Wolkite University
Mathematics Department
Calculus I
Worksheet 1

1. Using the $\epsilon - \delta$ definition of a limit, show the following:

   (a) $\lim_{x\to -2}(7x+2) = -12$

   (b) $\lim_{x\to 5}(x^2-3x) = 10$

   (c) $\lim_{x\to 1}\frac{x+1}{x+2} = \frac{2}{3}$

   (d) $\lim_{x\to 0}\frac{\sin 3x}{x} = 3$

2. If it exists, evaluate the following limits:

   (a) $\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$

   (b) $\lim_{y\to 1}\frac{y^3-1}{\sqrt{y}-1}$

   (c) $\lim_{x\to 8}\frac{\sqrt{7-\sqrt[3]{x}}-3}{x-8}$

   (d) $\lim_{y\to 8}\frac{y^{\frac{1}{3}}-2}{y-8}$

   (e) $\lim_{x\to -2}\frac{x^2-4}{x^3-8}$

   (f) $\lim_{x\to 0}\frac{x^2}{\sec x - 1}$

   (g) $\lim_{x\to 0}\frac{2x^2+x}{\sin x}$

   (h) $\lim_{x\to 0}x^2(1+\cot^2 3x)$

   (i) $\lim_{x\to \pi}\frac{\sin x\cos x}{x^2-\pi^2}$

   (j) $\lim_{x\to \infty}\frac{2+\sin^2 x}{x^2}$

   (k) $\lim_{x\to \infty}\left(1-\frac{1}{3x}\right)^{2x+1}$

   (l) $\lim_{x\to \infty}\frac{e^x-e^{-x}}{e^x+e^{-x}}$

   (m) $\lim_{x\to -\infty}\frac{3x+4}{\sqrt{2x^2-21}}$

   (n) $\lim_{t\to \infty}\left(\sqrt{t^2+t}-\sqrt{t^2+4}\right)$

   (o) $\lim_{x\to \infty}\frac{(x^3+1)^{\frac{1}{3}}}{\sqrt{x^2-3}}$

   (p) $\lim_{x\to \infty}\frac{3^x-5^x}{3^x+5^x}$

   (q) $\lim_{x\to -1}\frac{|x+2|-1}{1-|x|}$

   (r) $\lim_{x\to 0}\frac{|x|^3-x^2}{x^3+x^2}$

3. Using the squeezing theorem, evaluate:

   (a) $\lim_{x\to \pi}(x-\pi)\cos^2\left(\frac{1}{x-\pi}\right)$

   (b) $\lim_{x\to 0}\sin x\cos\left(\frac{1}{x}\right)$

   (c) $\lim_{x\to 0}x^4\sin\left(\frac{1}{\sqrt[5]{x^3}}\right)$

 Question 3 continues on the next page...

(d) $\lim_{x\to -2} g(x)$, if $|g(x)-3| < 5(x+2)^2$ for all x

(e) $\lim_{x\to \frac{-\pi}{2}} f(x)$, if $4 - 2\sin x \le 2f(x) + 4 \le 8 + 2\sin x$ for all $x \in (-\pi, 0)$

4. Find all vertical asymptotes of the graph of f:

 (a) $f(x) = \dfrac{x^2 - 5x + 6}{x^3 - 8}$

 (b) $f(x) = \dfrac{8x - 2x^2}{x^2 - 16}$

 (c) $f(x) = \dfrac{\sin(x^2 - 1)}{x^3 - x}$

5. Find all horizontal asymptotes of the graph of f:

 (a) $f(x) = \dfrac{x^2 - 5x + 6}{x^3 - 8}$

 (b) $f(x) = \dfrac{(1 - x^2)(x + 1)}{x^2(1 - 2x)}$

 (c) $f(x) = \dfrac{x}{\sqrt{x^2 + 1}}$

6. Show each of the following:

 (a) $\lim_{x\to a} f(x) = 0$ if and only if $\lim_{x\to a} |f(x)| = 0$.

 (b) If $\lim_{x\to a} f(x) = L$, then $\lim_{x\to a} |f(x)| = |L|$.

7. (a) Suppose $\lim_{x\to a} f(x) = L \in \mathbb{R}$ and $\lim_{x\to a}(f(x)g(x)) = 1$. What can we say about $\lim_{x\to a} g(x)$?

 (b) If $\lim_{x\to a}(f(x) + g(x))$ and $\lim_{x\to a} f(x)$ exist, what can we say about $\lim_{x\to a} g(x)$?

8. Prove that if there is a number M such that $\left|\dfrac{f(x) - L}{x - a}\right| \le M$ for all $x \ne a$, then $\lim_{x\to a} f(x) = L$.

9. If $\lim_{x\to a} f(x) = 0$ and $g(x) \le M$ for all $x \ne a$, then show that $\lim_{x\to a}(f(x)g(x)) = 0$.

10. Let

$$f(x) = \begin{cases} ax + b & \text{if } x \le -2 \\ x^2 + 3 & \text{if } -2 < x < 1 \\ bx - a & \text{if } x \ge 1 \end{cases}$$

 Find numbers a and b such that f is continuous on $\mathbb{R}$.

11. Find the value of k so that $f(x) = \begin{cases} k & \text{if } x = 0 \\ \dfrac{\cos x - 1}{\sin 2x} & \text{if } x \ne 0 \end{cases}$ is continuous at 0.

12. Determine for which of the following functions we can define $f(a)$ so as to make f continuous at a:

    (a) $f(x) = \dfrac{\tan 3x}{2x^2 - 5x}$, $a = 0$

    (b) $f(x) = \dfrac{x^2 - 1}{|x - 1|}$, $a = 1$

    (c) $f(x) = \begin{cases} x + 1 & \text{if } x > -1 \\ x & \text{if } x < -1 \end{cases}$, $a = -1$