Learning Objectives

- **<u>Compare</u>** *independent t-test* vs. *paired t-test* vs. *single sample t-test*

- **<u>Describe</u>** conditions under which we would select an *independent t-test*

- **<u>Conduct</u>** an independent *t*-test
  - With effect size!

Which test to use?

What do we want?

Is our mean different from a specific population mean (μ)?

Do we know the population standard deviation (σ)?

Yes

z-test

No

single sample t-test

Are these two sample means different from each other?

Are the data correlated or independent?

Correlated

Correlated samples t-test

Independent

Independent samples t-test

Independent *t*-test

- a.k.a., between-subjects *t*, Student's *t* (don't use), unpaired *t*

<u>Requirements</u>:

  - Compare 2 groups
  - DV is interval/ratio* (for parametric tests)
  - DV is approximately normal*
    - $n_1 > 30$ & $n_2 > 30$
  - Absence of outliers*
  - <u>Homogeneity of variance</u>*

*Note that the *t*-test is generally robust to violations of these requirements

Comparison of *simplified* Conceptual Formulas

$$t_{\text{obt}} = \frac{\bar{X}_{\text{obt}}}{s_{\bar{X}}}$$

where *usually:* $\mu = 0$

$$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}}}{s_{\bar{D}}}$$

where *usually*: $\mu_{D\,=\,0}$ & where: $\bar{D} = \bar{X}_{\text{pre}} - \bar{X}_{\text{post}}$

$$t_{\text{obt}} = \frac{\bar{X}_1 - \bar{X}_2}{s_w}$$

where *usually*: $\mu_{\bar{X}_1 - \bar{X}_2} = 0$ & where: $w =$ weighted estimate of s's

Comparison of Conceptual Formulas

$$t_{\text{obt}} = \frac{\bar{X}_{\text{obt}} - \mu}{s / \sqrt{N}}$$

$$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}} - \mu_D}{s_D / \sqrt{N}}$$

$$t_{\text{obt}} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_{\bar{X}_1 - \bar{X}_2})}{s_{(\bar{X}_1 - \bar{X}_2)} / \sqrt{n}}$$

Comparison of Computational Formulas

$$t_{\text{obt}} = \frac{\bar{X}_{\text{obt}} - \mu}{\sqrt{\frac{SS}{N(N-1)}}}$$

$$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}}}{\sqrt{\frac{SS_D}{N(N-1)}}}$$

$$t_{\text{obt}} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{SS_1 + SS_2}{n(n-1)}}}$$

Where n = sample size per condition & when $n_1 = n_2$

Computational formula for independent *t*-test

$$t_{\text{obt}} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_w^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$s_w^2 = \frac{df_1\, {s_1}^2 + df_2\, {s_2}^2}{df_1 + df_2}$$

$$s_w^2 = \frac{{s_1}^2 + {s_2}^2}{2}$$

only when $n_1 = n_2$

Example: *Independent t-test*

- Survey data
  - N = 8
  - IV: Early or Late PSYC 218 Section
  - DV: How much do you like spicy food?
    - 1: Not at all
    - 2: Somewhat
    - 3: A lot

Independent t-test

ID	Section	Spice Att.
1	2	3
2	2	2
3	1	2
4	1	1
5	1	3
6	2	2
7	1	1
8	2	3

$\Sigma X_{n_1} = 7$

$SS_1 = 2.75$

$$t_{\text{obt}} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_w^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\bar{X}_1 = \frac{\Sigma X_{n_1}}{n_1} = \frac{7}{4} = 1.75$$

$$s_w^2 = \frac{df_1\, s_1{}^2 + df_2\, s_2{}^2}{df_1 + df_2}$$

$$s_1{}^2 = \frac{\Sigma\left(X_{n_1} - \bar{X}_1\right)^2}{n_1 - 1} = \frac{SS_1}{4 - 1} = .9167$$

Independent t-test

ID	Section	Spice Att.
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$\Sigma X_{n_1} = 7$

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$$t_{\text{obt}} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_w^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\bar{X}_1 = \frac{\Sigma X_{n_1}}{n_1} = \frac{7}{4} = 1.75$$

$$s_w^2 = \frac{df_1\, s_1{}^2 + df_2\, s_2{}^2}{df_1 + df_2}$$

$$s_1{}^2 = \frac{\Sigma\left(X_{n_1} - \bar{X}_1\right)^2}{n_1 - 1} = \frac{SS_1}{4 - 1} = .9167$$

Independent t-test

$N = 8$

$n_1 \& n_2 = 4$

$df = n - 1 = 3$

$\bar{X}_1 = 1.75$

$s_1^2 = .9167$

$\bar{X}_2 = 2.50$

$s_2^2 = .3333$

$\bar{X}_2$ and s_2^2 Calculated same as for group 1

$$t_{\text{obt}} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_w^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$s_w^2 = \frac{df_1 s_1^2 + df_2 s_2^2}{df_1 + df_2}$$

Assumes:

Homogeneity of Variance

Computing an average only makes sense for estimates of the same thing

Independent t-test

$N = 8$

$n_1 \& n_2 = 4$

$df = n - 1 = 3$

$\bar{X}_1 = 1.75$

$s_1^2 = .9167$

$\bar{X}_2 = 2.50$

$s_2^2 = .3333$

$s_w^2 = \ldots$

$$t_{obt} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_w^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\boxed{s_w^2 = \frac{df_1 s_1^2 + df_2 s_2^2}{df_1 + df_2}}$$

$$\frac{\mathbf{3} \times \mathbf{.9167} + \mathbf{3} \times \mathbf{.3333}}{\mathbf{3} + \mathbf{3}}$$

Independent t-test

$N = 8$

n_1 & n_2 = 4

df_1 & df_1 = 3

$\bar{X}_1$ = **1.75**

$s_1^{\,2}$ = .9167

$\bar{X}_2$ = **2.50**

$s_2^{\,2}$ = .3333

s_w^2 = **.6250**

t_{obt} = …

$$\boxed{t_{\text{obt}} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_w^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}}$$

$$= \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_w^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\frac{-0.75}{\sqrt{.6250(0.5)}} =$$

Independent t-test

$N = 8$
n_1 & n_2 = 4
df_1 & df_1 = 3

$\bar{X}_1 = 1.75$
$s_1^2 = .9167$
$\bar{X}_2 = 2.50$
$s_2^2 = .3333$
$s_w^2 = .6250$

$t_{obt} = -1.34$
$\propto_{2-\text{tail}} = .05$
$t_{crit} = \ldots$

$t_{obt} = -1.34$
$t_{crit} = \pm 2.447$

$-t_{obt} > -t_{crit}$

Decision: Fail to reject H_0

APA reporting (*exact p-value calculated in *Jamovi*)

"We failed to reject H_0, $t(6) = -1.34$, $p = .228$*. Students in early vs. late sections of PSYC 218 did not detectably differ in their preference for spiciness."

Cohen's d_s

$N = 8$
n_1 & $n_2 = 4$
df_1 & $df_1 = 3$

$\bar{X}_1 = 1.75$
$s_1^2 = .9167$
$\bar{X}_2 = 2.50$
$s_2^2 = .3333$
$s_w^2 = .6250$

$t_{obt} = -1.34$
$\propto_{2-\text{tail}} = .05$
$t_{crit} = \pm 2.447$

$$\text{Cohen's } d_s = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_w^2}}$$

$$d_s = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_w^2}}$$

How large is this effect?