IEOR 240 Final Report:
Construction Plan of Lake Saddleback Development Corporation (LSDC)

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1. Introduction

The Lake Saddleback Development Corporation (LSDC) planned to build four series of products on a section of Lake Saddleback, Texas; the four series are (1) Grand Estates, (2) Glen Wood Collection, (3) Lakeview Patio Homes, and (4) Country Condominiums. This project is to help LSDC develop a community construction plan on the 300 acres land in such a way that maximizes the total profit while offering an appropriate variety of different home plans in different products. For each type of product, there are three to four different floor plans/models, varying on garage sizes, number of bedrooms, bathrooms, and stories. Some floor plans of certain products offer "exclusive" editions, such as houses "on the lake" and houses with premium sizes, which price higher than the normal edition. Different models have different requirements on ground area, yard size, and garage size, and it leads to different lot sizes (which includes Ground Area of House, Yard Size, and Garage Size). The setting requirements of outdoor parking space and roads/greenbelts areas should also match with the different floor plans of each model. To ensure an appropriate variety and meet government's requirement on affordable housing, there are also limitations on number of each type of houses to be built. Finally, the total occupied area of the project (which includes lot sizes of all houses built, outside parking, and roads/greenbelts areas) should not exceed 300 acres, which is equivalent to 13,06,000 sq.ft. Under these requirements, the goal is to find an optimal decision regarding the number of units of each model/floor plan to build, and each of these requirements will be discussed in detail in the model formulation section.

2. Model Formulation

2.1 Assumptions

For this project, we assume that:

1. All the numbers and equations given are correct. This includes the sizes, prices, percentages,

and calculation formula on lot sizes, ground area, and yard size, etc.

2. In order to get an accurate profit estimation, we assume that the given selling prices are the actual price for sell and all homes can be sold.

3. Non-integer results of the number of units of each floor plan are allowed, and they will be rounded to the nearest integer to deliver a feasible optimal decision.

4. All area/size are measured in square feet(s). We converted all acres to square feet to have consistent unit. The conversion relationship is 1 acre = 43560 sq.ft.

2.2 Decision Variables

We use $x_{i,j}$ to represent the number of each floor plan. The first index i ranges from 1 to 15, corresponding to the 15 floor plans in total. The second index j ranges from 1 to 2, representing whether the floor plan is exclusive or normal. In *Grand Estates* series, for example, j = 1 represents exclusive homes which will be built on the lake, and j = 2 represents the normal homes. For rand *Cypress* models (in *Glen Wood Homes* series), j = 1 represents "premium" homes and j = 2 represents standard homes. For models with only standard lots, $x_{i,j}$ equals to 0 when j = 1, and $x_{i,j}$ represents the actual number of units of the model when j = 2. The representations of each decision variable are summarized in Table 1.

2.3 Parameters

Constant parameters in this LP includes sales price, net profit percentage, lot size, parking, and road/greenbelt. Particularly, road/greenbelt area are fixed at 1000 sq.ft. for each house. The notation of each type of parameters is concluded in Table 2.

Parameters	Notation
Sales Price	$p_{i,j}$
Net Profit Percentage	$per_{i,j}$
Lot Size	$l_{i,j}$
Parking	$parking_{i,j}$

Table 2

Note that the index i,j in the notation of parameters has the same definition with the one in decision variables.

Amount of Units	Floor Plan Representation
	Grand Estates
$x_{1,1}$	The Trump Exclusive
$x_{1,2}$	The Trump Standard
$x_{2,1}$	The Vanderbilt Exclusive
$x_{2,2}$	The Vanderbilt Standard
$x_{3,1}$	The Hughes Exclusive
$x_{3,2}$	The Hughes Standard
$x_{4,1}$	The Jackson Exclusive
$x_{4.2}$	The Jackson Standard
	Glen Wood Collection
$x_{5,1}$	The Grand Cypress Premium
$x_{5,2}$	The Grand Cypress Standard
$x_{6,1}$	N/A
$x_{6,2}$	The Lazy Oak Standard
$x_{7,1}$	N/A
$x_{7,2}$	The Wind Row Standard
$x_{8,1}$	N/A
$x_{8,2}$	The Orangewood Standard
	Lakeview Patio Homes
$x_{9,1}$	The Bayview Premium
$x_{9,2}$	The Bayview Standard
$x_{10,1}$	N/A
$x_{10,2}$	The Storeline Standard
$x_{11,1}$	N/A
$x_{11,2}$	The Docks Edge Standard
$x_{12,1}$	N/A
$x_{12,2}$	The Golden Pier Standard
	Country Condominiums
$x_{13,1}$	N/A
$x_{13,2}$	The Country Stream Standard
$x_{14,1}$	N/A
$x_{14,2}$	The Weeping Willow Standard
$x_{15,1}$	N/A
$x_{15,2}$	The Picket Fence Standard

Table 1: Variable Representation

2.3.1 Sales Price and Net Profit Percentage

The sales price of each floor plan with normal edition is listed in the article. *Grand Estates* series, *Grand Cypress* models, and *Bayview* models offer "exclusive" editions, which price higher than the normal edition based on their special properties.

For *Grand Estates* series, the selling price of models built on the lake will be an additional 30% plus \$50, 000 more than the models not on the lake. For example, the \$700,000 *Trump* model would sell for $\$700{,}000 * (1 + 0.3) + 50{,}000$ = \$960,000 if on the lake.

For *Grand Cypress* models, the floor plan with quarter-acre lot size would sell for \$40,000 more than same models on standard lots. It means that the sell price of *Grand Cypress* models with quarter-acre lot size would be \$420,000 + \$40,000 = \$460,000.

For *Bayview* models, the floor plan with one-sixth acre lot size would sell for \$30,000 more than same models on standard lots. That is the sell price of *Bayview* models with one-sixth acre lot size would be \$300,000 + \$30,000 = \$330,000.

The sales price and net profit percentage of each floor plan are summarized in Table 3.1 and Table 3.2.

Floor Plan	Sales Price Notation ($p_{i,j}$)	Sales Price (in dollars)	Net Profit Percentage Notation ($per_{i,j}$)	Net Profit Percentage
Grand Estates				
Trump Exclusive	$p_{1.1}$	960,000	$per_{1.1}$	22%
Trump Standard	$p_{1,2}$	700,000	$per_{1,2}$	22%
Vanderbilt Exclusive	$p_{2,1}$	934,000	$per_{2,1}$	22%
Vanderbilt Standard	$p_{2,2}$	680,000	$per_{2,2}$	22%
Hughes Exclusive	$p_{3,1}$	895,000	$per_{3,1}$	22%
Hughes Standard	$p_{3,2}$	650,000	$per_{3,2}$	22%
Jackson Exclusive	$p_{4,1}$	817,000	$per_{4,1}$	22%
Jackson Standard	$p_{4,2}$	590,000	$per_{4,2}$	22%
Glen Wood Collection				
Grand Cypress Premium	$p_{5,1}$	460,000	$per_{5,1}$	18%
Grand Cypress Standard	$p_{5,2}$	420,000	$per_{5,2}$	18%
N/A	$p_{6,1}$	0	$per_{6,1}$	18%
Lazy Oak Standard	$p_{6,2}$	380,000	$per_{6,2}$	18%
N/A	$p_{7,1}$	0	$per_{7,1}$	18%
Wind Row Standard	$p_{7,2}$	320,000	$per_{7,2}$	18%
N/A	$p_{8,1}$	0	$per_{8,1}$	18%
Orangewood Standard	$p_{8,2}$	280,000	$per_{8,2}$	18%

Table 3.1

Floor Plan	Sales Price Notation ($p_{i,j}$)	Sales Price (in dollars)	Net Profit Percentage Notation ($per_{i,j}$)	Net Profit Percentage
Lakeview Patio Homes				
Bayview Premium	$p_{9,1}$	330,000	$per_{9,1}$	20%
Bayview Standard	$p_{9,2}$	300,000	$per_{9,2}$	20%
N/A	$p_{10,1}$	0	$per_{10,1}$	20%
Storeline Standard	$p_{10,2}$	270,000	$per_{10,2}$	20%
N/A	$p_{11,1}$	0	$per_{11,1}$	20%
Docks Edge Standard	$p_{11,2}$	240,000	$per_{11,2}$	20%
N/A	$p_{12,1}$	0	$per_{12,1}$	20%
Golden Pier Standard	$p_{12,2}$	200,000	$per_{12,2}$	20%
Country Condominiums				
N/A	$p_{13,1}$	0	$per_{13,1}$	25%
Country Stream Standard	$p_{13,2}$	220,000	$per_{13,2}$	25%
N/A	$p_{14,1}$	0	$per_{14,1}$	25%
Weeping Willow Standard	$p_{14,2}$	160,000	$per_{14,2}$	25%
N/A	$p_{15,1}$	0	$per_{15,1}$	25%
Picket Fence Standard	$p_{15,2}$	140,000	$per_{15,2}$	25%

