these number-specific representations appear early in human development, again demonstrating that the ANS develops prior to experience with number words or formal math education.

HOW IS THE ANS USED?

Beginning with the seminal finding that ANS acuity in adolescents correlates with their standardized math scores (35), in many subsequent studies, individual differences in ANS acuity correlate with achievement in symbolic math, even when controlling for other factors, including intelligence, working memory, and vocabulary size (see 36 and 37 for meta-analyses, including negative findings). In fact, ANS acuity in infancy—thus, before the start of formal math education—predicts later math achievement (16), suggesting a directional relation between ANS acuity and symbolic math achievement.

However, some investigators contend that the correlation between ANS acuity and symbolic mathematics reflects a relation between inhibitory control and symbolic math. According to this view, inhibitory control is required to respond based on number rather than nonnumerical features (e.g., area or density) in incongruent trials. Indeed, in some studies, only performance on incongruent trials correlates with math achievement (38). However, in other studies, performance on both congruent and incongruent trials correlates with math achievement (39). Furthermore, when numerical acuity and selective attention to number are measured independently, only numerical acuity predicts children's symbolic math ability (40).

We do not claim that only the ANS predicts math achievement, nor that it is the strongest predictor, because many cognitive and socioeconomic factors contribute to children's development in math (e.g., inhibitory control, visuospatial skills, the home environment). Rather, we argue that the ANS is a foundation on which symbolic number representations are constructed. For example, children with more precise ANS representations may find it easier to master the meaning of number words, which in turn may facilitate early arithmetic learning (41). Later in life, the ANS may continue to influence mathematical thinking by supporting ordinal representations of the relations between numerical magnitudes (42) or serving as an online error-detection system (43).

The ANS may also be an attractive target for interventions to improve children's math performance. For example, training preschoolers and college students on approximate arithmetic (e.g., as in Figure 1) improves their symbolic exact arithmetic performance (44), potentially because of a functional overlap between approximate and symbolic addition. Similarly, exercising the ANS or temporarily boosting children's confidence in their ANS acuity also improves performance on a subsequent math test (45, 46). Although these studies are not without limitations and more work is needed to test the scope and strength of these effects (47), the findings provide preliminary evidence that interventions targeting the ANS may help young children who have not mastered symbolic digits by providing an alternative avenue for them to practice the same types of mathematical operations they will learn to perform symbolically.

CONCLUSIONS

The ANS is of broad interest to cognitive, developmental, comparative, and computational psychologists because it suggests that the brain can automatically and efficiently represent one of the most complex properties in the external world. Although debates continue within the literature, we have argued that the ANS's characteristic signatures exist in most species of animals, develop and function independently of nonnumerical dimensions, have unique neural instantiations, and are used throughout life, even before children enter formal schooling. In essence, number—as represented by the ANS—may be a foundational building block of perception and thought, akin to our basic and universal representations of color, agency, and objects.

REFERENCES

1. Feigenson, L., Dehaene, S., & Spelke, E. S. (2004). Core systems of number. *Trends in Cognitive Sciences*, *8*, 307–314. https://doi.org/10.1016/j.tics.2004.05.002
2. Dehaene, S. (2009). Origins of mathematical intuitions. *Annals of the New York Academy of Sciences*, *1156*, 232–259. https://doi.org/10.1111/j.1749-6632.2009.04469.x
3. Halberda, J., Sires, S. F., & Feigenson, L. (2006). Multiple spatially overlapping sets can be enumerated in parallel. *Psychological Science*, *17*, 572–576. https://doi.org/10.1111/j.1467-9280.2006.01746.x

