



MASTER 1 ECONOMÉTRIE STATISTIQUES

2020

Monetary, credit and business cycle relationship: an empirical approach

Authors:

EL MEHDI AGUNAOU
STOJANOVIĆ JOVAN
VILLE SOLAL
NELSON CASTRO

Directed by:

PHILIPPE DE PERETTI

Programming in:

SAS/IML

June 2020

ABSTRACT

The relationship between business, monetary and credit cycles has different interpretation in economic theory. However, there have been very few attempts in economic literature to measure the relationship between the different cycles. The core of this paper is to discuss and test different theoretical propositions, based on the different definitions of the cycles and the methods used to analyze the relationship between them. We will mainly base our analysis on the Event Synchronization method. We will use empirical data from the United States available for each cycle to test our ideas.

Keywords: *Cycle extraction, Event synchronization, Cross Correlation, Time delay*

Contents

ABSTRACT	1
1 Introduction: defining the different cycles	4
1.1 Business cycle theory	4
1.2 Credit cycle theory	5
1.3 Monetary cycle theory	6
1.4 Our problematic	6
1.5 Our plan	7
2 Cycle Indicators	7
2.1 Chosen indicators	7
2.2 Precision on Data	8
3 Cycle extraction using Hodrick-Prescott filter	9
3.1 Theoretical overview	9
3.2 Cycle extraction from the chosen indicators	11
3.3 Cycles stationarity detection	12
4 Event synchronization method	15
4.1 The intuition behind the method	15
4.2 Mathematical implementation	16
4.2.1 The framework	16
4.2.2 The recording of linked events	16
4.2.3 The statistics	17
5 Interpretation of ES: testing the business, the monetary and the credit cycle	19
5.1 The link between the business and the credit cycle: IP and LTIR	19
5.2 The link between the business and the monetary cycle: IP and M3	22
5.3 The link between the monetary and the credit cycle: M3 and LTIR	23
5.4 ES results recap	25
6 Cross-Correlation	26
6.1 Measure of temporal similarity	26
6.2 Moving window cross-correlation	27
6.3 Comparison between the Cross correlation and the Event Synchronization method	28
7 Model Uncertainty and Robustness	30
7.1 Results sensitivity to the choice of indicators	30
7.1.1 The link between business and credit cycle : CLI and SHP	31
7.1.2 The link between business and credit cycle : CLI and FFR	33
7.1.3 The link between business and credit cycle : SHP and FFR	35

8 Conclusion	37
8.1 Stylized Facts	37
8.2 Methodological thoughts	37
REFERENCES	39
APPENDICES	40

1 Introduction: defining the different cycles

A cyclic pattern exists when data represents successive rises and falls of an indicator during time. In economics, this is usually defined as alternating contraction and expansion phases of economic activity measured by an economic indicator [13]. The duration of these fluctuations varies depending on the statistical indicator. If we take the classical example of the economic cycle, in most cases, four stages stand out : expansion, peak, contraction, and trough (Figure 1).

1.1 Business cycle theory

When it comes to economic (or business) cycle literature, various theories aim to explain their evolution. We will name some of them.

It was Joseph Kitchin, who in the 1920s established the idea of a short business cycle of about 40 months in length [8]. This cycle is based on the fact that imperfect information may provoke phases of over-supply of the market, and thus accumulation of goods (i.e. the inventory effect), leading to a contracting economic activity. By the price mechanism, the fall in the goods price regulates the cycle, because the demand exceeds supply again and production restarts to grow.

Clement Juglar [6] defined cycles as periods extending from 7 to 11 years. Economic history rules in Juglar's favour if we look over the frequency of some of the last crises in France and even in United States.

Nevertheless, by changing our point of view, if we look at the last few crisis, even if they are regular, they all never hit as hard. In this way, Simon Kuznets, probably influenced by the unprecedented 1929 october krach identified longer, 15 to 25 years cycles [9]. His main point was the impact of demographic processes on the economy, as immigrant flows may provoke great changes in the construction intensity thanks to cheap workforce. Of course, such structural and infrastructural changes could not impact the economy short-termly. That's why Kuznets swings were larger.

To conclude our short summary of economic cycle literature we can also introduce Kondratiev's supercycles [10]. During the 1920's decade, he identified a length of 50 to 60 years for each cycle. According to him, these cycles could be explained by the "innovation theory", meaning that these waves arise from the accumulation of basic innovations that launch consequently a technological revolution that in turn creates new leading industrial or commercial sectors (steam engine, railway, automobiles...).

It was Schumpeter, in his most famous work [16], who summarized all of these theories by demonstrating the coexistence of multiple business cycles inside the same time series:

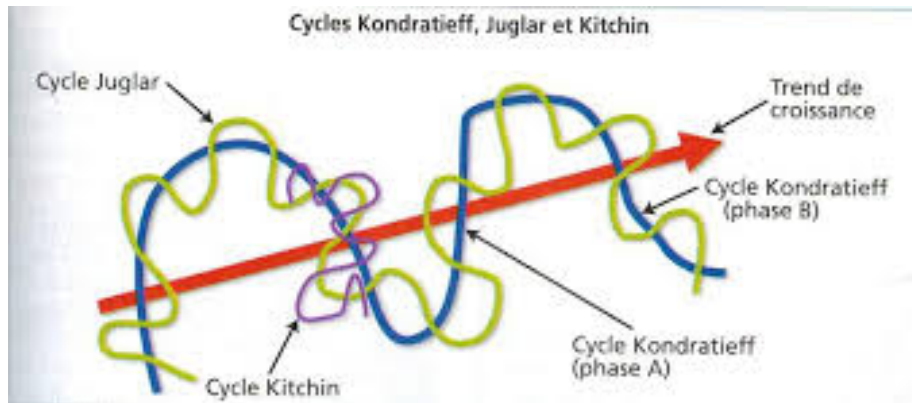


Figure 1: Cycles
[17]

1.2 Credit cycle theory

Along with the theories of cyclical economic activity, other fluctuations, such as in the access to credit by borrowers occur. Multiple theories describe these fluctuations also as cyclical.

This is called the credit cycle. The first period of the cycle is characterized by an easy access to borrowed funds, manifested by lower interest rates, lower lending requirements and a general increase in the amount of credit available. This period is followed by a contraction of the credit cycle characterized by a lower availability of credit, and where interest rates increase and lending requirements become more strict. The contraction period starts to decline when risks decrease for the lending institutions to such an extent where the cycle troughs out and starts over again.

One of the credit cycle theory that resurged after the 2008 crisis is Hyman Minsky's credit cycle model. According to him [12], the paradox of the credit cycle is that economic and financial stability are the root cause of future instability, brought by the movement of the credit cycle. Effectively, optimistic previsions of investors lead to a lowering of their risk aversion, as they start to borrow excessively and push up asset prices excessively high. The negative tendency starts to prevail at the moment when, with excessive credit, some of the investor's, known as "Ponzi borrowers" are incapable of repaying the debt even by borrowing again. The credit bubble thus explodes and a period of uncertainty reinstate itself, with an impact on the economy.

Therefore, theory often implies that there is a potential strong relationship between the credit and the business cycle, which we will attempt to identify in this paper.

1.3 Monetary cycle theory

Monetary policy is, in general, controlled by an independent central bank. However, the decision to fix interest rates at a certain level is often highly impacted by the economic situation. On the other hand, monetary policy may impact the economy directly by impacting the access to credit and the credit cycle, and therefore also indirectly the business cycle.

For instance, the Austrian business cycle theory (ABCT) [5], with Hayek as its most famous defender, is one of the reemerging theories on the link between the monetary and the business cycle. It suggests that when the interest rates (or other monetary instruments) fixed by monetary authorities are lower than what is considered to be the market rate (function of supply and demand), there is excessive borrowing. This credit boom, a source of imperfect allocation of resources, creates a downturn in economic activity. Structural economic adjustments, undermined by the central bank, are delayed but eventually accomplished in order to reverse the business cycle again on the path to growth.

Thus, we will test the potential causality existing between the monetary and the business cycle.

The link between the monetary and the credit cycle is more straightforward: lower interest rate should encourage more credit and inversely. The interest rate acts thus a regular market price for the loan, and there is no other major determinant for money creation. This gives a power to a central Bank (the Federal Reserve) to act by influencing the interest rate, and the credit cycle.

This vision was not shared by everyone. For instance, John Maynard Keynes introduces the notion of liquidity trap [7]. The liquidity trap appears when interest rates are below a minimum acceptable level, after which there is no effect of monetary policy on the economy. This is the case because of the liquidity preference of individuals, especially at a time of economic instability. In practical terms, the possibility is that economic agents will no longer want to borrow money, and will favour savings on investment, even though the interest rate is extremely favorable for credit. This liquidity trap is, consequently, producing a break of the common relationship between the monetary and the credit cycle. We will be very interest therefore in the level of synchronization between them.

1.4 Our problematic

Through this paper we aim to answer some of the following questions :

- Are Credit cycles pro-or-counter-cyclical ?
- Is the link between the monetary and the credit cycle always strong?
- Is there a link between the monetary and business cycle?

1.5 Our plan

In order to do that, we have to go through a process of, firstly, collecting data in the form of time series that represent the cycles we aim to analyse. After collecting the relevant series, we will extract and isolate the cycles using rigorous statistical techniques based on the Hodrick-Prescott filter. Afterwards, we need to post validate the extracted cycles using different stationarity tests.

Finally, we will measure the degree of synchronisation of the cycles to determine the direction of the causality (*here, the meaning of causality is not literal and depends on the technique used*), and we will be answering the questions that most interest us. To post-validate our results, comes robustness testing, as we need to test how sensitive our results and conclusions are to changes of methodology.

Many statistical methods have been used in order to describe cycle relationships, for instance non-linear methods based on the similarities of trajectories or phases, the coherence function. We will focus on two methodologies: the cross-correlation, used more traditionally in this type of analysis, and the Event Synchronization method [14], a method probably never applied before in economics, but very simple and intuitive. The comparison between the results of the methodologies will give us a clear insight on the limits and advantages of both of them.

We will dedicate a section in this paper to each step of the process and explicitly explain the theories behind and the techniques used.

2 Cycle Indicators

2.1 Chosen indicators

Economists and financial analysts are constantly looking for indications of future development of economic activity and the business sector. The most closely watched of these signs are financial and business indicators.

There are three types of indicators :

- **Leading indicators** are those who point towards future events,
- **Lagging indicators** are used to confirm patterns and regularities that are in progress,
- **Coincident indicators** are those who vary in real-time and are used to clarify the current state of the economy.

For our study we will analyse, firstly, **The Composite index of leading indicators**, which is designed to provide early signals of turning points in business cycles showing variations of the economic activity. We will also add **The Industrial production** to the previously mentioned indicator for economic activity ***Business cycle***, as it is a monthly available data in the OECD Database, and it refers to the output of industrial establishments and covers sectors such as mining, manufacturing, electricity, gas-and-steam and others.

In order to properly extract cycles from time series we need high variability in the series data. This is why we have chosen industrial production that is available on a monthly basis, instead of GDP, only available in a quarterly basis.

For the *Credit cycle* extraction we will use **Long-term interest rates** as an indicator, it refers to government bonds maturing in ten years, these rates are determined by the price charged by the lender and most importantly a sign of risk from the borrower. That is one of the reasons we think it's a good indicator for the credit cycle.

Lastly, we chose the monetary aggregate **Broad Money : M3** as an indicator for the *Monetary cycle*, it includes :currency, deposits with a maturity of up to two years, deposits redeemable at notice of up to three months and repurchase agreements and money market fund shares/units and debt securities up to two years. The variation of the available amount of broad money (M3) can be a good indicator of the monetary policy conducted by the central bank, thus it can show us where we stand in the monetary cycle.

Another representative indicator for the Monetary cycle would be the FED's policy rate or also known as the **Federal funds rate**, it refers to the interest rate to which banks charge other banks for lending to them excess cash from their reserve. These reserves are constrained by a law, the latter requires banks to maintain these Reserves as a certain percentage of their deposits in an account at the Federal Reserve bank. The amount of money that exceeds these reserves is available for lending to other banks. Fluctuations in this rate will indicate the policy conducted by the FED and its goals, which gives an indication to where we are located in the monetary cycle.

Indicators	Abbreviation	Objective cycle
Index of Composite leading indicators	CLI	Business cycle
Industrial production Index	IP	
Long-term interest rates	LTIR	Credit cycle
Share prices	SHP	
Broad Money	M3	Monetary cycle
Federal funds rate	FFR	

2.2 Precision on Data

We will apply the study of the cycles only in the case of the Unites-States because of the large availability of data.

Data used are monthly data, as we wish to have the most precise analysis of cycle available. Therefore, short term cycle, as well as long term may be analysed. The Index of Composite leading indicators was directly extracted from the OECD database, as it is directly computed by this organisation based on different patterns. The Long-term interest rate and the Share Prices were also available in the OECD database.

The monthly data for Industrial production Index in the US was extracted from the Federal Reserve Economic Data of St. Louis site. The data for Broad Money (M3) and Federal Funds Rate was also found on the publicly available data of the FRED, based on the publicly available Federal Reserve (the central bank of the US) data.