Table 3.2

Grand Estates		
Floor Plan	Lot Size Notation $l_{i,j}$	Lot Size (in Sq.ft)
Trump Exclusive	$l_{1.1}$	21,780
Trump Standard	$l_{1,2}$	21,780
Vanderbilt Exclusive	$l_{2,1}$	21,780
Vanderbilt Standard	$l_{2,2}$	21,780
Hughes Exclusive	$l_{3,1}$	21,780
Hughes Standard	$l_{3,2}$	21,780

Jackson Exclusive	$l_{4,1}$	21,780
Jackson Standard	$l_{4,2}$	21,780

Table 4

2.3.2 Lot Size

The formula to calculate Lot Size is

$$\textit{Lot size} = \textit{(Ground Area of House)} + \textit{(Yard Size)} + \textit{(Garage Size)} \quad \textbf{(1)}$$

For *Grand Estate* Series, all models in the *Grand Estate* series are built on one-half acre lots, which is equivalent to 21,780 sq.ft. It means that the lot size of all floor plans in Grand Estate Series is fixed at 21,780 sq.ft regardless of the size of the yard and garage (see Table 4).

For *Glen Wood Collection*,

1. The lot size of *Grand Cypress Premium* is fixed at quarter-acre lots, which is equivalent to 10,890 sq.ft.
2. For any standard models, the ground area of any single-story house is the advertised square footage of the house, and the ground area for two-story homes is 75% of the advertised square footage.
3. For any standard models, yard sizes 1,200 square feet for single-story homes. For two-story homes, the yard has the same size as the ground area of the house.
4. For any standard models, two-car garages occupy 500 square feet of the ground area, and three-car garages occupy 750 square feet of ground area space.
5. The minimum standard lot for homes (except for the premium models) is 0.1 acre, which means that the lot size of all standard models is the maximum of the lot size calculated by the formula **(1)** and 0.1 acre. That is the lot size of all standard models equals to

$$Max\ \ (\text{ Ground Area of House} + \text{Yard Size} + \text{Garage Size}, 4356\text{ })$$

   Note that 0.1 acre = 4,356 sq.ft.

The information of the yard size, parking area, and lot size of models in *Glen Wood Collection* is summarized in Table 5.

Glen Wood Collection								
Floor Plan	Stories	Garage Size	Ground Area (in Sq.ft)	Yard Size (in Sq.ft)	Garage Area (in Sq.ft)	Lot Size by formula (in Sq.ft)	Lot Size = Max(by formula, 0.1 acre) (in Sq.ft)	Lot Size Notation
Grand Cypress Premium	2	3	N/A	N/A	N/A	10,890	10,890	$l_{5,1}$
Grand Cypress Standard	2	3	2,800*0.75	2,800*0.75	750	4,950	4,950	$l_{5,2}$
N/A	0	0	0	0	0	0	0	$l_{6,1}$
Lazy Oak Standard	2	2	2,400*0.75	2,400*0.75	500	4,100	4,356	$l_{6,2}$
N/A	0	0	0	0	0	0	0	$l_{7,1}$
Wind Row Standard	2	2	2,200*0.75	2,200*0.75	500	3,800	4,356	$l_{7,2}$
N/A	0	0	0	0	0	0	0	$l_{8,1}$
Orangewood Standard	1	2	1,800	1,200	500	3,500	4,356	$l_{8,2}$

Table 5

For Lakeview Patio Homes,

1. The lot size of *Bayview Premium* is fixed at one-sixth acre lots, which is equivalent to 7,260 sq.ft.
2. For any standard models, the ground area of any single-story house is the advertised square footage of the house, and the ground area for two-story homes is 75% of the advertised square footage.
3. For any standard models, yard sizes are 900 square feet for single-story homes. For two-story homes, the yard size is 600 square feet + 50% of the ground area of the house.

4. For any standard models, two-car garages occupy 500 square feet of the ground area, and three-car garages occupy 750 square feet of ground area space.
5. The minimum standard lot for homes (except for the premium models) is 0.1 of an acre, which means that the lot size of all standard models is the maximum of the lot size calculated by the formula (1) and 0.1 acre. That is Lot size of all standard models equals to

$$Max\ (\text{Ground Area of House} + \text{Yard Size} + \text{Garage Size}, \quad 4{,}356\ )$$

Note that 0.1 acre = 4,356 sq.ft.

The information of the yard size, parking area, and lot size of models in *Lakeview Patio Homes* is summarized in Table 6.

Lakeview Patio Homes								
Floor Plan	Stories	Garage Size	Ground Area (in Sq.ft)	Yard Size (in Sq.ft)	Garage Area (in Sq.ft)	Lot Size by formula (in Sq.ft)	Lot Size = Max(by formula, 0.1 acre) (in Sq.ft)	Lot Size Notation
Bayview Premium	2	2	N/A	N/A	N/A	7,260	7,260	$l_{5,1}$
Bayview Standard	2	2	2,000*0.75	600 + 2,000*0.75*0.5	500	3,350	4,356	$l_{5,2}$
N/A	0	0	0	0	0	0	0	$l_{6,1}$
Storeline Standard	2	2	1,800*0.75	600 + 1,800*0.75*0.5	500	3,125	4,356	$l_{6,2}$
N/A	0	0	0	0	0	0	0	$l_{7,1}$
Docks Edge Standard	1	2	1,500	900	500	2,900	4,356	$l_{7,2}$
N/A	0	0	0	0	0	0	0	$l_{8,1}$
Golden Pier Standard	1	2	1,200	900	500	2,600	4,356	$l_{8,2}$

Table 6

For *Country Condominiums*, lot sizes are fixed at 1,500 square feet (see Table 7).

Country Condominiums		
Floor Plan	Lot Size Notation $l_{i,j}$	Lot Size (in Sq.ft)
N/A	$l_{13,1}$	0
Country Stream Standard	$l_{13,2}$	1,500
N/A	$l_{14,1}$	0
Weeping Willow Standard	$l_{14,2}$	1,500
N/A	$l_{15,1}$	0
Picket Fence Standard	$l_{15,2}$	1,500

Table 7

2.3.3 Parking

For all models in all series, one parking space per bedroom for each unit built is required. Each outside parking space will occupy 200 square feet of space, and all parking for the *Country Condominiums* is outside the house. The information of parking area of each floor plan is shown in Table 8.

Floor Plan	Bedrooms	Garage Size	Parking Area Needed (in Sq. ft)	Parking Area Notation
Grand Estates				
Trump Exclusive	5	3	400	$parking_{1,1}$
Trump Standard	5	3	400	$parking_{1,2}$
Vanderbilt Exclusive	4	3	200	$parking_{2,1}$
Vanderbilt Standard	4	3	200	$parking_{2,2}$
Hughes Exclusive	4	3	200	$parking_{3,1}$
Hughes Standard	4	3	200	$parking_{3,2}$
Jackson Exclusive	3	3	100	$parking_{4,1}$
Jackson Standard	3	3	100	$parking_{4,2}$

Glen Wood Collection				
Grand Cypress Premium	4	3	200	$parking_{5,1}$
Grand Cypress Standard	4	3	200	$parking_{5,2}$
N/A	0	0	0	$parking_{6,1}$
Lazy Oak Standard	4	2	400	$parking_{6,2}$
N/A	0	0	0	$parking_{7,1}$
Wind Row Standard	3	2	200	$parking_{7,2}$
N/A	0	0	0	$parking_{8,1}$
Orangewood Standard	3	2	200	$parking_{8,2}$
Lakeview Patio Homes				
Bayview Premium	4	2	400	$parking_{9,1}$
Bayview Standard	4	2	400	$parking_{9,2}$
N/A	0	0	0	$parking_{10,1}$
Storeline Standard	3	2	200	$parking_{10,2}$
N/A	0	0	0	$parking_{11,1}$
Docks Edge Standard	3	2	200	$parking_{11,2}$
N/A	0	0	0	$parking_{12,1}$
Golden Pier Standard	2	2	0	$parking_{12,2}$
Country Condominiums				
N/A	0	0	0	$parking_{13,1}$
Country Stream Standard	3	0	600	$parking_{13,2}$
N/A	0	0	0	$parking_{14,1}$
Weeping Willow Standard	2	0	400	$parking_{14,2}$
N/A	0	0	0	$parking_{15,1}$
Picket Fence Standard	2	0	400	$parking_{15,2}$

Table 8

2.4 Objective Function

The objective function is to maximize the net profit, which is the sum of multiplication of number of unit of each floor plan and its net profit. Particularly, the net profit of each floor plan is calculated by multiplying its selling price with its product type's net profit percentage. The selling prices also depend on the type of floor plan and whether it is exclusive/premium or not.

