ANOVA

- You <u>do not</u> need to know **most** calculations for ANOVA

- We'll focus on conceptual logic of ANOVA:
  - Why is it called "ANalysis Of VAriance"?
  - What are MS_{between}, MS_{within}, & F-ratio
  - What is the shape of the F-distribution?
  - What are *df* for ANOVA?
  - What conclusion follows from the ANOVA test?
    - What follow-up tests are generally performed?

ANOVA

- We use ANOVA when comparing more than 2 groups
  - But, ANOVA can also be used for exactly two groups
    - Decision will be the same, in fact $F = t^2$

- "One-way" ANOVA has one IV
  - Ex. Testing anti-anxiety Rx with 500mg, 1000mg, or placebo control
    - One IV with three levels

ANOVA: Factorial

- “Factorial” ANOVA has multiple IVs (Ch. 16)
  - Factor 1: Dose (500mg, 1000mg, placebo)
  - Factor 2: Schedule (once per day, twice per day)

Dose

Schedule

	500mg	**1000mg**	**Placebo**
Once	500mg/once	1000mg/once	Placebo/once
Twice	250mg/twice	500mg/twice	Placebo/twice

- “Fully” factorial = all combinations

ANOVA: Assumptions

1. Sampled populations are normally distributed*

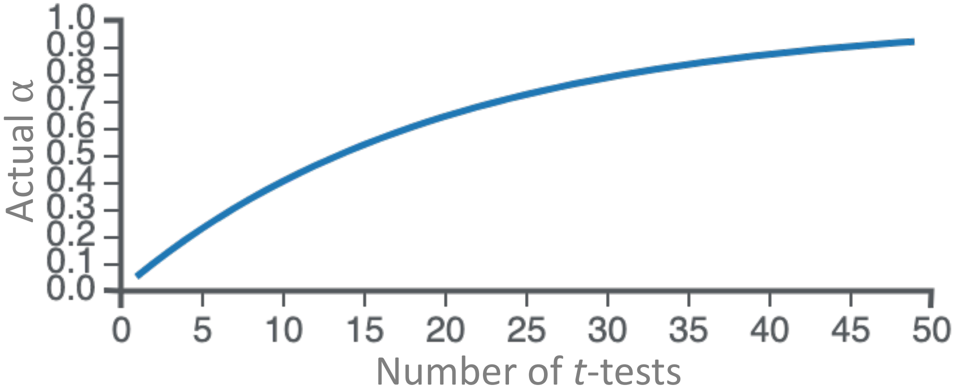
2. DV is interval or ratio*

3. Homogeneity of variance*
   – Like *t*-test, we will be averaging variance from different conditions

*ANOVA is generally robust to violations assuming that *N* is relatively large and *n*'s are relatively equal

Example: Memory SPAN

ID	Digit span	Letter span	Word Span
1	8	6	4
2	10	10	4
3	7	6	4
4	5	5	2
5	9	7	5
6	10	4	4
7	9	5	5
8	6	8	4
$\bar{X}$	**8.000**	**6.375**	**4.000**



- Why not just multiple *t*-tests?
  - Need 3 tests
  - Each comes with α
- Could we (α/3)?
  - Yes! But then we greatly increase β ☹

Example: Memory SPAN

ID	Digit span	Letter span	Word Span
1	8	6	4
2	10	10	4
3	7	6	4
4	5	5	2
5	9	7	5
6	10	4	4
7	9	5	5
8	6	8	4
$\bar{X}$	**8.000**	**6.375**	**4.000**

- ANOVA has *family wise* α
  - We know $\alpha = \alpha$,
    & maximize power!
- Limitation: ANOVA is always non-directional

ANOVA: Hypotheses

- H_0: $\mu_{\text{digits}} = \mu_{\text{letters}} = \mu_{\text{words}}$
- H_1: At least one mean differs

- Conclusions are also non-directional:
  – "A one-way ANOVA revealed a significant difference in memory SPAN between the three stimulus types."

