

Learning Objectives

- Review Models (central tendency, variability, correlation) and reframe them within context of prediction
- Describe goal of statistical inference
- Define new terminology related to probability
- Practice using the *addition* and *multiplication* rules of probability

Probability

Assessed in two ways:

- ‘*a priori*’: before data expectation

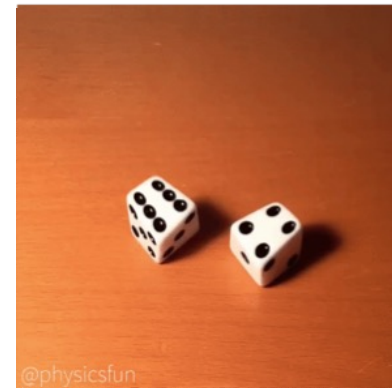
$$p(A) = \frac{\# \text{ of } A_{\text{possible events}}}{N_{\text{possible events}}}$$

- ‘*a posteriori*’: after data

$$p(A) = \frac{\# \text{ of } A_{\text{observed}}}{N_{\text{obs.}}}$$

if *priori* and *posteriori*
very mismatched;
revisit assumptions

Example: Dice



What is the probability of an odd roll with 1 dice*?

– *a priori probability*

$$p(odd) = \frac{\# \text{ of Odd}_{\text{possible}}}{N_{\text{possible events}}} = \boxed{3/6 = 1/2}$$

if we get way more than 50%, we may have trick dice

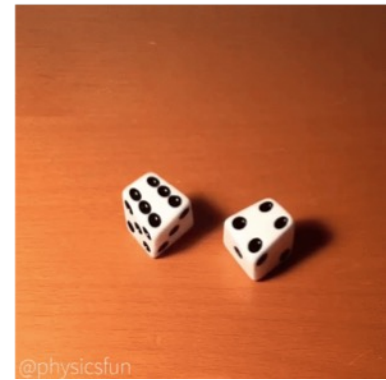
– *a posteriori probability*

- Data: 100 rolls, with 58 odd rolls

This is well within realm of possibility for true dice

vs. 70%, 100%; the greater the difference the more improbable it is that observation/method is well-founded

Example: Dice

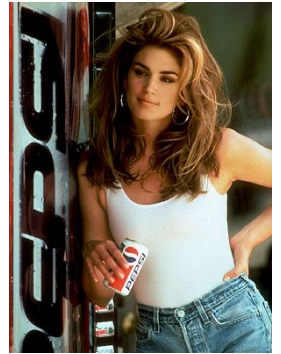


What is the probability of an odd roll with 1 dice?

- *a priori probability* = .50
- *a posteriori probability* = .58
- Goal of statistical inference:
 - What should we predict for future observations?
 - Odd = .50
 - Odd \neq .50
 - > .50? < .50?

Are the dice rigged, or did we observe randomness?

Example: Does advertising work?



What is the probability of choosing Pepsi?

– *a priori probability*

$$p(\text{pepsi}) = \frac{\# \text{ of } A_{\text{possible}}}{N_{\text{possible events}}} = \frac{1 \text{ (choose } \text{pepsi} \text{)}}{2 \text{ (choose. } \text{pepsi} \text{ or } \text{Coca-Cola} \text{)}} = .50$$

– *a posteriori probability*

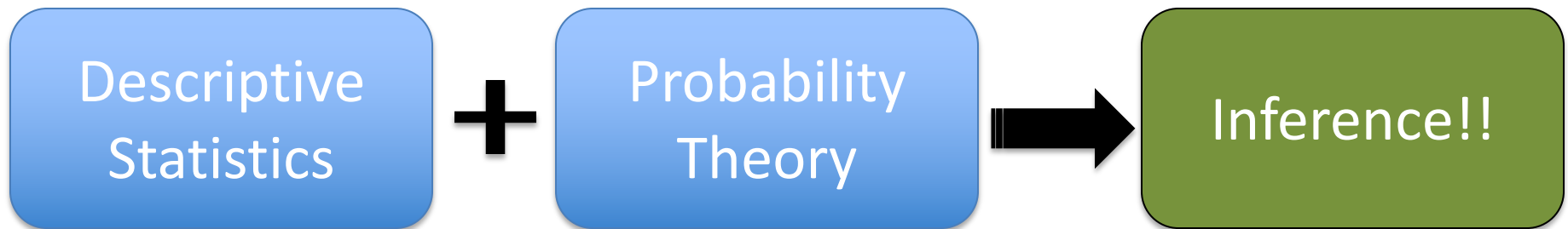
- Data: 100 participants, 58 choose pepsi

$$p(\text{pepsi}) = \frac{\# \text{ of } \text{pepsi} \text{ obs.}}{N_{\text{obs.}}} = \frac{58}{100} = .58$$

