

**Wolkite University**  
**Department of Mathematics**  
**Applied Mathematics III**

**Assignment 2**  
Vector Calculus and Complex Analysis

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## Instructions

- This is an **individual assignment**. Collaboration is not allowed.
- Attempt **all 10 questions**. Show all necessary working for full credit.
- Submit your solutions neatly written by **May 14, 2025**.

## Vector Calculus Questions

1. Find the domain of the following vector-valued functions.
  - (a)  $\mathbf{f}(t) = \ln t \mathbf{i} + 10t \mathbf{j} + 4t \mathbf{k}$
  - (b)  $\mathbf{g}(t) = \frac{1}{t+2} \mathbf{i} + 10 \mathbf{j} + e^{2t} \mathbf{k}$
  - (c)  $\mathbf{h}(t) = \frac{t+2}{t} \mathbf{i} + \ln(25-t^2) \mathbf{j} + \frac{1}{\ln t} \mathbf{k}$
2. Show that the curve  $\mathbf{r}(t) = t \mathbf{i} + 2\sqrt{t} \mathbf{j} + 10 \mathbf{k}$  is not smooth. Verify also that it is not piecewise smooth.
3. Find a parametric equation for the tangent line to the curve  $\mathbf{r}(t) = (\cos t, \sin t, e^{4t})$  at  $t = 2$ .
4. A particle has a position vector  $\mathbf{r}(t) = t^2 \mathbf{i} + 2t \mathbf{j} + 3 \sin(2t) \mathbf{k}$ . Find:
  - (a) Velocity  $\mathbf{v}(t)$
  - (b) Speed  $|\mathbf{v}(t)|$
  - (c) Acceleration  $\mathbf{a}(t)$
5. Using Green's Theorem evaluate  $\oint_C (y^2 dx + x dy)$  where  $C$  is the circle  $x^2 + y^2 = 1$  positively oriented.

## Complex Analysis Questions

6. Evaluate  $\oint_C \frac{1}{z^2 + 2z + 2} dz$  where  $C$  is the circle  $|z - 1| = 2$ .
7. Find
- (a) the principal value of  $i^i$ .
  - (b) all values of  $\log(-1 + i)$ .
8. Show that  $f(z) = z^2$  is differentiable everywhere and analytic everywhere.
9. Evaluate  $\int_C \frac{e^z}{z} dz$  where  $C$  is the circle  $|z| = 2$  traversed once counterclockwise.
10. Evaluate

$$I = \oint_C \frac{z^2 + 3}{(z - 1)(z - 3)(z + 2)^2} dz,$$

where  $C$  is the contour  $|z| = \frac{5}{2}$ , oriented counterclockwise.