Analysis of a Turn-based Strategy Game: Compact Conflict

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Abstract

We use the Markov Decision Processes to design a winning strategy for a Turn-based Strategy game.

1 Introduction

Section Owners:

2 Detailed Model Analysis

Section Owners:

2.1 Preliminaries

Section Owners:

- **Regions**
- **Faith Accumulation**
- **Temple**
- **Auxiliary State**
- **Additional Soldiers**

2.2 Engagement Dynamics

The Jakub's Rule of Engagement (JRE) considers a *round* that ends when the *total* number of casualties is equal to the minimum of the strength of the engaging entities.
Section Owners:

Consider two neighboring regions with n_a (resp. n_d) being the number of soldiers that are attacking (resp. defending). The auxiliary state of the two players is given as s_a and s_d. The transition probability is

$$p_(n_{a,s}, n_{a,t}, n_{d,t} | n_a, n_d, s_a, s_d) =$$

(Need to incorporate the remaining soldiers after engagement as well).
Draw the two-dimensional state space.
$n_{a,s}$ is the number of attacker's soldiers remaining in *source* region from which the attack is made.
$n_{a,t}$ is the number of attacker's soldiers remaining in *target* region on which the attack is made.
$n_{d,t}$ is the number of defender's soldiers remaining in *target* region on which the attack is made.

Under the logical assumption (derived from JRE) that attack is possible only when $n_a \geq n_d$)

$$p_(n_{a,s}, n_{a,t}, n_{d,t} | n_a, n_d, s_a, s_d) \to 0 \text{ as } n_a - n_d \to \infty$$

or, more precisely,

$$p_(n_{a,s}, n_{a,t}, n_{d,t} | n_a, n_d, s_a, s_d) \sim o(\frac{n_a - n_d}{n_a})????$$

Proof. □

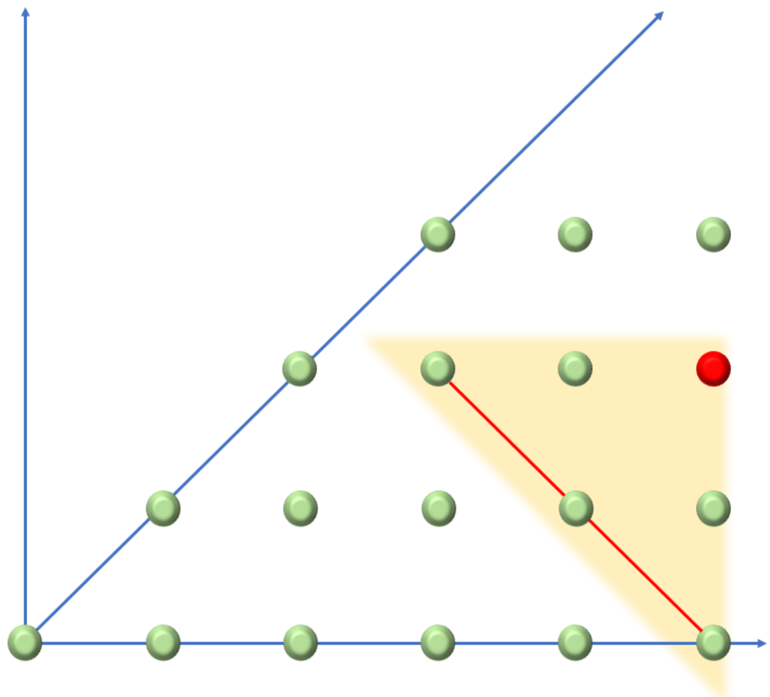

Figure 1: Possible transitions after an engagement under the Jakub Rule of Engagement.

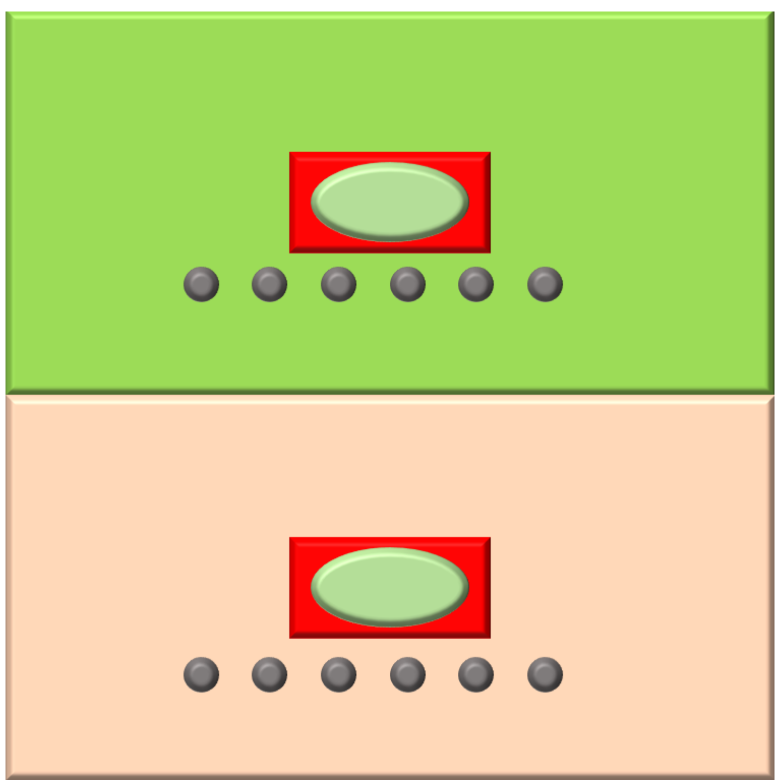

Figure 2: The setup showing 2 temples on two regions with equal number of initial soldiers.