ANOVA: Formula

$$F_{\text{obt}} = \frac{n\left[(\bar{X}_{\text{digit}} - \bar{X}_{\text{G}})^2 + (\bar{X}_{\text{letter}} - \bar{X}_{\text{G}})^2 + (\bar{X}_{\text{word}} - \bar{X}_{\text{G}})^2\right] / (k-1)}{\boxed{(SS_{\text{digit}} + SS_{\text{letter}} + SS_{\text{word}}) \big/ (N-k)}}$$

Homogeneity req.

Where $\bar{X}_{\text{G}}$ = Grand mean,

n = participants per level,

N = total participants

k = number of groups

ANOVA: Formula

- Conceptual elements:
  - **Numerator** is *SS* for difference between each group mean and the grand mean
    - This is known as MS_{between}
      - Remember *MS* (Mean Square) ~ variance (s^2)
  - **Denominator** is calculating *SS* for scores within each group
    - This is known as MS_{within}

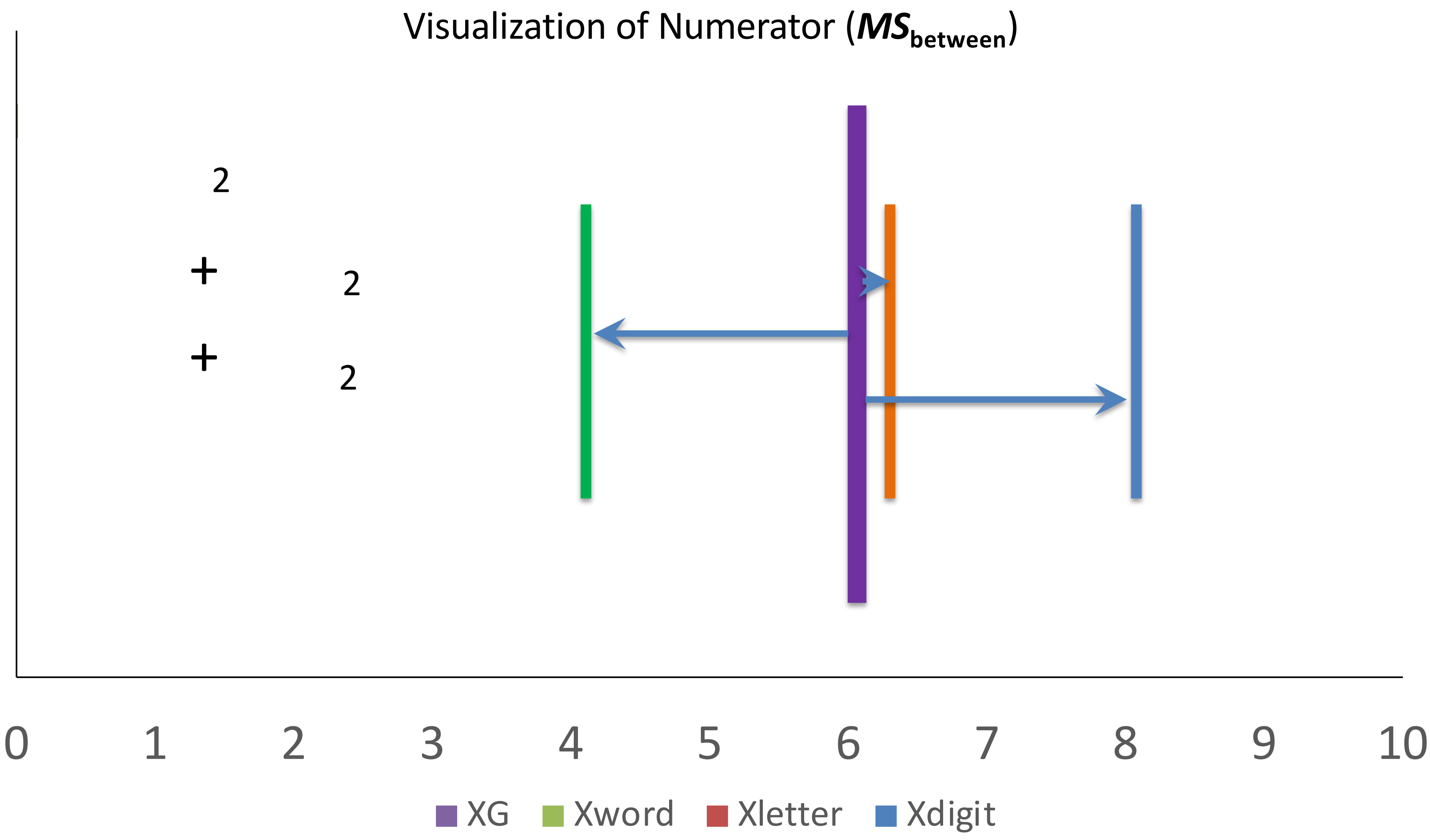
$$F_{\text{obt}} = \frac{MS_{\text{between}}}{MS_{\text{within}}} = \frac{\sigma^2_{\text{groups}}}{\sigma^2_{\text{observations}}}$$

