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EE 511: Simulation of Stochastic Processes

Spring 2018

Project#1

[A Few Coins]

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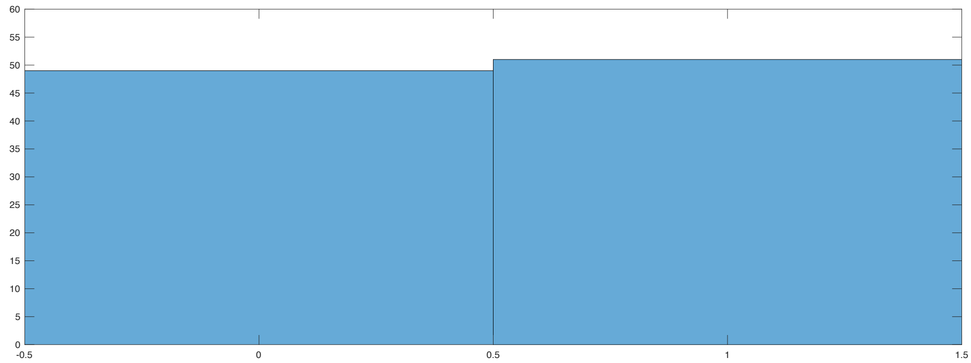
```
p=0.5;
u=zeros(1,100);
x=zeros(1,100);
for i=1:1:100
    u(i)=rand;
if u(i)>p
   x(i)=1;
end
end
histogram(x);
```



Histogram for 100 Bernoulli trials

Discussion:

We can easily see that from histogram heads and tails are mostly equally divided to two since our p is 0.5. Above code generates uniform random variable and we collect sample from it, if it is greater than our p value we take it as heads, if it is not we regard it as tails and put the data in a vector. Then we compute histogram of it to see how the data is distributed.

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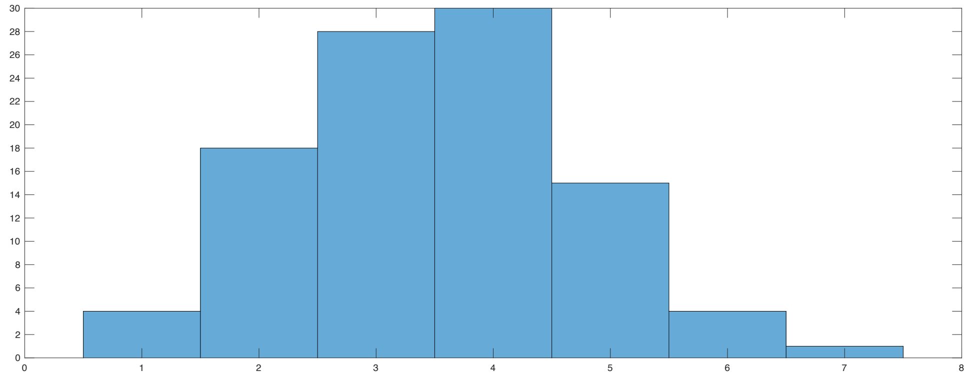
```
p=0.5;
u=zeros(1,100);
x=zeros(1,100);
for i=1:1:100
 for k=1:1:7
    u(k)=rand;
if u(k)>p
   x(i)=x(i)+1;
end
end
```

```
end
histogram(x);
```



Discussion:

This simulation is not much different from the first one. The probability distribution is also Bernouilli but since we are counting the “successes” within 7 bernouilli trials this makes binomial distribution. Since we know that the expected value of the binomial is np, when we collect 100 samples from it we expect the histogram to be centered around 3.5. We can see this from above graph easily. The code for this question uses the same code to generate and sample uniform random variable but I had to add two for loops on top of it. The first for loop is simply for generating 100 samples, the second for loop is for counting the successes among n trials.

