Themes: Building a definite integral, numerical integration with MATLAB/Mathematica on computer.

The Happy Holiday Candy Factory

Name: Solar Shao; Mike Wang

Prodfessor: Danhua Wang

Math 9:Introduction to Programming for Numerical Analysis [Matlab; Mathematica]

Date: December 8th

1 Introduction

In this project, we served as mathematical consultants to the *Happy Holiday Candy Factory* to design a production schedule for the wildly popular chocolate-covered-asparagus Holiday candies. The project is about the best production plan of producing two million candies in the last 81 days until the New Year. The following information is given: producing candies at a rate of R candies per day costs the factory $50 + \frac{R}{1000}$ cents per candy and storing A candies costs $\frac{A}{2}$ cents per day (to rent storage space, power the refrigerators, hire security guards, etc.)
This project investigates which production schedule should be used to make the total production cost as small and realistic as possible. Matlab is the main tool being used to address the problems.

2 Uniform Model

Since the total cost should be a sum of the cost of each day, integration was used to calculate the total sum. Giving the information shown in the introduction, the general total cost formula is

$$\int_{1}^{81} R * (50 + (\frac{R}{1000}) + \frac{Rt}{2} dt$$

where R is the number of candy produced each day and t is the number of days needed.
In the integral, we divided the time t into 81 pieces and set it as the upper and lower bound of the integral given above since there were 81 days left until the New Year when the project was assigned. Known R candies would be produced each day and it costs the factory $50 + \frac{R}{1000}$ cents to produce a candy, we multiplied these two variables to get the cost of producing one candy in a day. Since Rt is the total amount of candy produced in t days, $\frac{Rt}{2}$is the storing cost for this amount of candy. The integral showed the total cost of producing candies in 81 days.

2.1 Model with Constant R Rate

Now we set the production rate "R" constant, meaning candies are produced at a constant rate in the last 81 days before New Year. Knowing two millions of candies should be produced by New Year, we divided the total amount of candy by the number of days, to get:

Rate: R= $\frac{2{,}000{,}000}{81}$=24,691.358=24,692 candies per day

Since the amount of candy can only be an integer, we rounded the number up to the next largest integer, which made the production rate 24692 candies per day.

Then, to get the total cost, we could just plug the constant rate we got back into the integral written in the last section.

In this case, the total cost should be

$$\int_{1}^{81} 24691.36 * (50 + (\frac{24691.36}{1000}) + \frac{24691.36 * t}{2} dt = \$1.9038 * 10^{8}$$

2.2 Model with Changing R Rate

Now, a model with a changing production rate R was considered, in which we wanted a schedule that results a smaller cost compared to the uniform schedule.

Since there are 81 days, every day could be considered as an individual variable. In such a case, the total cost is a function that contains 81 different variables. Moreover, 2 million candies should be produced within 81 days. Thus, 81 individual variables add up to a total number of 2 million. Now, the situation was converted into a function with a constraint in which we had to figure out where the minimum is. In our case, the constraint is an equality constraint that satiates conditions for using the Lagrange Multipliers.
The method of Lagrange multipliers is a strategy for finding the local maximum and minimum of a function subject to equality constraints. So here, we are using the method of Lagrange Multiplier to help us find out the minimum. Therefore, we know the lowest cost of the production process and then we can form a schedule according to this function, with the constraint of producing 2 million candies by New Year. This is mathematically the lowest cost for our model.

Therefore, we set up the Lagrange model in Matlab with the conditions showing above.
And we run the code, figuring out the production rates of each day in the last 81 days.
The total cost for the uniform case is $1.9038 * 10^{8}$, that is, the total cost of producing the candies at a constant rate.

The total cost for the Lagrange case is $2.9554 * 10^{6}$, that is, the total cost of producing candies at a rate that minimizes the total cost.

