

Equations

Listed here are the computational equations used in this textbook. The page number refers to the page where the equation first appears.

Equation	Description	Equation First Occurs on Page:
$\sum_{i=1}^N X_i = X_1 + X_2 + X_3 + \cdots + X_N$	summation	27
$\text{cum } \% = \frac{\text{cum } f}{N} \times 100$	cumulative percentage	55
$\text{Percentile point} = X_L + (i/f_i)(\text{cum } f_p - \text{cum } f_L)$	equation for computing percentile point	58
$\text{Percentile rank} = \frac{\text{cum } f_L + (f_i/i)(X - X_L)}{N} \times 100$	equation for computing percentile rank	59
$\bar{X} = \frac{\sum X_i}{N}$	mean of a sample	81
$\mu = \frac{\sum X_i}{N}$	mean of a population of raw scores	81
$\bar{X}_{\text{overall}} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2 + \cdots + n_k\bar{X}_k}{n_1 + n_2 + \cdots + n_k}$	overall mean of several groups	84
$\text{Mdn} = P_{50} = X_L + (i/f_i)(\text{cum } f_p - \text{cum } f_L)$	median of a distribution	85
$\text{Range} = \text{Highest score} - \text{Lowest score}$	range of a distribution	89
$X - \bar{X}$	deviation score for sample data	90
$X - \mu$	deviation score for population data	90
$\sigma = \sqrt{\frac{SS_{\text{pop}}}{N}} = \sqrt{\frac{\sum (X - \mu)^2}{N}}$	standard deviation of a population of raw scores	91
$s = \sqrt{\frac{SS}{N - 1}} = \sqrt{\frac{\sum (X - \bar{X})^2}{N - 1}}$	standard deviation of a sample of raw scores	91

Equation	Description	Equation First Occurs on Page:
$SS = \sum X^2 - \frac{(\sum X)^2}{N}$	sum of squares	93
$s^2 = \frac{SS}{N - 1}$	variance of a sample of raw scores	95
$\sigma^2 = \frac{SS_{\text{pop}}}{N}$	variance of a population of raw scores	95
$z = \frac{X - \mu}{\sigma}$	z score for population data	106
$z = \frac{X - \bar{X}}{s}$	z score for sample data	106
$X = \mu + \sigma z$	equation for finding a population raw score from its z score	114
$Y = bX + a$	equation of a straight line	125
$b = \text{Slope} = \frac{\Delta Y}{\Delta X} = \frac{Y_2 - Y_1}{X_2 - X_1}$	slope of a straight line	125
$r = \frac{\sum z_X z_Y}{N - 1}$	computational equation for Pearson r using z scores	133
$r = \frac{\sum XY - \frac{(\sum X)(\sum Y)}{N}}{\sqrt{\left[\sum X^2 - \frac{(\sum X)^2}{N} \right] \left[\sum Y^2 - \frac{(\sum Y)^2}{N} \right]}}$	computational equation for Pearson r	133
$r_s = 1 - \frac{6 \sum D_i^2}{N^3 - N}$	computational equation for Spearman rho	141
$Y' = b_Y X + a_Y$	linear regression equation for predicting Y given X	162
$b_Y = \frac{\sum XY - \frac{(\sum X)(\sum Y)}{N}}{\sum X^2 - \frac{(\sum X)^2}{N}}$	regression constant b for predicting Y given X , computational equation with raw scores	163
$a_Y = Y - b_Y \bar{X}$	regression constant a for predicting Y given X	163
$s_{Y X} = \sqrt{\frac{SS_Y - \frac{[\sum XY - (\sum X)(\sum Y)/N]^2}{SS_X}}{N - 2}}$	computational equation for the standard error of estimate when predicting Y given X	170
$b_Y = r \frac{s_Y}{s_X}$	equation relating r to the b_Y regression constant	173
$R^2 = \frac{r_{YX_1}^2 + r_{YX_2}^2 - 2r_{YX_1}r_{YX_2}r_{X_1X_2}}{1 - r_{X_1X_2}^2}$	equation for computing the squared multiple correlation	176