3 Cycle extraction using Hodrick-Prescott filter

3.1 Theoretical overview

Hodrick-Prescott filter [15] is a popular tool in macroeconomics for fitting smooth trend to time series. It's a detrending method using the "deviation-from-trend" approach. This method suggests that our initial series can be decomposed between a trend component τ_t and a cyclical component c_t such as :

$$y_t = \tau_t + c_t$$

We can think this problem by seeing the trend as the non-stationary growth component of the series and conversely the cycle as the stationary component. HP filter aims to estimate the trend as a result of an optimization problem, based on the minimization of a penalized least square criterion in order to isolate the cycle c_t from y_t :

$$\min_{\tau_t} \sum_t (y_t - \tau_t)^2 + \lambda \sum_t (\tau_{t+1} - 2\tau_t + \tau_{t-1})^2$$

With lambda the trade-off between the two main goals of our problem :

- Minimization of distance between original series and its trend in the left part (detrending).
- Minimization of trend's curvature in the right part (smoothing). We can see λ as a penalty on the trend. If we fix it to 0, the trend will perfectly fit the observed series (and there won't be any cyclical component). Conversely, if λ goes to infinity, the growth component will approach a linear deterministic time trend. So λ has to be fixed to catch the non-stationary of the series. The risk of catching too much curvature on the estimated trend is lowering the explanatory power of the cycle, by potentially reducing its fluctuations because they are already caught in the trend.

Lambda can obviously take any value. However, a commonly known consensus consists in using a power rule of 2, yielding the original Hodrick and Prescott values for λ :

$$\lambda = \begin{cases} 100 & \text{for annual data} \\ 1600 & \text{for quarterly data} \\ 14400 & \text{for monthly data} \end{cases}$$

So, instead of observing the series in the time domain, we can treat the series as it was in the frequency domain, like a complex sinusoid, built from simple sine waves of different wave length. The trend part of the series is comprised by the low frequency (high wave length) sinusoids.

However, it's essential to understand trend's properties in order to filter it under the best possible conditions.

$$\begin{aligned}\tau_t &= \tau_{t-1} + \beta_{t-1} + \eta_t \\ \beta_t &= \beta_{t-1} + \zeta_t\end{aligned}$$

With τ_t the trend itself and β_t its slope, η_t and ζ_t respectively trend and slope disturbances. In this essay we will keep naming τ_t the trend even if this is a bit of a misuse of language, software like SAS prefers the term of "Level" because mathematically speaking, the trend is a composition of both this level component and a slope.

According to Hodrick-Prescott filter's founding article, local Linear Trend Model with integrated random walk case are characteristic of American time series. As we are working on such series, we will model that hypothesis by fixing $\sigma\zeta > 0$ et $\sigma\eta = 0$.

With this in mind, we understand that we need to eliminate the long wave of length trend component and how we can do it. We next to subtract the filtered trend from the original series in order to preserve short-term frequency fluctuation, in this way we get a circumstantial cycle. Now we have extracted the cycle of our indicators, we will be able to study event synchronization between them.

To do so we are mainly using SAS's *Proc UCM*. We could have used another procedure named *Proc Expand*. However, in the next part we will show that both of the series find very similar results. That sort of sensitivity analysis is justified by the fact that these procedures doesn't use same method. UCM is based on Kalman Filtering/Smoothing, while EXPAND computes HP filter by inverting a suitable matrix. As we are working on monthly data without seasonality effect, we will focus on only detrending our series by the consensus about using power rule of 2. It means that we are going to fix the variance of the level of our trend filter at zero, while we fix its variance's slope at 0.000625 in order to match American time series theory.

Mathematical theory behind Expand procedure is based on inverting suitable matrix, but as it isn't the main point of our essay we will simply outline the method.

Indeed, by taking derivatives of the minimization function, it can be shown that :

$$y_T = (\lambda * F_T + I_T)\tau_T$$

With F_T a known pentadiagonal positive definite ($T \times T$) matrix :

$$\begin{pmatrix} 1 & -2 & 1 & 0 & \dots & 0 \\ -2 & 5 & -4 & 0 & \dots & 0 \\ 1 & -4 & 6 & 0 & \dots & 0 \\ 0 & \dots & 0 & 6 & -4 & 1 \\ 0 & \dots & 0 & -4 & 5 & -2 \\ 0 & \dots & 0 & 1 & -2 & 1 \end{pmatrix}$$

Then, the trend component and the cyclical component can be identified as follows:

$$\hat{\tau}_T = \frac{1}{(\lambda * F_T + I_T)} * y_T$$

$$\hat{c}_T = y_T - \hat{\tau}_T$$

3.2 Cycle extraction from the chosen indicators

We previously mentioned the indicators we are going to extract cycles from, namely : (Industrial production (IP), Long-term interest rates (LTIR)/ Share prices (SHP), Broad money (M3)/ Federal funds rate (FFR)), We chose to use data from United-States and in a monthly frequency, all data was extracted from OECD Database except for Federal funds rate which was extracted from the FED's Database. As mentioned earlier, there are two ways to implement the Hodrick-Prescott filter in SAS (proc UCM and proc Expand, in this section we will try them both and compare the outcomes.

Here's how to implement both of the procedures in SAS :

```
proc expand data=ip_us out=filtered_ip
method=none;
id time;
Convert ip_us=hpc/transformout=(hp_c 14400);
run;
```

We use the hpc to call the Hodrick-Prescott filtered cycle in the expand procedure.

```
proc ucm data=ip_us;
id time interval=month;
irregular print=smooth;
level var=0 noest ;
slope var=0.00006944444 noest;
model ip_us;
run;
```

Same thing with Ucm procedure, we specify the slope variance as $\frac{1}{14400} = 0.00006944444$.

The two procedures give arguably the same output, in our case the cycles extracted from the same data (*Industrial production in the United States*) using the two procedures are shown in the figure below.

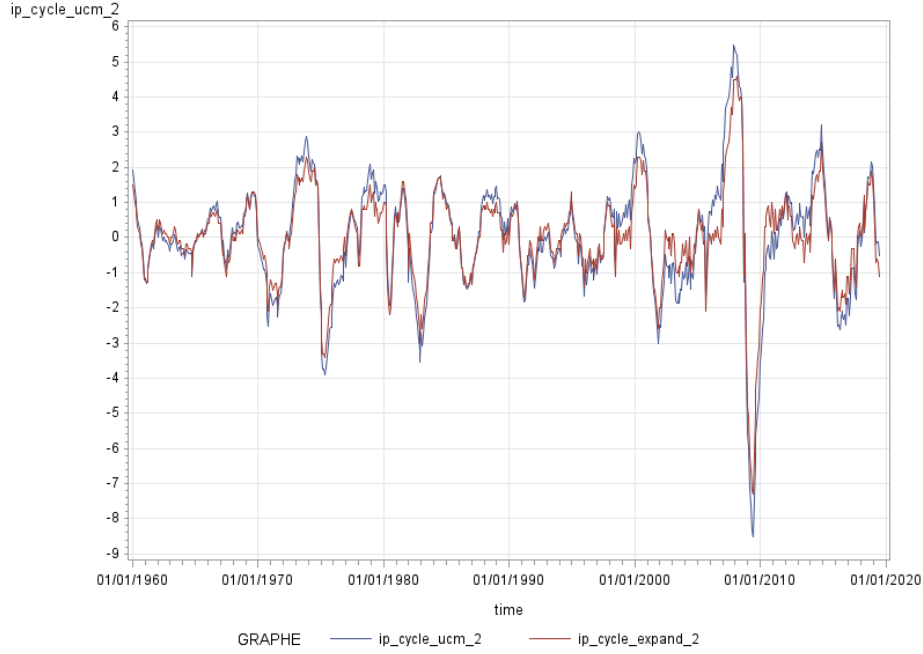


Figure 2: Ucm and Expand procedures

The figure show us that the two procedures produce the same cycle and that the differences of output are not significant.

Using the Ucm procedure we applied Hodrick-Prescott filter to extract the cycles of all the indicators mentioned above. The resultant data of all indicators has a very cycle like form but with different magnitudes, meaning that the scales of the extracted cycles are different. This is not really a problem for the upcoming analysis as our main measure of synchronisation is based on the relative timing of events, defined as local maximas, therefore doesn't rely on the actual value of indicators.

3.3 Cycles stationarity detection

To validate our extraction cycle, we must look for the stationarity of our series. As we saw previously, the Hodrick-Prescott filter had been used to extract our cycles by eliminating the non-stationarity induced. With this filter, we should have removed the prominent trends and the increasing variance that can be

observed.

After filtering, cycles must be stationary, which means that both the expected values of the series and its autocovariance function are independent of time.

The standard way to check for non-stationarity is to plot the series and its autocorrelation function. You can visually examine the autocorrelation function (ACF) and partial autocorrelation function (PACF) of our series, to identify a visible trend, as well as variability changes over time. Non-stationary series can be identified by a slow decay on the autocorrelation function.

To guarantee the stationarity of our series, we will use the Augmented Dickey-Fuller's Unit Root tests and the Phillips-Perron Unit Root tests. With the Augmented Dickey-Fuller (ADF)¹ test, we will be able to ascertain, with high probability, that a series is generated by a stationary process. It will be useful to detect the presence of a unit root for AR(p) type processes. The Phillips and Perron (PP)² test statistics can be viewed as Dickey-Fuller statistics that have been made robust to serial correlation by using the Newey-West (1987) heteroskedasticity and autocorrelation consistent covariance matrix estimator. We will use it in addition, to confirm our assumption on the stationarity of our cycles.

Augmented Dickey-Fuller Unit Root Tests							
Type	Lags	Rho	Pr Rho <	Tau	Pr Tau <	F	Pr F >
Zero Mean	0	-22.4025	0.0007	-3.32	0.0010		
	1	-33.1013	< 0.0001	-4.02	< 0.0001		
Single Mean	0	-22.3987	0.0064	-3.32	0.0150	5.51	0.0219
	1	-33.0963	0.0018	-4.01	0.0015	8.06	0.0010
Trend	0	-22.3882	0.0414	-3.31	0.0652	5.53	0.0893
	1	-33.0748	0.0038	-4.01	0.0089	8.08	0.0050
Phillips-Perron Unit Root Tests							
Type	Lags	Rho	Pr Rho <	Tau	Pr Tau <		
Zero Mean	0	-22.4025	0.0007	-3.32	0.0010		
	1	-26.5354	0.0002	-3.62	0.0003		
Single Mean	0	-22.3987	0.0064	-3.32	0.0150		
	1	-26.5308	0.0025	-3.61	0.0060		
Trend	0	-22.3882	0.0414	-3.31	0.0652		
	1	-26.5168	0.0169	-3.61	0.0298		

Table 1: ADF and PP tests on Ip cycle

¹Cf, Appendix : Presentation of the Augmented Dickey-Fuller test

²Cf, Appendix : Presentation of the Phillips-Perron test

There is no obvious time trend in the series, and the mean is close to zero. The descriptive statistics and stationarity test results are shown in table 1, for the Ip indicator. The three types of models are listed in the "Type" column. As shown in the Adf test section and the PP test section, there is no obvious time trend. For each type of model, two rows of test statistics are reported: 0 lags and 1 lag. This is set by the ADF=1 option in the STATIONARITY= option of the ARIMA procedure. The row of 0 lags shows test statistics for an AR(1) model, where zero lagged differences are used in the test regression. The row of 1 lag shows test statistics for the AR(2) model, where one lagged differenced term is used in the test regression.

To reject the null hypothesis, we focus on the Tau test statistic. The output structure of the PP tests is similar to that of the ADF tests, except that there is no F test. The PP tests also report the rho test statistic (shown in the Rho column) and the tau test statistic, but they are calculated differently from the ADF tests

In Table 1, the p-values of the tau test statistics in both the AR(1) and AR(2) models for both the ADF and PP tests, are smaller than 0.1, except for the Tau statistic of trend(0). The null hypothesis of non-stationary (presence of a unit root) is rejected at the 5% level, 10% level for the trend of an Ar(1) of Ip.

Augmented Dickey-Fuller Unit Root Tests							
Type	Lags	Rho	$Pr < Rho$	Tau	$Pr < Tau$	F	$Pr > F$
Zero Mean	0	-110.577	0.0001	-7.75	< 0.0001		
	1	-229.978	0.0001	-10.70	< 0.0001		
Single Mean	0	-110.579	0.0001	-7.74	< 0.0001	29.96	0.0010
	1	-229.980	0.0001	-10.69	< 0.0001	57.18	0.0010
Trend	0	-110.588	0.0001	-7.74	< 0.0001	29.93	0.0010
	1	-229.981	0.0001	-10.69	< 0.0001	57.10	0.0010
Phillips-Perron Unit Root Tests							
Type	Lags	Rho	$Pr < Rho$	Tau	$Pr < Tau$		
Zero Mean	0	-110.577	0.0001	-7.75	< 0.0001		
	1	-140.414	0.0001	-8.65	< 0.0001		
Single Mean	0	-110.579	0.0001	-7.74	< 0.0001		
	1	-140.417	0.0001	-8.65	< 0.0001		
Trend	0	-110.588	0.0001	-7.74	< 0.0001		
	1	-140.423	0.0001	-8.64	< 0.0001		

Table 2: ADF and PP tests on Ltir cycle

Augmented Dickey-Fuller Unit Root Tests							
Type	Lags	Rho	$Pr < Rho$	Tau	$Pr < Tau$	F	$Pr > F$
Zero Mean	0	-140.843	0.0001	-8.81	< 0.0001		
	1	-225.085	0.0001	-10.52	< 0.0001		
Single Mean	0	-140.839	0.0001	-8.81	< 0.0001	38.75	0.0010
	1	-225.093	0.0001	-10.51	< 0.0001	55.22	0.0010
Trend	0	-140.825	0.0001	-8.79	< 0.0001	38.69	0.0010
	1	-225.110	0.0001	-10.50	< 0.0001	55.14	0.0010
Phillips-Perron Unit Root Tests							
Type	Lags	Rho	$Pr < Rho$	Tau	$Pr < Tau$		
Zero Mean	0	-140.843	0.0001	-8.81	< 0.0001		
	1	-163.288	0.0001	-9.42	< 0.0001		
Single Mean	0	-140.839	0.0001	-8.80	< 0.0001		
	1	-163.283	0.0001	-9.41	< 0.0001		
Trend	0	-140.825	0.0001	-8.79	< 0.0001		
	1	-163.266	0.0001	-9.41	< 0.0001		

Table 3: ADF and PP tests on M3 cycle

In table 2 and 3, we have the same conclusions. Both tests of unit root for zero mean, single mean and trend, rejected the null hypothesis of non-stationarity, with highly significant; ie < 0.0001. Furthermore, the results of our stationary tests remains unchanged with greater number of lagged. We can conclude that we succeed in our cycles extraction.

