Wolkite University
Mathematics Department
Applied Mathematics III
Worksheet 1

1. Determine the order and degree of
   (a) $(y'')^3 - 10y' + y^{11} = 0$
   (b) $x^6 y'' - x^8 y' - 12 = 0$
   (c) $y'' - \cos y = 3$
   (d) $y' - y^3 = x$
   (e) $(y')^{\frac{2}{3}} = \sqrt{y'^5 + x}$

2. Verify that $y = e^{-2x}(c_1 \cos 2x + c_2 \sin 2x)$ is a solution of $y'' + 4y' + 8y = 0$.

3. Find the constants m, n and k such that $x^m y^3 y' + 7ax^4 y^n = 0$ is exact.

4. Find a differential equation whose solution is $y = \dfrac{4}{x}$.

5. Solve the following separable first order differential equations.
   (a) $9yy' - 4x = 0$
   (b) $y' = \cos x$
   (c) $y' = \dfrac{y^2 + 1}{x}$
   (d) $x(\ln x)y' = y$
   (e) $y' \sin 2x = y \cos 2x$
   (f) $\dfrac{dy}{dx} = e^{2x} \cos^2 y$

6. Solve the following homogeneous first order differential equations.
   (a) $y' - 4y = 0$
   (b) $4y' = -8y$

7. Solve the following reducible first order differential equations.
   (a) $\dfrac{dy}{dx} = \dfrac{y - x + 1}{y - x + 5}$
   (b) $2xyy' - y^2 + x^2 = 0$
   (c) $y' - y^2 - 2xy - x^2 = 0$

8. Solve the following exact first order differential equations.
   (a) $(x^3 + 3xy^2)dx + (3yx^2 + y^3)dy = 0$
   (b) $2xy^3 dx + 3x^2 y^2 = 0$
   (c) $\dfrac{dy}{dx} 3x^2 y^2 = -1 - 2xy^3$
   (d) $y^2 dx + x^2 dy = 0$

9. Solve the following linear first order differential equations.

   (a) $y' - 5y = 6$

   (b) $\dfrac{dy}{dx} + 4y = 4$

   (c) $y' - 12 = 0$

10. The acceleration a of an object is given by $a = \dfrac{dv}{dt}$. Find the velocity v in terms of t given that $v(0) = v_0$.

11. Determine the value of k, given that $x^3 \dfrac{dy}{dx} = k - x$ and $y(2) = 0$ and when $x = 6$.

12. Find the equation of the curve which satisfies $xy = (1 + x^2)\dfrac{dy}{dx}$ and passes through the point $(0, 1)$.

13. The equation $\dfrac{dv}{dt} + av = bt$, where a and b are constants, represents an equation of motion when a particle moves in a resisting medium. Solve the equation for v given that $v(0) = 1$.

14. Solve the following Bernoulli's equations.

   (a) $y' + 2xy = -xy^4$

   (b) $2y' + y = \frac{x}{y}$

   (c) $y' + y = (1 - 2x)y^2$

15. Solve the following initial value problems using appropriate methods.

   (a) $(y - x)e^x dx + (1 + e^x)dy = 0, \quad y(1) = 1$

   (b) $x^2 y' = x^2 + y^2 - xy, \quad y(1) = 2$

   (c) $y' + y^2 - y = 0, \quad y(0) = \dfrac{1}{3}$