Wolkite University
Mathematics Department
Calculus I
Worksheet 4

1. Evaluate the following indefinite integrals:

   (a) $\displaystyle\int \frac{1}{x+\sqrt{x}}\,dx$

   (b) $\displaystyle\int x\sinh x\,dx$

   (c) $\displaystyle\int \cos x\ln(\sin x)\,dx$

   (d) $\displaystyle\int \cos(\ln x)\,dx$

   (e) $\displaystyle\int x\tan^{-1}x\,dx$

   (f) $\displaystyle\int x^5e^{x^2}\,dx$

   (g) $\displaystyle\int \log_5(2x+3)\,dx$

   (h) $\displaystyle\int x^n\ln x\,dx$

   (i) $\displaystyle\int \sin x\tan^{-1}(\cos x)\,dx$

   (j) $\displaystyle\int \frac{(\ln x)^2}{x^2}\,dx$

2. Evaluate the following integrals:

   (a) $\displaystyle\int x^2\sin 10x\,dx$

   (b) $\displaystyle\int x^5\sqrt{x^3+1}\,dx$

   (c) $\displaystyle\int \frac{x+2}{\sqrt[3]{x-3}}\,dx$

   (d) $\displaystyle\int \frac{2}{x-3\sqrt{x+10}}\,dx$

3. Evaluate the following trigonometric integrals:

   (a) $\displaystyle\int \cos x\sin^5 x\,dx$

   (b) $\displaystyle\int \sin^5 x\,dx$

   (c) $\displaystyle\int \sin^6 x\cos^3 x\,dx$

   (d) $\displaystyle\int \cos(15x)\cos(4x)\,dx$

   (e) $\displaystyle\int \sec^9 x\tan^5 x\,dx$

   (f) $\displaystyle\int \tan^3 x\,dx$

   (g) $\displaystyle\int \sin^6 x\cos^8 x\,dx$

4. Evaluate the following integrals using trigonometric substitution:

   (a) $\displaystyle\int x\sqrt{25x^2-4}\,dx$

   (b) $\displaystyle\int \frac{1}{x^4\sqrt{9-x^2}}\,dx$

   (c) $\displaystyle\int \frac{x^5}{(\sqrt{x^2+1})}\,dx$

Question 4 continues on the next page...

(d) $\displaystyle\int \frac{x^2}{(x^2+x+1)}\,dx$

(e) $\displaystyle\int e^{4x}\sqrt{1+e^{2x}}\,dx$

5. Evaluate the following integrals using the partial fraction method:

(a) $\displaystyle\int \frac{3x+11}{x^2-x-6}\,dx$

(b) $\displaystyle\int \frac{x^2+4}{3x^3+4x^2-4x}\,dx$

(c) $\displaystyle\int \frac{x^3+10x^2+3x+36}{(x-1)(x^2+4)^2}\,dx$

(d) $\displaystyle\int \frac{x^4-5x^3+6x^2-18}{x^3-3x^2}\,dx$

(e) $\displaystyle\int \frac{x^2}{x^2-1}\,dx$

(f) $\displaystyle\int \frac{1-x}{x^2(x^2+2x-1)}\,dx$

(g) $\displaystyle\int \frac{x}{x^3+1}\,dx$

(h) $\displaystyle\int \frac{-x^3+x^2+x+3}{(x-1)(x^2+1)^2}\,dx$

6. Evaluate the following definite integrals:

(a) $\displaystyle\int_1^2 (x^2-\frac{1}{x^2})^2\,dx$

(b) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \sec x(\tan x+\sec x)\,dx$

(c) $\displaystyle\int_1^0 |2x-1|\,dx$

(d) $\displaystyle\int_1^6 f(x)\,dx$ where $f(x)=\begin{cases}2x & \text{for } 4\le x\le 5\\ 20-2x & \text{for } 5<x\le 6\end{cases}$

(e) $\displaystyle\int_0^1 x5^x\,dx$

(f) $\displaystyle\int_{\frac{\sqrt{\pi}}{2}}^{\sqrt{\pi}} x^3\cos(x^2)\,dx$

(g) $\displaystyle\int_{-1}^2 |x^2-4|\,dx$

7. Let m and n be positive integers. Show that:

(a) $\displaystyle\int_{-\pi}^{\pi} \sin mx\cos nx\,dx = 0$

(b) $\displaystyle\int_{-\pi}^{\pi} \cos mx\cos nx\,dx = \begin{cases}0, & \text{if } m\neq n\\ \pi, & \text{if } m=n\end{cases}$

(c) $\displaystyle\int_{-\pi}^{\pi} \sin mx\sin nx\,dx = \begin{cases}0, & \text{if } m\neq n\\ \pi, & \text{if } m=n\end{cases}$

8. Find $\int x^n \ln x^m \, dx$

9. Prove the following reduction formula:
   (a) $\int (\ln x)^n \, dx = x(\ln x)^n - n \int (\ln x)^{n-1} \, dx$, and evaluate $\int (\ln x)^4 \, dx$
   (b) $\int x^n e^x \, dx = x^n e^x - n \int x^{n-1} e^x \, dx$

10. Show whether the following improper integrals converge or diverge:
    (a) $\int_0^1 \frac{1}{x} \, dx$
    (b) $\int_0^1 \frac{dx}{(2x-1)^2}$
    (c) $\int_0^\infty x e^{-x} \, dx$
    (d) $\int_2^3 \frac{1}{(x-2)(x-3)} \, dx$
    (e) $\int_{-\infty}^\infty \frac{dx}{1+x^2}$
    (f) $\int_0^\infty \frac{1}{x} \, dx$

11. Determine the area of the region enclosed by $f(x) = x^2$ and $f(x) = \sqrt{x}$.

12. Determine the area of the region bounded by $y = xe^{-x^2}$, $y = x + 1$, $x = 2$, and the y-axis.

13. Determine the area of the region bounded by $y = 2x^2 + 10$, $y = 4x + 16$, $x = -2$ and $x = 5$.

14. Find the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$, and the x-axis about the y-axis.

15. What is the area (in square roots) of the region bounded by $y = x^3 - 4x$ and the x-axis between $x = -2$ and $x = 2$?

16. Evaluate the following integrals:
    (a) $\int \left(\frac{x}{x+1}\right)^2 \, dx$
    (b) $\int \left(\frac{1}{e^x} + \sqrt{e^x}\right) \, dx$
    (c) $\int (e^{-x} + 7\cos x) \, dx$
    (d) $\int \frac{x+2}{x+1} \, dx$
    (e) $\int x \sin x \, dx$
    (f) $\int \left(\frac{1}{\sqrt{x+1}} - e^{2x}\right) \, dx$
    (g) $\int_0^1 \frac{3t^2 - 1}{e^{t^3 - t}} \, dt$
    (h) $\int \frac{6x}{(x^2+1)^2} \, dt$

17. If $f(x) = \frac{x^2}{x^2+2}$ and $f(1) = 2$, then find the antiderivative of f.