4 Event synchronization method

4.1 The intuition behind the method

Event synchronization method consists in measuring the level of synchronization and the time delay (or which series leads the other) between two discrete time series. The method was first presented by Quiroga, Kreuz and Grassberger in an article published in 2002 [15]. The main advantage of this method is the simplicity with which the analysis is made, based on defining, recording and then computing events.

The method works best when clear events can be defined (i.e. best when spikes, i.e. strong magnitude changes, are included in the series). For instance, stock market crashes, big monetary policy changes, they can all be taken into account. In order to respond to our problem, we would want to define events based on important changes in our indicators evolution. This would be defined mathematically as local maxima, and the relationship between these events, or maxima, is at the heart of our analysis.

The process will consist of the following stages:

- Defining the events. Events are defined as points that meet the following conditions:

$$x(t_i) > x(t_{i+k}) , \text{ for } k = -K + 1, \dots, 0, \dots, K + 1 \quad (1)$$

$$x(t_i) > x(t_{i \pm k}) + h \quad (2)$$

The same applies for all series taken into account. Equation (1) represents the fact that a local maxima is superior to its neighboring points. Equation (2) notes the fact that this difference needs to be significative enough to be taken into account. The level of K, the time lag of events with which the optimum candidate is compared and h, a parameter, are both defined based on the case, and such that all significant events are eventually taken into account.

- Defining the counter. Counting the number of events that are close enough to each other.
- Computing the statistics that describe the phenomena of synchronization and time-delay. Use the counter to compute the following statistics (see equation (3) below).
- Interpretation of the statistics by focusing on potential time evolution.

In the next section we will focus on the mathematical implementation of the previously described methodology.

4.2 Mathematical implementation

4.2.1 The framework

We are working on a pair of time series cycles, x_n and y_n with $n = 1, \dots, N$. First, we need to define what signals (or events) we will be analysing to evaluate synchronization between these variables. We will take local maxima, based on equation (1) and (2), a good time event measure to do what we want because they represent turnarounds in the cycle's phase. Consequently, if series are synchronized, most of local maxima should be close in time. With this in mind, we define t_i^x and t_j^y , the events time, with $i=1, \dots, m_x$ and $j=1, \dots, m_y$, the number of events.

4.2.2 The recording of linked events

However, we need to ask ourselves how much time lag should be allowed between local maxima of both time series ? We define this parameter $\pm\tau$, as the authorized time lag between two maxima (or events). As we can see in equation (4) below, the level of this parameter will have a great impact on our conclusions, because if this time lag is too short we won't be able to identify synchronization between our variables. In this case only equally timed events

will be counted, but we know that synchronization between two series is mostly not at the exact same time (so as we are working on monthly data, at the exact same month). Grassberger et al. [14] give this unique condition that this parameter τ "should be smaller than half the minimum inter-event distance, to avoid double counting", so we decided to begin with 1.75 as a fixed τ .

We next need to count the number of times the chosen time event appears in both time series within the predefined time lag τ :

$$c^\tau(x|y) = \sum_i^{m_x} \sum_j^{m_y} J_{ij}^\tau \quad (3)$$

With J_{ij}^τ a counter permitting to compute the number of time an event appears in our x variable after its happens in y .

$$J_{ij}^\tau = \begin{cases} 1 & \text{if } 0 < t_i^x - t_j^y < \tau \\ 1/2 & \text{if } t_i^x = t_j^y \\ 0 & \text{else} \end{cases} \quad (4)$$

4.2.3 The statistics

Next we need to define the synchronization and time delay coefficients.

$$Q_\tau = \frac{c^\tau(x|y) + c^\tau(y|x)}{\sqrt{m_x m_y}} \quad (5)$$

$$q_\tau = \frac{c^\tau(y|x) - c^\tau(x|y)}{\sqrt{m_x m_y}} \quad (6)$$

Denominator makes normalization possible in order to get $Q_\tau \in [0; 1]$ and $q_\tau \in [-1; 1]$.

Interpretation keys:

Q_τ representing synchronization between our series, it equals to 1 if they are perfectly synchronized, and zero otherwise. As for q_τ , it equals 1 when events in x always precede those in y , and vice versa with -1.

However, in those last lines we made an important supposition : That the hypothetical coefficients (time delay and synchronization) and the time lag τ doesn't change over time. Even if it gives us a great intuition about the event synchronization characteristic between two series, it's important to remember that we are not currently working on some deterministic cycles, so these results could change other time. For example, we could expect crisis having an impact on time series synchronization (upwards or downwards, it's too soon on the essay to give the answer).

With this in mind, we now need to determine a local τ_{ij} evolving over time according to the frequency of event appearing in a certain time scale.

$$\tau_{ij} = \min\{t_{i+1}^x - t_i^x, t_i^x - t_{i-1}^x, t_{i+1}^y - t_i^y, t_i^y - t_{i-1}^y\}/2 \quad (7)$$

With $1/2$ permitting to avoid double counting of a time event, for example when two events in x are too close to an event in y , this would cause bias in our final results. The other way is to take $\min[\tau, \tau_{ij}]$

$$c_n^\tau(x|y) = \sum_i \sum_j J_{ij} \Theta(n - t_i^x) \quad (8)$$

With Θ a step function, it means a mathematical function of a single real variable that remains constant within each of a series of adjacent intervals but changes in value from one interval to the next. It will give scale graphs to study the evolution of the events synchronization across time. It's necessary to put that step function in there because synchronization can't be analyzed in a continuous way (because of the definition of τ_{ij}).

$$\Theta(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases} \quad (9)$$

J_{ij} is computed like J_{ij}^τ , except that τ_{ij} is replacing τ .

Now we can compute again our synchronization and time delay coefficient, they will be evolving over time with n .

$$Q(n) = c_n^\tau(x|y) + c_n^\tau(y|x) \quad (10)$$

$$q(n) = c_n^\tau(y|x) - c_n^\tau(x|y) \quad (11)$$

$Q(n)$ is *the strength of event synchronization across time* [14], its results are shaped by the existence of the step function, it assures that $Q(n)$ will increase one step if and only if an event is found in both time series within the time lag. Same logic for $q(n)$, which is a special random walk stepping up each time an event in x precedes an event in y within the time lag, and vice versa. If events appear simultaneously in both time series $q(n)$ doesn't move. However, like $Q(n)$, its results are deduced from the step function.

As the domain $Q(n)$ and $q(n)$ is discrete, we may compute discrete derivatives from it to obtain event synchronization at time averaged depending on the last Δn time steps.

$$Q'(n) = \frac{Q(n) - Q(n - \Delta n)}{\sqrt{n_x n_y}} \quad (12)$$

We denote n_x (respectively n_y) the number of time events appearing in x (respectively in y) in the interval $\{n, n - \Delta n\}$ we choose.

The interval we choose will have an important consequence on $Q'(n)$ and its graph because of its role to play on potential number of steps of our function. Unlike Grassberger et al. [14], who work on sets of intracranial EEG recordings, we are applying that algorithm to economic time series. Those intracranial recordings are by nature subject to a lot of local maximas while there are less turnarounds in economic series. So we need a large interval to avoid cases where no time event appears in a shorter time scale. The consequence of a too short

interval would be a lot of n for $Q'(n)$ is null, so our results and graphs would be unreadable and uninterpretable because of fractures in our function.

5 Interpretation of ES: testing the business, the monetary and the credit cycle

After presenting in the last section the method used to evaluate the link between different cycles, we will now test the parameter chosen in section 2, and present our results.

5.1 The link between the business and the credit cycle: IP and LTIR

According to some, lowering interest rates is an expansionary credit policy which should favour money supply and therefore positive business results [4]. However, others indicate that it could also be at the source of future business instability [12].

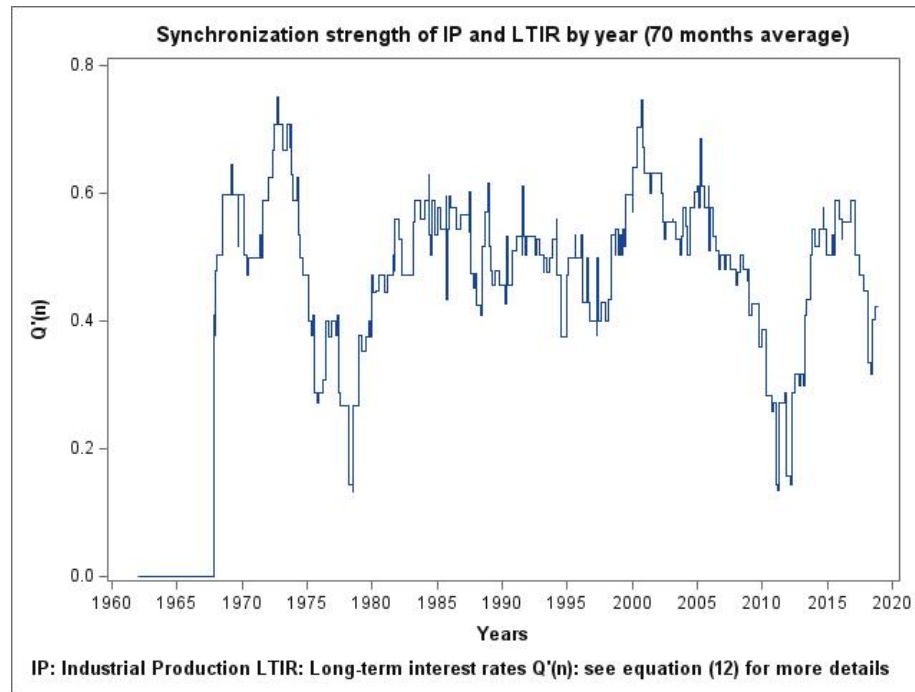


Figure 3

Let's first observe the strength of synchronization between Industrial Production

and Long-term interest rates from 1960-2020 in the United States, in Figure 3. The synchronization level may show us the strength between the relationship of the business and the credit cycle in the US in the last 60 years. We consider a regular level of synchronization to be between 0.3 and 0.6, while values below and above are irregularly low or high values. We can deduce 5 distinct periods:

- 1. From the 1960s to the 1973 oil crisis, when the synchronization between IP and LTIR was relatively strong.
- 2. From 1973 to 1980, where there is a strong fall in the synchronization between the credit and the business cycle.
- 3. From 1980 to 2000, a stable period of synchronization between the two cycles.
- 4. From 2000 to 2008, where there is again a sudden rise in the synchronization level between IP and LTIR. From 2008 to 2014, this transforms into a big drop in the level of synchronization with the global economic crisis.
- 5. From 2014, it seems that we are again at a stable level, with a potential hard fall in 2020 (still not verified).

The pattern that we can deduce is clear: in a period of big economic instability, it seems as if the two cycle disconnect and are relatively less synchronized (period 2 and 4). However, to be noted is a strongly increasing relationship right before the fall.

Thus, period 2 is characterised by two important oil crises (1973 and 1979) which has provoked a downturn in the business cycle (and IP in particular). Period 4 marks the beginning of the 2008 global financial crisis, which has become very quickly a global business crisis.

To complete our analysis, we have another statistic available to analyze: time-delay relationship in Figure 4. We will note that the 70 months average was chosen manually, after numerous attempts to obtain correct results, as the average needs to be on a period that is sufficiently big to be precise.

The following Figure 4 represents a walk that takes a step up if an event in LTIR precedes IP, and a step down as IP precedes LTIR.

There is a very interesting result, suggesting that before 1971, the Industrial Production was the leading cycle. That is, it seems as if the business cycle was moving first, thus determining the long term interest rates in the U.S. However, this tendency was completely reversed after 1971, where the long-term interest rate is starting to impact Industrial Production.

Finance was more strongly regulated after the second World War. Therefore, the business cycle had an impact on the interest rates, which were much less subject to financial volatility. However, in August 1971, was the end of the Bretton Woods system, a system of fixed exchange rates and relatively strong financial regulation (as we can see, the time delay evolution changes precisely around that year).