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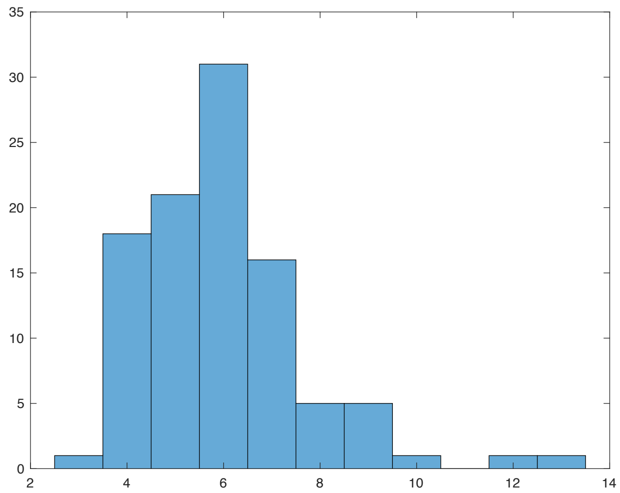
```
p=0.5;
u=zeros(1,100);
b=zeros(1,100);

for k=1:1:100
   x=zeros(1,100);
for i=1:1:100
    u(i)=rand;
if u(i)>p
   x(i)=1;
end
end
b(k)=max( diff( [0 (find( ~ (x > 0) ) ) numel(x) + 1] ) - 1);

end
histogram(b);
```



Discussion:

For the histogram, the question does not state how many sample to use. Since we are counting maximum occurrence within each 100 binomial trials, it will output a single number. To generate a histogram, we need more sample than this. Thus, I assume to have 100 samples of 100 Bernoulli trials. From the histogram we see that the most occurred number is 6, 31 times in 100 samples. As the number of seeing consecutive heads increases, the probability of seeing that event decreases thus number of occurrence is also decreases. Also, as the number decreases the probability seeing maximum number heads decreases as well.

For the coding part, we use vector u to generate uniform rvs for each bernouilli trial and we use vector x to store heads and tails. Vector b is used to store the longest run of heads for each sample k.

[Counting Successes]

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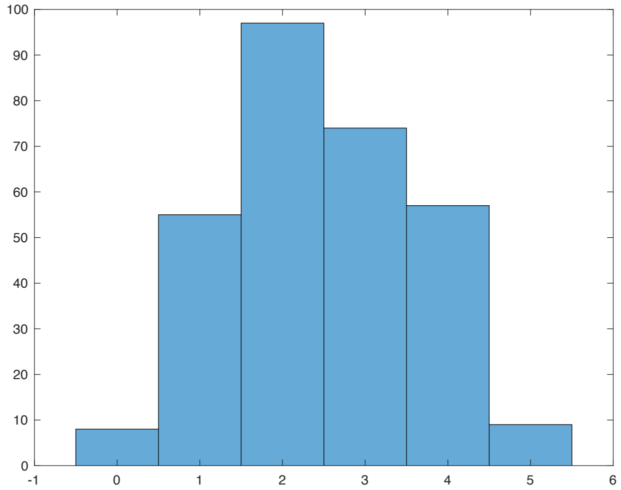
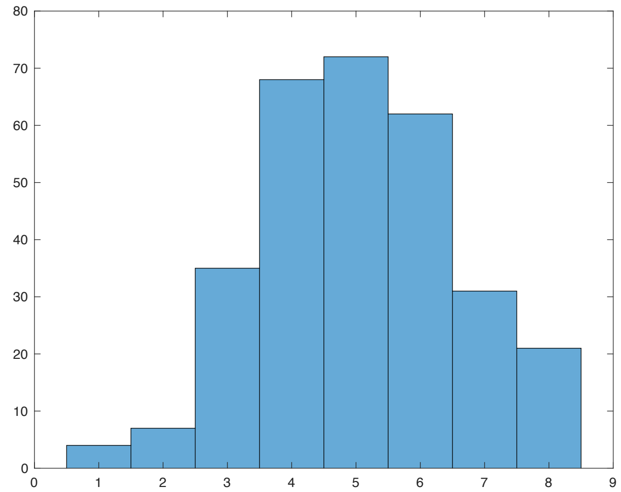
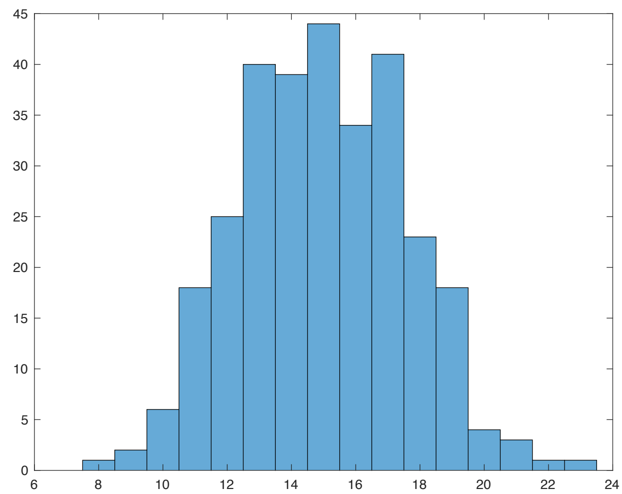
```
p=0.5;
u=zeros(1,300);
x=zeros(1,300);
for i=1:1:300
 for k=1:1:5
    u(k)=rand;
if u(k)>p
   x(i)=x(i)+1;
end
end
end
histogram(x);
```



