

Learning Objectives

- **Describe** the four possible decisions in the *Null Hypothesis Statistical Testing (NHST)* framework
- **Identify** factors that increase the likelihood of rejecting the null hypothesis (H_0)
- **Justify** decisions to manage error
- **Visualize** statistical power

Announcements

- Midterm exam reviews ongoing
 - See corrected item in announcements
 - Even if you missed the midterm
- SPSS/Jamovi 4 is available (Sign Test)
- No video for Monday
- Ch. 10 is important (for Ch. 11 focus on lecture)

Inferential Statistics

- Quantify confidence in whether our pattern of observations will replicate
 - **Goal:** Rule out random chance as potential cause
 - a.k.a, reject H_0
 - “My estimates will fluctuate but the pattern is reliable; (it would replicate if I sampled again)”

Possible conclusions:

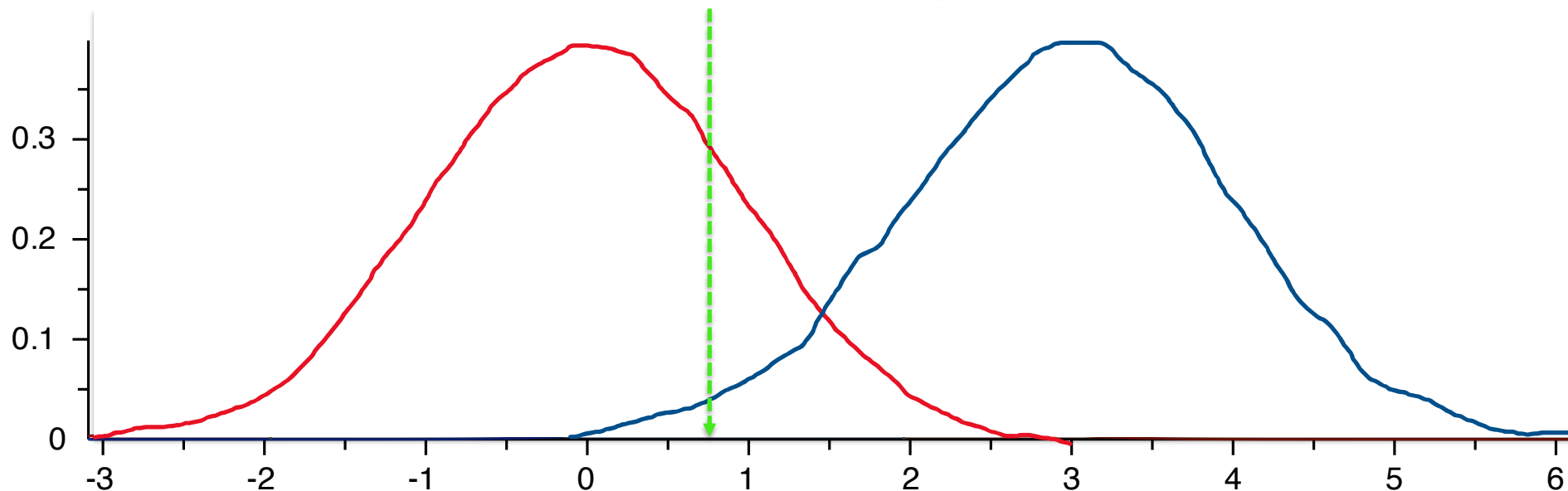
1. We can rule out random chance
 - “The relationship is ***probably*** real”
2. We cannot rule out random chance*
 - “The relationship ***could easily*** be random”

Note:** We do not conclude that chance is the cause, but it ***could be

Inference: Which distribution did our observation come from?