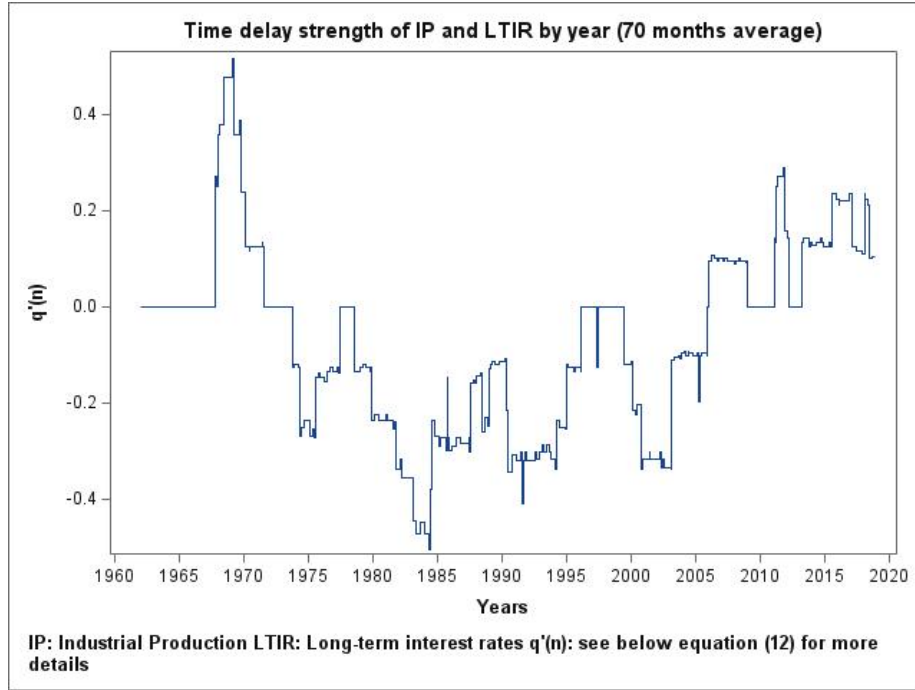


Figure 4

The second period (after 1971) is marked by a leading credit cycle compared to the business cycle of Industrial Production. The period from 1975-1983 is marked by smaller synchronisation, and the time-delay is therefore not particularly pertinent. Even so, the fall of $q(n)$ is, interestingly, coming at the exact moment when there was an expansion of "financial innovations" in the US, along with the new situation of flexible exchange rates. Finance was deregulated, leading to financial innovation, particularly at the end of the 1970s and in the 1980s. After the two oil crisis, synchronization between the two cycles are again at a strong level and it seems as if the financial indicators have become much more powerful in determining economic activity. The business cycle, on the other hand became less relevant and more dependant, compared to the credit cycle. However, the difficulty in this type of analysis is that events are not defined precisely enough. Thus, negative and positive impacts of an indicator are both taken into account, and there is hardly a possibility to precisely describe causality without exploring more in details the cycles.

5.2 The link between the business and the monetary cycle: IP and M3

The link between the monetary and the business cycle is strongly discussed in economic theory (see section 1.3.). M3 is an indicator of money supply, mostly influenced by the policy of the Federal Reserve, in the United States. We will search for its relationship with the Industrial production from 1960 onwards.

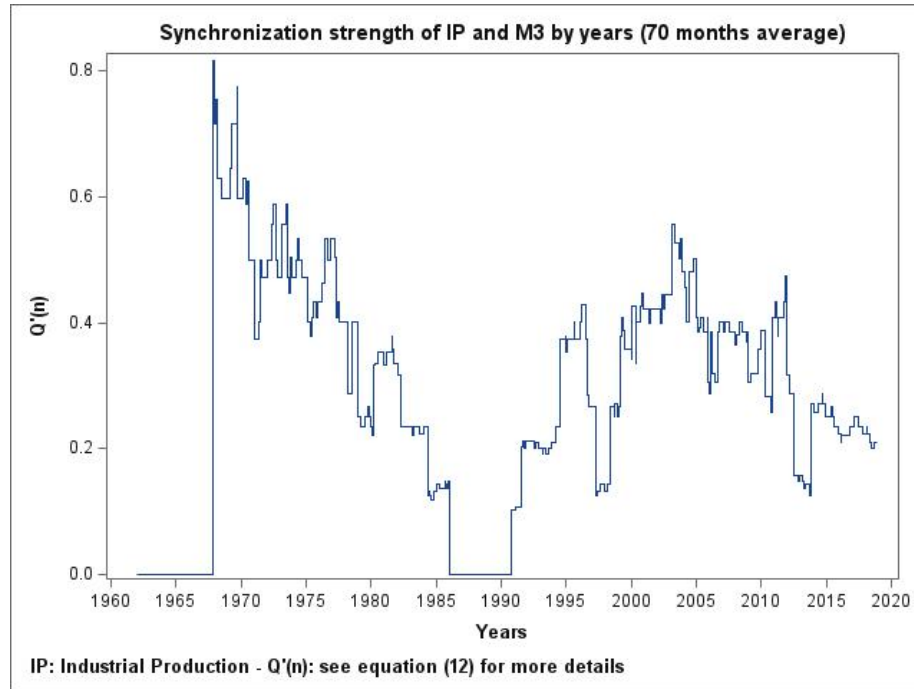


Figure 5

Figure 5 will show us the level of synchronization between our monetary and business cycle indicator. We will thus establish the periodical difference in the relationship between the two. There seems to be the following periods:

- Firstly, from the beginning of our period, when the synchronization is very strong, there is a decreasing tendency up to 1986, when there is no synchronization at all.
- The events are then increasingly synchronized from the start of the 1990s, to 2005, with a falling break in 1997-1998.
- A mild synchronization from 2005, with another fall from 2012-2013.

In general, the synchronization relationship seems to be much less straightforward than in the previous case, of the credit cycle. The synchronization level

seems also to be much less than in the previous case.

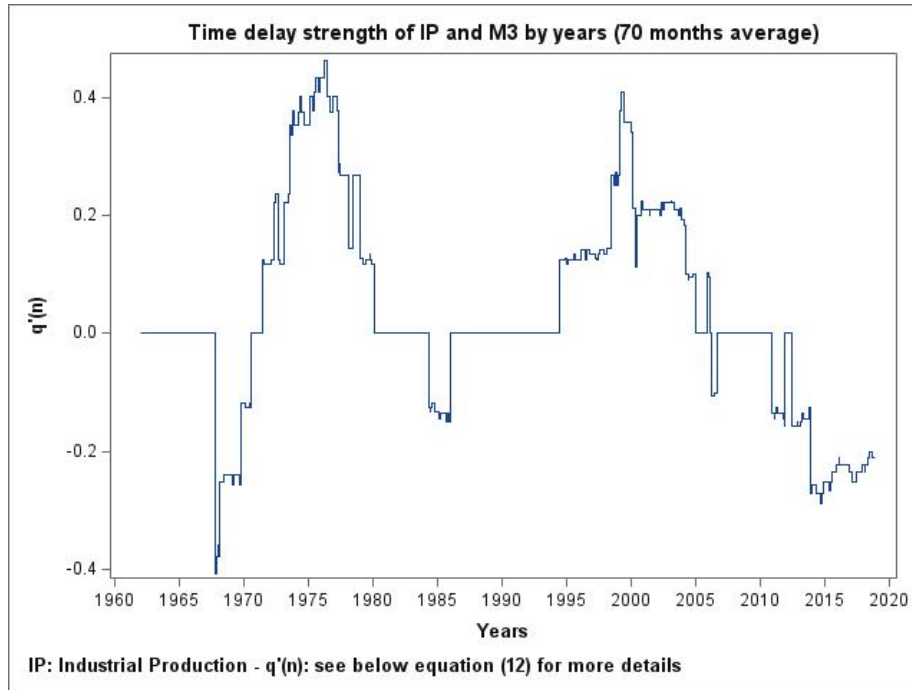


Figure 6

Figure 6 shows a walk that takes a step up if an event in M3 precedes those in IP, and a step down if IP precedes M3.

5.3 The link between the monetary and the credit cycle: M3 and LTIR

The relationship between the credit and the monetary cycle seems, at first, as the most obvious one. Interest rates have a direct impact on the level of borrowing as lower interest rates favour more and higher interest rates favour less credits. The interest rate is considered, in a way, the price of the loan. Figure 7 shows us this important synchronization by years, in the United States.

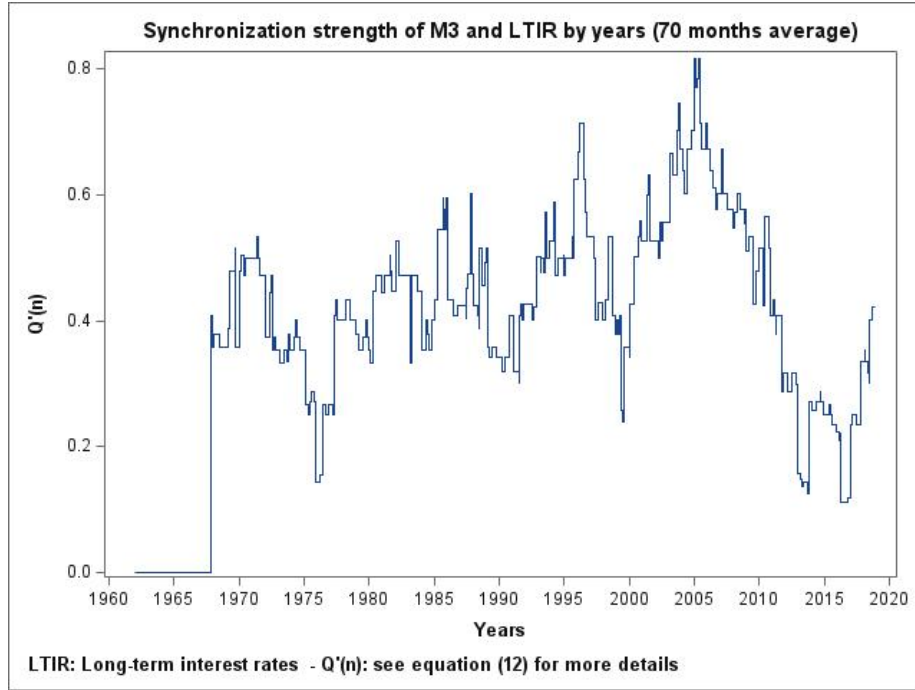


Figure 7

Unlike the two previous cases, there seems not to be a big difference in synchronization strength through the periods.

To be noted is maybe only the reaction in synchronization strength to crises. Firstly, the 1970s oil crisis in the U.S. provoked the first drop in synchronization between the two cycles. Secondly the 2008 crisis has provoked a second, much more significant, free fall in the synchronization level between the monetary and the credit cycle. This is, as we explained in section 1, possible by the phenomenon known as liquidity trap. The liquidity trap is a concept developed by J.M. Keynes in which he explains the possibility of having extremely low interest rates, and, at the same time, consumers which are uninterested in borrowing at these rates. Credit creation, and M3 as an indicator of the amount of money in the economy, are not reacting to such changes of monetary conditions. According to many authors (e.g. Paul Krugman [11]), this has been precisely the case after the 2008 crisis, which this synchronization fall may represent well.

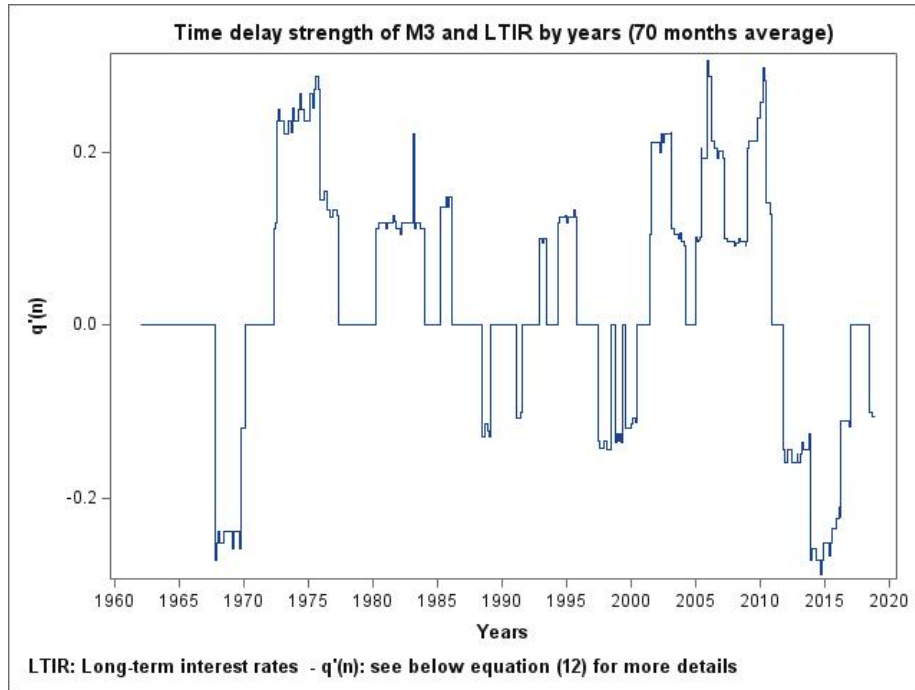


Figure 8

Finally, Figure 8 shows us the time delay between M3 and LTIR. In this case, the figure shows a walk that takes a step up if an event in M3 precedes those in LTIR, and a step down if M3 precedes LTIR.

The relationship between the two seems to be rather unstable. There is no clear tendency amongst the periods, and the time delay is not strong as in the previous two cases (the maximum goes only beyond 0.2, and before it was going beyond 0.4). With these results, we are thus incapable of giving the leading indicator between the monetary and the credit cycle, and in particular, it will be impossible to deduce a certain causality.

5.4 ES results recap

In conclusion, we were able to have some indication on the relationship strength between cycles with this analysis. This is, however, not enough to conclude a certain causality.

6 Cross-Correlation

6.1 Measure of temporal similarity

In time series analysis, cross-correlation corresponds to a measure of similarity of two time series. A function of time-lag is applied to one of them, by estimating the correlation between one time series at time t and the other at time $t \pm x$ lags.

In our analysis, cross correlations are used to determine whether a series of data leads another series, giving us an estimation of the lag (or time-delay) between our cycles. This method allows us to observe the long-term pattern that binds the series and not just short-term anomalies.

Before, it is necessary to assure that there is no auto-correlation in the two series. The auto-correlation usually causes difficulty in identifying meaningful relationships between the two-time series.

Let us consider two simultaneously measured univariate time series $x(t)$ and $y(t)$, where t is the time; in this case discrete time points $t = 1, \dots, T$ and with lags $h \in \{-H, \dots, 0, \dots, H\}$.