Translations:

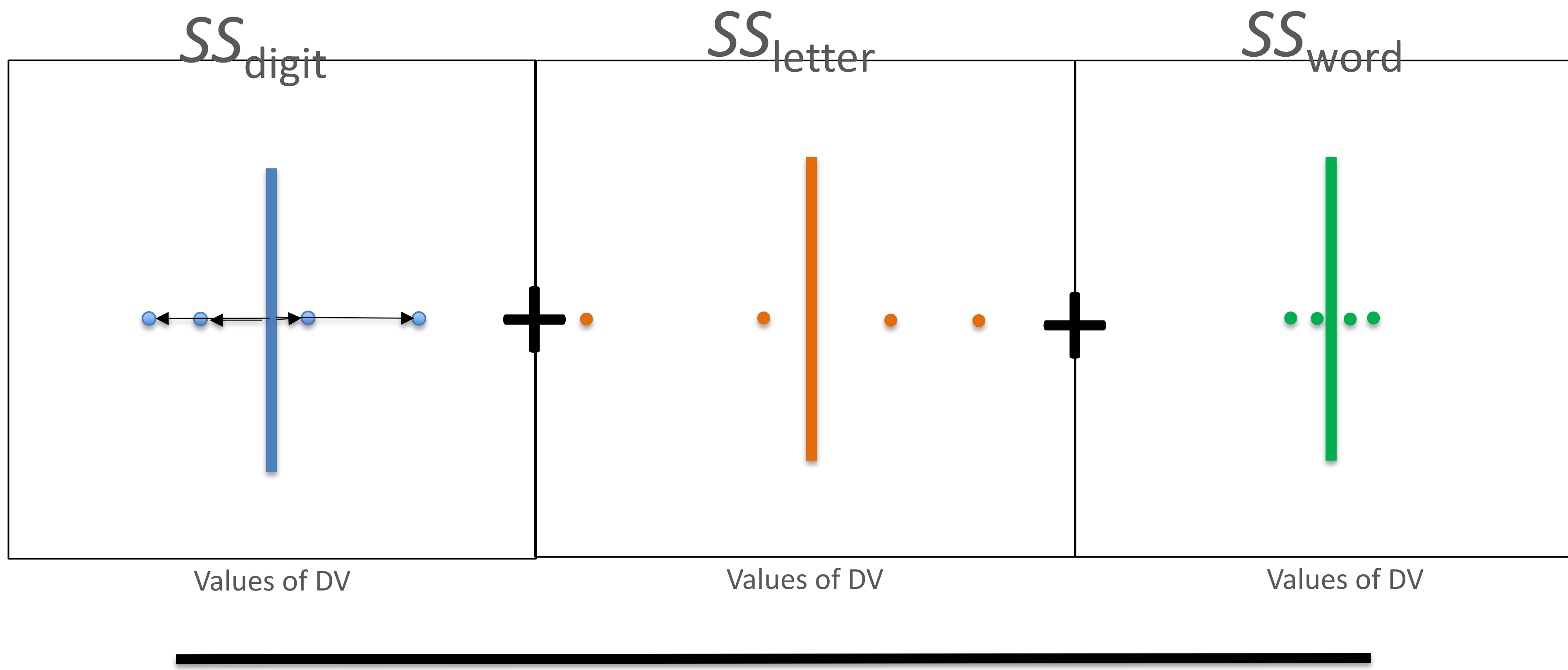
- MS_{between}
  - **Effect**, systematic, groups
    - Quantifies how much DV varied as function of IV