Equation	Description	Equation First Occurs on Page:
$p(A) = \frac{\text{Number of events classifiable as } A}{\text{Total number of possible events}}$	<i>a priori</i> probability	193
$p(A) = \frac{\text{Number of times } A \text{ has occurred}}{\text{Total number of occurrences}}$	<i>a posteriori</i> probability	194
$p(A \text{ or } B) = p(A) + p(B) - p(A \text{ and } B)$	addition rule for two events, general equation	196
$p(A \text{ or } B) = p(A) + p(B)$	addition rule when <i>A</i> and <i>B</i> are mutually exclusive	196
$p(A \text{ or } B \text{ or } C \text{ or } \dots \text{ or } Z) = p(A) + p(B) + p(C) + \dots + p(Z)$	addition rule with more than two mutually exclusive events	200
$p(A) + p(B) + p(C) + \dots + p(Z) = 1.00$	when events are exhaustive and mutually exclusive	200
$P + Q = 1.00$	when two events are exhaustive and mutually exclusive	201
$p(A \text{ and } B) = p(A)p(B A)$	multiplication rule with two events—general equation	201
$p(A \text{ and } B) = 0$	multiplication rule with mutually exclusive events	201
$p(A \text{ and } B) = p(A)p(B)$	multiplication rule with independent events	202
$p(A \text{ and } B \text{ and } C \text{ and } \dots \text{ and } Z) = p(A)p(B)p(C) \dots p(Z)$	multiplication rule with more than two independent events	206
$p(A \text{ and } B) = p(A)p(B A)$	multiplication rule with dependent events	207
$p(A \text{ and } B \text{ and } C \text{ and } \dots \text{ and } Z) = p(A)p(B A)p(C AB) \dots p(Z ABC \dots)$	multiplication rule with more than two dependent events	210
$p(A) = \frac{\text{Area under curve corresponding to } A}{\text{Total area under curve}}$	probability of <i>A</i> with a continuous variable	214
$(P + Q)^N$	binomial expansion	229
Number of <i>Q</i> events = <i>N</i> – Number of <i>P</i> events	relationship between number of <i>Q</i> events, number of <i>P</i> events, and <i>N</i>	235
$\mu = NP$	mean of the normal distribution approximated by the binomial distribution	239
$\sigma = \sqrt{NPQ}$	standard deviation of the normal distribution approximated by the binomial distribution	239
Beta = 1 – Power	relationship between beta and power	285
$\mu_{\bar{X}} = \mu$	mean of the sampling distribution of the mean	305
$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}}$	standard deviation of the sampling distribution of the mean or stan- dard error of the mean	305

Equation	Description	Equation First Occurs on Page:
$z_{\text{obt}} = \frac{\bar{X}_{\text{obt}} - \mu}{\sigma_{\bar{X}}}$	z transformation for \bar{X}_{obt}	312
$z_{\text{obt}} = \frac{\bar{X}_{\text{obt}} - \mu}{\sigma/\sqrt{N}}$	z transformation for \bar{X}_{obt}	315
$N = \left[\frac{\sigma(z_{\text{crit}} - z_{\text{obt}})}{\mu_{\text{real}} - \mu_{\text{null}}} \right]^2$	determining N for a specified power	321
$t_{\text{obt}} = \frac{\bar{X}_{\text{obt}} - \mu}{s/\sqrt{N}}$	equation for calculating the t statistic	328
$t_{\text{obt}} = \frac{\bar{X}_{\text{obt}} - \mu}{s_{\bar{X}}}$	equation for calculating the t statistic	328
$s_{\bar{X}} = \frac{s}{\sqrt{N}}$	estimated standard error of the mean	328
$df = N - 1$	degrees of freedom for t test (single sample)	331
$t_{\text{obt}} = \frac{\bar{X}_{\text{obt}} - \mu}{\sqrt{\frac{SS}{N(N-1)}}}$	equation for calculating the t statistic from raw scores	334
$d = \frac{ \text{mean difference} }{\text{population standard deviation}}$	general equation for size of effect	339
$d = \frac{ \bar{X}_{\text{obt}} - \mu }{\sigma}$	conceptual equation for size of effect, single sample t test	339
$\hat{d} = \frac{ \bar{X}_{\text{obt}} - \mu }{s}$	computational equation for size of effect, single sample t test	340
$\mu_{\text{lower}} = \bar{X}_{\text{obt}} - s_{\bar{X}}t_{0.025}$	lower limit for the 95% confidence interval	342
$\mu_{\text{upper}} = \bar{X}_{\text{obt}} + s_{\bar{X}}t_{0.025}$	upper limit for the 95% confidence interval	342
$\mu_{\text{lower}} = \bar{X}_{\text{obt}} - s_{\bar{X}}t_{\text{crit}}$	general equation for the lower limit of the confidence interval	343
$\mu_{\text{upper}} = \bar{X}_{\text{obt}} + s_{\bar{X}}t_{\text{crit}}$	general equation for the upper limit of the confidence interval	343
$\mu_{\text{lower}} = \bar{X}_{\text{obt}} - s_{\bar{X}}t_{0.005}$	lower limit for the 99% confidence interval	344
$\mu_{\text{upper}} = \bar{X}_{\text{obt}} + s_{\bar{X}}t_{0.005}$	upper limit for the 99% confidence interval	344
$t_{\text{obt}} = \frac{r_{\text{obt}} - p}{s_r}$	t test for testing the significance of r	346