The cross-correlation function (CCF) is then given by:

$$\gamma_{x,y}(h) = \begin{cases} \frac{1}{T} \sum_{t=h+1}^T (x_t - \bar{x})(y_{t-h} - \bar{y}) & 0 \leq h < T \\ \frac{1}{T} \sum_{t=|h|+1}^T (x_{t-|h|} - \bar{x})(y_t - \bar{y}) & -T < h < 0 \end{cases} \quad (13)$$

$$\rho_{x,y}(h) = \hat{\gamma}_{x,y}(h) / \sqrt{\hat{\gamma}_x(0)\hat{\gamma}_y(0)} \quad (14)$$

In this equation, $\gamma_{x,y}(h)$ corresponds to the autocovariance function of two jointly processes, \bar{x} and \bar{y} are respectively the empirical mean of the series $x(t)$ and $y(t)$ and h a time lag. With cross-correlation, the absolute value of CCF ranges from zero, no synchronization, to one, perfect synchronization.

After estimating the CCF, we look at the time lag for which the cross-correlation function estimation reaches its maximum value (in absolute value), to identify a leading or lagging relation. This time lag correspond to the time delay between the two series.

Notice that: $\rho_{x,y}(h) = \rho_{y,x}(-h)$ but $\rho_{x,y}(h) \neq \rho_{x,y}(-h)$

The main disadvantage of the cross-correlation method is expose in the article of J. Arnhold, P. Grassberger, K. Lehnertz, C.E. Elger [1]. The Cross-correlation method measures only linear dependencies. In contrast to them, the event synchronization method, developed in section 4, is non-linear and depends on recording events and not on lag. With cross-correlation, causal relationships can be detected only if they are associated with time delays. Another interesting approach is to evaluate the moving window cross-correlation.

6.2 Moving window cross-correlation

By computing the cross-correlation with the moving window method, we will be able to observe the time delay between two curves by years. Therefore, we will have a possibility to compare the Event synchronization method with the cross-correlation.

The method consists of taking for each iteration a different sample of observations, and compute the cross correlation of all lags in the series. Thus, for each sample, we will be able to maximise the cross-correlation (in absolute value), and determine the time delay of the series.

Suppose we have N observations, or months for us, and $n = 1, \dots, N$ a variable changing at each iteration. In our case, to keep consistency with the time delay measurement in the Event Synchronisation, we will take into account a 70 month period. The interval for which is computed the cross-correlation is $[n, n + 70]$, with n moving at each iteration from 1 to $N - 70$. We maximise for each interval the cross correlation (in absolute value) and we have a time series of lags for which the cross-correlation was maximized.

6.3 Comparison between the Cross correlation and the Event Synchronization method

This is what we have done obtaining the following results for IP and LTIR:

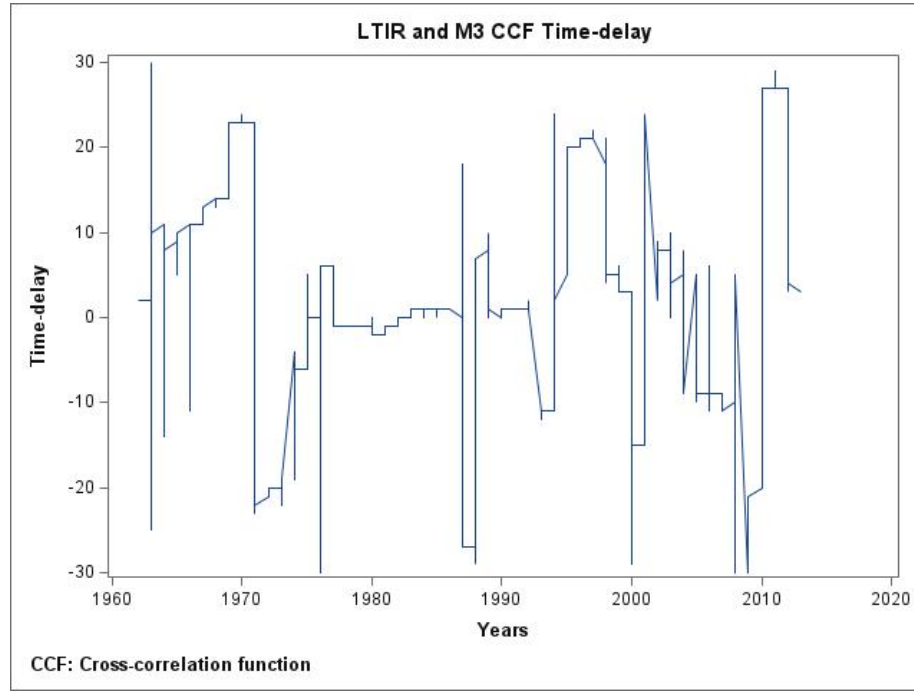


Figure 9

For Figure 9, a positive time-delay means that IP precedes LTIR and a negative means the opposite.

Up to 1970, the figure shows a positive time-delay, meaning that the Industrial Production is leading the cycle progression. This is coherent to the results obtained in Figure 4, with the Event Synchronization method. Moreover, the break in 1971 is also clear in this figure as in the previous one. However, the time-delay becomes then extremely low, and gravitates towards zero (1975-1988). This appears at the same period where, in Figure 3, we have concluded a smaller synchronization level between the two variables. For the last period, especially beginning in 2000, there is an unclear relationship between the cycles, and there is no possibility to tell which one leads the other.

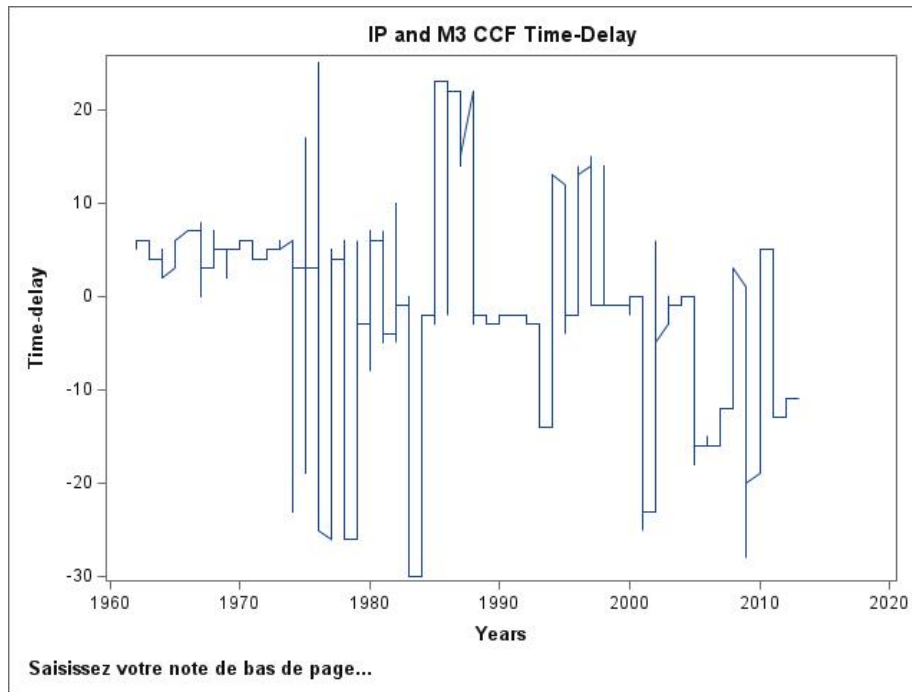


Figure 10

In this Figure 10, positive time-delays means that the Industrial Production precedes M3.

As we can see, there is no clear tendency, as the relationship between the two indicators is very unstable. To be noted is also the difficulty, as for the last period in the previous example, compared with the Event Synchronization Method (here, Figure 6), to draw any conclusion when time-delay fluctuates a lot. The power of the Event Synchronization method, especially compared to the more traditionally used Cross-correlation method, is that it divides synchronization between the variables and time-delay, which is not done here. Again, there are a lot of time-delays around 0, meaning as in the previous example, probably just a lack of synchronization. This is something easily concluded in the Event Synchronization method.

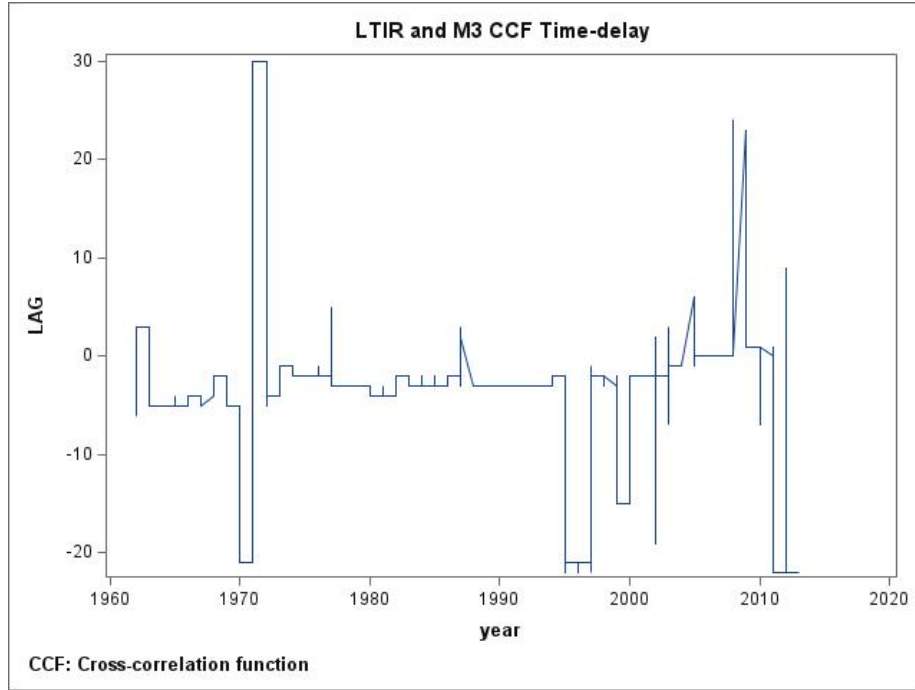


Figure 11

Finally, Figure 11, comparing Long term interest rates and M3, should be positive if LTIR precedes M3.

We observe, on the contrary, that, almost constantly, M3 very slightly precedes LTIR, according to the Cross-correlation method. From the year 2000, the relationship becomes very unstable, which is also observed in Figure 8, according to the Event Synchronization Method. The only conclusion we can make is that, again, the tendency is much more easily perceived with the Event Synchronization method than the Cross-correlation, when fluctuations increase.

7 Model Uncertainly and Robustness

7.1 Results sensitivity to the choice of indicators

In Section 2.1 we discussed the indicators we chose to represent the different cycles, and we decided to choose 2 different indicators to represent each cycle so that we can assess variations in conclusions regarding those chosen indicators. Therefore, and as mentioned above, in this section we will apply the Event synchronisation method to measures synchronization and time delay on the alternative indicators of cycles, namely : CLI as Composite Leading Indicator, SHP as Share Prices and FFR for Federal funds rate. (*See sections 2.1 for details*). Then, we will compare results with those obtained in Section 5 using

the other indicators to assess the results sensitivity to the choice of indicators.

7.1.1 The link between business and credit cycle : CLI and SHP

The following Figures represent, respectively, the Synchronisation degree and time delay in a 70 months windowed average.

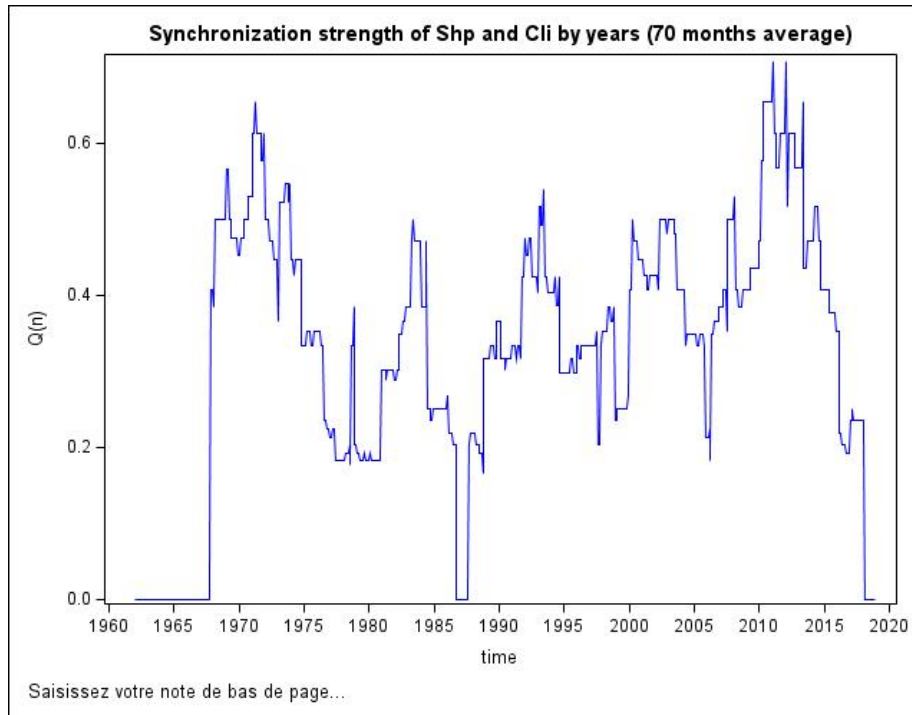


Figure 12

We can notice, comparing with Figure 3, that by using new indicators we get the same timing of peaks and the same overall trend of synchronisation degree as before. But, we also notice some differences in its variation: that it is much more volatile using the new indicators.