Distribution | H_0 : Red curve
Distribution | H_1 : Blue curve

Decision threshold
(we choose this value)



What happiness scores look
like if treatment did not work

What happiness scores look
like if treatment worked

Truth/Reality

Our Decision

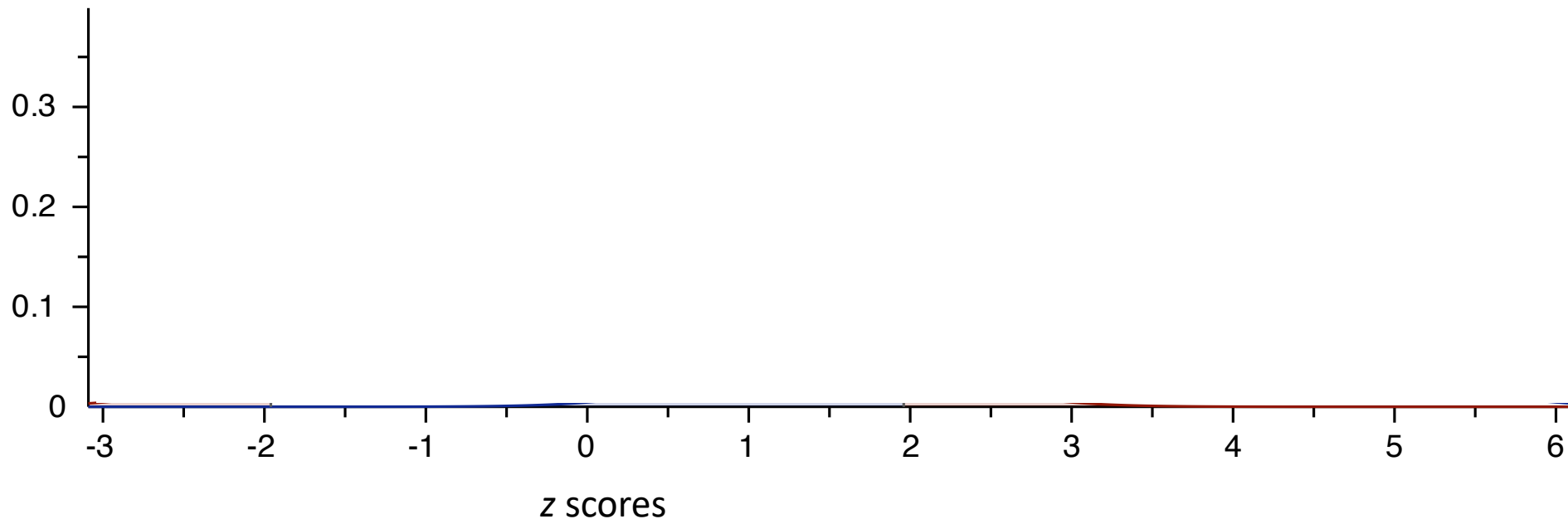
	No effect: Relationship is random	Real effect: Relationship is real
Don't reject chance: "Relationship could easily be random"	Correct! $(1 - \alpha)$	Type II Error β
Reject chance: "Relationship seems real"	Type I Error α	Correct! $(1 - \beta)$

Inference: Which distribution did our observation come from?

Distribution | H_0 : Red curve
Distribution | H_1 : Blue curve

Chosen value
for α

	No effect	Yes effect
Don't reject H_0	Correct! (1 - α)	Type II Error β
Reject H_0	Type I Error α	Correct! (1 - β)



Alpha and Beta

Beta (β) is the probability of **Type II error**

- We say “there is no effect”, but there is an effect

Alpha (α) is the probability of **Type I error**

- We say “there is an effect”, but there is not an effect

Mathematical Formulae

When H_0 is true, in reality there is **no effect**

- We reject H_0 , a *Type I* error, or
- We correctly retain H_0

$$1 = \alpha + (1 - \alpha)$$

When H_0 is not true, in reality there is **an effect**

- We retain H_0 , a *Type II* error, or
- We correctly reject H_0

$$1 = \beta + (1 - \beta)$$

Error Management

- **Goal:** Be correct all the time!
 - Impossible goal
- **Reality:** Manage errors in a systematic way
- Which errors are worse? Type I or Type II?
 - **Verdict:** Guilty vs. not guilty
 - **Medicine:** New Coronavirus test
 - Type I error: Test says you are sick, but you aren't sick
 - Type II error: Test says you aren't sick, but you are

Controlling Error rates

Controlling Type II (or β) errors:

