Learning Objectives

- **Review** Models (central tendency, variability, correlation) and reframe them within context of prediction
- **Describe** goal of statistical inference
- **Define** new terminology related to probability
- **Practice** using the *addition* and *multiplication* rules of probability

Probability

Assessed in two ways:

– ***'a priori'***: before data

$$p(A) = \frac{\#\text{ of } A_{\text{possible events}}}{N_{\text{possible events}}}$$

– ***'a posteriori'***: after data

$$p(A) = \frac{\#\text{ of } A_{\text{observed}}}{N_{\text{obs.}}}$$

Example: Dice

<u>What is the probability of an odd roll with 1 dice</u>*?

– ***a priori probability***

$$p(odd) = \frac{\#\text{ of Odd}_{\text{possible}}}{N_{\text{possible events}}} =$$

– ***a posteriori probability***

- Data: 100 rolls, with 58 odd rolls

Example: Dice

<u>What is the probability of an odd roll with 1 dice?</u>

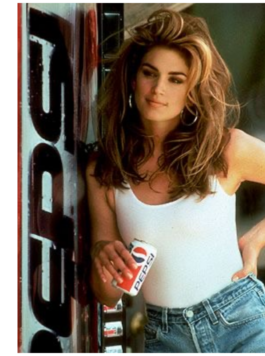
- ***a priori probability*** = .50
- ***a posteriori probability*** = .58

- Goal of statistical inference:
  - What should we predict for <u>future</u> observations?
    - Odd = .50
    - Odd ≠ .50

      > .50? < .50?

Are the dice rigged, or did we observe randomness?

Example: Does advertising work?



What is the probability of choosing Pepsi?

– ***a priori probability***

$p($$) = \dfrac{\#\ of\ A_{\text{possible}}}{N_{\text{possible events}}} = \dfrac{1\ (\text{choose}}{2\ (\text{choose.}}$  $)$ /  or  $)$ $= .50$

– ***a posteriori probability***

- Data: 100 participants, 58 choose pepsi

$p($$) = \dfrac{\#\ of}{N_{\text{obs.}}}$  $_{\text{obs.}} = \dfrac{58}{100} = .58$

Inferential Statistics

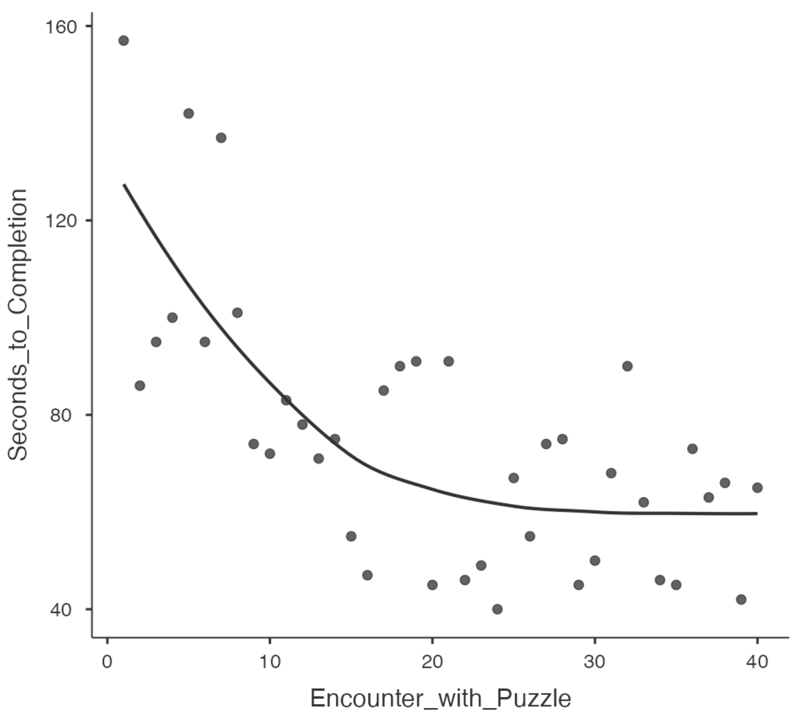
Did we observe randomness or something real?