Inferential Statistics

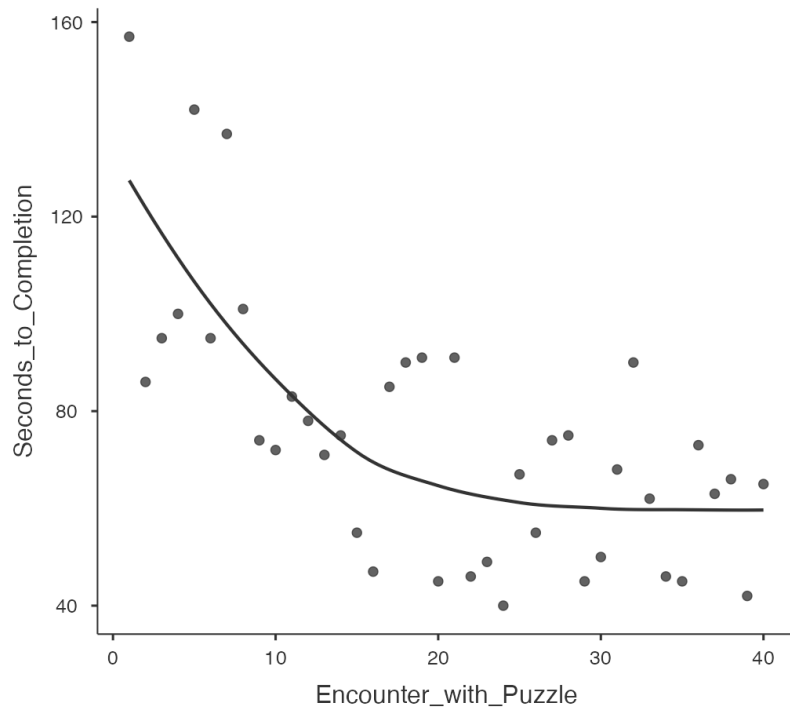
Did we observe randomness or something real?

- Did the ad work or did we accidentally sample from more pepsi people?
- In Psychology, we will always observe ‘random’ processes: We quantify how much is random and how much is real.

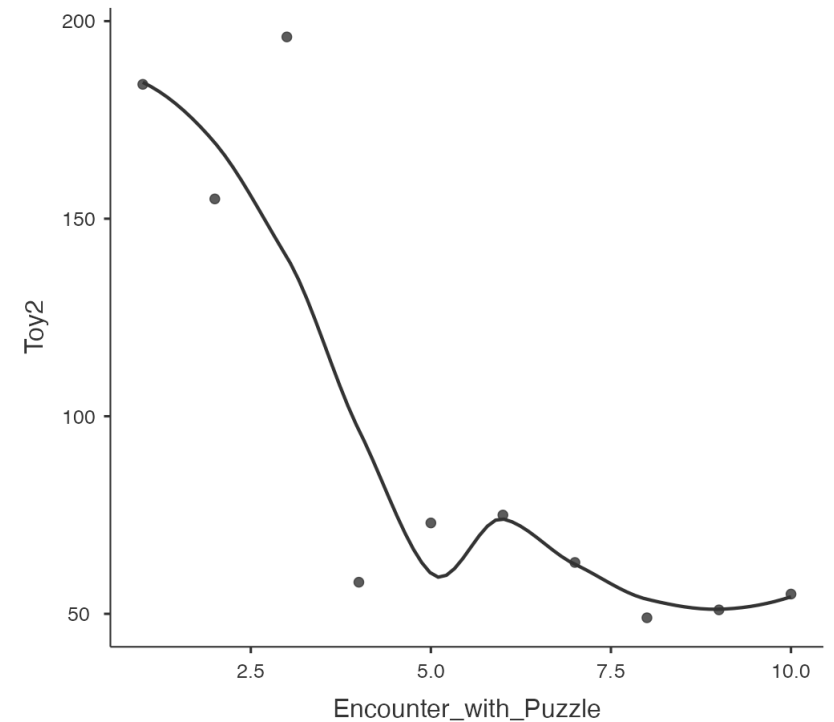


Was it random or real?