Equation	Description	Equation First Occurs on Page:
$t_{\text{obt}} = \frac{r_{\text{obt}}}{\sqrt{\frac{1 - r_{\text{obt}}^2}{N - 2}}}$	<i>t</i> test for testing the significance of <i>r</i>	346
$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}} - \mu_D}{s_D / \sqrt{N}}$	<i>t</i> test for correlated groups	360
$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}} - \mu_D}{\sqrt{\frac{SS_D}{N(N - 1)}}}$	<i>t</i> test for correlated groups	360
$SS_D = \Sigma D^2 - \frac{(\Sigma D)^2}{N}$	sum of squares of the difference scores	360
$d = \frac{ \bar{D}_{\text{obt}} }{\sigma_D}$	conceptual equation for size of effect, correlated groups <i>t</i> test	364
$\hat{d} = \frac{ \bar{D}_{\text{obt}} }{s_D}$	computational equation for size of effect, correlated groups <i>t</i> test	364
$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2$	mean of the difference between sample means	368
$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\sigma^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$	standard deviation of the difference between sample means	368
$t_{\text{obt}} = \frac{(\bar{X}_1 - \bar{X}_2) - \mu_{\bar{X}_1 - \bar{X}_2}}{\sqrt{\left(\frac{SS_1 + SS_2}{n_1 + n_2 - 2} \right) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$	computational equation for t_{obt} , independent groups design	371
$t_{\text{obt}} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left(\frac{SS_1 + SS_2}{n_1 + n_2 - 2} \right) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$	computational equation for t_{obt} assuming the independent variable has no effect	371
$SS_1 = \Sigma X_1^2 - \frac{(\Sigma X_1)^2}{n_1}$	sum of squares for group <i>x</i>	373
$SS_2 = \Sigma X_2^2 - \frac{(\Sigma X_2)^2}{n_2}$	sum of squares for group <i>x</i>	373
$t_{\text{obt}} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{SS_1 + SS_2}{n(n - 1)}}}$	computational equation for t_{obt} when $n_1 = n_2$	373
$d = \frac{ \bar{X}_1 - \bar{X}_2 }{\sigma}$	conceptual equation for size of effect, independent groups <i>t</i> test	377
$\hat{d} = \frac{ \bar{X}_1 - \bar{X}_2 }{\sqrt{s_w^2}}$	computational equation for size of effect, independent groups <i>t</i> test	377