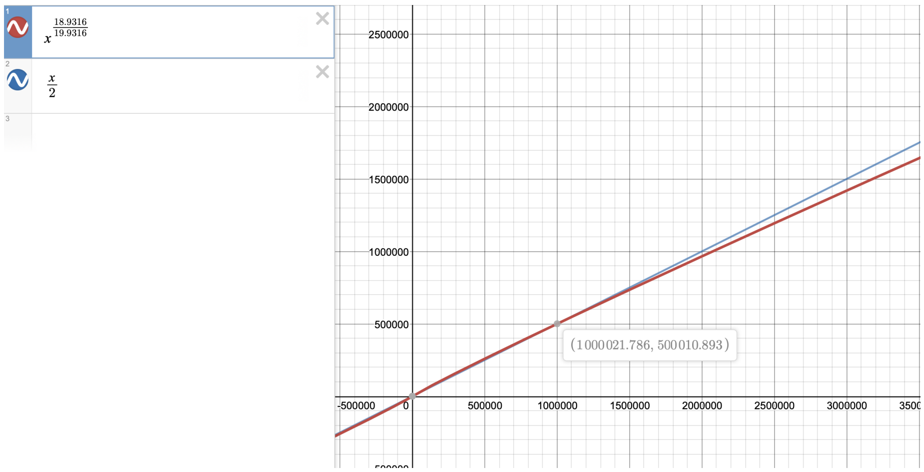
3 More Realistic Model

3.1 Realistic Storage Cost

In the previous model, the storage cost for candies is a linear function, which is $\frac{A}{2}$. However, in the real-life scenario, the storage cost should not be linearly related to the number of candies produced, due to the reason of diminishing return. ("The law of diminishing marginal returns states that, at some point, adding a factor of production results in smaller increases in output. In the short run, the law of diminishing returns states that as we add more units of a variable input to fixed amounts of land and capital, the change in total output will at first rise and then fall.")
Therefore, the marginal cost for the storage cost should decrease as more candies being stored. This made the storage cost function in a more realistic scenario a function that approximately equals to $\frac{A}{2}$ for A small, but starts to deviate somewhat from $\frac{A}{2}$ when A is around half of the total amount, which is 1 million. That is due to the reason that when the amount of candy reaches 1 million, the marginal cost of storing a unit of candy start to decrease. Giving the fact of a fixed storage capacity, the cost to power the refrigerators, hire security guards would not change a lot with one unit of candy added.

We found a piecewise function that satisfies the conditions mentioned above.

When A is small, the storage cost function is linear, that is, f(x)=$\frac{x}{2}$, for $x <= 1,000,000$. When A is bigger than 1 million, the function starts to deviate and becomes a power function, that is, f(x)=$x^{\frac{18.9316}{19.9316}}$, for $x > 1,000,000$. We were thinking about a power function when we solved this problem is because we knew the function for storage cost should be an increasing diminishing function after 1 million. Among all types of functions we had already learned, there are only a few types of function that have a downward concavity. And, if we want a function to be a concave down second function, its second derivative should be negative so the power should be a proper fraction. Giving all these constraints, we fixed the function to be a power function with a proper fraction power.



Graph of the more realistic storage cost

And, the total cost equation for this model is

$$\int_1^{81} R*(50+(\frac{R}{1000})+Rt^{\frac{18.9316}{19.9316}}dt$$

We took the variable storage into consideration in a new model. The result of the total cost is $1.8742*10^8$. Comparing to the model with a constant rate of storage which the total cost is $1.9038*10^8$, we can see a certain amount of deviation of storage cost from $\frac{A}{2}$, which is relatively cheaper than the model before.

Previously, we had known that the result for the total cost with the Lagrange method is $2.9554*10^6$. Now, we took the same method into the new model, the one with variable storage cost. The total cost turned out to be $2.9381*10^6$. Combining this result with the second part, it is clear to see that this new model of storage could result a lower cost after the production number reaches 1 million. Thus, we could claim that the result was consistent with the graph that we had for the storage cost model. Also, it was closer to realistic. Since in real life, when the production process became mature, it was easier for manufacturers to gradually lower the cost of the production cycle including the storage cost. This could also be explained economically that "In the long run, all the factors of production can be varied."

3.2 New Factor Being Introduced - Logistics Cost

In the previous model, logistics was not considered. However, it is a huge amount of cost that cannot be ignored when calculating the total cost in producing chocolate.

The reason we took logistics cost, or shipping cost into consideration is the fact that chocolate needs to be conserved in a cool temperature, otherwise it would melt. According to the article *High Cost of Keeping Chocolate Cool Tempers Online Sales*, written by Robbie Whelan, "Chocolate often melts during shipping in the summer months and warmer climes, or arrives covered in "bloom," a white film that coats the chocolate when sugar or fat rises to the surface." And, "Keeping a small shipment of chocolate cool can cost more than the product itself."

Therefore, we have conducted some research and found one Refrigerated cargo van rental rate that would fit our needs the best. We planned to rent the truck for 12 weeks in total (last 81 days till New Year). Knowing that the rental rate is \$1,600 per four weeks and we would need to rent this truck 3 times in the last 12 weeks till New Year, the total amount of money we pay would be $3 * \$1,600 = \$4,800$.

