
Wolkite University
Mathematics Department
Calculus I
Worksheet 3

1. Explain why the Mean Value Theorem (MVT) does not apply for $f(x) = x^{2/3}$ on $[-1, 1]$.
2. Using the MVT show that
 - (a) If $|f'(x)| \leq 1, \forall x$ in some interval I , then $|f(x_1) - f(x_2)| \leq |x_1 - x_2|, \forall x_1, x_2 \in I$.
 - (b) $|\sin x| \leq |x|, \forall x \in \mathbb{R}$.
3. Find the asymptote(s), intervals of monotonicity, critical points, the local extreme points, intervals of concavity, and inflection point(s) of the following functions:
 - (a) $f(x) = -3x^4 + 4x^3$
 - (b) $f(x) = |x^2 + x - 2|$
 - (c) $f(x) = (x - 2)^{2/3}$
 - (d) $f(x) = x^2 + \frac{2}{x}$
 - (e) $f(x) = \frac{x^2}{x - 2}$
 - (f) $f(x) = \frac{x^2 - 6x}{(x + 1)^2}$
 - (g) $f(x) = x^2 e^{-x}$
 - (h) $f(x) = |x - 2| + |x - 4|$
 - (i) $f(x) = x\sqrt{1 - x}$
4. Of all the triangles that pass through the point $(1, 1)$ and have sides lying on the coordinate axes, one has the smallest area. Determine the lengths of its sides.
5. A ladder is to reach over a fence 8 feet high to a wall 1 foot behind the fence. What is the length of the shortest ladder that can be used?
6. A rectangle of the greatest possible area is inscribed in a triangle whose base is a and altitude h . Determine the area of the rectangle.
7. Find the dimensions of the right circular cylinder of the largest volume that can be inscribed in a sphere of radius 10 units.