Wolkite University
Mathematics Department
Group Assignment 1: Applied Mathematics III

1. **Order and Degree of a Differential Equation:** Determine the order and degree of the following differential equation: $(y'')^3 - 10y' + y^{11} = 0$

2. **Separable Differential Equations:** Solve the following separable differential equation: $y' = \dfrac{y^2 + 1}{x}$

3. **Exact Differential Equations:** Solve the following exact differential equation: $(x^3 + 3xy^2)dx + (3yx^2 + y^3)dy = 0$

4. **Bernoulli Equations and Stability:** Solve the Bernoulli equation $y' + \dfrac{1}{x}y = xy^3$.

   (a) Find the general solution.

5. **Second-Order Linear ODEs with Variable Coefficients:** Solve the differential equation $x^2y'' + xy' - 4y = x^3$.

   (a) Identify the type of equation and explain the method used.

   (b) Find the general solution.

6. **Cauchy-Euler Equations and Initial Value Problems:** Solve the Cauchy-Euler equation $x^2y'' - 3xy' + 4y = \ln(x)$, with $y(1) = 0$ and $y'(1) = 1$.

7. **Laplace Transform and Nonhomogeneous ODEs:** Solve the differential equation $y'' + 4y' + 4y = te^{-2t}$, with $y(0) = 1$ and $y'(0) = -1$, using Laplace transforms.

8. **Laplace Transform and Piecewise Functions:** Solve the differential equation
$y'' + 9y = f(t)$, where $f(t) = \begin{cases} 1, & 0 \leq t < \pi \\ 0, & t \geq \pi \end{cases}$, with $y(0) = 0$ and $y'(0) = 0$, using Laplace transforms.

9. **First-Order Linear ODEs and Applications:** A tank contains 100 liters of brine with 20 kg of dissolved salt. Brine containing 0.1 kg of salt per liter flows into the tank at a rate of 2 liters per minute. The well-mixed solution flows out at the same rate.

   (a) Set up a differential equation for the amount of salt $A(t)$ in the tank at time t.

   (b) Solve the differential equation.

   (c) Find the amount of salt in the tank after 1 hour.

10. **Second-Order Linear ODEs:** Solve the differential equation $y'' + 4y = \cos(2t)$.