- Type II errors less likely with more statistical power ($1 - \beta$)
 - **Larger N** more likely to reject the null hypothesis
 - **Smaller σ_{pop}**
 - **Larger difference between means**
 - **Larger α , or 1-tail test**

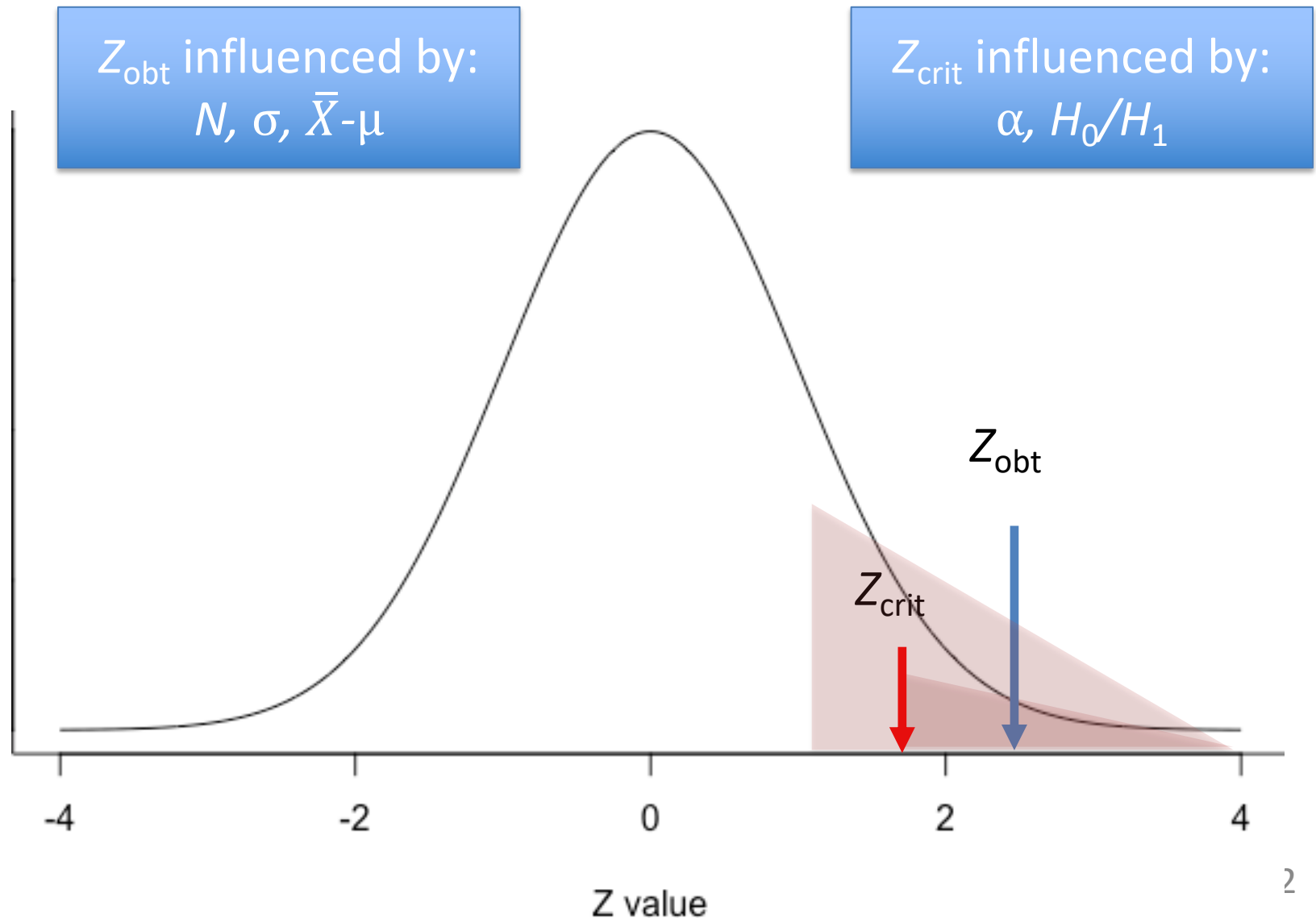
$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}}$$

$$z_{\text{obt}} = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}$$

When do we reject the null?

