Wolkite University
Mathematics Department
Applied Mathematics III
Worksheet 2

1. Solve the following homogeneous second order differential equations with constant coefficients.
   (a) $y'' - 2y' + y = 0$
   (b) $y'' - 3y' + 2y = 0$
   (c) $y'' - 2y' + 2y = 0$
   (d) $64y'' - 48y' - 7y = 0$

2. Solve the following nonhomogeneous second order differential equations with constant coefficients.
   (a) $y'' - 5y + 6y = 10e^{3x}$
   (b) $y'' + 4y = e^x + x$
   (c) $y'' + 6y' + 8y = 2\sin x - 8\cos x$
   (d) $y'' + 4y' + 5y = -\cos hx$

3. The motion of a vibrating mass is given by

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 10y = 90\sin 2t.$$

   Show that the general solution of the differential equation is given by

$$y = e^{-2t}(A\cos t + B\sin t) - 6\cos 2t + 3\sin 2t.$$

4. Find the general solution of the following differential equations.
   (a) $x^2y'' + 4xy' + 2y = 0$
   (b) $(x-3)^2y'' + 3(x-3)y' + y = 0, \quad x > 3$
   (c) $(2x+1)^2y'' + 4(2x+1)y' - 24y = 0, \quad x > \dfrac{1}{2}$
   (d) $x^2y'' + xy' + y = 0, \quad y(1) = 1, \quad y'(1) = 2$
   (e) $x^2y'' + y' = x, \quad y(1) = 1, \quad y'(1) = -\dfrac{1}{2}$

5. Find the values of m and n for which
   $y'' + my' + ny = 0$ has a general solution $y = c_1e^{-3x}\cos 2x + c_2e^{-3x}\sin 2x$.

6. Solve the initial value problem $y'' - 5y' + 4y = e^{2x}, \quad y(0) = 1, \; y'(0) = 0.$

7. Solve $y'' + 2y' + 2y = e^t \sin t$.

8. Find a general solution of $y'' + 15y' + 50y = 0$ by converting it to a system of linear first order differential equations.

9. A particle moves along the x-axis according to the law $\dfrac{d^2x}{dt^2} + 6\dfrac{dx}{dt} + 25x = 0$. If the particle is started at $x = 0$ with an initial velocity of 12 ft/sec to the left, determine its displacement.

10. Suppose y_h is a solution of
$$y'' + ay' + by = 0$$
where a and b are positive constants. Show that
$$\lim_{x\to\infty} y_h(x) = 0.$$