- Did the ad work or did we accidentally sample from more pepsi people?
- In Psychology, we will always observe ‘random’ processes: We quantify how much is random and how much is real.

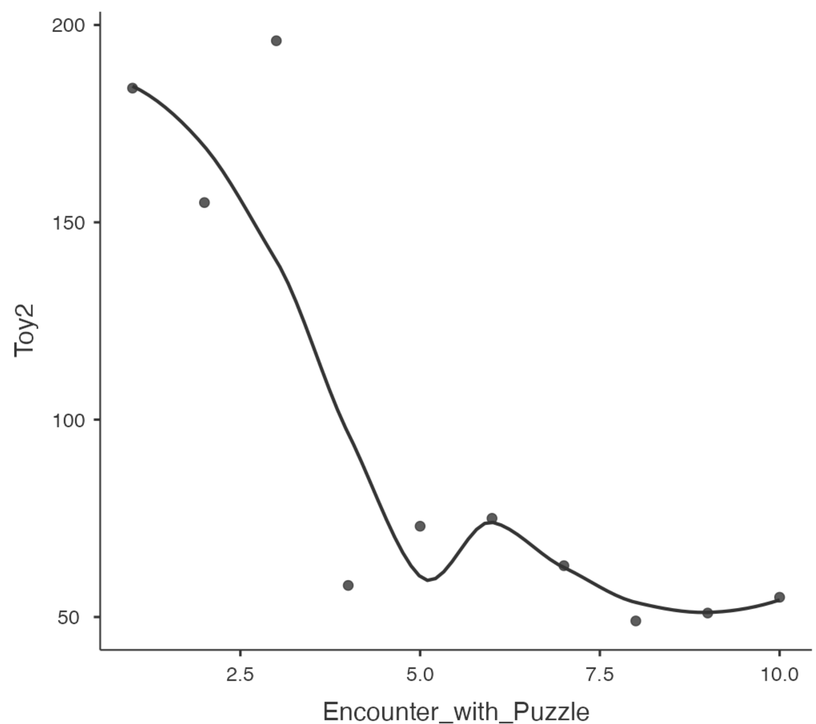
Descriptive Statistics + Probability Theory → Inference!!

Was it random or real?

Scatterplot



Scatterplot



Definitions

- **Mutually exclusive** events cannot happen at the same time
  - Dice cannot land on even *and* odd
    - p(A & B) = 0
    - *Statisticians suck*: When representing 2 mutually exclusive outcomes, use P and Q

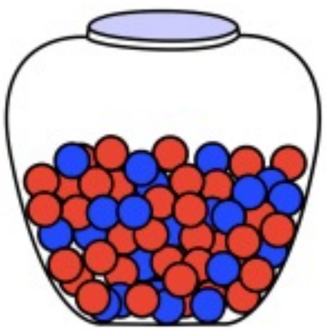
$$\boxed{P = 1 - Q}$$

Definitions

- **Independent** events have no influence on each other
  - 1^{st} roll of dice does not influence 2^{nd} roll
    - Correlation between dice rolls, $\rho = .000$
- **Exhaustive sets** of events describe all possible events
  - We know whether a dice roll is “*A*” or is “not *A*”
    - e.g., “even” or “not even”

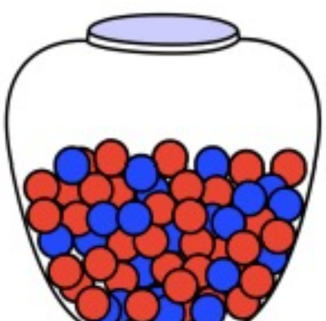
Comparing Probabilities

- Jar #1 and Jar #2 both contain blue and red marbles. While blindfolded, you draw one marble from one jar. If it is red, you win $10.