- Reject when z_{obt} gets large enough
 - Specifically when ± 1.96 (for 2-tail tests)
 - Because z_{crit} is always ± 1.96 or ± 1.645 (for 1-tail)
 - Critical values change for other test statistics (e.g., t , F)
- **Larger N** increases z_{obt} (because it reduces $\sigma_{\bar{X}}$)
- **Smaller σ_{pop}** increases z_{obt} (b/c it reduces $\sigma_{\bar{X}}$)
- **Larger difference** ($\bar{X} - \mu$)
- **Larger α** , b/c it reduces z_{crit}
(z_{obt} does not need to be as extreme to reject H_0)
- **Directional H_0/H_1** , b/c it reduces z_{crit}
(z_{obt} does not need to be as extreme to reject H_0)

z_{obt} & z_{crit} are the moving parts:



Type I vs. Type II errors

Reducing Type I errors increases Type II errors

Note: Reducing Type II errors does not increase Type I errors

- We need to balance errors:
 - What are the consequences of **failing to find a real effect?**
 - What are the consequences of **“finding” an effect that is not real?**

We choose acceptable rates of Type I error:

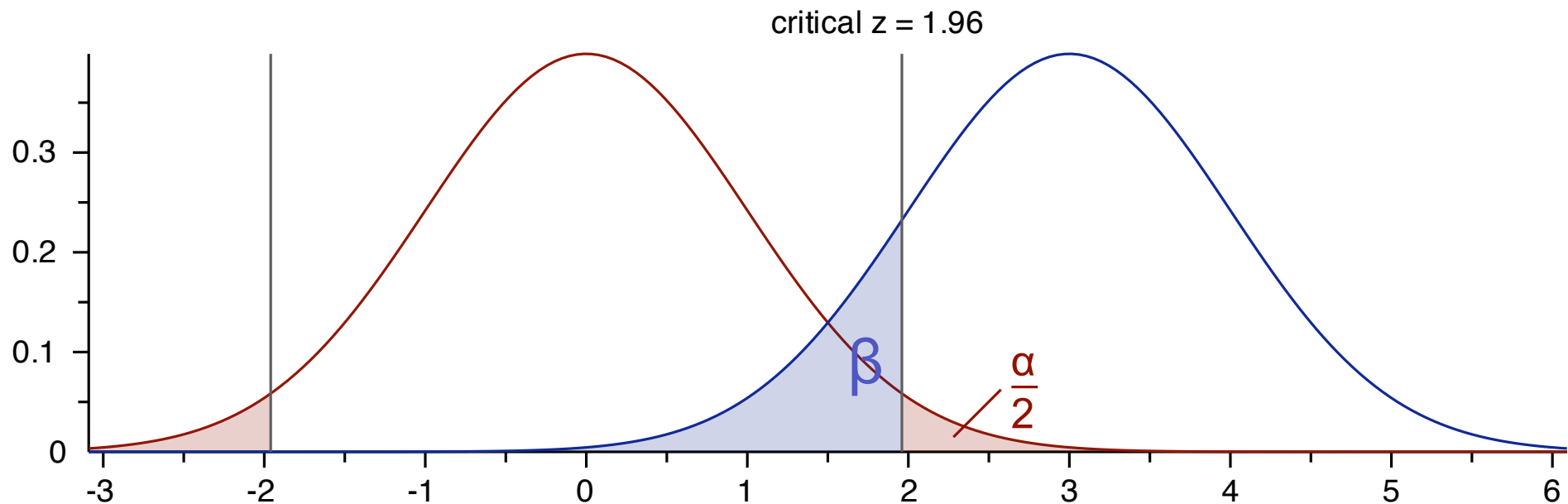
- Psychology: $\alpha = .05$
- Physics: $\alpha = .000000003$
- Biology (macro): $\alpha = .05$
- Biology (micro): $\alpha = .0000001$

$$(1 - \beta) = \underline{\text{Power}}$$

- Research seeks at least 80% power
 - Not 40-60%!!
- Visualize in ***G*Power*** [*\(or Laken's Shiny\)*](#)
 - Most common tool for calculating power
 - Normal distribution (or z-distribution)
 - What is/are the z-score for: $\alpha_{2\text{-tail}} = .05$