- MS_{within}
  - **Error**, residual, observations
    - Quantifies how much DV varied as function of individual differences

$$F_{\text{obt}} = \frac{n\left[(\bar{X}_{digit} - \bar{X}_G)^2 + (\bar{X}_{\text{letter}} - \bar{X}_G)^2 + (\bar{X}_{\text{word}} - \bar{X}_G)^2\right]/2}{(SS_{\text{digit}} + SS_{\text{letter}} + SS_{\text{word}})\Big/_{(N-k)}}$$



Visualization of Denominator (MS_{within})

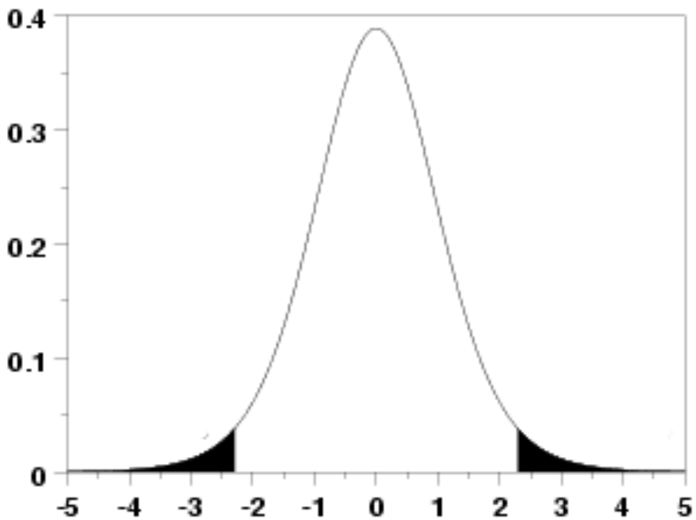


$N - k$

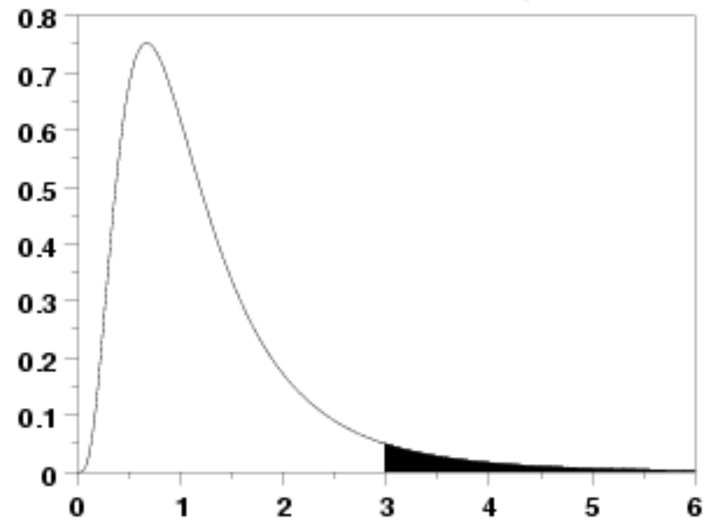
F-ratio

- $F_{\text{obt}} = \frac{MS_{\text{between}}}{MS_{\text{within}}} = \frac{\text{Effect } \sigma^2 + \text{Error } \sigma^2}{\text{Error } \sigma^2}$

