Learning Objectives

- **Select** the proper inferential test when comparing two groups

- **Describe** conditions under which we would select a *paired-samples t-test*

- **Conduct** a paired *t*-test
  - With effect size!

Which test to use?

What do we want?

Is our mean different from a specific population mean (μ)?

Are these two sample means different from each other?

Paired-Samples *t*-test

- a.k.a., repeated-measures *t*, dependent *t*

<u>Requirements</u>:

- Compare exactly 2 groups
- DV is interval/ratio (for parametric tests)
- DV is approximately normal
  - Or, $N > 30$ (making distribution of $\bar{X}$ approx. normal)
- Absence of outliers

However, *t*-tests are relatively ***robust***, meaning violations of these assumptions are often not problematic

Example: *Paired t-test*

- Stroop Color-Word task
  - N = 10
  - DV = ms until correct response

Green = rtSame

Blue = rtDifferent

Equivalent Paired t-test formulae

$$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}}}{s_{\bar{D}}}$$

$$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}} - \mu_D}{s_{\bar{D}}}$$

$$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}}}{s_D / \sqrt{N}}$$

$$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}}}{\sqrt{\frac{SS_D}{N(N-1)}}}$$

Equivalent Paired t-test formulae

$$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}}}{s_{\bar{D}}}$$

Where, $s_{\bar{D}} = {}^{s_D}/_{\sqrt{N}}$

$$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}} - \mu_D}{s_{\bar{D}}}$$

Where, $\mu_D = 0$

$$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}}}{{}^{s_D}/_{\sqrt{N}}}$$

Where, $s_D = \sqrt{\frac{SS_D}{N-1}}$

$$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}}}{\sqrt{\frac{SS_D}{N(N-1)}}}$$

This N is related to converting s_D to $s_{\bar{D}}$

This N is related to converting SS_D to s_D

Paired t-test

rtDiff	rtSame
659	782
1183	577
1032	780
871	950
711	488
854	658
915	327
765	822
982	456
1092	539

$$t_{\text{obt}} = \frac{\overline{D}_{\text{obt}}}{\sqrt{\frac{SS_D}{N(N-1)}}}$$

$$\overline{D}_{\text{obt}} = \frac{\Sigma D}{N} = \frac{2685}{10} = 268.5$$

$$SS_D = \Sigma D^2 - \frac{(\Sigma D)^2}{N}$$

Paired t-test

$\boldsymbol{N} = \mathbf{10}$

$\boldsymbol{\Sigma D} = \mathbf{2{,}685}$

$\boldsymbol{\Sigma D^2} = \mathbf{1{,}471{,}733}$

$\bar{D}_{\text{obt}} = 268.5$

$SS_D = \ldots$

$$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}}}{\sqrt{\frac{SS_D}{N(N-1)}}}$$

$$SS_{\text{D}} = \Sigma D^2 - \frac{(\Sigma D)^2}{N} = \ldots$$

$$= 1{,}471{,}733 - \frac{2{,}685^2}{10} = 750{,}810.5$$

Paired t-test

N = 10

ΣD = 2,685

ΣD^2 = 1,471,733

$\bar{D}_{\text{obt}}$ = 268.5

SS_D = 750,810.5

t_{obt} = …

$$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}}}{\sqrt{\frac{SS_D}{N(N-1)}}}$$

$$t_{\text{obt}} = \frac{268.5}{\sqrt{\frac{750{,}810.5}{90}}}$$

Paired t-test

$N = 10$

$\Sigma D = 2{,}685$

$\Sigma D^2 = 1{,}471{,}733$

$\bar{D}_{\text{obt}} = 268.5$

$SS_D = 750{,}810.5$

$t_{\text{obt}} = \ldots$

$$t_{\text{obt}} = \frac{\bar{D}_{\text{obt}}}{\sqrt{\frac{SS_D}{N(N-1)}}}$$

$$t_{\text{obt}} = \frac{268.5}{91.3364}$$

Paired t-test

$N = 10$

$\Sigma D = 2{,}685$

$\Sigma D^2 = 1{,}471{,}733$

$\overline{D}_{\text{obt}} = 268.5$

$SS_D = 750{,}810.5$

$t_{\text{obt}} = 2.940$

$$t_{\text{obt}} = \frac{\overline{D}_{\text{obt}}}{\sqrt{\frac{SS_D}{N(N-1)}}}$$

$$t_{\text{obt}} = 2.940$$

Paired t-test

$N = 10$

$\Sigma D = 2{,}685$

$\Sigma D^2 = 1{,}471{,}733$

$\bar{D}_{obt} = 268.5$

$SS_D = 750{,}810.5$

$\boldsymbol{t_{obt} = 2.940}$

$\boldsymbol{t_{crit} = \ldots}$

$\boldsymbol{\propto_{2-tail} = .05}$

$\boldsymbol{df = N - 1}$

$t_{obt} = 2.940$

$t_{crit} = \pm 2.262$

$t_{obt} > t_{crit}$

Decision: Reject H_0

APA reporting with p-value calculated in Jamovi:

"People responded slower to mismatching- vs. matching-Stroop trials, $t(9) = 2.940$, $p = .017$."

Cohen's d_z

$N = 10$

$\Sigma D = 2685$

$\Sigma D^2 = 1471733$

$\bar{D}_{\text{obt}} = 268.5$

$SS_D = 750{,}810.5$

$s_D = 288.83$

$t_{\text{obt}} = 2.940$

$t_{\text{crit}} = \pm 2.262$

$\propto_{2-\text{tail}} = .05$

$df = 9$

$$\text{Cohen's } d_z = \frac{\bar{D}_{\text{obt}}}{s_D}$$

$$s_D = \sqrt{\frac{SS_D}{N-1}} = \sqrt{\frac{750{,}810.5}{9}} = 288.83$$

$$d_z = \frac{\bar{D}_{\text{obt}}}{s_D} = \frac{268.5}{288.83} = .930$$

How large is this effect?

Comparing *paired t-test* vs. *sign test*

rtDiff	rtSame	*D*	*Sign*
659	782	-123	-
1183	577	606	+
1032	780	252	+
871	950	-79	-
711	488	223	+
854	658	196	+
915	327	588	+
765	822	-57	-
982	456	526	+
1092	539	553	+

$t = 2.940$, $p = .017$

$P(\geq 7) = .172$*

When N=10

**This is for 1-tail test. For non-directional test, multiply by 2 to account for P(≤3); which is the other tail of binomial dist.*

$P_{2\text{-tail}}$ for our result or even more extreme (positive or negative) is $p = .344$