Red = H_0 Distribution (If H_0 were true)

Blue = H_1 Distribution (If H_1 were true)



$$\alpha_{2\text{-tail}} = .05$$

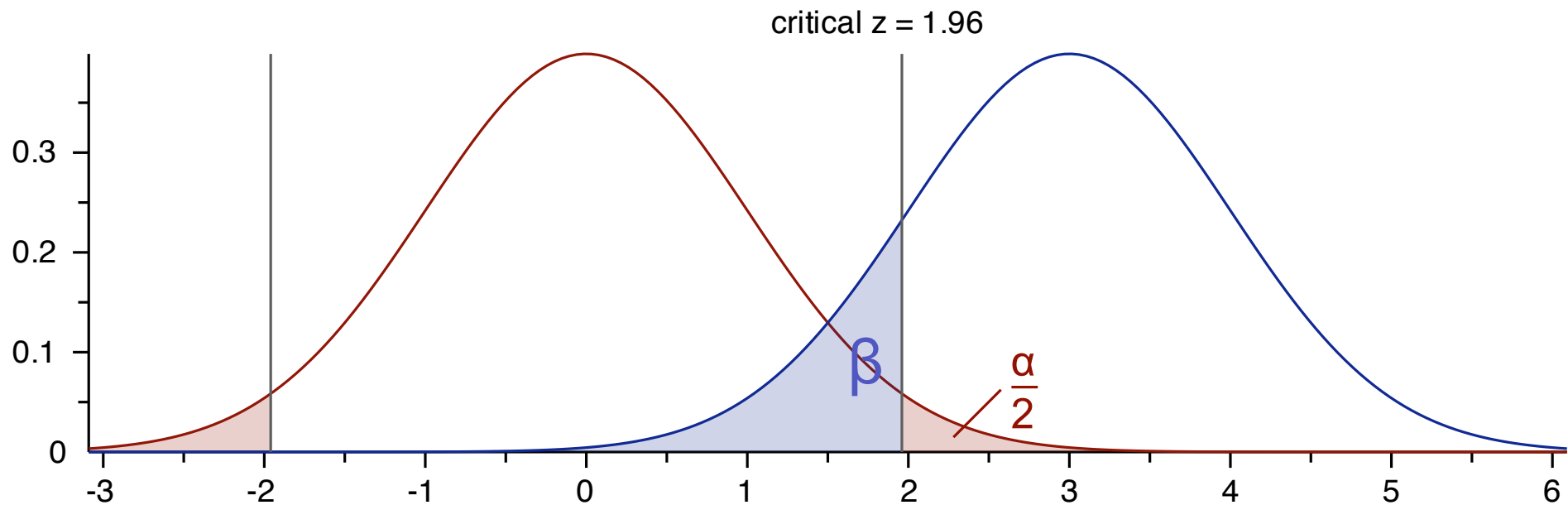
$$\beta = .15$$

$$\text{Power} = .85$$

$$\bar{X} - \mu = 3 \text{ units}$$

$$\sigma_{\bar{X}} = 1 \text{ unit}$$

Red = If H_0 is true
Blue = If H_1 is true



$$\alpha_{2\text{-tail}} = .05$$

$$\beta = .15$$