4. McCrink, K., & Wynn, K. (2004). Large-number addition and subtraction by 9-month-old infants. *Psychological Science*, *15*, 776–781. https://doi.org/10.1111/j.0956-7976.2004.00755.x
5. Odic, D. (2017). Children's intuitive sense of number develops independently of their perception of area, density, length, and time. *Developmental Science*. https://doi.org/10.1111/desc.12533
6. Carey, S. (2009). *The origin of concepts*. New York, NY: Oxford University Press.
7. Cordes, S., Gelman, R., Gallistel, C. R., & Whalen, J. (2001). Variability signatures distinguish verbal from nonverbal counting for both large and small numbers. *Psychonomic Bulletin & Review*, *8*, 698–707. https://doi.org/10.3758/BF03196206
8. Piazza, M., Izard, V., Pinel, P., Le Bihan, D., & Dehaene, S. (2004). Tuning curves for approximate numerosity in the human intraparietal sulcus. *Neuron*, *44*, 547–555. https://doi.org/10.1016/j.neuron.2004.10.014
9. Halberda, J., & Odic, D. (2014). The precision and internal confidence of our approximate number thoughts. In D. C. Geary, D. Berch, & K. Koepke (Eds.), *Evolutionary origins and early development of number processing* (pp. 305–335). Cambridge, MA: Academic Press.
10. Izard, V., Sann, C., Spelke, E. S., & Streri, A. (2009). Newborn infants perceive abstract numbers. *Proceedings of the National Academy of Sciences of the United States of America*, *106*, 10382–10385. https://doi.org/10.1073/pnas.0812142106
11. Frank, M. C., Everett, D. L., Fedorenko, E., & Gibson, E. (2008). Number as a cognitive technology: Evidence from Piraha language and cognition. *Cognition*, *108*, 819–824. https://doi.org/10.1016/j.cognition.2008.04.007
12. Vallortigara, G. (2017). An animal's sense of number. In J. Adams, P. Barmby, & A. Mesouid (Eds.), *The nature and development of mathematics: Cross disciplinary perspectives on cognition, learning, and culture* (pp. 43–66). London, UK: Taylor & Francis.
13. Spaepen, E., Coppola, M., Spelke, E. S., Carey, S., & Goldin-Meadow, S. (2011). Number without a language model. *Proceedings of the National Academy of Sciences of the United States of America*, *108*, 3163–3168. https://doi.org/10.1073/pnas.1015975108
14. Kanjlia, S., Lane, C., Feigenson, L., & Bedny, M. (2016). Absence of visual experience modifies the neural basis of numerical thinking. *Proceedings of the National Academy of Sciences of the United States of America*, *113*, 11172–11177. https://doi.org/10.1073/pnas.1524982113
15. Halberda, J., Ly, R., Wilmer, J. B., Naiman, D. Q., & Germine, L. (2012). Number sense across the lifespan as revealed by a massive internet-based sample. *Proceedings of the National Academy of Sciences of the United States of America*, *109*, 11116–11120. https://doi.org/10.1073/pnas.1200196109
16. Starr, A., Libertus, M. E., & Brannon, E. M. (2013). Number sense in infancy predicts mathematical abilities in childhood. *Proceedings of the National Academy of Sciences of the United States of America*, *110*, 18116–18120. https://doi.org/10.1073/pnas.1302751110
17. Tosto, M. G., Petrill, S. A., Halberda, J., Trzaskowski, M., Tikhomirova, T. N., Bogdanova, O. Y., . . . Kovas, Y. (2014). Why do we differ in number sense? Evidence from a genetically sensitive investigation. *Intelligence*, *43*, 35–46. https://doi.org/10.1016/j.intell.2013.12.007
18. Piazza, M., Pica, P., Izard, V., Spelke, E. S., & Dehaene, S. (2013). Education enhances the acuity of the nonverbal approximate number system. *Psychological Science*, *24*, 1037–1043. https://doi.org/10.1177/0956797612464057
19. Gebuis, T., & Reynvoet, B. (2012). The interplay between nonsymbolic number and its continuous visual properties. *Journal of Experimental Psychology: General*, *141*, 642–648. https://doi.org/10.1037/a0026218
20. Leibovich, T., Katzin, N., Harel, M., & Henik, A. (2017). From "sense of number" to "sense of magnitude"—The role of continuous magnitudes in numerical cognition. *Behavioral and Brain Sciences*, *40*, 1–62. https://doi.org/10.1017/S0140525X16000960
21. Gebuis, T., Kadosh, R. C., & Gevers, W. (2016). Sensory-integration system rather than approximate number system underlies numerosity processing: A critical review. *Acta Psychologica*, *171*, 17–35. https://doi.org/10.1016/j.actpsy.2016.09.003
22. Barth, H., La Mont, K., Lipton, J., & Spelke, E. S. (2005). Abstract number and arithmetic in preschool children. *Proceedings of the National Academy of Sciences of the United States of America*, *102*, 14116–14121. https://doi.org/10.1073/pnas.0505512102
23. Arrighi, R., Togoli, I., & Burr, D. C. (2014). A generalized sense of number. *Proceedings of the Royal Society of London B: Biological Sciences*, *281*, 20141791. https://doi.org/10.1098/rspb.2014.1791
24. Kadosh, R. C., Kadosh, K. C., Linden, D. E., Gevers, W., Berger, A., & Henik, A. (2007). The brain locus of interaction between number and size: A combined functional magnetic resonance imaging and event-related potential study. *Journal of Cognitive Neuroscience*, *19*, 957–970. https://doi.org/10.1162/jocn.2007.19.6.957
25. Ferrigno, S., Jara-Ettinger, J., Piantadosi, S. T., & Cantlon, J. F. (2017). Universal and uniquely human factors in spontaneous number perception. *Nature Communications*, *8*, 13968. https://doi.org/10.1038/ncomms13968
26. DeWind, N. K., Adams, G. K., Platt, M. L., & Brannon, E. M. (2015). Modeling the approximate number system to quantify the contribution of visual stimulus features. *Cognition*, *142*, 247–265. https://doi.org/10.1016/j.cognition.2015.05.016
27. Park, J., DeWind, N. K., Woldorff, M. G., & Brannon, E. M. (2015). Rapid and direct encoding of numerosity in the visual stream. *Cerebral Cortex*, *26*, 748–763. https://doi.org/10.1093/cercor/bhv017
28. Starr, A., & Brannon, E. M. (2015). Evidence against continuous variables driving numerical discrimination in infancy. *Frontiers in Psychology*, *6*. https://doi.org/10.3389/fpsyg.2015.00923
29. Nieder, A., & Miller, E. K. (2004). A parieto-frontal network for visual numerical information in the monkey. *Proceedings of the National Academy of Sciences of the United States of America*, *101*, 7457–7462. https://doi.org/10.1073/pnas.0402239101
30. Piazza, M., & Eger, E. (2016). Neural foundations and functional specificity of number representations. *Neuropsychologia*, *83*, 257–273. https://doi.org/10.1016/j.neuropsychologia.2015.09.025
31. Hyde, D. C., Boas, D. A., Blair, C., & Carey, S. (2010). Near-infrared spectroscopy shows right parietal specialization for number in pre-verbal infants. *NeuroImage*, *53*, 647–652. https://doi.org/10.1016/j.neuroimage.2010.06.030
32. Cantlon, J. F., Brannon, E. M., Carter, E. J., & Pelphrey, K. A. (2006). Functional imaging of numerical processing in adults and 4-year-old children. *PLoS Biology*, *4*. https://doi.org/10.1371/journal.pbio.0040125
33. Piazza, M., Mechelli, A., Price, C. J., & Butterworth, B. (2006). Exact and approximate judgements of visual and auditory numerosity: An fMRI study. *Brain Research*, *1106*, 177–188. https://doi.org/10.1016/j.brainres.2006.05.104
34. Cantlon, J. F., Libertus, M. E., Pinel, P., Dehaene, S., Brannon, E. M., & Pelphrey, K. A. (2009). The neural development of an