Hence, the objective function in general is:

Max $Number\ of\ unit * Net\ profit$

$$= Number\ of\ unit * sales\ price * net\ profit\ percentage$$

$$= \sum_{i=1}^{15} \sum_{j=1}^{2} x_{i,j} * p_{i,j} * per_{i,j}$$

2.5 Constraints

2.5.1 Construction Amount Constraints

There are requirements on the unit number of particular floor plans.

1. Each *Grand Estate* series plans must have at least eight units on the lake, which is formulated as: **For $\boldsymbol{i}$ in range 1 to 4, $\boldsymbol{x_{i,1} \geq 8}$**
2. 50 half-acre lots on the lake are to be used exclusively by the *Grand Estate* Series homes, which can be formulated as: $\boldsymbol{\sum_{i=1}^{4} x_{i,1} \leq 50}$
3. No more than 25% of the total *Grand Cypress* models may be built on the premium lots, which can be formulated as: $\boldsymbol{x_{5,1} \leq 0.25 * \sum_{j=1}^{2} x_{5,j}}$
4. No more than 25% of the total *Bayview* models may be built on the premium lots, which can be formulated as: $\boldsymbol{x_{9,1} \leq 0.25 * \sum_{j=1}^{2} x_{9,j}}$

2.5.2 Parking Constraints

Parking space refers to the outside parking space. Each outside parking space occupies 200 sq.ft. and no more than 15 acres should be used for outside parking. This requirement can be formulated as: $\boldsymbol{\sum_{i=1}^{15} \sum_{j=1}^{2} x_{i,j} * parking_{i,j}}$

2.5.3 Total Land Space Constraints

For this project, we were given a total 300 acres of land to build the communities. Thus, the total land space we use for construction should not exceeded 300 acres. Since the project required 1,000 sq.ft space for each home to be set aside for roads, greenbelts and small parks, the total land space for this project should be $\boldsymbol{\sum_{i=1}^{15} \sum_{j=1}^{2} x_{i,j} * (l_{i,j} * parking_{i,j} + 1000)}$.

Hence, the total land space constraint is

$$\sum_{i=1}^{15}\sum_{j=1}^{2} \boldsymbol{x_{i,j}} * \left(\boldsymbol{l_{i,j}} * \boldsymbol{parking_{i,j}} + \mathbf{1000}\right) \leq \mathbf{300} * \mathbf{43560}$$

2.5.4 Variety Constraints

There is a limitation to the number of floor plans with a different number of bedrooms. For example, the total number of homes having 4 bedrooms should hold 40% to 25% of the total number of homes. There is one plan offering 1 bedroom, five plans offering 4 bedrooms, six plans offering 3 bedrooms, and three plans offering 2 bedrooms.

This requirement can be formulated as:

1. For five-bedroom homes:

$$\sum_{j=1}^{2} x_{1,j} \leq 0.15 * \sum_{i=1}^{15} \sum_{j=1}^{2} x_{i,j}$$
$$\sum_{j=1}^{2} x_{1,j} \geq 0.05 * \sum_{i=1}^{15} \sum_{j=1}^{2} x_{i,j}$$

2. For four-bedroom homes:

$$\sum_{j=1}^{2} x_{2,j} + x_{3,j} + x_{5,j} + x_{6,j} + x_{9,j} \leq 0.4 * \sum_{i=1}^{15} \sum_{j=1}^{2} x_{i,j}$$
$$\sum_{j=1}^{2} x_{2,j} + x_{3,j} + x_{5,j} + x_{6,j} + x_{9,j} \geq 0.25 * \sum_{i=1}^{15} \sum_{j=1}^{2} x_{i,j}$$

3. For three-bedroom homes:

$$\sum_{j=1}^{2} x_{4,j} + x_{7,j} + x_{8,j} + x_{10,j} + x_{11,j} + x_{13,j} \leq 0.4 * \sum_{i=1}^{15} \sum_{j=1}^{2} x_{i,j}$$
$$\sum_{j=1}^{2} x_{4,j} + x_{7,j} + x_{8,j} + x_{10,j} + x_{11,j} + x_{13,j} \geq 0.25 * \sum_{i=1}^{15} \sum_{j=1}^{2} x_{i,j}$$

4. For two-bedroom homes:

$$\sum_{j=1}^{2} x_{12,j} + x_{14,j} + x_{15,j} \leq 0.25 * \sum_{i=1}^{15} \sum_{j=1}^{2} x_{i,j}$$
$$\sum_{j=1}^{2} x_{12,j} + x_{14,j} + x_{15,j} \geq 0.15 * \sum_{i=1}^{15} \sum_{j=1}^{2} x_{i,j}$$

In addition, none of the four products is to make up more than 35% or less than 15% of the units built in the development. The formulation comes as：

1. Grand Estates:

$$\sum_{i=1}^{4} \sum_{j=1}^{2} x_{i,j} \leq 0.35 * \sum_{i=1}^{15} \sum_{j=1}^{2} x_{i,j}$$
$$\sum_{i=1}^{4} \sum_{j=1}^{2} x_{i,j} \geq 0.15 * \sum_{i=1}^{15} \sum_{j=1}^{2} x_{i,j}$$

2. Glen Wood Collection:
$\sum_{i=5}^{8}\sum_{j=1}^{2} x_{i,j} \leq 0.35 \sum_{i=1}^{15}\sum_{j=1}^{2} x_{i,j}$
$\sum_{i=5}^{8}\sum_{j=1}^{2} x_{i,j} \geq 0.15 \sum_{i=1}^{15}\sum_{j=1}^{2} x_{i,j}$

3. Lakeview Patio Homes:
$\sum_{i=9}^{12}\sum_{j=1}^{2} x_{i,j} \leq 0.35 * \sum_{i=1}^{15}\sum_{j=1}^{2} x_{i,j}$
$\sum_{i=9}^{12}\sum_{j=1}^{2} x_{i,j} \geq 0.15 * \sum_{i=1}^{15}\sum_{j=1}^{2} x_{i,j}$

4. Country Condominiums:
$\sum_{i=13}^{15}\sum_{j=1}^{2} x_{i,j} \leq 0.35 * \sum_{i=1}^{15}\sum_{j=1}^{2} x_{i,j}$
$\sum_{i=13}^{15}\sum_{j=1}^{2} x_{i,j} \geq 0.15 * \sum_{i=1}^{15}\sum_{j=1}^{2} x_{i,j}$

We use $product_i$ to represent the four product series with i ranges from 1 to 4. Unit number of each product series should equal to the sum of unit number of each floor plan in that series. That is:

$product_1 = \sum_{i=1}^{4}\sum_{j=1}^{2} x_{i,j}$

$product_2 = \sum_{i=5}^{8}\sum_{j=1}^{2} x_{i,j}$
$product_3 = \sum_{i=9}^{12}\sum_{j=1}^{2} x_{i,j}$
$product_4 = \sum_{i=13}^{15}\sum_{j=1}^{2} x_{i,j}$

Furthermore, within each product series, each plan must occupy between 20% and 35% of the total units of that products. For *Grand Estate* series:

1. The Trump:
$\sum_{j=1}^{2} x_{1,j} \leq 0.35 * product_1$
$\sum_{j=1}^{2} x_{1,j} \geq 0.20 * product_1$

2. The Vanderbilt:
$\sum_{j=1}^{2} x_{2,j} \leq 0.35 * product_1$
$\sum_{j=1}^{2} x_{2,j} \geq 0.20 * product_1$

3. The Hughes:
$\sum_{j=1}^{2} x_{3,j} \leq 0.35 * product_1$
$\sum_{j=1}^{2} x_{3,j} \geq 0.20 * product_1$

4. The Jackson:
$\sum_{j=1}^{2} x_{4,j} \leq 0.35 * product_1$
$\sum_{j=1}^{2} x_{4,j} \geq 0.20 * product_1$

Similarly, the formulation for the remaining three series (*Glen Wood Collection*, *Lakeview Patio Homes*, and *Country Condominiums*) comes as:

Glen Wood Collection:
For i in range 5 to 8: $\sum_{j=1}^{2} x_{i,j} \leq 0.35 * product_2$
For i in range 5 to 8: $\sum_{j=1}^{2} x_{i,j} \geq 0.20 * product_2$

Lakeview Patio Homes:
For i in range 9 to 12: $\sum_{j=1}^{2} x_{i,j} \leq 0.35 * product_3$
For i in range 9 to 12: $\sum_{j=1}^{2} x_{i,j} \geq 0.20 * product_3$

Country Condominiums:
For i in range 13 to 15: $\sum_{j=1}^{2} x_{i,j} \leq 0.35 * product_4$
For i in range 13 to 15: $\sum_{j=1}^{2} x_{i,j} \geq 0.20 * product_4$

For appearance's sake, there should be no more than 70% of the single-family homes (except for *Country Condominiums* series) may be two-story homes. Thus, the total units for the first three products should make up no more than 70% of the total units of two-story homes.

$$0.7 * \sum_{i=1}^{3} product_i \geq \sum_{j=1}^{2} x_{1,j} + x_{2,j} + x_{5,j} + x_{6,j} + x_{7,j} + x_{9,j} + x_{10,j}$$

2.5.5 Affordable Housing Constraints

Affordable Housing is defined as a house with price $200,000 or below. The federal government requires at least 15% of the project to be affordable. Thus, we need to filter out the homes priced at or under $200,000 and the total units of these homes should occupy at least 15% of the total units of the homes in this project. Three models price at or under $200,000, which are Golden Pier, Weeping Willow, and Picket Fence. This requirement is formulated as:

$$\sum_{j=1}^{2} x_{12,j} + x_{14,j} + x_{15,j} \geq 0.15 * \sum_{i=1}^{15} \sum_{j=1}^{2} x_{i,j}$$

or similarly,

$$\sum_{j=1}^{2} x_{12,j} + x_{14,j} + x_{15,j} \geq 0.15 * \sum_{i=1}^{4} product_i$$

3. Optimal Construction Plan

3.1 Optimal Construction Plan

We solved the original problem using AMPL. The code was attached in the Appendix. The optimal units are rounded down to the nearest integer for each floor plan if the optimal solution is not an integer. The optimal construction plan is shown in Table 9. According to the objective function, $\sum_{i=i}^{15} \sum_{j=1}^{2} x_{i,j} * per_{i,j} * p_{i,j}$ (where $x_{i,j}$ are the optimal units shown in Table 9, $per_{i,j}$ and $p_{i,j}$ are the net profit percentage and selling price parameters as mentioned in the parameter section), the maximized net profit is $121,904,360.

Floor Plan	Notation of Number of Units	Optimal Value (units)
The Trump Exclusive	$x_{1,1}$	26
The Trump Standard	$x_{1,2}$	66
The Vanderbilt Exclusive	$x_{2,1}$	8
The Vanderbilt Standard	$x_{2,2}$	58
The Hughes Exclusive	$x_{3,1}$	8
The Hughes Standard	$x_{3,2}$	44
The Jackson Exclusive	$x_{4,1}$	8
The Jackson Standard	$x_{4,2}$	44
Grand Cypress Premium	$x_{5,1}$	0
Grand Cypress Standard	$x_{5,2}$	216
Lazy Oak Standard	$x_{6,2}$	154
Wind Row Standard	$x_{7,2}$	123
Orangewood Standard	$x_{8,2}$	123

Bayview Premium	$x_{9,1}$	0
Bayview Standard	$x_{9,2}$	102
Storeline Standard	$x_{10,2}$	67
Docks Edge	$x_{11,2}$	64
Golden Pier	$x_{12,2}$	58
Country Stream	$x_{13,2}$	205
Weeping Willow	$x_{14,2}$	205
Picket Fence	$x_{15,2}$	176

Table 9

3.2 Thoughts

Unit numbers of *The Vanderbilt Exclusive*, *The Hughes Exclusive*, and *The Jackson Exclusive* just meet the requirement on the number of houses built on the lake. *Grand Cypress Premium* and *Bayview Premium* are not included in the optimal construction plan. This observation may coincide with performance of the net profit percentage of each product series: *Glen Wood* and *Lakeview* have relatively lower net profit percentages; *Country Condominiums* has the highest net profit percentage and hence a relatively higher number of units to be built. Even though they price much higher than other series, the net profit generated by *Grand Estates* and *Glen Wood* is not as high as the resources consumed in construction. Therefore, the optimal construction plan may include more floor plans that have a higher net profit percentage and do not occupy a lot of construction resources.

	Constraint	Shadow Price	Slack	Upper Bound	Lower Bound
1	amount_c11	0	18	26	-1e+20
2	amount_c12	-1320	0	26	0
3	amount_c13	-3300	0	26	0
4	amount_c14	-7260	0	26	0
5	amount_c2	0	54.0283	1e+20	-54.0283
6	amount_c3	0	25.7278	1e+20	-25.7278
7	lot_size_c	2.62626	0	2539980	696960
8	parking_c	0	94151.7	1e+20	559248
9	total_land_space_c	9.17083	0	15268100	3668410
10	five_bedroom_max_c	0	172.009	1e+20	-172.009
11	five_bedroom_min_c	0	4.41048	4.41048	-1e+20
12	four_bedroom_max_c	0	113.202	1e+20	-113.202
13	four_bedroom_min_c	0	151.426	151.426	-1e+20
14	three_bedroom_max_c	0	67.6273	1e+20	-67.6273
15	three_bedroom_min_c	0	197.001	197.001	-1e+20
16	two_bedroom_max_c	33424.2	2.84217e-14	13.3098	-26.155
17	two_bedroom_min_c	0	176.419	176.419	-1e+20
18	products_definition1	61932.5	-4.26326e-14	5.17142	-7.18539
19	products_definition2	-14825.8	-2.84217e-14	5.13641	-7.25409
20	products_definition3	-2259.96	-7.10543e-15	5.13601	-20.3939
21	products_definition4	19806.5	0	16.5805	-26.8937
22	product1_max_c	0	352.838	1e+20	-352.838
23	product1_min_c	-59347.7	3.55271e-14	19.8532	-12.8021
24	product2_max_c	16454.8	1.98952e-13	20.3857	-24.0158
25	product2_min_c	0	352.838	352.838	-1e+20
26	product3_max_c	0	323.435	1e+20	-323.435
27	product3_min_c	0	29.4032	29.4032	-1e+20
28	product4_max_c	0	29.4032	1e+20	-29.4032
29	product4_min_c	0	323.435	323.435	-1e+20
30	variety_c1	2565.83	0	13.2287	-4.41077
31	variety_c2	0	39.6943	39.6943	-1e+20
32	variety_c3	0	26.4629	1e+20	-26.4629
33	variety_c4	0	13.2314	13.2314	-1e+20
34	variety_c5	0	39.6943	1e+20	-39.6943
35	variety_c6	-600	0	5.88063	-8.82095
36	variety_c7	0	39.6943	1e+20	-39.6943
37	variety_c8	-11965.8	0	5.88116	-8.81976
38	variety_c9	3586.69	0	30.8446	-61.8618
39	variety_c10	0	92.62	92.62	-1e+20
40	variety_c11	0	61.7467	1e+20	-61.7467
41	variety_c12	0	30.8733	30.8733	-1e+20
42	variety_c13	0	92.62	1e+20	-92.62
43	variety_c14	-8965.83	0	30.8879	-61.6884
44	variety_c15	0	92.62	1e+20	-92.62
45	variety_c16	-10165.8	0	5.88116	-8.81976
46	variety_c17	4165.83	-1.42109e-14	8.81976	-35.3029
47	variety_c18	0	44.1048	44.1048	-1e+20
48	variety_c19	0	35.2838	1e+20	-35.2838
49	variety_c20	0	8.82095	8.82095	-1e+20
50	variety_c21	0	38.2241	1e+20	-38.2241
51	variety_c22	0	5.88063	5.88063	-1e+20
52	variety_c23	0	44.1048	1e+20	-44.1048
53	variety_c24	-39590	0	4.8015	-13.3071
54	variety_c25	51590	0	13.3071	-26.1654
55	variety_c26	0	88.2095	88.2095	-1e+20
56	variety_c27	5000	0	58.8063	-29.4032
57	variety_c28	0	88.2095	88.2095	-1e+20
58	variety_c29	0	29.4032	1e+20	-29.4032
59	variety_c30	0	58.8063	58.8063	-1e+20
60	appearence_c	-6000	2.84217e-14	8.82095	-5.88063
61	affordable_c	0	176.419	176.419	-1e+20

Table 10

4. Sensitivity Analysis

4.1 Sensitivity Analysis on Constraints

The result of sensitivity analysis of constraints is presented in Table 10. The table gives us information about how the change in right-hand side value of the constrains affect objective function values and optimal solutions. The first column *Constraint* names each constraint in the linear program and 61 rows in Table correspond to 61 constraints. *Upper Bound* and *Lower Bound* give a range within which the right-hand value can vary to keep the shadow price of the constraint constant. If the right-hand value changes outside this range, the linear program needs to be reformulated and new optimal solutions would be produced. *Slack* describes the closeness or distance between the optimal solution and constraints. Positive slack means that optimal construction plan has met and exceeded the construction constraints and vice versa for negative slack. When slack equals to zero, the constraint is binding, and the optimal construction plan just meet the production constraint.