$$\text{Power} = .85$$

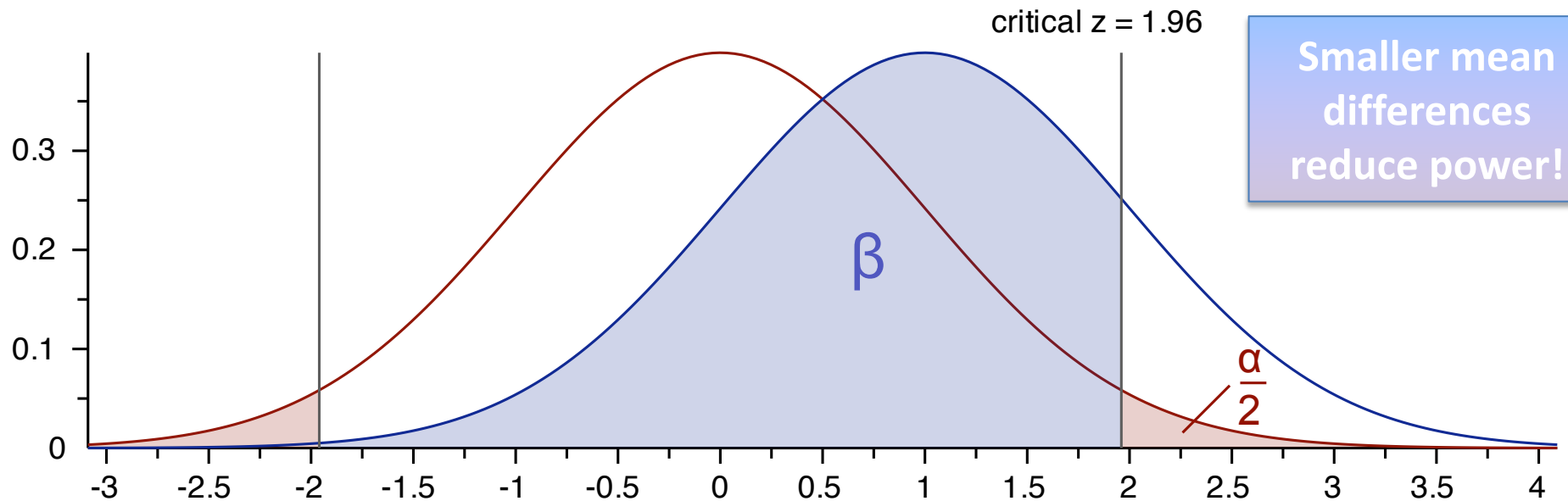
$$\bar{X} - \mu = 3 \text{ units}$$

$$\sigma_{\bar{X}} = 1 \text{ unit}$$

Power when effect size is smaller!

Red = If H_0 is true

Blue = If H_1 is true



$$\alpha_{2\text{-tail}} = .05$$

$$\beta = .83$$

$$\text{Power} = .17$$

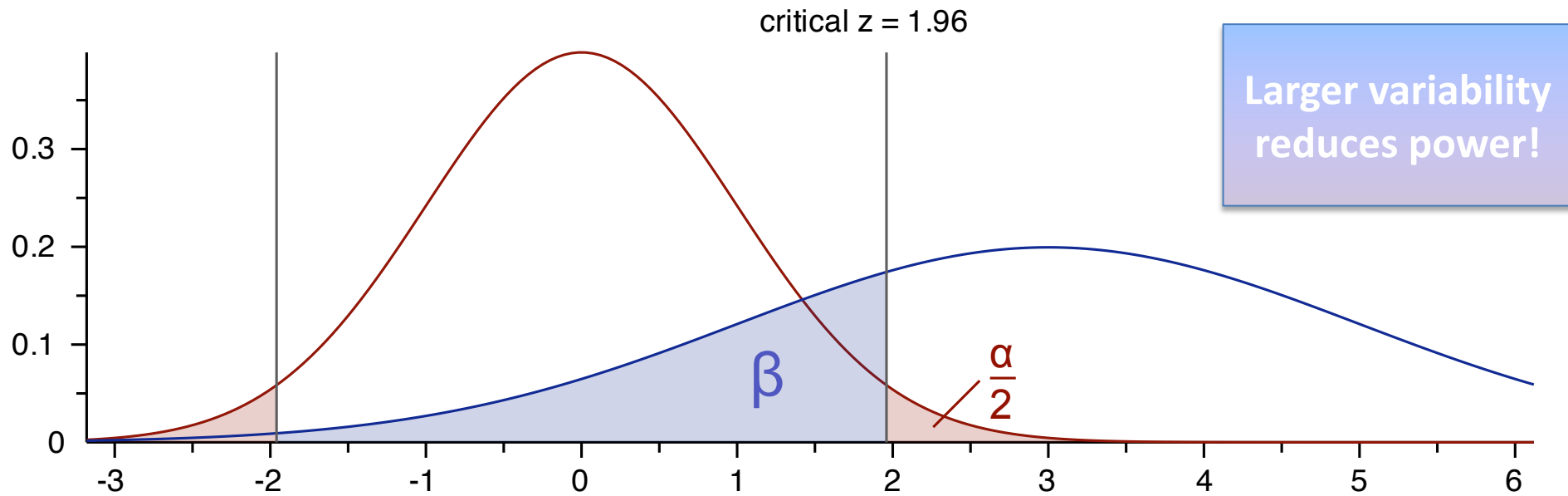
$$\bar{X} - \mu = \text{1 unit (was 3)}$$