3 red marbles
2 blue marbles

Which jar is the better choice?



51 red marbles
34 blue marbles

Addition Rule

- Use addition when one of *several* possible outcomes will occur
- *What is the probability of rolling an odd number* ***<u>or</u>*** *a multiple of 3?*
  - Not mutually exclusive, both outcomes can occur at the same time!

$$p(A \text{ or } B) = p(A) + p(B) - p(A \text{ and } B)$$

p(odd **<u>or</u>** multiple of 3) = p(odd) + p(multiple of 3) – p(both)

Addition Rule

- Use addition when one of several possible outcomes will occur
- *What is the probability of rolling odd number **or** a 6?*
  - *These are mutually exclusive events!*

    $p(A \text{ and } B) = 0$

Mutually exclusive

$$p(A \text{ or } B) = p(A) + p(B) - p(A \text{ and } B)$$

$$p(\text{odd } \underline{\textbf{or}} \text{ } 6) = p(\text{odd}) + p(6)$$

Test yourself

- In a seminar class containing 10 students, 7 are female, 6 are Psychology majors, and 5 are both female AND psychology majors.

What is the probability that a randomly-selected student is either female OR a psychology major?

Multiplication Rule

- Quantifies probability of **successive** events
- *What is the probability of rolling 'snake eyes' in craps?*
  - What is the probability of rolling 1 **and then** another 1?
  - **Important:** These are independent events!

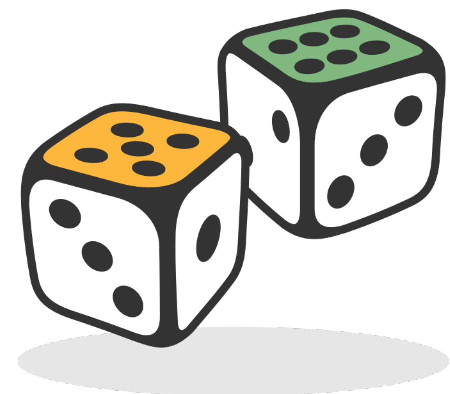
$$p(A \text{ and } B) = p(A) \times p(B|A)$$

$$p(1 \text{ and } 1) = p(1) \times p(1|1^*)$$

When events are independent, $p(1 \text{ and } 1) = p(1) \times p(1)$

Combining Rules

- *What is the probability of rolling '**yo-leven**'?*
  - Definition: Sum of two dice = 11



Two ways to reach ***yo-leven***:

1. $p(5) \times p(6|5) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

+

2. $p(6) \times p(5|6) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

=

Way #1 + Way #2, or $\frac{1}{36} + \frac{1}{36}$

=

$\mathbf{\frac{2}{36}}$ or .056

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

Dependent Successive Events

- What is the probability of being dealt "Cowboys" in poker?
  - Two cards that are both kings

N = 52 cards

4 kings

$$p(\text{king}) = \frac{4}{52}$$

$$p(\text{cowboys}) = \frac{4}{52} \times \frac{3}{51} = \frac{12}{2652} = .0045$$

Dependent Successive Events

- What is the probability of being dealt "pocket pairs" in poker?
  - Two cards that match in number

N = 52 cards

13 different numbers

4 cards for each number

$$p(\text{pairs}) = \frac{12}{2652} + \frac{12}{2652} \dots \frac{12}{2652} \text{ (13 times)}$$

$$p(\text{pairs}) = \frac{12 \times 13}{2652} = \frac{156}{2652} = .0588$$

Test yourself

- What is the probability of being dealt <u>two hearts</u> in poker?
  - **Now, what about same suit of <u>any</u> suit?**

N = 52 cards

4 different suits (hearts, spades, clubs, diamonds)

13 cards in each suit

$$p(\text{2 hearts}) = \frac{13}{52} \times \frac{12}{51}$$

$$p(\text{2 hearts}) = \frac{13 \times 12}{2652} = \frac{156}{2652} = .0588$$