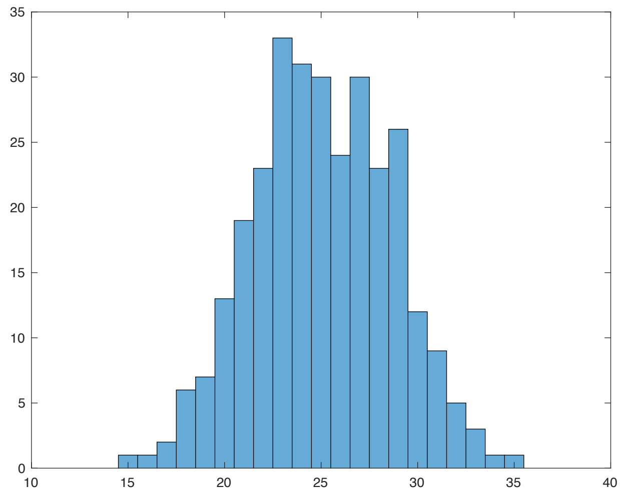
100
90
80
70
60
50
40
30
20
10
0
-1
0
1
2
3
4
5
6



80
70
60
50
40
30
20
10
0
0
1
2
3
4
5
6
7
8
9



45
40
35
30
25
20
15
10
5
0
6
8
10
12
14
16
18
20
22
24



Discussion:

The above histograms are for k={5, 10, 30, 50}. We can see that as k increases, the mean number of success is also increases. We know and stated in previous questions that expected number for success is np. We can see this result from the histograms since all of them is centered around their np values (this case it is kp).

If we would increase the sample sizes more. We see that the histograms are going to be closer to bell-shape.