$$\sigma_{\bar{X}} = 1 \text{ unit}$$

Power when variability is greater!

Red = If H_0 is true

Blue = If H_1 is true



$$\alpha_{2\text{-tail}} = .05$$

$$\beta = .29$$

$$\text{Power} = .71$$

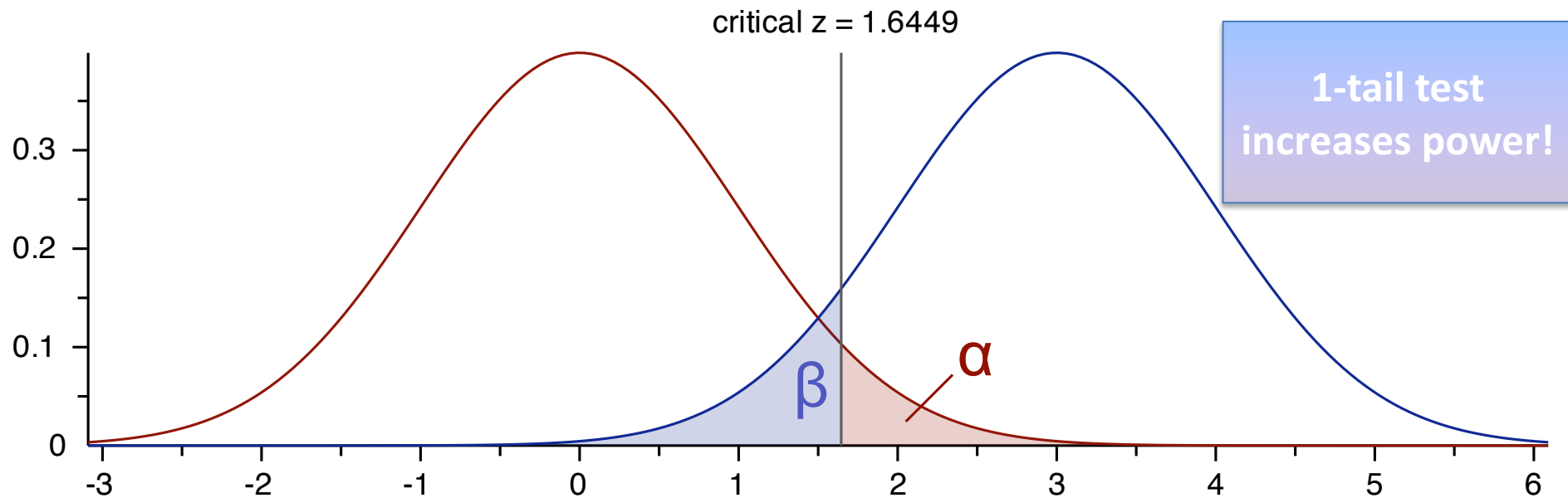
$$\bar{X} - \mu = 3 \text{ units}$$

$$\sigma_{\bar{X}} = \mathbf{2 \text{ units}} \text{ (was 1)}$$

Power when 1-tail (vs. 2-tail) test!