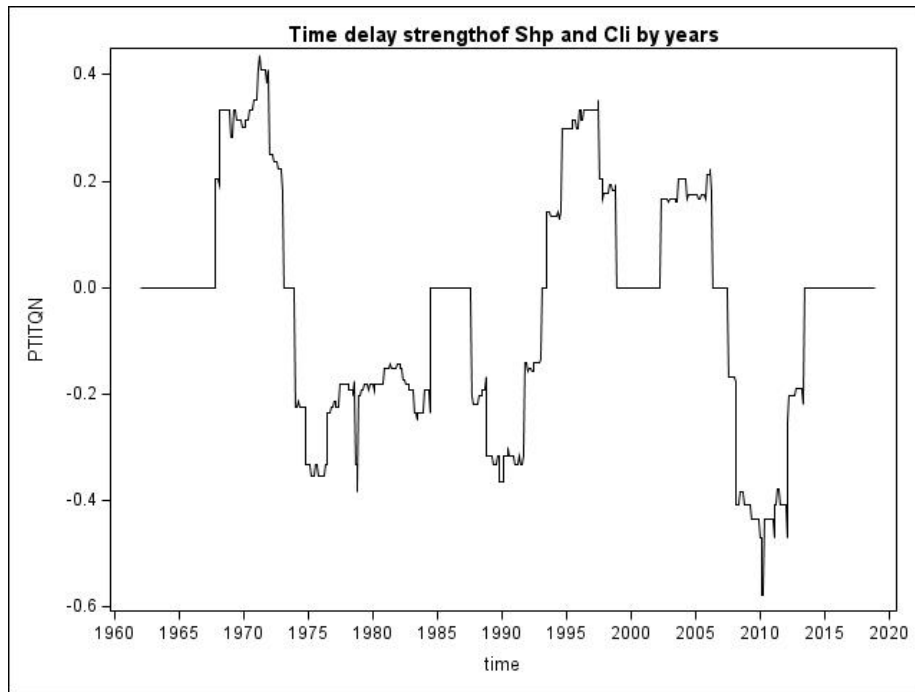


Figure 13

When it comes to time delay (comparing with Figure 4), we notice more difference in results. The new indicators give the same order in the relative timing of events in the first period (1967 - 1995) but the opposite one in the second period (1995 - 2020).

7.1.2 The link between business and credit cycle : CLI and FFR

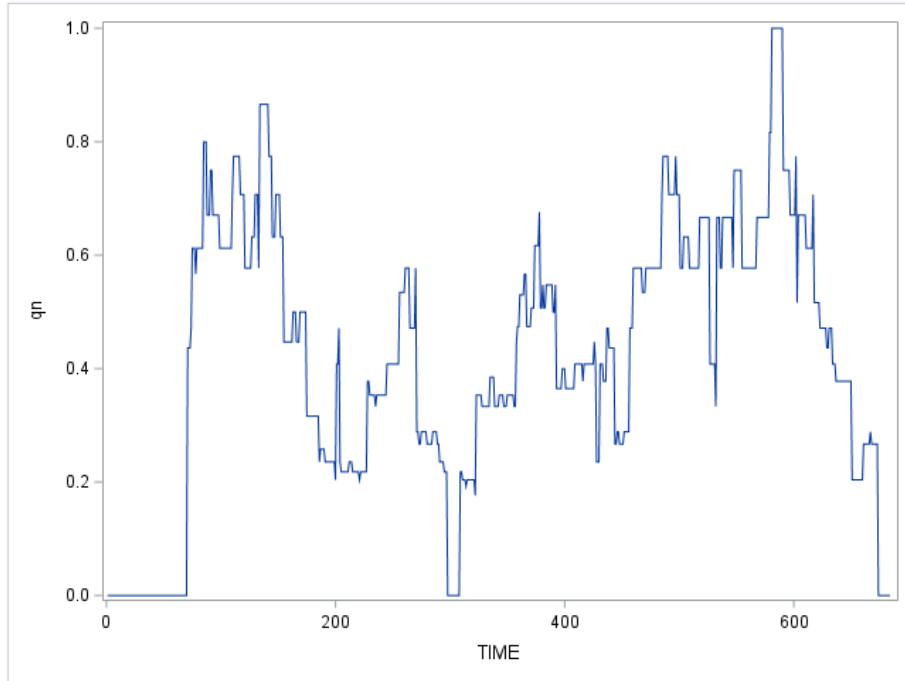


Figure 14: **Synchronization strength of CLI and FFR by years (70 months average)**

We notice that, compared to Figure 5, the new indicators give the exact same tendency in the degree of synchronization and the same timing of peaks. The most important difference we can notice is that CLI and FFR capture a more intense synchronisation during the Subprime crisis than that captured by IP and M3.

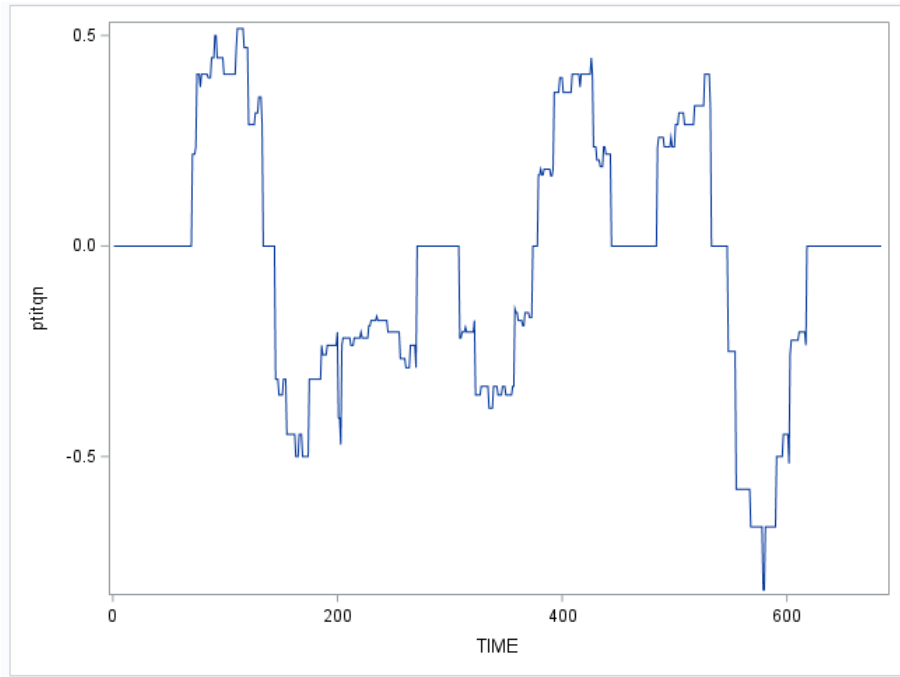


Figure 15: **Time delay of CLI and FFR by years**

The time delay analysis in Figure 10 using the new indicators (compared to Figure 6) give the same results except for the period between (1967 - 1972) where it gives completely opposite results.

7.1.3 The link between business and credit cycle : SHP and FFR

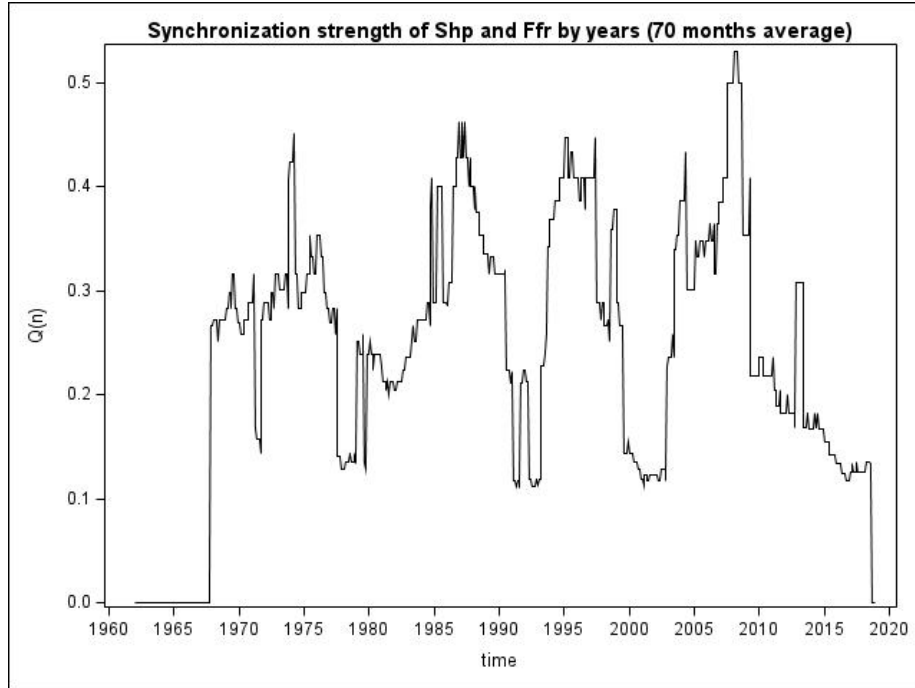


Figure 16

Using SHP and FFR, compared with Figure 7 of M3 and LTIR, we get the exact same results when it comes to the degree of synchronisation. Synchronization is consistent, relatively high and has peaks which can be explained by major monetary decisions made by the FED, those decisions directly impacting the Federal funds rate (FFR) which then impact the share prices (SHP) because of the relative abundance of money in the markets. We can validate this conclusion by analysing the time delay, hence the following Figure 14.

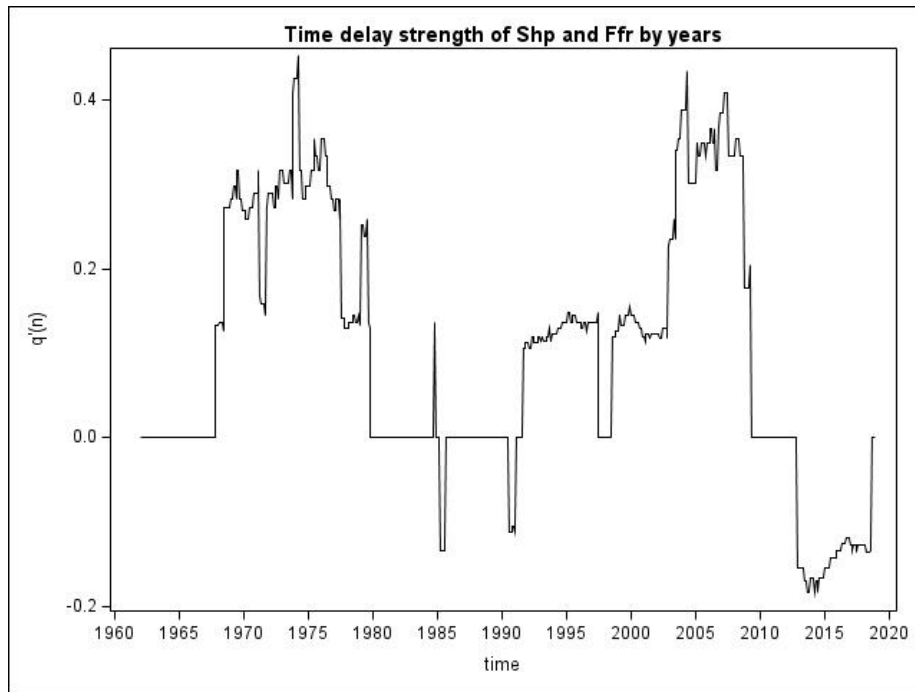


Figure 17

We notice that Using FFR and SHP to analyse time delay gives the same conclusions as using M3 and LTIR (Figure 8). Namely, almost always, the monetary cycle moves before the credit cycle, or put differently, events appear in the monetary cycle before the credit cycle.

8 Conclusion

8.1 Stylized Facts

Event Synchronization tests showed us some stylized facts that can be interpreted, but not generalized. For instance, while a drop of synchronization was seen right before the 2008 crisis between the business and credit cycle indicators, and a sharp fall when the crisis began, there is no such phenomenon for the business and monetary cycle. In the same way, when it comes to time-delay, its strength and the hypothetical causality vary across time.

Synchronization strength and time delay tests on new indicators led us to almost the same results, which is comforting us in our conclusions. The Event Synchronization method seems therefore to capture well the mutual influence different cycles have.

Cross-correlation's time delay analysis was not revealing and demonstrative in results compared to Event Synchronization, but we found some patterns close to the ES time delay strength, that gives confirmation to our first results.

To be noted is that all results found in section 5 are pertinent, but does not mean necessary an existing causality between the variables. To prove causality, future works should be able to integrate more factors into the research (for instance variables that both impact business and credit cycles) and test if the leading cycle plays the role that our results suggest.

When it comes to comparing the Event Synchronization method with the Cross-correlation, our main result is that we have demonstrated the superiority of the first method. The biggest advantages are that the synchronization level is an important factor to be taken into account when analyzing time delay, which can only be done with the first method. Cross-correlation, thus, shows results difficult to interpret when synchronization is low and/or time-delay has no clear tendency and fluctuates a lot.

8.2 Methodological thoughts

Some things in this essay could have been done differently. Indeed, the sensitivity analysis only focused on testing other statistical indicators. We could have implemented other cycle extraction methods to verify if they have an impact on our results. We have already mentioned that several approaches exist to extract a cycle from time series. We arbitrarily chose to use Hodrick-Prescott filter consisting in resolving a minimization problem in two parts : detrending and smoothing initial series in order to get its cycle.

However, we are well aware of some weaknesses the HP filter have. This method postulates that a series is nothing more than :

$$y_t = \tau_t + c_t$$

There isn't any error term, it means that this approach considers that a series is only characterised by a long term and a short term kind of trend (remember

the sinusoid angle of approach). However, even if such equation is theoretically effective to explain series construction, we can't just summarize an economic series to this, by regrouping cycle and error term. Since its introduction in 1981, the filter has been regularly used in statistical literature, but quickly voices raised to point out the risk of extracting spurious cycles with this method [3] because of smooth and slowly changing trend on some American time series, like durables consumption. In consequence, such method could affect stochastic properties of the data and simultaneously the explanatory power of variance and covariance of the filtered series. It could have been interesting to use other approach : for instance, the stochastic trend model [2] based on the Kalman filter, supposed to respect those stochastic properties. This would have been an other way to test the sturdiness of our conclusions.