- $t_{\text{obt}} = \frac{\bar{X}_1 - \bar{X}_2}{\text{Error } \sigma_{\overline{\text{X}}}} = \frac{\text{Effect}}{\text{Error}}$



t distribution (symmetrical)



F distribution (asymmetrical)

df for ANOVA

$$F_{\text{obt}} = \frac{MS_{\text{between}}}{MS_{\text{within}}}$$

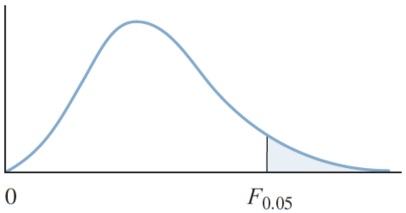
Two sources of *df:*

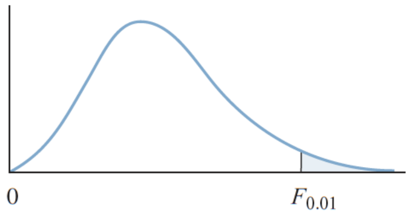
- Numerator *df* corrects *SS* between groups
  - Calculate one s^2 (of *k* groups around grand mean)
    - $df = k - 1$
- Denominator *df* corrects *SS* within each group
  - Calculate *k* number of s^2 (one s^2 per group)
    - $df = N - k$

Finding F_{crit}

table F Critical values of the F distribution for $\alpha = 0.05$ (Roman type) and $\alpha = 0.01$ (boldface type)

The values listed in the table are the critical values of F for the degrees of freedom of the numerator of the F ratio (column headings) and the degrees of freedom of the denominator of the F ratio (row headings). To be significant, $F_{obt} \geq F_{crit}$.