Red = If H_0 is true

Blue = If H_1 is true



$$\alpha_{1\text{-tail}} = .05$$

$$\beta = .09$$

$$\text{Power} = .91$$

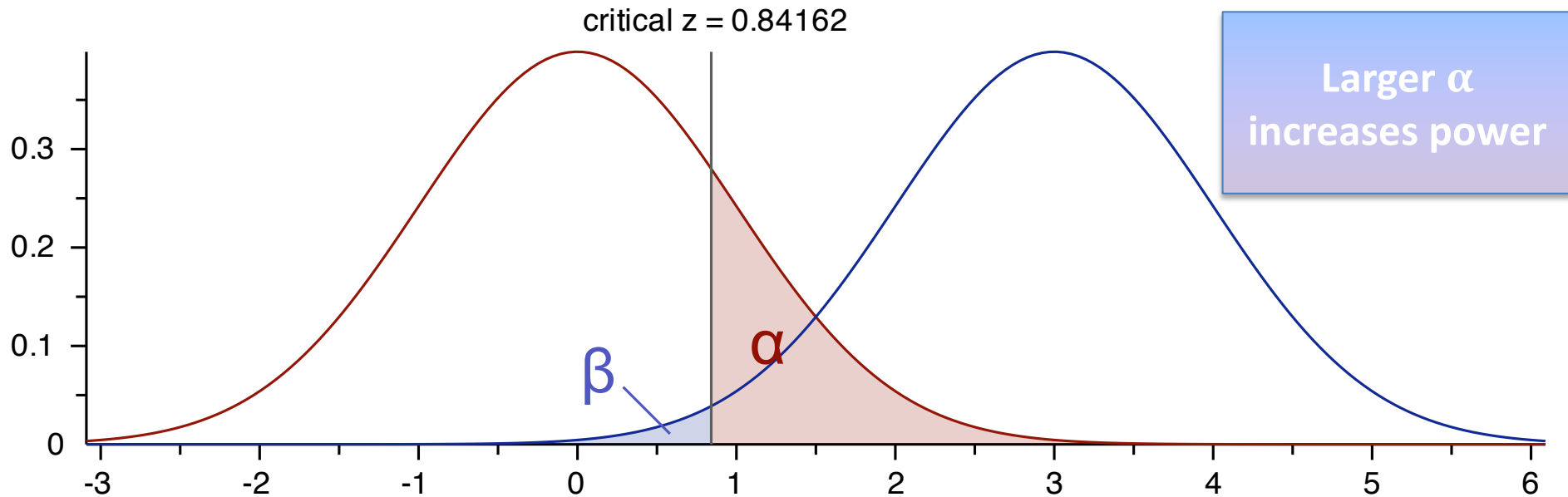
$$\bar{X} - \mu = 3 \text{ units}$$

$$\sigma_{\bar{X}} = 1 \text{ unit}$$

Power when α is large!

Red = If H_0 is true

Blue = If H_1 is true



$$\alpha_{1\text{-tail}} = .20$$

$$\beta = .01$$

$$\text{Power} = .99$$

$$\bar{X} - \mu = 3 \text{ units}$$

$$\sigma_{\bar{X}} = 1 \text{ unit}$$

3 Common Types of Power

- 1. *a priori*** – Before data, find **N** given:
 - α , $(1 - \beta)$, expected effect size
- 2. *post hoc*** – After data, find **power** given:
 - α , N , **observed effect size**
- 3. *Sensitivity*** – Before/after data, find **detectable effect size** given:
 - α , $(1 - \beta)$, N

Setting Power

- **Exp. 1:** “We sought to collect 80 participants... Sensitivity analysis indicated with power set at .80, we could detect an effect size as small as $d_z = .317$ ”
3. Sensitivity power b/c we don't know the size of the effect
- $\alpha = .05$
 - $(1 - \beta) = .80$
 - $N = 80$

Setting Power

- **Exp. 2:** “Based on the observed effect size of $d_z = .430$ in Experiment 1, we sampled from 48 participants to set power at .80”
 1. Use *a priori* power after getting an estimate of effect size in Exp. 1
 - $\alpha = .05$
 - $(1 - \beta) = .80$
 - Effect size = $d_z = .430$