References

- [1] J. Arnhold et al. “A Robust Method for Detecting Interdependences: Application to intracranially recorded EEG”. In: *Physica D: Non linear Phenomena* 134 (Dec. 1999), pp. 419–430.
- [2] Laurence Boone. “Estimation du cycle à l’aide d’un modèle à tendance stochastique et application au Royaume-Uni”. In: *CEPII* (1997).
- [3] Andrew Harvey and A Jaeger. “Detrending, Stylized Facts and the Business Cycle”. In: *Journal of Applied Econometrics* 8.3 (1993), pp. 231–47. URL: <https://EconPapers.repec.org/RePEc:jae:japmet:v:8:y:1993:i:3:p:231-47>.
- [4] John Hicks. “IS-LM: An Explanation”. In: *Journal of Post Keynesian Economics* 3.2 (1980), pp. 139–154.
- [5] Tempelman Jerry and Cfa. “Austrian Business Cycle Theory and the Global Financial Crisis: Confessions of a Mainstream Economist”. In: 13 (Jan. 2010).
- [6] Juglar and Clément. “Des crises commerciales et de leur retour périodique en France, en Angleterre et aux États-Unis”. In: (1862). URL: <https://gallica.bnf.fr/ark:/12148/bpt6k1060720>.
- [7] J. M. Keynes. *The General Theory of Employment, Interest and Money*. 1936.
- [8] Joseph Kitchin. “Cycles and Trends in Economic Factors”. In: *The Review of Economics and Statistics* 5.1 (1923), pp. 10–16. ISSN: 00346535, 15309142. URL: <http://www.jstor.org/stable/1927031>.
- [9] Kittredge and Dorothea D. “Secular Movements in Production and Prices: Their Nature and Their Bearing upon Cyclical Fluctuations, by Simon S. Kuznets. Boston and New York: Houghton Mifflin Company. 1930. p. xxiv, 536”. In: *American Journal of Agricultural Economics* 13.1 (Jan. 1931), pp. 177–179. ISSN: 0002-9092. DOI: 10.2307/1230399. eprint: <https://academic.oup.com/ajae/article-pdf/13/1/177/190399/13-1-177.pdf>. URL: <https://doi.org/10.2307/1230399>.
- [10] Nikolai Kondratieff. “The Long Wave Cycle”. In: (1925).
- [11] Paul Krugman. “How much of the world is in a liquidity trap?” In: *New York Times* (2010). URL: <https://krugman.blogs.nytimes.com/2010/03/17/how-much-of-the-world-is-in-a-liquidity-trap>.
- [12] Minsky and Hyman P. “Stabilizing an unstable economy”. English. In: (1986), xiv, 353 p. :
- [13] OCDE. “Glossary”. In: *OCDE Leading Indicator Website* (2001). URL: <https://stats.oecd.org/glossary/index.htm>.

- [14] Quian et al. “Event Synchronization: A simple and fast method to measure synchronicity and time delay patterns”. In: *Physical review. E, Statistical, nonlinear, and soft matter physics* 66 (Nov. 2002), p. 041904. DOI: 10 . 1103/PhysRevE.66.041904.
- [15] Hodrick Robert and Prescott E.C. “Post-war US business cycles: An empirical investigation”. In: 8 (Jan. 1981), pp. 231–247.
- [16] Schumpeter and Joseph Alois. “Business Cycles: A Theoretical, Historical, and Statistical Analysis of the Capitalist Process”. In: (1939).
- [17] Guy Vassallucci. “Cycles”. In: *Lycée Sévigné. Guy Vassallucci* (2014). URL: <http://eloge-des-ses.com/wp-content/uploads/2015/09/Pourquoi-la-croissance-est-elle-instable-2014-GV.pdf>.

Appendix 1

**Augmented Dickey-Fuller's test :
inspired from Econometric Analysis, William H Greene**

The Augmented Dickey Fuller Test (ADF) is unit root test for stationarity. Unit roots can cause unpredictable results in your time series analysis. The ADF test can handle more complex models than the Dickey-Fuller test, and it can be used with serial correlation. Likewise, with a greater number of regressors, in order to bleach the residues for example (regressors which will be here the first delayed differences) an ADF test can be used.

In the Dickey Fuller test, the null hypothesis is that there is a unit root. The alternate hypothesis differs slightly according to which equation you're using, mostly is that the time series is stationary (or trend-stationary).

The random walk version model is like :

$$y_t = \gamma y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N[0, \sigma^2] \text{ and } \text{Cov}[\varepsilon_t, \varepsilon_s] = 0 \quad \forall t \neq s$$

Under the null hypothesis that $\gamma = 1$, there are two approaches to carrying out the test. In order to detect the presence of a unit root :

The conventional t ratio

$$DF_t = \frac{\hat{\gamma} - 1}{\text{Est.Std.Error}(\hat{\gamma})}$$

With the revised set of critical values may be used for a one-sided test.

The second approach is based on the statistic

$$DF_\gamma = T(\hat{\gamma} - 1)$$

The augmented Dickey-Fuller test is the same one as above, carried out in the context of the model :

$$y_t = \mu + \beta t + \gamma y_{t-1} + \gamma_1 \Delta y_{t-1} + \dots + \gamma_p \Delta y_{t-p} + \varepsilon_t$$

The random walk form is obtained by imposing $\mu = 0$ and $\beta = 0$; the random walk with drift has $\beta \neq 0$; and the trend stationary model leaves both parameters free. The two test statistics are exactly as constructed before with :

$$DF_\tau = \frac{\hat{\gamma} - 1}{\text{Est.Std.Error}(\hat{\gamma})}$$

and

$$DF_\gamma = \frac{T(\hat{\gamma} - 1)}{1 - \hat{\gamma}_1 - \dots - \hat{\gamma}_p}$$

The advantage of this formulation is that it can accommodate higher-order autoregressive processes in ε_t . An alternative formulation may prove convenient. By subtracting y_{t-1} from both sides of the equation, we obtain :

$$\Delta y_t = \mu + \gamma^* y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta y_{t-j} + \varepsilon_t$$

with

$$\phi_j = -\sum_{k=j+1}^p \gamma_k \text{ and } \gamma^* = (\sum_{i=1}^p \gamma_i) - 1$$

Phillips and Perron's test :

Many alternatives to the Dickey-Fuller tests have been suggested, in some cases to improve on the finite sample properties and in others to accommodate more general modeling frameworks.

Phillips-Perron (1988) propose a non-parametric method for correcting the effect of the presence of auto-correlation, without having to add endogenous delayed as in the ADF method (more robust method in cases of MA errors in particular). The test procedure consists in testing the unit root hypothesis $H_0: \varphi = 0$ in the following models:

- (1) $\Delta X_t = \alpha + Bt + \varphi X_{t-1} + \varepsilon_t$
- (2) $\Delta X_t = \alpha + \varphi X_{t-1} + \alpha + \varepsilon_t$
- (3) $\Delta X_t = \varphi X_{t-1} + \varepsilon_t$

The Phillips-Perron's (PP) test statistic is a corrected Student statistic of the presence of auto-correlation by considering an estimate of the long-term variance of (ε_t) , robust to the presence of auto-correlation and heteroscedasticity. The critical thresholds are the critical thresholds of Dickey Fuller.

Appendix 2: Code for the Event Synchronization method

```

/* ***** WRITING DATA INTO MATRIX ***** */

/* Suppose lf, a matrix containing our six variables : ip, ltir,
   ↪ m3, cli, shp and ffr */

/****** LOCAL OPTIMA WITH THE EVENT SYNCHRONIZATION
   ↪ METHOD *****/

x=j(nrow(lf),ncol(lf)-1,0);
tmp= insert(lf,x, 0,ncol(lf)+1);

/* See the Quiroga,Kreuz, and Grassberger article for more
   ↪ details */
do t=3 to nrow(tmp)-3;
    do s=2 to 7;
if tmp[t,s]>tmp[t-1,s] & tmp[t,s]>tmp[t+1,s] &
   ↪ tmp[t,s]>tmp[t-2,s] & tmp[t,s]>tmp[t+2,s] then tmp[t,s+6]=1;
else tmp[t,s+6]=0;
        end;
    end;

print tmp;
/* tmp is a matrix containing data from lf and defined events (1
   ↪ if event, 0 if not) */

/* Identify Event times */
tx=loc(tmp[,8]=1)`;
ty=loc(tmp[,9]=1)`;
tz=loc(tmp[,10]=1)`;
txx=loc(tmp[,11]=1)`;
tyy=loc(tmp[,12]=1)`;
tzz=loc(tmp[,13]=1)`;
*print tx ty tz txx tyy tzz;
/* We now have event times. */

/* ***** LOCAL OPTIMA WITH THE SAS LOCPEAKS METHOD
   ↪ *****/

/* Defining the Module */
/* Source:
   ↪ https://blogs.sas.com/content/iml/2013/08/28/finite-diff-estimate-maxi.html
   ↪ */

```

```

/* Return the location of the local maxima of a vector (including
↪ end points).
   If there are duplicate values at a maximum, the index
   of the first and last maximum value is returned. */
start locpeaks(x);
  small = min(x)-1; /* smaller than any point in
↪ the series */
  y = small // colvec(x) // small; /* original values are now in
↪ the interior */
  difSgn = dif(sign(dif(y))); /* dif of sign of slopes */
  /* subtract 2: -1 for inserting 'small' at the beginning, and
↪ -1
   for the fact that negative values occurs AFTER the maximum
↪ */
  return(loc(element(difSgn, {-1 -2})) - 2);
finish;

/* Applying the module */
optip=locpeaks(ip);
optltir=locpeaks(ltir);
optm3=locpeaks(m3);
optcli=locpeaks(cli);
optshp=locpeaks(shp);
optffr=locpeaks(ffr);

print optip optltir optm3 optcli optshp optffr;

/* We preferred using the first method, as more events were
↪ identified */

/***** TAU PARAMETER
↪ *****/
/* Fixed tau, smaller normally than the half of the minimum
↪ inter-event distance. */
tau=1.75;

/* Time variable Tau */
do t=2 to nrow(ty)-1;
  do n=2 to nrow(tx)-1;
min_vect = tx[n+1] - tx[n] // tx[n] - tx[n-1] // ty[t+1] - ty[t]
↪ // ty[t] - ty[t-1] ;
min_tau = min(min_vect)/2;
taus = min_tau // tau;
v_tau = min(taus);
v_taus = v_taus // v_tau ;

```

```

end;
end;
*print v_taus;
tau2=j(1,1,tau);
v_taus2=insert(v_taus,tau2,1);
v_taus2=insert(v_taus,tau2,nrow(v_taus)+1);
print v_taus2;

/***** THE COUNTER
→ *****/

/* J(i,j) from the paper of Quiroga, Kreuz, and Grassberger. */
/* We establish the counter (i.e number of times an event appears
→ in x shortly after y */

/* ***** IP AND LTIR COMPARISON *****/

c1r=j(nrow(tmp), 1,0); /* Ct(x/y) in the paper */

do x=1 to nrow(tmp);
    do t=1 to nrow(ty);
        do n=1 to nrow(tx);
if (tx[n]-ty[t])>0 & tx[n]-ty[t]<=tau & x-tx[n]>0 then
→ c1r[x]=c1r[x]+1;
else if tx[n]=ty[t] & x-tx[n]>0 then c1r[x]=c1r[x]+0.5;
else c1r[x]=c1r[x];
end;
end;

end;

c2r=j(nrow(tmp), 1,0); /* Ct(y/x) in the paper */

do x=1 to nrow(tmp);
    do n=1 to nrow(tx);
        do t=1 to nrow(ty);

if (ty[t]-tx[n])>0 & ty[t]-tx[n]<=tau & x-ty[t]>0 then
→ c2r[x]=c2r[x]+1;
else if tx[n]=ty[t] & x-ty[t]>0 then c2r[x]=c2r[x]+0.5;
else c2r[x]=c2r[x];
end;
end;

```



```

end;

print c1r c2r;

/****** IP AND M3 COMPARISON *****/

c3r=j(nrow(tmp), 1,0); /* Ct(x/z) in the paper */

do x=1 to nrow(tmp);
    do t=1 to nrow(tz);
        do n=1 to nrow(tx);
if (tx[n]-tz[t])>0 & tx[n]-tz[t]<=tau & x-tx[n]>0 then
    ↪ c3r[x]=c3r[x]+1;
else if tx[n]=tz[t] & x-tx[n]>0 then c3r[x]=c3r[x]+0.5;
else c3r[x]=c3r[x];
end;
end;

end;

c4r=j(nrow(tmp), 1,0); /* Ct(z/x) in the paper */

do x=1 to nrow(tmp);
    do n=1 to nrow(tx);
        do t=1 to nrow(tz);

if (tz[t]-tx[n])>0 & tz[t]-tx[n]<=tau & x-tz[t]>0 then
    ↪ c4r[x]=c4r[x]+1;
else if tx[n]=tz[t] & x-tz[t]>0 then c4r[x]=c4r[x]+0.5;
else c4r[x]=c4r[x];
end;
end;

end;

print c3r c4r;

/****** LTIR AND M3 COMPARISON *****/

c5r=j(nrow(tmp), 1,0); /* Ct(x/z) in the paper */