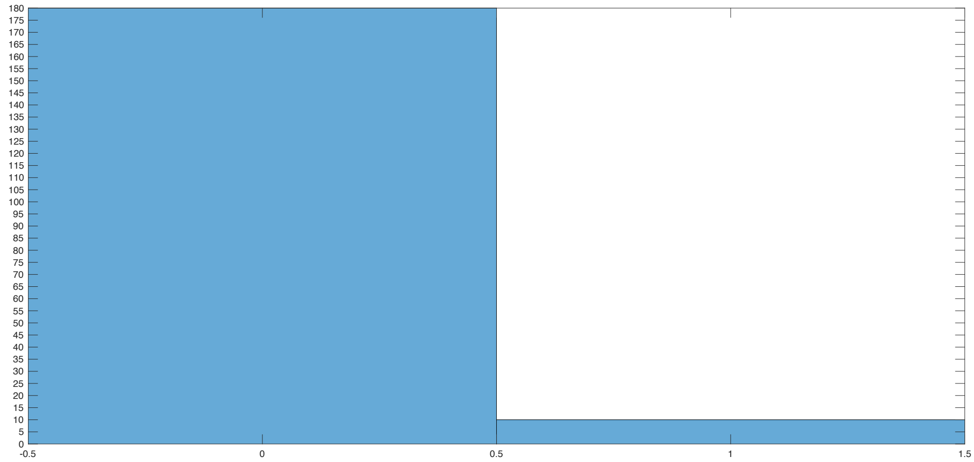
[Networking: part 1]

```
n=20;
p=0.05;
number_of_edges=nchoosek(n,2);
u=zeros(1,number_of_edges);
select=zeros(1,number_of_edges);
for i=1:1:number_of_edges
u(i)=rand;
if u(i)<p
    select(i)=1;
end
end
histogram(select);
```

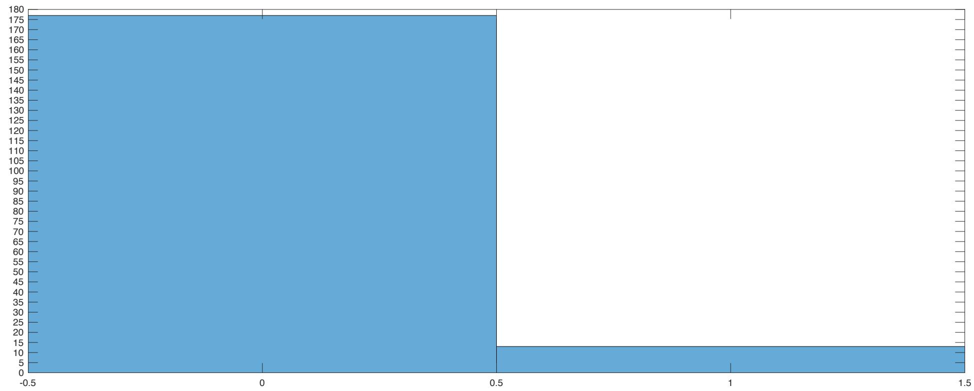
Discussion:

Since an edge represents a connection between two people among 20 people, to count this number of connections we can imagine it as if we are selecting 2 people out of 20 people and the total number of unique pairs among these 20 people. This implies $\binom{N}{2}$. So, total number of connections is $(20\times19)/2 = 190$.

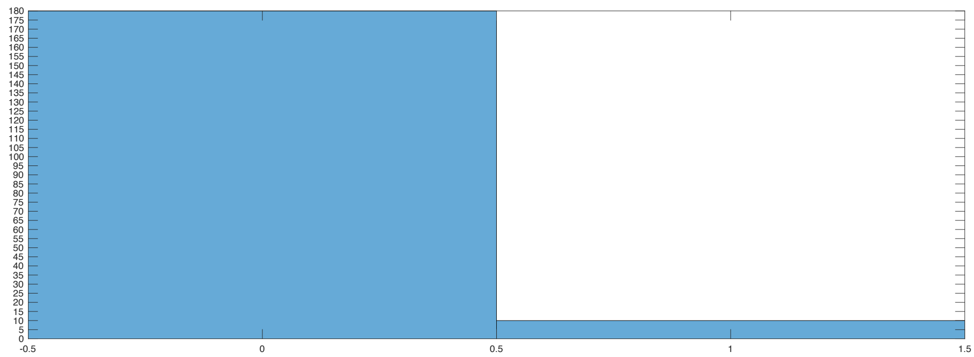
The distribution is binomial since it is as if we are selecting a possible connection between two people from 190 possible connections in total with probability 0.05. Thus, we would expect to have $0.05 \times 190 = 9.5$ connections. The below histograms supports this idea. There is total of 10 successes among 190 connections.



Histogram with 10 successes



Histogram with 13 successes



Histogram with 10 successes