Equation	Description	Equation First Occurs on Page:
$\mu_{\text{lower}} = (\bar{X}_1 - \bar{X}_2) - s_{\bar{X}_1 - \bar{X}_2} t_{0.025}$	lower limit for the 95% confidence interval for $\mu_{\bar{X}_1 - \bar{X}_2}$	383
$\mu_{\text{upper}} = (\bar{X}_1 - \bar{X}_2) + s_{\bar{X}_1 - \bar{X}_2} t_{0.025}$	upper limit for the 95% confidence interval for $\mu_{\bar{X}_1 - \bar{X}_2}$	383
$\mu_{\text{lower}} = (\bar{X}_1 - \bar{X}_2) - s_{\bar{X}_1 - \bar{X}_2} t_{0.005}$	lower limit for the 99% confidence interval for $\mu_{\bar{X}_1 - \bar{X}_2}$	385
$\mu_{\text{upper}} = (\bar{X}_1 - \bar{X}_2) + s_{\bar{X}_1 - \bar{X}_2} t_{0.005}$	upper limit for the 99% confidence interval for $\mu_{\bar{X}_1 - \bar{X}_2}$	385
$F = \frac{\text{Variance estimate 1 of } \sigma^2}{\text{Variance estimate 2 of } \sigma^2}$	basic definition of F	402
$F_{\text{obt}} = \frac{\text{Between-groups variance estimate}}{\text{Within-groups variance estimate}} = \frac{MS_{\text{between}}}{MS_{\text{within}}}$	F equation for the analysis of variance	406
$s_w^2 = \frac{SS_1 + SS_2}{(n_1 - 1) + (n_2 - 1)}$	t test, only 2 groups	407
$MS_{\text{within}} = s_w^2$	both are estimates of σ^2	407
$MS_{\text{within}} = \frac{SS_1 + SS_2 + SS_3 + \cdots + SS_k}{(n_1 - 1) + (n_2 - 1) + (n_3 - 1) + \cdots + (n_k - 1)}$	conceptual equation for MS_{within}	407
$MS_{\text{within}} = \frac{SS_1 + SS_2 + SS_3 + \cdots + SS_k}{N - k}$	simplified equation for MS_{within}	407
$MS_{\text{within}} = \frac{SS_{\text{within}}}{df_{\text{within}}}$	within-groups variance estimate	407
$SS_{\text{within}} = SS_1 + SS_2 + SS_3 + \cdots + SS_k$	within-groups sum of squares	407
$df_{\text{within}} = N - k$	within-groups degrees of freedom	407
$SS_{\text{within}} = \sum_{\text{all scores}} X^2 - \left[\frac{(\sum X_1)^2}{n_1} + \frac{(\sum X_2)^2}{n_2} + \frac{(\sum X_3)^2}{n_3} + \cdots + \frac{(\sum X_k)^2}{n_k} \right]$	computational equation for within-groups sum of squares	408
$MS_{\text{between}} = \frac{n[(\bar{X}_1 - \bar{X}_G)^2 + (\bar{X}_2 - \bar{X}_G)^2 + (\bar{X}_3 - \bar{X}_G)^2 + \cdots + (\bar{X}_k - \bar{X}_G)^2]}{k - 1}$	conceptual equation for between-groups variance estimate	409
$MS_{\text{between}} = \frac{SS_{\text{between}}}{df_{\text{between}}}$	between-groups variance estimate	409
$SS_{\text{between}} = n[(\bar{X}_1 - \bar{X}_G)^2 + (\bar{X}_2 - \bar{X}_G)^2 + (\bar{X}_3 - \bar{X}_G)^2 + \cdots + (\bar{X}_k - \bar{X}_G)^2]$	between-groups sum of squares	409
$df_{\text{between}} = k - 1$	between-groups degrees of freedom	409
$SS_{\text{between}} = \left[\frac{(\sum X_1)^2}{n_1} + \frac{(\sum X_2)^2}{n_2} + \frac{(\sum X_3)^2}{n_3} + \cdots + \frac{(\sum X_k)^2}{n_k} \right] - \frac{\left(\sum_{\text{all scores}} X \right)^2}{N}$	computational equation for between-groups sum of squares	409
$SS_{\text{total}} = SS_{\text{within}} + SS_{\text{between}}$	equation for checking SS_{within} and SS_{between}	412

Equation	Description	Equation First Occurs on Page:
$SS_{total} = \sum_{\text{all scores}} X^2 - \frac{\left(\sum_{\text{all scores}} X\right)^2}{N}$	equation for calculating the total variability	412
$\hat{\omega}^2 = \frac{SS_{between} - (k - 1) MS_{within}}{SS_{total} + MS_{within}}$	computational equation for estimating $\hat{\omega}^2$	419
$\eta^2 = \frac{SS_{between}}{SS_{total}}$	conceptual and computational equation for eta squared	420
$F_{obt} = \frac{MS_{between}}{MS_{within}} = \frac{n[(\bar{X}_1 - \bar{X}_G)^2 + (\bar{X}_2 - \bar{X}_G)^2 + (\bar{X}_3 - \bar{X}_G)^2]/2}{(SS_1 + SS_2 + SS_3)/(N - 3)}$	F equation for three-group experiment	421
$t_{obt} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{MS_{within} \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$	general equation for t equation for <i>planned</i> comparisons,	422
$t_{obt} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{2MS_{within}/n}}$	t equation for <i>planned</i> comparisons with equal n in the two groups	423
$Q_{obt} = \frac{\bar{X}_i - \bar{X}_j}{\sqrt{MS_{within}/n}}$	equation for calculating Q_{obt}	424
$SS_{between \text{ (groups } i \text{ and } j)} = \left[\frac{(\sum X_i)^2}{n_i} + \frac{(\sum X_j)^2}{n_j} \right] - \frac{\left(\sum_{\text{groups } i \text{ and } j} X \right)^2}{n_i + n_j}$	computational equation for $SS_{between \text{ (groups } i \text{ and } j)}$	426
$MS_{between \text{ (groups } i \text{ and } j)} = \frac{SS_{between \text{ (groups } i \text{ and } j)}}{df_{between \text{ (entire ANOVA)}}$	conceptual equation for $MS_{between \text{ (groups } i \text{ and } j)}$	426
$F_{Scheffé} = \frac{MS_{between \text{ (groups } i \text{ and } j)}}{MS_{between \text{ (entire ANOVA)}}$	equation for $F_{Scheffé}$	426
$F_{crit} = F_{crit \text{ (entire ANOVA)}}$	F_{crit} for Scheffé test	426
$MS_{within - cells} = \frac{SS_{within - cells}}{df_{within - cells}}$	conceptual equation for equation for within-cells variance estimate	451
$SS_{within - cells} = SS_{11} + SS_{12} + \dots + SS_{rc}$	conceptual equation for within-cells sum of squares	451
$SS_{within - cells} = \sum_{\text{all scores}} X^2 - \left[\frac{\left(\sum_{\text{cell } 11} X \right)^2 + \left(\sum_{\text{cell } 12} X \right)^2 + \dots + \left(\sum_{\text{cell } rc} X \right)^2}{n_{\text{cell}}} \right]$	computational equation for within-cells sum of squares	452
$df_{within - cells} = rc(n - 1)$	within-cells degrees of freedom	452
$MS_{rows} = \frac{SS_{rows}}{df_{rows}}$	conceptual equation for the row variance estimate	452
$SS_{rows} = n_{row}[(\bar{X}_{row 1} - \bar{X}_G)^2 + (\bar{X}_{row 2} - \bar{X}_G)^2 + \dots + (\bar{X}_{row r} - \bar{X}_G)^2]$	conceptual equation for the row sum of squares	453

