
Wolkite University
Mathematics Department
Calculus I
Worksheet 2

1. Find the set of all points where the derivative exists for:

(a) $f(x) = |\sin x|$

(b) $f(x) = [x]$

(c) $f(x) = \begin{cases} \sqrt{x} & \text{for } 0 \leq x \leq 1 \\ x^2 & \text{elsewhere} \end{cases}$

(d) $f(x) = \begin{cases} x \cos(\frac{1}{x}), & x \neq 0 \\ 0, & x = 0 \end{cases}$

2. If f and g are two functions with $f(a) = 1$, $f'(a) = 2$, $g(a) = -1$ and $g'(a) = 3$, then find $(fg + f')(a)$ and $\left(\frac{f}{f-g}\right)'(a)$.

3. Let f be a differentiable function such that $f'(x) = e^x$, then find

$\frac{d}{dx}(f(\ln(\ln(2x))))$.

4. Find the equation of the tangent line and the normal line to the graph of $f(x) = 1 - (x+1)^{\frac{1}{3}}$ at $(-1, f(-1))$.

5. Find the points where the tangent line is parallel to the x -axis for the curve $25y^2 + 12xy + 4x^2 = 1$.

6. Find the point on the curve $y = x^3 + x^2 + x$ where the tangent line is parallel to $y = 2x + 3$.

7. Find the equations for the tangent line to the ellipse $4x^2 + y^2 = 72$ that are perpendicular to the line $x + 2y + 3 = 0$.

8. Find the equation for the normal line to the hyperbola $4x^2 - y^2 = 36$ that are parallel to the line $2x + 5y - 4 = 0$.

9. Let $f(x) = x^2|x|$. Show that $f''(x)$ is continuous at 0 but not differentiable at 0.

10. Discuss the continuity and differentiability of each function at the given point a :

(a) $f(x) = \sqrt[3]{x} - x$ at $a = 0$

(b) $f(x) = \begin{cases} \frac{\sqrt{x-1}}{x-1}, & \text{for } x \neq 1 \\ \frac{1}{2}, & \text{for } x = 1 \end{cases}$ at $a = 1$

(c) $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ at $a = 1$.

11. Show that the derivative of an even function is odd and vice versa.

12. Assuming $f'(a)$ exists, express the following in terms of $f'(a)$:

(a) $\lim_{h \rightarrow 0} \frac{f(a-h) - f(a)}{h}$

(b) $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a-h)}{h}$

(c) $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{\sqrt{x} - \sqrt{a}}, a > 0$

13. Find $f'(x)$ at each point where the derivative exists:

(a) $f(x) = \frac{-x}{(x-1)^2}$

(b) $f(x) = xe^{-\ln x}$

(c) $f(x) = x + |x|$

(d) $f(x) = (x-7)|x-7|$

(e) $f(x) = \frac{e^x}{\sqrt{3x^2-1}}$

(f) $f(x) = \log_{x^2}(x^2 e^x)$

(g) $f(x) = (\ln x)^x$

(h) $f(x) = 3x^2 \log_3(x^2 + 3)$

(i) $f(x) = x^{x^2}$

14. Given $f(x) = \begin{cases} \cos x, & x \geq 0 \\ ax + b, & x < 0 \end{cases}$, find the values of a and b such that $f'(0)$ exists.

15. Given $f(x) = \begin{cases} x^3, & \text{if } x < 1 \\ ax^2 + bx + c, & \text{if } x \geq 1 \end{cases}$, find the values of a and b such that $f''(1)$ exists.

16. Find the third derivative of the following functions:

(a) $f(x) = x^3 e^{-3x}$

(b) $f(x) = \ln(\ln(5x))$

(c) $f(x) = \cos^2 x - e^{\frac{\tan x}{x}}$

17. Find the value of the constant A so that $y = A \sin 3t$ satisfies the equation

$$\frac{d^2 y}{dt^2} + 2y = 4 \sin 3t.$$