The second column *Shadow Price* shows the amount of increase or decrease in the current optimal objective function value (the total net profit) for each unit increase in the right-hand side value. Shadow price in Table 10 can be divided into 3 types: zero, positive, and negative. Shadow price equals to 0 indicating that alternations in right-hand side value of the constraint does not affect the current objective function value (the total net profit). For example, the shadow price of constraint $amount\_c_{11}$ ($x_{1,1} \geq 8$) is 0; this constraint requires that the floor plan *Trump* must have at least eight units on the lake. The optimal solution suggests that 26 units of *Trump Exclusive* should be built, and this number exceeds the production requirement by 18 units. That is increasing the required number of *Trump Exclusive* would not contribute extra profit (which means that the objective function value would not change) as long as the alternation of the required number happens within its upper and lower bounds, and hence the shadow price is 0.

On the other hand, negative and positive shadow price indicates the amount of decrease and increase in the optimal objective function value for each unit increase in the right-hand side value respectively. The shadow price of constraint $amount\_c_{12}$ ($x_{2,1} \geq 8,$ which requires that the floor plan *Vanderbilt* must have at least eight units on the lake) is -1320, which means that the total net profit will decrease by \$1320 if we require one more unit of *Vanderbilt* built on the lake. On the contrary, the total net profit would increase if we required less unit of *Vanderbilt Exclusive*.

Apparently, constructing *Vanderbilt Exclusive* cannot contribute a lot in maximizing net profit. If there is no minimum requirement of units of *Vanderbilt Exclusive*, the optimal production solution for this floor plan may be less than 8 units. The optimal solution suggests that 8 units of *Vanderbilt Exclusive* should be built, and it produces a binding constraint.

The shadow price of constraint *total_land_space_c* ($\sum_{i=1}^{15}\sum_{j=1}^{2} x_{i,j} * \left(l_{i,j} * parking_{i,j} + 1000\right) \leq 300 * 43560,$ which requires that the total land space should not be exceeded 300 acres) is 9.17083, which means that the total net profit will decrease by $9.17083 if we have one more square feet of land can be used for construction plan.

4.2 Sensitivity Analysis on Coefficients

Table 11 shows the lower bound and the upper bound between which the profit (selling price * net profit percentage) of each floor plan could vary while keeping the optimal floor plan schedule unchanged. Thus, this Table 11 can be considered as a pricing guide. Ideally, selling prices or the net profit percentages could be increased to their upper bounds for a greater total profit without changing the construction plan. In other words, we can adjust selling prices and net profit percentage within the range provided by the lower bounds and upper bounds in Table 11 to increase total profit without changing construction plans, as long as such price changes are still acceptable to consumers and all the houses built can be sold.

For example, for *The Trump Exclusive* model, the upper bound and lower bound of the selling profit for this floor plan is infinity and $209,880. It means that if the company increases either selling price or profit percentage, the profit for *The Trump Exclusive* homes will increase and so does the total profit while the number of unit to be built remains at 26. Theoretically, we can keep increasing the price of *The Trump Exclusive* since the upper bound of its profit parameter (selling price * net profit percentage) is infinity. However, it is not always guaranteed that all homes built would be sold out at higher prices. It's worth mentioning that if market sentiment is high and demand for luxury homes increases for certain factors, it may be feasible to increase the selling price to make a higher profit.

Also, the largest net profit can be calculated by the following formula if selling prices or net profit percentages are increased to their upper bounds.

$$maximal\ profit =$$

$\sum_{all\ floor\ plan} upper\ bound\ of\ each\ floor\ plan\ \times$
$current\ optimal\ units\ of\ each\ floor\ plan$

The increased profit can also be calculated by:

$increased\ profit =$
$current\ optimal\ units\ of\ the\ floor\ plan\ \times\ (new\ selling\ price\ or\ percentage\ value$
$-\ current\ selling\ price\ or\ percentage\ value)$

Floor Plan	Notation	Optimal Units	Current $p_{i,j}$ * $per_{i,j}$	Upper bound	Lower bound
The Trump Exclusive	$x_{1,1}$	26	211200	infinity	209880
The Trump Standard	$x_{1,2}$	66	154000	155320	151431
The Vanderbilt Exclusive	$x_{2,1}$	8	205480	206800	-infinity
The Vanderbilt Standard	$x_{2,2}$	58	149600	151952	149000
The Hughes Exclusive	$x_{3,1}$	8	196900	200200	-infinity
The Hughes Standard	$x_{3,2}$	44	143000	143600	139700
The Jackson Exclusive	$x_{4,1}$	8	179740	187000	-infinity
The Jackson Standard	$x_{4,2}$	44	129800	132740	122540
Grand Cypress Premium	$x_{5,1}$	0	82800	130075	-infinity
Grand Cypress Standard	$x_{5,2}$	216	75600	76320.1	71989.8
Lazy Oak Standard	$x_{6,2}$	154	68400	69408.2	59412.9
Wind Row Standard	$x_{7,2}$	123	57600	58860.2	41070.6
Orangewood Standard	$x_{8,2}$	123	50400	51660.2	41394.6

Bayview Premium	$x_{9,1}$	0	66000	86632.1	-infinity
Bayview Standard	$x_{9,2}$	102	60000	61512.2	55827.6
Storeline Standard	$x_{10,2}$	67	54000	54600	48000
Docks Edge Standard	$x_{11,2}$	64	48000	50405.8	47400
Golden Pier Standard	$x_{12,2}$	58	40000	67896.3	39947.1
Country Stream Standard	$x_{13,2}$	205	55000	55756.1	27355.4
Weeping Willow Standard	$x_{14,2}$	205	40000	40756.1	35000
Picket Fence Standard	$x_{15,2}$	176	35000	35882.1	16961.9

Table 11

4.3 Sensitivity Analysis: New Assumptions

In the original problem, the lot size of the *Grand Estates Premium* is half-acre; the lot size of the *Grand Cypress Exclusive* is ¼ acres; the lot size of the *Bayview Premium* is ⅙ acre. As we discussed in Section 3, Optimal Construction Plan, profit rate of *Grand Estates* and *Glen Wood* may be not high enough to justify the amount of resources consumed in construction. Therefore, the optimal construction plan includes more floor plans that have a higher net profit percentage and do not occupy a lot of construction resources.

How will the total profit change if we limit the amount of construction resources or lost sizes of *Grand Estates Premium, Grand Cypress Premium*, and *Bayview Premium*? Now, we assume that the lot size of *Grand Estates Premium* is ¼ acres; the lot size of the *Grand Cypress Exclusive* is ⅕ acres; the lot size of the *Bayview Premium* is ⅕ acres.

Table 12 gives a new set of optimal construction plan and new lower and upper bound of each profit parameter under the new assumptions.

