Wolkite University
Mathematics Department
Applied Mathematics III
Worksheet 4

1. Suppose $\int_C (4x+3)ds = 12\pi$, where C is a circle centered at the origin. Find the radius of C.

2. Evaluate
   (a) $\int_C 2xyz\ ds$, where C is parameterized by $r(t) = e^t i + e^{-t} j + \sqrt{2}tk$, for $0 \le t \le 1$.
   (b) $\int_C (y^2 i + z^2 j + x^2 k)\ dr$, where C the helix $r(t) = 3\cos ti + 3\sin tj + 2tk$, $0 \le t \le 8\pi$
   (c) $\int_{(2,3,0)}^{(0,1,2)} (ze^{xz}\ dx + dy + xe^{xz}\ dz)$
   (d) $\int_C yzdx - xzdy + xydz$, where $C : r(t) = e^t i + e^{3t} j + e^{-t} k, \quad 0 \le t \le 1$

3. Using Green's theorem evaluate $\int_C y^2 dx + xdy$, where C is
   (a) the square with vertices $(0,0), (2,0), (2,2), (0,2)$.
   (b) the circle of radius 1 centered at the origin.
   (c) the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ parametrized by $x = a\cos t$ and $y = b\sin t$.

4. Using Green's theorem, evaluate
   (a) $\int_C (5 - xy - y^2)dx + (x^2 - 2xy)dy$, where C is the boundary of a square with vertices $(0,0),\ (2,0),\ (2,2)$ and $(0,2)$ oriented counter clockwise.
   (b) $\int_C -3x^2 ydx + 3xy^2 dy$, where C is the boundary of the region in the first quadrant bounded between the coordinate axes and the circle $x^2 + y^2 = 4$.
   (c) $\int_C (e^x \ln y - 4xy)dx + \frac{e^x}{y}dy$, where C is the boundary of the region bounded above by $y = 3 - x^2$ and below by $y = x^4 + 1$.
   (d) $\int_C (x^3 - x^2 y)dx + xy^2 dy$, where C is the boundary of the region between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 16$.

5. Find the flux of a vector field F across the surface where Σ, where $F(x, y, z) = xi + yj + 4zk$ and Σ is the portion of the cone $z^2 = x^2 + y^2$ between the planes $z = 1$ and $z = 2$ oriented by upward normal.

6. Evaluate $\displaystyle\int_c F.dr$

   (a) $F(x, y, z) = (3x^2 + 6y)i - 14yzj + 20xz^2k$ and $C : r(t) = ti + t^2j + t^3k$, $0 \leq t \leq 1$

   (b) $F(x, y, z) = xi + y^2j + zk$ over the triangle determined by the plane $x + y + z = 1$, and the coordinate planes.

7. Using Stokes's theorem evaluate $\displaystyle\int_C F.dr$, where

   (a) $F(x, y, z) = (z, x, y)$, S defined by $z = 4 - x^2 - y^2, \quad z \geq 0$.

   (b) $F(x, y, z) = (x^2 + y)i + yzj + (x - z^2)k$ and S is the triangle defined by the plane $2x + y + 2z = 2$ and $x, y, z \geq 0$

   (c) $F(x, y, z) = (-y, x, x)$ and the surface is the part of the paraboloid $z = x^2 + y^2$ inside the cylinder $x^2 + y^2 = 4$.

   (d) $F(x, y, z) = (x - y, x + z, z^2)$ and the surface is the part of the cone $z^2 = x^2 + y^2$ between the planes $z = 0$ and $z = 1$.

8. Using the Divergence theorem evaluate $\displaystyle\iint_\Sigma F.n \, d\sigma$, where

   (a) $F(x, y, z) = (yz, xz, xy)$ over the cube centered at the origin and sides of length 2.

   (b) $F(x, y, z) = (x + y, y + z, x + z)$ over the surface bounded by the paraboloid $z = 4 - x^2 - y^2$ and the disc of radius 2 centered at the origin in the xy plane.

   (c) $F(x, y, z) = (x, y, z)$ over the surface bounding the region enclosed by the paraboloid $z = x^2 + y^2$, the cylinder $x^2 + y^2 = 9$ and the plane $z = 0$.

9. Evaluate $\displaystyle\int_S 3xdydz + 2ydxdz - 5zdxdy$, where S is a smooth surface bounding an arbitrary volume V.