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$df_{rows} = r - 1$	row degrees of freedom	453
$SS_{rows} = \left[\frac{\left(\sum_1^{row} X \right)^2 + \left(\sum_2^{row} X \right)^2 + \cdots + \left(\sum_r^{row} X \right)^2}{n_{row}} \right] - \frac{\left(\sum_{all\ scores} X \right)^2}{N}$	computational equation for the row sum of squares	453
$MS_{columns} = \frac{SS_{columns}}{df_{columns}}$	column variance estimate	454
$SS_{columns} = n_{column} \left[(\bar{X}_{column\ 1} - \bar{X}_G)^2 + (\bar{X}_{column\ 2} - \bar{X}_G)^2 + \cdots + (\bar{X}_{column\ c} - \bar{X}_G)^2 \right]$	conceptual equation for the column sum of squares	454
$df_{columns} = c - 1$	column degrees of freedom	454
$SS_{columns} = \left[\frac{\left(\sum_1^{column} X \right)^2 + \left(\sum_2^{column} X \right)^2 + \cdots + \left(\sum_c^{column} X \right)^2}{n_{column}} \right] - \frac{\left(\sum_{all\ scores} X \right)^2}{N}$	computational equation for the column sum of squares	454
$MS_{interaction} = \frac{SS_{interaction}}{df_{interaction}}$	interaction variance estimate	455
$SS_{interaction} = n_{cell} \left[(\bar{X}_{cell\ 11} - \bar{X}_G)^2 + (\bar{X}_{cell\ 12} - \bar{X}_G)^2 + \cdots + (\bar{X}_{cell\ rc} - \bar{X}_G)^2 \right] - SS_{rows} - SS_{columns}$	conceptual equation for the interaction sum of squares	455
$SS_{interaction} = \left[\frac{\left(\sum_{11}^{cell} X \right)^2 + \left(\sum_{12}^{cell} X \right)^2 + \cdots + \left(\sum_{rc}^{cell} X \right)^2}{n_{cell}} \right] - \frac{\left(\sum_{all\ scores} X \right)^2}{N} - SS_{rows} - SS_{columns}$	computational equation for the interaction sum of squares	455
$df_{interaction} = (r - 1)(c - 1)$	interaction degrees of freedom	455
$\chi_{obt}^2 = \sum \frac{(f_o - f_e)^2}{f_e}$	equation for calculating χ_{obt}^2	485
$U_{obt} = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1$	general equation for calculating U_{obt} or U'_{obt}	503
$U_{obt} = n_1 n_2 + \frac{n_2(n_2 + 1)}{2} - R_2$	general equation for calculating U_{obt} or U'_{obt}	503
$H_{obt} = \left[\frac{12}{N(N + 1)} \right] \left[\sum_{i=1}^k \frac{(R_i)^2}{n_i} \right] - 3(N + 1)$	equation for computing H_{obt}	509