Floor Plan	Notation	Optimal Units	Current $p_{i,j}$ * $per_{i,j}$	Upper bound	Lower bound
The Trump Exclusive	$x_{1,1}$	26	-1.18234e-11	209880	1e+20
The Trump Standard	$x_{1,2}$	75.4779	-4.09273e-11	151606	155320
The Vanderbilt Exclusive	$x_{2,1}$	8	1.45519e-11	-1e+20	206800
The Vanderbilt Standard	$x_{2,2}$	64.4842	-4.36557e-11	149000	151988
The Hughes Exclusive	$x_{3,1}$	8	1.45519e-11	-1e+20	200200
The Hughes Standard	$x_{3,2}$	49.9874	-4.36557e-11	139700	143600
The Jackson Exclusive	$x_{4,1}$	8	1.81899e-12	-1e+20	187000
The Jackson Standard	$x_{4,2}$	49.9874	1.81899e-12	122540	141580
Grand Cypress Premium	$x_{5,1}$	0	-77353.1	-1e+20	160153
Grand Cypress Standard	$x_{5,2}$	236.782	-7.27596e-12	56659.2	154550
Lazy Oak Standard	$x_{6,2}$	169.13	-3.63798e-12	59586.8	88063.8
Wind Row Standard	$x_{7,2}$	135.304	3.63798e-12	36208.4	66372.3
Orangewood Standard	$x_{8,2}$	135.304	1.81899e-12	36494	60369.8
Bayview Premium	$x_{9,1}$	0	-31932.8	-1e+20	97932.8
Bayview Standard	$x_{9,2}$	112.753	-3.00133e-11	56002.7	85620.8
Storeline Standard	$x_{10,2}$	74.095	-1.81899e-12	48000	54600
Docks Edge Standard	$x_{11,2}$	70.8735	-2.72848e-12	47400	54000
Golden Pier Standard	$x_{12,2}$	64.4304	0	34927.8	57570.2
Country Stream Standard	$x_{13,2}$	225.507	0	34871.9	137898
Weeping Willow Standard	$x_{14,2}$	225.507	-5.45697e-12	35000	122898
Picket Fence Standard	$x_{15,2}$	193.291	1.81899e-12	18631	40000

Table 12

The new total profit is \$134,071,000, which increased by \$134,071,000 - \$121,904,360 = \$12,166,640. When the lot size of the premium lots decreases and keep selling prices constant, the total profit increases. It means that the yields of the floor plan become higher relative to the cost of construction resources. Hence, the company can consider finding a suitable tradeoff between the construction cost, such as lot sizes, and the selling price of floor plans to ensure the profit under different market conditions.

5. Modified Problem

5.1 Introduction

Based on the original problem presented in the previous four sections, four modifications on construction assumptions are proposed when two months after providing the original construction report. This modified problem is a mixed integer linear program because some decision variables should be defined as integers. In the following sections, we demonstrate the processing of solving the modified problem in four parts: Assumption, Decision Variables, Parameters, and Modification.

5.2 Formulation

5.2.1 Assumption

For this project, we assume that:

1. The additional profit by building a 10-acre sports/recreational complex on the property is to the net profit determined in the original problem, which would not affect the selling price. So, whether to build sports/recreational complex will not affect the taxes payment.
2. All numbers and equations given are correct. This included the sizes, prices, percentages, and calculation formula on lot sizes, ground area, and yard size, etc.
3. In order to get an accurate profit estimation, we assume that the given selling prices are the actual price for sell and all homes can be sold.
4. Non-integer results of the number of units of each floor plan are allowed, and they will be rounded to the nearest integer to deliver a feasible optimal decision.
5. All area/size are measured in square feet(s). We converted all acres to square feet to have consistent unit. The conversion relationship is 1 acre = 43560 sq.ft.

5.2.2 Decision Variables

In addition to the decision variables in the original problem, several new decision variables are added in this modified problem. The notation of each type of decision variable is concluded in Table 13.

Variable Notation	Definition
X [i,j]	The number of units of homes in each plan i and type j, for i = 1,..,15, j = 1, 2. j = 1 represents “premium” homes, j = 2 represents standard homes.
Products [i]	The total unit of homes in each series i, for i = 1, 2, 3, 4.
TaxC	The Luxury Tax on total units sold of the Country Condominiums.
Net_Profit	The total net profit the corporation gains from selling houses except for taxes and sports complex costs.
Z[i]	Binary variables. If Z[i] = 1, the total units sold of Country Condominiums is larger than products[i], otherwise 0, for i = 1, 2, 3.
Rank	Integer. The rank of total units sold of country condominiums in four products If Rank = 1, the total units sold of country condominiums is the highest. If Rank = 2, the total units sold of country condominiums is the second highest among all four series.
position[i]	Binary variables. If Z[i] = 1, the total units sold of Country Condominiums is larger than products[i], otherwise 0, for i = 1, 2, 3.
sports	Binary variables. If sports = 1, the corporation builds the sports/recreational complex, otherwise 0.
W[i]	Binary variables. If W[i] = 1, the number of units of the plan i built on the lake is greater than 8 units, otherwise 0, for i = 1, 2, 3, 4.

Table 13

5.2.3 Parameters

Constant parameters in the original problem are sales price, net profit percentage, lot size, and parking. In the Modified problem, we add big M and additional profit to fit the mixed integer linear programming model. The notation of each type of parameters is concluded in Table 14.

Parameter Notation	definition
P [i,j]	Sales price
Per [i,j]	Net profit percentage
Lot_size [i,j]	Lot size
Parking [i,j]	Parking
M	Big M = 999999999999, constant
Addition_profit [i,j]	Additional profit percentage by building a sports complex.

Table 14

5.2.4 Modification

We treat the modified problem as a situation two months after providing the report of the original problem. We were informed four modifications described in the Modified Problem Statement.

5.2.4.1 Modification 1

In the Variety section, ignore the table of the maximum and minimum percentages. Keep all the other requirements in this section. Regarding the first modification, we remove the constraints of maximum and minimum percentages limit. Thus, requirement on unit number of different floor plans with different number of bedrooms is removed. It is free to build different types of homes no matter how many bedrooms they have. The company have more freedom to build more profitable home plans instead considering the limitations on bedroom numbers that the floor plan has.

5.2.4.2 Modification 2

Ignore the requirement in the Affordable Housing section. Instead, the county government imposes a "Luxury Tax" on each unit sold of 8% of the selling price per unit. However, based on the number of *Country Condominiums* sold, the company have to pay the following Luxury Tax on each unit sold of the *Country Condominiums*: Let *i* be the position of the number of *Country Condominiums* units sold compared to the other three types. Then, the Luxury Tax paid on each unit of the *Country Condominiums* type is 2*i*%.

Thus, we remove the constraints in the Affordable Housing section, and add the "Luxury Tax" on the net profit. The taxes for *Grand Estates*, *Glen Wood Collection*, *Lakeview Patio Homes* equals to 8% * selling price * units. For the Luxury Tax paid on each unit of the *Country Condominiums* type is 2*i*%.

We set the binary variable **Z[i]** , subjecting to the number of *Country Condominiums* units sold compared to the other three types. If Z[i] = 1, the total units sold of Country Condominiums is larger than products[i], otherwise 0, for i = 1, 2, 3.

Also, we set another variable **Rank** = 4 - Z[1] - Z[2] - Z[3] to show the rank of *Country Condominiums* in the total four product series. So, we write the following formula:

1. Products[4] - Products[1] $\leq$ M * Z[1]
2. Products[4] - Products[1] $\geq$ M * (Z[1]-1)
3. Products[4] - Products[2] $\leq$ M * Z[2]
4. Products[4] - Products[2] $\geq$ M * (Z[2]-1)
5. Products[4] - Products[3] $\leq$ M * Z[3]
6. Products[4] - Products[3] $\geq$ M * (Z[3]-1)
7. Rank = 4 - Z[1] - Z[2] - Z[3]

Then, we could use *Rank * 2% * Units* to calculate the "Luxury Tax" on *Country Condominiums*. However, both Rank and Units are decision variables and in order to make a linear program we need to use other binary variables **Position[i]** and **Big M** to transform the integer value of the Rank variable to binary variables. If rank of *Country Condominiums* = 1, Position[1] = 1. If rank of *Country Condominiums* = 2, Position[2] = 1, etc. Then, we can set constraints using binary variables and Big M to calculate the "Luxury Tax" on *Country Condominiums*. The formulation comes as:

1. Rank $\leq$ Position[1] + M * (1 - Position[1])
2. Rank $\geq$ Position[1] - M * (1 - Position[1])
3. Rank $\leq$ 2 * Position[2] + M * (1 - Position[2])
4. Rank $\geq$ 2 * Position[2] - M * (1 - Position[2])
5. Rank $\leq$ 3 * Position[3] + M * (1 - Position[3])
6. Rank $\geq$ 3 * Position[3] - M * (1 - Position[3])
7. Rank $\leq$ 4 * Position[4] + M * (1 - Position[4])
8. Rank $\geq$ 4 * Position[4] - M * (1 - Position[4])
9. Position[1] + Position[2] + Position[3] + Position[4] = 1
10. 0.02 * $\sum_{i=13}^{15}\sum_{j=1}^{2} x[i,j]$ * sale_price [i,j] - TaxC $\leq$ M * (1- position[1])
11. 0.04 * $\sum_{i=13}^{15}\sum_{j=1}^{2} x[i,j]$ * sale_price [i,j] - TaxC $\leq$ M * (1- position[2])
12. 0.06 * $\sum_{i=13}^{15}\sum_{j=1}^{2} x[i,j]$ * sale_price [i,j] - TaxC $\leq$ M * (1- position[3])
13. 0.08 * $\sum_{i=13}^{15}\sum_{j=1}^{2} x[i,j]$ * sale_price [i,j] - TaxC $\leq$ M * (1- position[4])

5.2.4.3 Modification 3

Modify the requirement in the Lot Sizing section that "Each of the *Grand Estate* series plan must have at least eight units on the lake" to "At least three of the *Grand Estate* series plans must have at least eight units on the lake." In the original problem we limited the number of four exclusive plans in the *Grand Estate* series to be at least eight units. So, in the original problem, we simply set X[i,1]$\geq$ 8, for i = 1, 2, 3, 4. However in the modified problem, we just need at least three exclusive plans in the *Grand Estate*. So, we introduce a binary variable W[i], for i = 1, 2, 3, 4. Table 15 shows the definition of W[i].

W[1]	=1 if the Trump Exclusive is sold at least 8 units.
	=0 otherwise
W[2]	=1 if the Vanderbilt Exclusive is sold at least 8 units.
	=0 otherwise
W[3]	=1 if the Hughes Exclusive is sold at least 8 units.
	=0 otherwise
W[4]	=1 if the Jackson Exclusive is sold at least 8 units.
	=0 otherwise

Table 15

Then, we can formulate the constraints as :

1. $X[i,1] \geq 8 * W[i]$ for $i = 1, 2, 3, 4$
2. $W[1] + W[1] + W[1] + W[1] \geq 3$

5.2.4.4 Modification 4

LSDC can build a 10-acre sports/recreational complex on the property, this would reduce the available land by 10 acres and cost $8 million. However, LSDC can then raise the cost of the homes as follows:

Grand Estates	5%
Glen Wood Collection	3%
Lakeview Patio Homes	2%
Country Condominiums	3%

First, we set parameters for the percentage increase in profit of each floor plan, which summarized in Table 16:

Floor Plan	Raise Profit Percentage Notation	Percentage
Grand Estates		
Trump Exclusive	Addition_profit [1,1]	5%
Trump Standard	Addition_profit [1,2]	5%
Vanderbilt Exclusive	Addition_profit [2,1]	5%
Vanderbilt Standard	Addition_profit [2,2]	5%
Hughes Exclusive	Addition_profit [3,1]	5%
Hughes Standard	Addition_profit [3,2]	5%
Jackson Exclusive	Addition_profit [4,1]	5%
Jackson Standard	Addition_profit [4,2]	5%
Glen Wood Collection		
Grand Cypress Premium	Addition_profit [5,1]	3%
Grand Cypress Standard	Addition_profit [5,2]	3%
N/A	Addition_profit [6,1]	3%
Lazy Oak Standard	Addition_profit [6,2]	3%
N/A	Addition_profit [7,1]	3%
Wind Row Standard	Addition_profit [7,2]	3%
N/A	Addition_profit [8,1]	3%
Orangewood Standard	Addition_profit [8,2]	
Lakeview Patio Homes		
Bayview Premium	Addition_profit [9,1]	2%
Bayview Standard	Addition_profit [9,2]	2%
N/A	Addition_profit [10,1]	2%
Storeline Standard	Addition_profit [10,2]	2%
N/A	Addition_profit [11,1]	2%
Docks Edge Standard	Addition_profit [11,2]	2%

N/A	Addition_profit [12,1]	2%
Golden Pier Standard	Addition_profit [12,2]	2%
Country Condominiums		
N/A	Addition_profit [13,1]	3%
Country Stream Standard	Addition_profit [13,2]	3%
N/A	Addition_profit [14,1]	3%
Weeping Willow Standard	Addition_profit [14,2]	3%
N/A	Addition_profit [15,1]	3%
Picket Fence Standard	Addition_profit [15,2]	3%

Table 16

In addition, we need to set a decision variable **'sports'** as a binary variable. If sports = 1, we choose to build a 10-acre sports/recreational complex on the property. If sports = 0, we will not build the sports complex. So, there are two situations.

1. If LSDC chooses not to build the complex:
   Net_Profit = The original net profit = $\sum_{i=1}^{15}\sum_{j=1}^{2} x[i,j]$ * sale price [i,j] * percentage [i,j]
2. If LSDC chooses to build the complex:
   Net_Profit = The new net profit = $\sum_{i=1}^{15}\sum_{j=1}^{2} x[i,j]$* sale price [i,j] * percentage [i,j] * (1 + Addition_profit [i,j])

Parameter **Big M** is introduced to set the constraints on Net_Profit. We set parameter M = 999999999999. 10 acres of land is occupied per recreational complex. There are three constraints under this modification:

1. Net_Profit - $\sum_{i=1}^{15}\sum_{j=1}^{2} x[i,j]$ * p [i,j] * per [i,j] $\leq$ M * sports
2. Net_Profit - $\sum_{i=1}^{15}\sum_{j=1}^{2} x[i,j]$ * p [i,j] * per [i,j] * (1 + Addition_profit [i,j]) $\leq$ M * (1- sports)
3. $\sum_{i=1}^{15}\sum_{j=1}^{2} x[i,j]$* (lot_size[i, j] + parking[i, j] + 1000) $\leq$ 300 * 43560 - 10 * 43560 * sports

5.2.5 Objective Function

Maximize Profit: Net_Profit - 0.08 * $\sum_{i=1}^{15}\sum_{j=1}^{2}(x[i,j] * p[i,j])$ - TaxC - 8,000,000 * sports

In the modified problem, the company should pay Luxury taxes. If the company wants to build a sports complex, it will cost \$ 8,000,000. So we minus the Luxury taxes and sports complex cost from the net profit.

6. Solution of Modified Problem

We solved the modified problem using AMPL. The code is attached in the Appendix. The optimal number of units is rounded down to the nearest integer for each floor plan, which is shown in Table 17 and the maximum profit is \$82,278,900. We choose *Trump Exclusive, Vanderbilt Exclusive*, and *Hughes Exclusive* to satisfy the modified requirement that at least three of the *Grand Estate* series plans must have at least eight units on the lake. For the "Luxury Tax", the taxes paid for *Country Condominiums* is \$2,173,930, and the total taxes paid for *Grand Estate*, *Glen Wood Collection*, and *Lakeview Patio Homes* is \$38,539,360. Additionally, we choose not to build the sports or recreational complex.

Floor Plan	Notation x_{i.j}	Optimal Value (units)
The Trump Exclusive	x_{1,1}	34
The Trump Standard	x_{1,2}	59
The Vanderbilt Exclusive	x_{2,1}	8
The Vanderbilt Standard	x_{2,2}	58
The Hughes Exclusive	x_{3,1}	8
The Hughes Standard	x_{3,2}	45
The Jackson Exclusive	x_{4,1}	0
The Jackson Standard	x_{4,2}	53

Grand Cypress Premium	x_{5,1}	0
Grand Cypress Standard	x_{5,2}	217
Lazy Oak Standard	x_{6,2}	155
Wind Row Standard	x_{7,2}	124
Orangewood Standard	x_{8,2}	124
Bayview Premium	x_{9,1}	0
Bayview Standard	x_{9,2}	93
Storeline Standard	x_{10,2}	57
Docks Edge	x_{11,2}	62
Golden Pier	x_{12,2}	53
Country Stream	x_{13,2}	217
Weeping Willow	x_{14,2}	217
Picket Fence	x_{15,2}	186

Table 17

7. Discussion

After removing the requirement on maximum and minimum percentages in variety constraints of the original problem, LSDC has more choices in the unit number of each floor plan to be built and reaches higher profits. By comparing the luxury tax rate of *Country Condominiums* with the fixed luxury tax rate of the other three home series, *Country Condominiums* is more economical because the tax paid reduces when the number of units sold increases. Hence, *Country Condominiums* becomes the most profitable product series under the tax rule, and LSDC may build more *Country Condominiums* units to make more profit due to the saving on luxury tax.