The load capacity of the truck was also considered to make sure the truck we chose would be able to fit all the candies produced in four weeks.
We checked the weight.
Chocolate-Covered asparagus is very similar to a Lunch Bar, either on their weight or shape. According to the *"How much your chocolate bar weighs"* written by Kevin Lancaster on the website *BusinessTecho*, a Lunch Bar weighs around 50 grams. Supposed we would produce our candy in a constant rate in 81 days, by dividing the total amount of candy we need to produce in 81 days, which is 2millions, by the number of days, which is 81 days.

We will get a constant rate of candy production of 24,691 candies per day.

$$\frac{2000000}{81} = 24691$$

To find out how much this amount of candy weighs, we timed the amount of candy produced in a day by the unit weights of one single candy, which, in this case, is around 50 grams.

$$24,691 * 50 = 1,234,550$$

The unit should be converted from "grams" to the unit same as the capacity load of the truck, which is "lb". Knowing $1g = 0.00220462262185lb$, the calculation steps could be found as followed.

$$\frac{1,234,550grams}{453.59237} = 2,721.7168578lb$$

To get the total cost for previous models, we could simply add the cost of renting the truck, which is a constant, to the production cost we got. For the uniform model with a constant R rate, if logistics cost was considered, our total cost would become:

$$\$4,800 + \$1.93038 * 10^8 = \$1.903848 * 10^8$$

For the uniform model with a changing R rate, if logistics cost was considered, our total cost would become:

$$\$4,800 + \$1.8742 * 10^8 = \$1.874248 * 10^8$$

For the model with a stable storage cost and lagrange method, if logistics cost was considered, our total cost would become:

$$\$4,800 + \$2.9554 * 10^6 = \$2.9602 * 10^6$$

For the model with a realistic storage cost and lagrange method, if logistics cost was considered, our total cost would become:

$$\$4,800 + \$2.9381 * 10^6 = \$2.9429 * 10^6$$

4 Conclusion

In conclusion, we have achieved multiple things in this project and we are proud of the results shown from each section. First of all, we had managed to come up with a standard model that was based on the given conditions. With that model in mind, we have calculated the Model with Constant rate of production, in which we have come up with the results the total cost is $1.9 * 10^8$ dollars. However, to make the model as close to reality as possible, we have also figured out a way to lower the production cost. In this case, we used Matlab as our tool, adapted Lagrange Multipliers, set the standard model as a function that contains 81 different variables so that we can directly find a local minimum of the function and in turn to decide the production schedule. With this new production schedule, we managed to lower the cost to $2.9554 * 10^6$ dollars.

Moreover, in the previous models, we used $\frac{x}{2}$ as our storage cost. After researching and trying with different numbers, we finally figured out the equation $x^{\frac{18.9316}{19.9316}}$ as our storage cost equation because this equation intersects with $\frac{x}{2}$ when 1,000,000 candies are produced. So, we replaced $\frac{x}{2}$ by $x^{\frac{18.9316}{19.9316}}$ as the storage function to best simulate the storage cost as described in the conditions. Previously, we have the results indicating the total cost with the model($\frac{x}{2}$) is $2.9554 * 10^6$. Comparing to the total cost with a variable storage cost being $2.9381 * 10^6$, the total cost decreased with the storage cost decreases when x is bigger than one million.

In the last part, to make the model even more realistic, we considered logistics. We have discussed multiple possibilities and finally decided that the company should rent a refrigerated cargo van that can be used to ship candies from the factory to the storage. With the information we gathered from online, the final cost of logistic is a constant rate of 4,800 dollars for renting the equipment we need. And, the final cost of the whole production turned out to be $\$2.9429 * 10^6$

With this project, it is certain for us to say that we have learned a lot during the process. Even though we have encountered with numerous problems, we have utilized our resources and knowledge to solve them. This experience was truly irreplaceable and inspiring. With this model, there were certain things that we haven't take into consideration, but we are happy with the results and hopefully this project can provide a little bit of insight views on candy manufacturing.

5 References

Whelan, Robbie. *"High Cost of Keeping Chocolate Cool Tempers Online Sales"* The Wall Street Journal. 03 2016. 01 2019 ¡https://www.wsj.com/articles/high-cost-of-keeping-chocolate-cool-tempers-online-sales-1458860341¿.

Lancaster, Kevin . *"How much your chocolate bar really weighs"* BUSINESSTECH. 03 2015. 01 2019 ¡https://www.wsj.com/articles/high-cost-of-keeping-chocolate-cool-tempers-online-sales-1458860341¿.

Kenton, Will. *"Law of Diminishing Marginal Returns"* Investopedia. 07 2019. 01 2019 ¡https://www.investopedia.com/terms/l/lawofdiminishingmarginalreturn.asp¿.