Degrees of Freedom: Denominator	Degrees of Freedom: Numerator 1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞
1	161	200	216	225	230	234	237	239	241	242	243	244	245	246	248	249	250	251	252	253	253	254	254	254
	4,052	**4,999**	**5,403**	**5,625**	**5,764**	**5,859**	**5,928**	**5,981**	**6,022**	**6,056**	**6,082**	**6,106**	**6,142**	**6,169**	**6,208**	**6,234**	**6,258**	**6,286**	**6,302**	**6,323**	**6,334**	**6,352**	**6,361**	**6,366**
2	18.51	19.00	19.16	19.25	19.30	19.33	19.36	19.37	19.38	19.39	19.40	19.41	19.42	19.43	19.44	19.45	19.46	19.47	19.47	19.48	19.49	19.49	19.50	19.50
	98.49	**99.00**	**99.17**	**99.25**	**99.30**	**99.33**	**99.34**	**99.36**	**99.38**	**99.40**	**99.41**	**99.42**	**99.43**	**99.44**	**99.45**	**99.46**	**99.47**	**99.48**	**99.48**	**99.49**	**99.49**	**99.49**	**99.50**	**99.50**
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84	8.81	8.78	8.76	8.74	8.71	8.69	8.66	8.64	8.62	8.60	8.58	8.57	8.56	8.54	8.54	8.53
	34.12	**30.82**	**29.46**	**28.71**	**28.24**	**27.91**	**27.67**	**27.49**	**27.34**	**27.23**	**27.13**	**27.05**	**26.92**	**26.83**	**26.69**	**26.60**	**26.50**	**26.41**	**26.35**	**26.27**	**26.23**	**26.18**	**26.14**	**26.12**
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.93	5.91	5.87	5.84	5.80	5.77	5.74	5.71	5.70	5.68	5.66	5.65	5.64	5.63
	21.20	**18.00**	**16.69**	**15.98**	**15.52**	**15.21**	**14.98**	**14.80**	**14.66**	**14.54**	**14.45**	**14.37**	**14.24**	**14.15**	**14.02**	**13.93**	**13.83**	**13.74**	**13.69**	**13.61**	**13.57**	**13.52**	**13.48**	**13.46**
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.78	4.74	4.70	4.68	4.64	4.60	4.56	4.53	4.50	4.46	4.44	4.42	4.40	4.38	4.37	4.36
	16.26	**13.27**	**12.06**	**11.39**	**10.97**	**10.67**	**10.45**	**10.27**	**10.15**	**10.05**	**9.96**	**9.89**	**9.77**	**9.68**	**9.55**	**9.47**	**9.38**	**9.29**	**9.24**	**9.17**	**9.13**	**9.07**	**9.04**	**9.02**
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.96	3.92	3.87	3.84	3.81	3.77	3.75	3.72	3.71	3.69	3.68	3.67
	13.74	**10.92**	**9.78**	**9.15**	**8.75**	**8.47**	**8.26**	**8.10**	**7.98**	**7.87**	**7.79**	**7.72**	**7.60**	**7.52**	**7.39**	**7.31**	**7.23**	**7.14**	**7.09**	**7.02**	**6.99**	**6.94**	**6.90**	**6.88**
7	5.59	4.47	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.63	3.60	3.57	3.52	3.49	3.44	3.41	3.38	3.34	3.32	3.29	3.28	3.25	3.24	3.23
	12.25	**9.55**	**8.45**	**7.85**	**7.46**	**7.19**	**7.00**	**6.84**	**6.71**	**6.62**	**6.54**	**6.47**	**6.35**	**6.27**	**6.15**	**6.07**	**5.98**	**5.90**	**5.85**	**5.78**	**5.75**	**5.70**	**5.67**	**5.65**
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.34	3.31	3.28	3.23	3.20	3.15	3.12	3.08	3.05	3.03	3.00	2.98	2.96	2.94	2.93
	11.26	**8.65**	**7.59**	**7.01**	**6.63**	**6.37**	**6.19**	**6.03**	**5.91**	**5.82**	**5.74**	**5.67**	**5.56**	**5.48**	**5.36**	**5.28**	**5.20**	**5.11**	**5.06**	**5.00**	**4.96**	**4.91**	**4.88**	**4.86**
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.13	3.10	3.07	3.02	2.98	2.93	2.90	2.86	2.82	2.80	2.77	2.76	2.73	2.72	2.71
	10.56	**8.02**	**6.99**	**6.42**	**6.06**	**5.80**	**5.62**	**5.47**	**5.35**	**5.26**	**5.18**	**5.11**	**5.00**	**4.92**	**4.80**	**4.73**	**4.64**	**4.56**	**4.51**	**4.45**	**4.41**	**4.36**	**4.33**	**4.31**
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.97	2.94	2.91	2.86	2.82	2.77	2.74	2.70	2.67	2.64	2.61	2.59	2.56	2.55	2.54
	10.04	**7.56**	**6.55**	**5.99**	**5.64**	**5.39**	**5.21**	**5.06**	**4.95**	**4.85**	**4.78**	**4.71**	**4.60**	**4.52**	**4.41**	**4.33**	**4.25**	**4.17**	**4.12**	**4.05**	**4.01**	**3.96**	**3.93**	**3.91**
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.86	2.82	2.79	2.74	2.70	2.65	2.61	2.57	2.53	2.50	2.47	2.45	2.42	2.41	2.40
	9.65	**7.20**	**6.22**	**5.67**	**5.32**	**5.07**	**4.88**	**4.74**	**4.63**	**4.54**	**4.46**	**4.40**	**4.29**	**4.21**	**4.10**	**4.02**	**3.94**	**3.86**	**3.80**	**3.74**	**3.70**	**3.66**	**3.62**	**3.60**
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85	2.80	2.76	2.72	2.69	2.64	2.60	2.54	2.50	2.46	2.42	2.40	2.36	2.35	2.32	2.31	2.30
	9.33	**6.93**	**5.95**	**5.41**	**5.06**	**4.82**	**4.65**	**4.50**	**4.39**	**4.30**	**4.22**	**4.16**	**4.05**	**3.98**	**3.86**	**3.78**	**3.70**	**3.61**	**3.56**	**3.49**	**3.46**	**3.41**	**3.38**	**3.36**
13	4.67	3.80	3.41	3.18	3.02	2.92	2.84	2.77	2.72	2.67	2.63	2.60	2.55	2.51	2.46	2.42	2.38	2.34	2.32	2.28	2.26	2.24	2.22	2.21
	9.07	**6.70**	**5.74**	**5.20**	**4.86**	**4.62**	**4.44**	**4.30**	**4.19**	**4.10**	**4.02**	**3.96**	**3.85**	**3.78**	**3.67**	**3.59**	**3.51**	**3.42**	**3.37**	**3.30**	**3.27**	**3.21**	**3.18**	**3.16**
14	4.60	3.74	3.34	3.11	2.96	2.85	2.77	2.70	2.65	2.60	2.56	2.53	2.48	2.44	2.39	2.35	2.31	2.27	2.24	2.21	2.19	2.16	2.14	2.13
	8.86	**6.51**	**5.56**	**5.03**	**4.69**	**4.46**	**4.28**	**4.14**	**4.03**	**3.94**	**3.86**	**3.80**	**3.70**	**3.62**	**3.51**	**3.43**	**3.34**	**3.26**	**3.21**	**3.14**	**3.11**	**3.06**	**3.02**	**3.00**