Scatterplot



Scatterplot



Definitions

- **Mutually exclusive** events cannot happen at the same time
 - Dice cannot land on even *and* odd
 - $p(A \& B) = 0$
 - *Statisticians suck*: When representing 2 mutually exclusive outcomes, use P and Q

$$P = 1 - Q$$

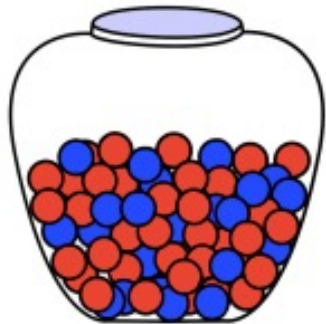
Definitions

- **Independent** events have no influence on each other
 - 1st roll of dice does not influence 2nd roll
 - Correlation between dice rolls, $\rho = .000$
- **Exhaustive sets** of events describe all possible events
 - We know whether a dice roll is “A” or is “not A”
 - e.g., “even” or “not even”



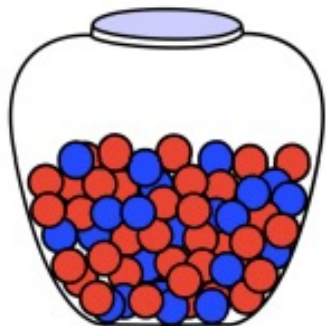
Comparing Probabilities

- Jar #1 and Jar #2 both contain blue and red marbles. While blindfolded, you draw one marble from one jar. If it is red, you win \$10.



3 red marbles
2 blue marbles

Probabilities are
the same



51 red marbles
34 blue marbles

Which jar is the
better choice?

Addition Rule

- Use addition when one of *several* possible outcomes will occur
- *What is the probability of rolling an odd number or a multiple of 3?*
 - Not mutually exclusive, both outcomes can occur at the same time!

$$p(A \text{ or } B) = p(A) + p(B) - p(A \text{ and } B)$$

$$p(\text{odd } \underline{\text{or}} \text{ multiple of } 3) = p(\text{odd}) + p(\text{multiple of } 3) - p(\text{both})$$

Addition Rule

- Use addition when one of several possible outcomes will occur
- *What is the probability of rolling odd number or a 6?*
 - *These are mutually exclusive events!*

$$p(A \text{ and } B) = 0$$

*Mutually
exclusive*

$$p(A \text{ or } B) = p(A) + p(B) - p(A \text{ and } B)$$

$$p(\text{odd } \underline{\text{or}} \text{ } 6) = p(\text{odd}) + p(6)$$

Test yourself

- In a seminar class containing 10 students, 7 are female, 6 are Psychology majors, and 5 are both female AND psychology majors.

What is the probability that a randomly-selected student is either female OR a psychology major?

student is female: $7/10$, student is psych major: $6/10 - 5/10$
both = $8/10$, **80%**

Multiplication Rule



- Quantifies probability of **successive** events
- *What is the probability of rolling 'snake eyes' in craps?*
 - What is the probability of rolling 1 **and then** another 1?
 - **Important:** These are independent events!

$$p(A \text{ and } B) = p(A) \times p(B|A)$$

$$p(1 \text{ and } 1) = p(1) \times p(1|1^*)$$

When events are independent, $p(1 \text{ and } 1) = p(1) \times p(1)$

Combining Rules

- *What is the probability of rolling 'yo-leven'?*
 - Definition: Sum of two dice = 11

Two ways to reach **yo-leven**:

$$\underline{1.} \ p(5) \times p(6|5) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

+

$$\underline{2.} \ p(6) \times p(5|6) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

=

$$\text{Way \#1} + \text{Way \#2, or } \frac{1}{36} + \frac{1}{36}$$

=

$$\frac{2}{36} \text{ or } .056$$



2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

Dependent Successive Events

- What is the probability of being dealt “Cowboys” in poker?
 - Two cards that are both kings

$N = 52$ cards
4 kings

$$p(\text{king}) = \frac{4}{52}$$

$$p(\text{cowboys}) = \frac{4}{52} \times \frac{3}{51} = \frac{12}{2652} = .0045$$



Dependent Successive Events

- What is the probability of being dealt “pocket pairs” in poker?
 - Two cards that match in number

$N = 52$ cards

13 different numbers

4 cards for each number

$$p(\text{pairs}) = \frac{12}{2652} + \frac{12}{2652} \cdots \frac{12}{2652} \text{ (13 times)}$$

$$p(\text{pairs}) = \frac{12 \times 13}{2652} = \frac{156}{2652} = .0588$$



Test yourself

- What is the probability of being dealt two hearts in poker?
 - **Now, what about same suit of any suit?**

$N = 52$ cards

4 different suits (hearts, spades, clubs, diamonds)

13 cards in each suit

$$p(2 \text{ hearts}) = \frac{13}{52} \times \frac{12}{51}$$

$$p(2 \text{ hearts}) = \frac{13 \times 12}{2652} = \frac{156}{2652} = .0588$$

