
Wolkite University
Mathematics Department
Applied Mathematics III
Worksheet 5

1. Simplify each of the following

(a) $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$

(b) $\frac{5i}{(3-i)(1-i)(2-i)}$

2. Show that

(a) $\overline{(2-i)^2} = 3-4i$

(b) $|(2\bar{z}+5)(\sqrt{2}-i)| = \sqrt{3}|2z+5|$

(c) z is real if and only if $\bar{z} = z$.

(d) $|e^{i\theta}| = 1$

3. Write in the form $x+iy$

(a) $(1-i)^{10}$

(b) $\frac{2}{2-i}$

(c) $\frac{1-i}{(1+i)^2}$

(d) $\sqrt[7]{-4-4i}$

4. Represent the following in polar form

(a) $4i$

(b) $-2+2i$

(c) $\frac{\sqrt{20}}{4+2i}$

5. Find the principal value of

(a) i^i

(b) $(1-i)^{1+i}$

(c) $\ln(-1)$

(d) $\ln\left(\frac{1-i}{1+i}\right)$

6. Check for analyticity and find $f(z) = u(x, y) + iv(x, y)$

(a) $u = x^2 - 2xy - y^2$

(b) $v = e^{-3x} \sin 3y$

(c) $u = \cos 2x \cosh 2y$

7. Are the following expressions harmonic? If so, find a harmonic conjugate.

(a) $x^3 y^3$

(b) $x^2 + y^2$

(c) $e^{-\frac{x}{2}} \cos\left(\frac{1}{2}y\right)$

8. Find the values of

(a) $\cos(3 + 4\pi i)$

(b) $\tan(3 + 3\pi i)$

(c) $\cosh(l + \pi i)$

9. Show that $f(z) = \begin{cases} |z|^2, & \text{if } z \neq 0 \\ 0, & \text{if } z = 0. \end{cases}$ is continuous at $z = 0$ but not differentiable at $z = 0$.

10. Prove the parallelogram law

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2).$$

11. Show that $f(z) = |z|^2$ is differentiable but not analytic at $z = 0$.