```

```

do x=1 to nrow(tmp);
    do t=1 to nrow(tz);
        do n=1 to nrow(ty);
if (ty[n]-tz[t])>0 & ty[n]-tz[t]<=tau & x-ty[n]>0 then
    ↪ c5r[x]=c5r[x]+1;
else if ty[n]=tz[t] & x-ty[n]>0 then c5r[x]=c5r[x]+0.5;
else c5r[x]=c5r[x];
end;
end;

end;

c6r=j(nrow(tmp), 1,0); /* Ct(z/x) in the paper */

do x=1 to nrow(tmp);
    do n=1 to nrow(ty);
        do t=1 to nrow(tz);

if (tz[t]-ty[n])>0 & tz[t]-ty[n]<=tau & x-tz[t]>0 then
    ↪ c6r[x]=c6r[x]+1;
else if ty[n]=tz[t] & x-tz[t]>0 then c6r[x]=c6r[x]+0.5;
else c6r[x]=c6r[x];
end;
end;

end;

print c5r c6r;

/* ***** CLI AND SHP COMPARISON ***** */

c7r=j(nrow(tmp), 1,0); /* Ct(x/y) in the paper */

do x=1 to nrow(tmp);
    do t=1 to nrow(tyy);
        do n=1 to nrow(txx);
if (txx[n]-tyy[t])>0 & txx[n]-tyy[t]<=tau & x-txx[n]>0 then
    ↪ c7r[x]=c7r[x]+1;
else if txx[n]=tyy[t] & x-txx[n]>0 then c7r[x]=c7r[x]+0.5;
else c7r[x]=c7r[x];
end;
end;

```

```

end;

c8r=j(nrow(tmp), 1,0); /* Ct(x/y) in the paper */

do x=1 to nrow(tmp);
  do n=1 to nrow(txx);
    do t=1 to nrow(tyy);

if (tyy[t]-txx[n])>0 & tyy[t]-txx[n]<=tau & x-tyy[T]>0 then
  ↪ c8r[x]=c8r[x]+1;
else if txx[n]=tyy[t] & x-tyy[t]>0 then c8r[x]=c8r[x]+0.5;
else c8r[x]=c8r[x];
end;
end;

end;

*print c7r c8r;

***** CLI AND FFR COMPARISON *****

c9r=j(nrow(tmp), 1,0);

do x=1 to nrow(tmp);
  do t=1 to nrow(tzz);
    do n=1 to nrow(txx);
if (txx[n]-tzz[t])>0 & txx[n]-tzz[t]<=tau & x-txx[n]>0 then
  ↪ c9r[x]=c9r[x]+1;
else if txx[n]=tzz[t] & x-txx[n]>0 then c9r[x]=c9r[x]+0.5;
else c9r[x]=c9r[x];
end;
end;

end;

c10r=j(nrow(tmp), 1,0);

do x=1 to nrow(tmp);
  do n=1 to nrow(txx);
    do t=1 to nrow(tzz);

if (tzz[t]-txx[n])>0 & tzz[t]-txx[n]<=tau & x-tzz[T]>0 then
  ↪ c10r[x]=c10r[x]+1;

```

```

else if txx[n]=tzz[t] & x-tzz[t]>0 then c10r[x]=c10r[x]+0.5;
else c10r[x]=c10r[x];
end;
end;

end;

print c9r c10r;

/***** SHP AND FFR COMPARISON *****/

c11r=j(nrow(tmp), 1,0);

do x=1 to nrow(tmp);
    do t=1 to nrow(tzz);
        do n=1 to nrow(tyy);
if (tyy[n]-tzz[t])>0 & tyy[n]-tzz[t]<=tau & x-tyy[n]>0 then
    ↪ c11r[x]=c11r[x]+1;
else if tyy[n]=tzz[t] & x-tyy[n]>0 then c11r[x]=c11r[x]+0.5;
else c11r[x]=c11r[x];
end;
end;

end;

c12r=j(nrow(tmp), 1,0);

do x=1 to nrow(tmp);
    do n=1 to nrow(tyy);
        do t=1 to nrow(tzz);

if (tzz[t]-tyy[n])>0 & tzz[t]-tyy[n]<=tau & x-tzz[t]>0 then
    ↪ c12r[x]=c12r[x]+1;
else if tyy[n]=tzz[t] & x-tzz[t]>0 then c12r[x]=c12r[x]+0.5;
else c12r[x]=c12r[x];
end;
end;

end;

print c11r c12r;

/* **** IP AND FFR COMPARISON *****/

```

```

c13r=j(nrow(tmp), 1,0);

do x=1 to nrow(tmp);
    do t=1 to nrow(tzz);
        do n=1 to nrow(tx);
if (tx[n]-tzz[t])>0 & tx[n]-tzz[t]<=tau & x-tx[n]>0 then
    ↪ c13r[x]=c13r[x]+1;
else if tx[n]=tzz[t] & x-tx[n]>0 then c13r[x]=c13r[x]+0.5;
else c13r[x]=c13r[x];
end;
end;

end;

c14r=j(nrow(tmp), 1,0);

do x=1 to nrow(tmp);
    do n=1 to nrow(tx);
        do t=1 to nrow(tzz);

if (tzz[t]-tx[n])>0 & tzz[t]-tx[n]<=tau & x-tzz[t]>0 then
    ↪ c14r[x]=c14r[x]+1;
else if tx[n]=tzz[t] & x-tzz[t]>0 then c14r[x]=c14r[x]+0.5;
else c14r[x]=c14r[x];
end;
end;

end;

print c13r c14r;

/* We have the count. We will now use it to compute the pertinent
   ↪ statistics Q and q */

/***** COMPUTING THE STATISTICS
   ↪ *****/

/***** SYNCHRO IP AND LTIR *****/

q1=j(nrow(tmp), 1,0);
q2=j(nrow(tmp), 1,0);

```

```

q1=c1r+c2r; /* Q */
q2=c1r-c2r; /* q */

print q1 q2;

time = (1:nrow(tmp))`;
call series(time,q1);
call series(time,q2); /* Positive if LTIR events goes before IP,
→ negative if not */

/***** SYNCHRO IP AND M3 *****/

q3=j(nrow(tmp), 1,0);
q4=j(nrow(tmp), 1,0);

q3=c3r+c4r; /* Q */
q4=c3r-c4r; /* q */

print q3 q4;

time = (1:nrow(tmp))`;
call series(time,q3);
call series(time,q4);

/***** SYNCHRO LTIR AND M3 *****/

q5=j(nrow(tmp), 1,0);
q6=j(nrow(tmp), 1,0);

q5=c5r+c6r; /* Q */
q6=c5r-c6r; /* q */

print q5 q6;

time = (1:nrow(tmp))`;
call series(time,q5);
call series(time,q6);

/***** SYNCHRO CLI AND SHP *****/

```

```

q11=j(nrow(tmp), 1,0);
q22=j(nrow(tmp), 1,0);

q11=c7r+c8r; /* Q */
q22=c7r-c8r; /* q */

print q11 q22;

time = (1:nrow(tmp))`;
call series(time,q11);
call series(time,q22);

/***** SYNCHRO CLI AND FFR *****/

q33=j(nrow(tmp), 1,0);
q44=j(nrow(tmp), 1,0);

q33=c9r+c10r; /* Q */
q44=c9r-c10r; /* q */

print q33 q44;

time = (1:nrow(tmp))`;
call series(time,q33);
call series(time,q44);

/***** SYNCHRO SHP AND FFR *****/

q55=j(nrow(tmp), 1,0);
q66=j(nrow(tmp), 1,0);

q55=c11r+c12r; /* Q */
q66=c11r-c12r; /* q */

print q55 q66;

time = (1:nrow(tmp))`;
call series(time,q55);
call series(time,q66);

/***** SYNCHRO IP AND FFR *****/

```

```

q111=j(nrow(tmp), 1,0);
q222=j(nrow(tmp), 1,0);

q111=c13r+c14r; /* Q */
q222=c13r-c14r; /* q */

print q111 q222;

time = (1:nrow(tmp))`;
call series(time,q111);
call series(time,q222);

***** COMPUTING Q'(n) and q'(n) *****
→ *****

***** FOR IP AND LTIR *****

/* Q'(n) with Delta(n)=71*/

qn=j(nrow(q1), 1, 0);
do n=71 to nrow(q1);
  nx=0;
  ny=0;
  do x=1 to 70;
    if tmp[n-x,8]=1 then nx=nx+1;
    if tmp[n-x,9]=1 then ny=ny+1;
  end;
  if (nx=0 & ny=0) then qn[n]= (q1[n]-q1[n-70])/(sqrt(nx*ny)) ;
  else qn[n] = qn[n-1] ;
end;

time = (1:nrow(tmp))`;
call series(time,qn);

/* Resolved assymetry q'(n) */

ptitqn=j(nrow(q2), 1, 0);
do n=71 to nrow(q2);
  nx=0;
  ny=0;
  do x=1 to 70;
    if tmp[n-x,8]=1 then nx=nx+1;
    if tmp[n-x,9]=1 then ny=ny+1;

```



```

        end;
    if (nx~=0 & ny~=0) then ptitqn[n]= (q2[n]-q2[n-70])/(sqrt(nx*ny))
        ↪ ;
    else ptitqn[n] = ptitqn[n-1] ;
    end;

time = (1:nrow(tmp))`;
call series(time,ptitqn);

/****** FOR IP AND M3 *****/

/* Q'(n) with Delta(n)=71*/

qn2=j(nrow(q3), 1, 0);
do n=71 to nrow(q3);
    nx=0;
    ny=0;
    do x=1 to 70;
        if tmp[n-x,8]=1 then nx=nx+1;
        if tmp[n-x,10]=1 then ny=ny+1;
    end;
    if (nx~=0 & ny~=0) then qn2[n]= (q3[n]-q3[n-70])/(sqrt(nx*ny)) ;
    else qn2[n] = qn2[n-1] ;
    end;

time = (1:nrow(tmp))`;
call series(time,qn2);

/* Resolved assymetry q'(n) */

ptitqn2=j(nrow(q4), 1, 0);
do n=71 to nrow(q4);
    nx=0;
    ny=0;
    do x=1 to 70;
        if tmp[n-x,8]=1 then nx=nx+1;
        if tmp[n-x,10]=1 then ny=ny+1;
    end;
    if (nx~=0 & ny~=0) then ptitqn2[n]=
        ↪ (q4[n]-q4[n-70])/(sqrt(nx*ny)) ;
    else ptitqn2[n] = ptitqn2[n-1] ;
    end;

time = (1:nrow(tmp))`;

```

```

call series(time,ptitqn2);

/***** FOR LTIR AND M3 *****/

/* Q'(n) with Delta(n)=71*/

qn3=j(nrow(q5), 1, 0);
do n=71 to nrow(q5);
    nx=0;
    ny=0;
    do x=1 to 70;
        if tmp[n-x,9]=1 then nx=nx+1;
        if tmp[n-x,10]=1 then ny=ny+1;
    end;
    if (nx=0 & ny=0) then qn3[n] = (q5[n]-q5[n-70])/(sqrt(nx*ny)) ;
    else qn3[n] = qn3[n-1] ;
end;

time = (1:nrow(tmp))^;
call series(time,qn3);

/* Resolved assymetry q'(n) */

ptitqn3=j(nrow(q6), 1, 0);
do n=71 to nrow(q6);
    nx=0;
    ny=0;
    do x=1 to 70;
        if tmp[n-x,9]=1 then nx=nx+1;
        if tmp[n-x,10]=1 then ny=ny+1;
    end;
    if (nx=0 & ny=0) then ptitqn3[n]=
        ↪ (q6[n]-q6[n-70])/(sqrt(nx*ny)) ;
    else ptitqn3[n] = ptitqn3[n-1] ;
end;

time = (1:nrow(tmp))^;
call series(time,ptitqn3);

/***** FOR IP AND FFR *****/

/* Q'(n) with Delta(n)=71*/

```

```

qn4=j(nrow(q111), 1, 0);
do n=71 to nrow(q111);
    nx=0;
    ny=0;
    do x=1 to 70;
        if tmp[n-x,8]=1 then nx=nx+1;
        if tmp[n-x,13]=1 then ny=ny+1;
    end;
    if (nx^=0 & ny^=0) then qn4[n]=
        → (q111[n]-q111[n-70])/(sqrt(nx*ny)) ;
    else qn4[n] = qn3[n-1] ;
end;

time = (1:nrow(tmp))^;
call series(time,qn4);

/* Resolved assymetry q'(n) */

ptitqn4=j(nrow(q222), 1, 0);
do n=71 to nrow(q222);
    nx=0;
    ny=0;
    do x=1 to 70;
        if tmp[n-x,8]=1 then nx=nx+1;
        if tmp[n-x,13]=1 then ny=ny+1;
    end;
    if (nx^=0 & ny^=0) then ptitqn3[n]=
        → (q222[n]-q222[n-70])/(sqrt(nx*ny)) ;
    else ptitqn4[n] = ptitqn4[n-1] ;
end;

time = (1:nrow(tmp))^;
call series(time,ptitqn4);

```

Appendix 3: Code for the Cross correlation method

```

proc timeseries data=cycles out=Tscycles;
var ip ltir m3 cli shp ffr;
run;

options nonotes nosource nosource2 errors=0;
%macro cc2(start,end);
    %do i=&start %to &end;
        %let j=%eval(&i+70-1);
        proc timeseries data=Tscycles (firstobs=&i obs=&j) out=ccl_&i outcrosscorr=cc_&i;
            var ip;
            crossvar ltir;
            crosscorr lag n ccov / NLAG=30;

            run;
            data cc_&i;
            set cc_&i;
            ccov2=ccov**2;
            run;

        %end;
    %mend cc2;

%cc2(1,614)

%macro order(start,end);
    %do i=&start \%to &end;
        proc sort data=cc_&i;
        by descending ccov2 ;
        run;
    %end;
%mend order;

%order(1,614)

%macro final(start,end);
%do n=&start \%to &end;
data ccl_&n;
    set cc_&n(obs=1);
    run;
proc delete data = cc_&n;
run;
%end;
%mend final;

%final(1,614)

```

```
data all ;  
    set ccl_1 - ccl_614 ;  
run;  
  
proc timeseries data=all out=all2;  
var ccov2 lag;  
run;  
  
symbol i=join v=none;  
proc gplot data=all2;  
plot (lag)*time / overlay grid legend=legend;  
run;
```