ANOVA Output:

$\frac{SS_{\text{between}}}{SS_{\text{within}}}$

$\frac{SS}{df} = MS$

Probability of *F* this or more extreme $|H_0$

Repeated ANOVA

Within Subjects Effects

	Sum of Squares	df	Mean Square	F	p	η²
Span Type	64.750	2	32.375	15.857	<.001	0.537
Residual	28.583	14	2.042			

Note. Type 3 Sums of Squares

$df_{\text{b}} = k - 1$
$df_{\text{w}} = N - k$

$\frac{MS_{\text{between}}}{MS_{\text{error}}} = F$

Standardized Effect Size

<u>Conclusion</u>: *"A one-way ANOVA revealed a reliable difference in memory SPAN between the three stimulus types, F*(2,14) = 15.86, *p* < 001, $\eta^2 = .537$."

Planned Comparisons vs. Corrected Post Hoc Tests

$\bar{X}_1 - \bar{X}_2$

$df = n - 1$

Uncorrected *p*-values

Conservatively corrected *p*'s

Post Hoc Tests

Post Hoc Comparisons - Span Type

Comparison									
Span Type		Span Type	Mean Difference	SE	df	t	p	p_{tukey}	$p_{scheffe}$
Digit	-	Letter	1.625	0.885	7.000	1.836	0.109	0.227	0.253
	-	Word	4.000	0.500	7.000	8.000	<.001	<.001	<.001
Letter	-	Word	2.375	0.706	7.000	3.366	0.012	0.028	0.034

$\frac{s}{\sqrt{N}} = s_{\bar{X}}$

$\frac{\bar{X}_1 - \bar{X}_2}{s_{\bar{X}}} = t$

Modestly corrected *p*-values

Conclusion: *"**Tukey-corrected** post hoc tests showed that digit span and letter span did not reliably differ* ($t(7) = 1.84$, $p = .227$)*, but that both digit span and letter span were reliably greater than word span* ($t(7) = 8.00$, $p < .001$; $t(7) = 3.37$, $p = .028$, respectively).*"*

Effect Size in ANOVA

- Effect size in ANOVA is more similar to effect size in regression than to the *t*-test
  - R^2 & r^2 are used in regression
  - η^2 & ω^2 are used in ANOVA

- Eta, η, is a biased estimator (it is too high)
- Omega, ω, is much better
  - Most people still use η^2?!???
    - Possibly bc SPSS added ω^2 in 2020

Effect Size in ANOVA

- When η^2 & ω^2 =
  - 0.01-0.05, "small"
  - 0.06-0.13, "medium"
  - 0.14+, "large"

- Most often you will encounter "partial Eta"
  - η_p^2 estimates the strength of different effects in factorial designs
    - η_p^2 for main effect of IV1
    - η_p^2 for main effect of IV2
    - η_p^2 for interaction of IV1 & IV2