2.3 Jakub's Algorithm

how the current algorithm decides on next steps

Q.1 How to select the next attack regions

Q.2 How to select the movement of soldiers (can a balance be created)

Q.3 How to decide on the upgrade

2.4 Faith Accumulation and Purchases

Section Owners:

3 Analysis

We will limit ourselves to threshold-based policies which will take into account the following: total strength difference
total size difference

Section Owners
In this paper we will be limiting ourselves to a two player setup.
There are total of 3 rings of regions
Each ring has N regions
The two players' initial temples are equidistant in the middle ring
We will perform analysis with different scenario. Three different assumptions, and three analysis for each group. I will be working with each group separately on the analysis part.
These assumptions could be, for example, soldier upgrades available along with

3.1 Only air temple

Section Owners:
(unlimited amount of additional moves possible, with additional cost per additional move)

3.2 Only Earth temple

Section Owners:
(unlimited amount of additional kills possible, with additional cost per additional kill upgrade)

3.3 Only water temple

Section Owners:
(unlimited amount of additional faith percentage possible, with additional cost per additional percentage)

3.4 Actions that can be ruled out

A region with lesser number of soldiers would clearly not be attacking a region with higher number of soldiers. This is because of the JRE.

4 Casting the Problem as an MDP

The Figure 3 provides the description of the Markov Decision Process that we are considering in this paper. The following modifications are assumed in the original game of Jakub.

1. No upgrades are possible

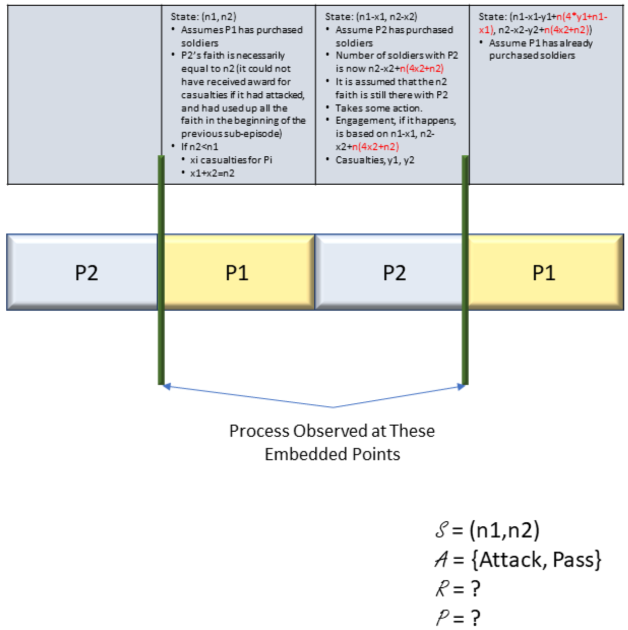

Figure 3: The Embedded Markov Chain and the various Assumptions.

2. The players exhaust the faith available to them in buying as many soldiers as possible, and the remaining faith vanishes

3. The strategy of the AI is fixed, as provided in the code of the game

State: (n1, n2) Assumes P1 has purchased soldiers P2's faith is necessarily equal to n2 (it could not have received award for casualties if it had attacked, and had used up all the faith in the beginning of the previous sub-episode) If n2¡n1 xi casualties for Pi x1+x2=n2

State: (n1-x1, n2-x2) Assume P2 has purchased soldiers Number of soldiers with P2 is now n2-x2+n(4x2+n2) It is assumed that the n2 faith is still there with P2 Takes some action. Engagement, if it happens, is based on n1-x1, n2-x2+n(4x2+n2) Casualties, y1, y2

State: (n1-x1-y1+n(4*y1+n1-x1), n2-x2-y2+n(4x2+n2)) Assume P1 has already purchased soldiers

4.1 Asymptotic Policy

As (n_1, n_2) increase, the incentive for attacking decreases. This is seen as per the following discussion.

For a given (n_1, n_2), let $p(n_1, n_2)$ be the probability that one soldier of the defending player dies in the Binomial experiment. Thus, the average number of kills of the opponent is

$$n_2 p(n_1, n_2).$$

This implies that the defender will gain, on an average, $4n_2$ faith.

$$p(n_1, n_2) = \frac{0.88 * (120 + 100 \left(\frac{n_1}{n_2}\right)^{1.6}) - 120}{0.76 * (120 + 100 \left(\frac{n_1}{n_2}\right)^{1.6})}.$$

Further, the probability that the attacker (Player 1) wins (or, the defender is Extinct) is

$$P_E(n_1, n_2) = p^{n_2}(n_1, n_2),$$

i.e., all the casualties are from the defender. Note that $P_E(n_1, n_2) \to 0$ as $n_2 \to \infty$ for a given ratio $\frac{n_1}{n_2}$, or also for any given n_1. In such a case, if the player 1 attacks, there will be a tendency to have $n_2 p(n_1, n_2)$ soldiers killed (see Large Deviations results, Thus, after the round, the player 2 would be left with

$$n(n_2 + 3n_2 p(n_1, n_2))$$

soldiers, while the player 1 will be left with

$$n(n_1 - n_1 p(n_1, n_2))$$

soldiers (after purchasing using its faith). These two will quickly catch up if player 1 keeps on attacking as player 1 will loose without enough replenishment, while player 2 will be able to compensate for its losses more.

Thus, for a given ratio of $\frac{n_1}{n_2} = r$, we can assume that there is a threshold $n_2^*(r)$ after which the policy is to not attack. Clearly, the reward function for the MDP formulation needs to be adapted to ensure that this state is not reached.

Reward set:

5 Numerical Results

6 The Case of Three Players

In our experiments, we let the two players use the

7 Self-Learning

References