Wolkite University
Mathematics Department
Applied Mathematics III
Worksheet 3

1. Find the domain of the following vector valued functions.
   (a) $f(t) = \ln t\,i + 10tj + \sqrt{4-t}\,k$
   (b) $g(t) = \dfrac{1}{t+2}i - 10j + e^{2t}k$
   (c) $h(t) = \sqrt{t-2}\,i + \ln(25-t^2)j - \sqrt{1-\ln t}$

2. Show that the curve parametrized by $C : r(t) = ti + 2t^{\frac{1}{2}}j + 10k$ is not smooth. Verify also that C is not piecewise smooth.

3. Find a parametric equation for the tangent line to the following curves at the given point.
   (a) $(\cos t, \sin t, e^{4t})$ at $t = \dfrac{\pi}{2}$.
   (b) $(e^t, e^{-2t}, \sqrt{t})$ at $t = 1$.
   (c) (t, t^3, t^4) at the point $(2, 8, 16)$.

4. A particle has a position vector at time t given by $r(t) = t^2i + 2t + 3\sin 2tk$. Find its
   (a) velocity
   (b) speed
   (c) acceleration

5. Find the arc length along the circular helix $r(t) = 3\cos t\,i + 3\sin t\,j + 4tk$ between the points corresponding to $t = \pi$ and $t = \dfrac{3}{2}\pi$.

6. Find the arc length of
   (a) $r(t) = \cos 2ti + \sin 2tj + 3tk, \quad 1 \le t \le 3$
   (b) $r(t) = e^{3t}i + e^{-3t}j + 3\sqrt{2}tk, \quad 0 \le t \le \dfrac{1}{3}$
   (c) $r(t) = (\cos 4t,\ \sin 4t,\ 1)$ between $t = 0$ and $t = \dfrac{\pi}{2}$

7. Find the curvature of
   (a) $r(t) = (t^2, 2t)$.
   (b) $r(t) = (a\cos t, a\sin t, b)$, where a and b are constants.
   (c) the parabola $y = x^2$.

8. Find the normal and binormal vector for $r(t) = 4\sin t i + 3tj - 4\cos tk$.

9. Find a and b, if $F(x, y, z) = axi + 4yz^2 j + by^2 zk$ is conservative.

10. Show that $F(x, y, z)$ is conservative and find its potential function where
    (a) $F(x, y, z) = (2x,\ 3y,\ 4z)$
    (b) $F(x, y, z) = (y + z,\ x + z,\ x + y)$
    (c) $F(x, y, z) = y\sin zi + x\sin zj + xy\cos zk$
    (d) $F(x, y, z) = (e^{y+2z},\ xe^{y+2z},\ 2xe^{y+2z})$
    (e) $F(x, y, z) = z^2 i + 2yj + 2xzk$
    (f) $F(x, y, z) = (yz\cos xy,\ xz\cos xy,\ \sin xy)$