abstract concept of number. *Journal of Cognitive Neuroscience*, *21*, 2217–2229. https://doi.org/10.1162/jocn.2008.21159

35. Halberda, J., Mazzocco, M. M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, *455*, 665–668. https://doi.org/10.1038/nature07246
36. Chen, Q., & Li, J. (2014). Association between individual differences in non-symbolic number acuity and math performance: A meta-analysis. *Acta Psychologica*, *148*, 163–172. https://doi.org/10.1016/j.actpsy.2014.01.016
37. Schneider, M., Beeres, K., Coban, L., Merz, S., Susan Schmidt, S., Stricker, J., & De Smedt, B. (2017). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: A meta-analysis. *Developmental Science*, *20*. Retrieved from http://onlinelibrary.wiley.com/doi/10.1111/desc.12372/full
38. Gilmore, C. K., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N., … Inglis, M. (2013). Individual differences in inhibitory control, not non-verbal number acuity, correlate with mathematics achievement. *PLoS ONE*, *8*, e67374. https://doi.org/10.1371/journal.pone.0067374
39. Keller, L., & Libertus, M. (2015). Inhibitory control may not explain the link between approximation and math abilities in kindergarteners from middle class families. *Frontiers in Psychology*, *6*. https://doi.org/10.3389/fpsyg.2015.00685
40. Starr, A., DeWind, N. K., & Brannon, E. M. (2017). The role of numerical acuity and non-numerical stimulus features in the development of the number sense and symbolic math achievement. *Cognition*, *168*, 222–233. https://doi.org/10.1016/j.cognition.2017.07.004
41. vanMarle, K., Chu, F. W., Li, Y., & Geary, D. C. (2014). Acuity of the approximate number system and preschoolers' quantitative development. *Developmental Science*, *17*, 492–505. https://doi.org/10.1111/desc.12143
42. Lyons, I. M., & Beilock, S. L. (2011). Numerical ordering ability mediates the relation between number-sense and arithmetic competence. *Cognition*, *121*, 256–261. https://doi.org/10.1016/j.cognition.2011.07.009
43. Feigenson, L., Libertus, M. E., & Halberda, J. (2013). Links between the intuitive sense of number and formal mathematics ability. *Child Development Perspectives*, *7*, 74–79. https://doi.org/10.1111/cdep.12019
44. Park, J., Bermudez, V., Roberts, R. C., & Brannon, E. M. (2016). Non-symbolic approximate arithmetic training improves math performance in preschoolers. *Journal of Experimental Child Psychology*, *152*, 278–293. https://doi.org/10.1016/j.jecp.2016.07.011
45. Hyde, D. C., Khanum, S., & Spelke, E. S. (2014). Brief non-symbolic, approximate number practice enhances subsequent exact symbolic arithmetic in children. *Cognition*, *131*, 92–107. https://doi.org/10.1016/j.cognition.2013.12.007
46. Wang, J. J., Odic, D., Halberda, J., & Feigenson, L. (2016). Changing the precision of preschoolers' approximate number system representations changes their symbolic math performance. *Journal of Experimental Child Psychology*, *147*, 82–99. https://doi.org/10.1016/j.jecp.2016.03.002
47. Szűcs, D., & Myers, T. (2017). A critical analysis of design, facts, bias and inference in the approximate number system training literature: A systematic review. *Trends in Neuroscience and Education*, *6*, 187–203. https://doi.org/10.1016/j.tine.2016.11.002