User-based Rating Prediction

User-based neighborhood recommendation methods predict the rating r_{ui} of a user u for a new item i using the ratings given to i by users most similar to u, called nearest-neighbors. Suppose we have for each user $v \neq u$ a value w_{uv} representing the preference similarity between u and v. The k-nearest-neighbors (k-NN) of u, denoted by $N(u)$, are the k users v with the highest similarity w_{uv} to u.

Only the users who have rated item i can be used in the prediction of r_{ui}, and we instead consider the k users most similar to u that have rated i. We write this set of neighbors as $N_i(u)$. The rating r_{ui} can be estimated as the average rating given to i by these neighbors:

$$\hat{r}_{ui} = \frac{1}{|N_i\left(u\right)|} \sum r_{vi}$$

A problem is that is does not take into account the fact that the neighbors can have different levels of similarity. A common solution to this problem is to weigh the contribution of each neighbor by its similarity to u. However, if these weights do not sum to 1, the predicted ratings can be well outside the range of allowed values. Consequently, it is customary to normalize these weights, such that the predicted rating becomes

$$\hat{r}_{ui} = \frac{\sum w_{uv} r_{vi}}{\sum |w_{uv}|}$$

$|w_{uv}|$ is used instead of w_{uv} because negative weights can produce ratings outside the allowed range. Also, w_{uv} can be replaced by w_{uv}^{α} , where $\alpha > 0$ is an amplification factor. When $\alpha > 1$, as is it most often employed, an even greater importance is given to the neighbors that are the closest to u.

The fact that users may use different rating values to quantify the same level of appreciation for an item. For example, one user may give the highest rating value to only a few outstanding items, while a less difficult one may give this value to most of the items he likes. This problem is usually addressed by converting the neighbors' ratings r_{vi} to normalized ones $h(r_{vi})$, giving the following prediction:

$$\hat{r}_{ui} = h^{-1}\left(\frac{\sum w_{uv} h\left(r_{vi}\right)}{\sum |w_{uv}|}\right)$$

Note that the predicted rating must be converted back to the original scale, hence the h^{-1} in the equation.