Also, after removing the "at least eight units on the lake" rule, LSDC can build more profitable floor plans and reduced less-nonprofitable floor plans to increase the total net profit. For example, the optimal unit number of The Jackson Exclusive is changed from 8 to 0.

Furthermore, LSDC may not choose to build 10- acre sports/recreational complex due to the high cost and reduction in land resources used for house construction brought by the recreational complex. It shows that the extra profit from building the recreational complex cannot cover the cost of building it.

8. Appendix

8.1 Original Problem Model Code

```
reset;
option solver cplex;
option cplex_options 'sensitivity';
option presolve 0;

# initialize row, col index
set row;
set col;

# parameters
param per{row, col}; #percentage of selling price to profit
param p{row, col}; #selling price, calculation detail showing in the report
param lot_size{row, col}; #lot size constant, detail showing in report
param parking{row, col}; #parking constant, detail showing in report

# decision variable
var x{row, col} >= 0;
var products{i in 1..4} >= 0; #the total unit of homes in each series

# objective function
maximize profit: sum{i in row} sum{j in col} per[i, j] * p[i,j] * x[i,j];

# constraint
# construction amount constraint
subject to amount_c11:
    x[1,1] >= 8;
subject to amount_c12:
    x[2,1] >= 8;
    subject to amount_c13:
    x[3,1] >= 8;
    subject to amount_c14:
    x[4,1] >= 8;
subject to amount_c2:
    x[5,1] <= 0.25 * (x[5,1] + x[5,2]);
subject to amount_c3:
    x[9,1] <= 0.25 * (x[9,1] + x[9,2]);

# lot size constraint
subject to lot_size_c:
    (x[1,1] + x[2,1] + x[3,1] + x[4,1]) * 0.5 * 43560 <= 50 * 0.5 * 43560;

# parking constraint
subject to parking_c:
    sum{i in row} sum{j in col} x[i,j] * parking[i,j] <= 15 * 43560;

# total land space constraint
subject to total_land_space_c:
    sum{i in row} sum{j in col} x[i,j] * (lot_size[i, j] + parking[i, j] + 1000) <= 300*43560;

# bedroom variety constraints
subject to five_bedroom_max_c:
```

```
    x[1,1] + x[1,2] <= 0.15 * sum{i in row} sum{j in col} x[i, j];
subject to five_bedroom_min_c:
    x[1,1] + x[1,2] >= 0.05 * sum{i in row} sum{j in col} x[i, j];
subject to four_bedroom_max_c:
    sum{j in col} (x[2,j] + x[3,j] + x[5,j] + x[6,j] + x[9,j]) <= 0.4 * sum{i in row} sum{j in col} x[i, j];
subject to four_bedroom_min_c:
    sum{j in col} (x[2,j] + x[3,j] + x[5,j] + x[6,j] + x[9,j]) >= 0.25 * sum{i in row} sum{j in col} x[i, j];
subject to three_bedroom_max_c:
    sum{j in col} (x[4,j] + x[7,j] + x[8,j] + x[10,j] + x[11,j] + x[13,j]) <= 0.4 * sum{i in row} sum{j in col} x[i, j];
subject to three_bedroom_min_c:
    sum{j in col} (x[4,j] + x[7,j] + x[8,j] + x[10,j] + x[11,j] + x[13,j]) >= 0.25 * sum{i in row} sum{j in col} x[i, j];
subject to two_bedroom_max_c:
    sum{j in col} (x[12,j] + x[14,j] + x[15,j]) <= 0.25 * sum{i in row} sum{j in col} x[i, j];
subject to two_bedroom_min_c:
    sum{j in col} (x[12,j] + x[14,j] + x[15,j]) >= 0.15 * sum{i in row} sum{j in col} x[i, j];

subject to products_definition1:
    products[1] = sum{i in 1..4} sum{j in col} x[i,j];
subject to products_definition2:
    products[2] = sum{i in 5..8} sum{j in col} x[i,j];
subject to products_definition3:
    products[3] = sum{i in 9..12} sum{j in col} x[i,j];
subject to products_definition4:
    products[4] = sum{i in 13..15} sum{j in col} x[i,j];

# product variety constraints
subject to product1_max_c:
    products[1] <= 0.35 * sum{i in row} sum{j in col} x[i, j];
subject to product1_min_c:
    products[1] >= 0.15 * sum{i in row} sum{j in col} x[i, j];
subject to product2_max_c:
    products[2] <= 0.35 * sum{i in row} sum{j in col} x[i, j];
subject to product2_min_c:
    products[2] >= 0.15 * sum{i in row} sum{j in col} x[i, j];
subject to product3_max_c:
    products[3] <= 0.35 * sum{i in row} sum{j in col} x[i, j];
subject to product3_min_c:
    products[3] >= 0.15 * sum{i in row} sum{j in col} x[i, j];
subject to product4_max_c:
    products[4] <= 0.35 * sum{i in row} sum{j in col} x[i, j];
subject to product4_min_c:
    products[4] >= 0.15 * sum{i in row} sum{j in col} x[i, j];

# constraints for each series
# Grand Estate
subject to variety_c1:
    x[1,1] + x[1,2] <= 0.35 * products[1];
subject to variety_c2:
    x[1,1] + x[1,2] >= 0.2 * products[1];
subject to variety_c3:
    x[2,1] + x[2,2] <= 0.35 * products[1];
```

```
subject to variety_c4:
    x[2,1] + x[2,2] >= 0.2 * products[1];
subject to variety_c5:
    x[3,1] + x[3,2] <= 0.35 * products[1];
subject to variety_c6:
    x[3,1] + x[3,2] >= 0.2 * products[1];
subject to variety_c7:
    x[4,1] + x[4,2] <= 0.35 * products[1];
subject to variety_c8:
    x[4,1] + x[4,2] >= 0.2 * products[1];

# Glen Wood Collection
subject to variety_c9:
    x[5,1] + x[5,2] <= 0.35 * products[2];
subject to variety_c10:
    x[5,1] + x[5,2] >= 0.2 * products[2];
subject to variety_c11:
    x[6,1] + x[6,2] <= 0.35 * products[2];
subject to variety_c12:
    x[6,1] + x[6,2] >= 0.2 * products[2];
subject to variety_c13:
    x[7,1] + x[7,2] <= 0.35 * products[2];
subject to variety_c14:
    x[7,1] + x[7,2] >= 0.2 * products[2];
subject to variety_c15:
    x[8,1] + x[8,2] <= 0.35 * products[2];
subject to variety_c16:
    x[8,1] + x[8,2] >= 0.2 * products[2];

# Lakeview Patio Homes
subject to variety_c17:
    x[9,1] + x[9,2] <= 0.35 * products[3];
subject to variety_c18:
    x[9,1] + x[9,2] >= 0.2 * products[3];
subject to variety_c19:
    x[10,1] + x[10,2] <= 0.35 * products[3];
subject to variety_c20:
    x[10,1] + x[10,2] >= 0.2 * products[3];
subject to variety_c21:
    x[11,1] + x[11,2] <= 0.35 * products[3];
subject to variety_c22:
    x[11,1] + x[11,2] >= 0.2 * products[3];
subject to variety_c23:
    x[12,1] + x[12,2] <= 0.35 * products[3];
subject to variety_c24:
    x[12,1] + x[12,2] >= 0.2 * products[3];

# Country Condominiums
subject to variety_c25:
    x[13,1] + x[13,2] <= 0.35 * products[4];
subject to variety_c26:
```

```
    x[13,1] + x[13,2] >= 0.2 * products[4];
subject to variety_c27:
    x[14,1] + x[14,2] <= 0.35 * products[4];
subject to variety_c28:
    x[14,1] + x[14,2] >= 0.2 * products[4];
subject to variety_c29:
    x[15,1] + x[15,2] <= 0.35 * products[4];
subject to variety_c30:
    x[15,1] + x[15,2] >= 0.2 * products[4];

subject to appearence_c:
    0.7 * sum{i in 1..3} products[i] >= sum{j in col} (x[1,j] + x[2,j] + x[5,j] + x[6,j] + x[7,j] + x[9,j] + x[10,j]);

subject to affordable_c:
    sum{j in col} (x[12,j] + x[14,j] + x[15,j]) >= 0.15 * sum{i in row} sum{j in col} x[i,j];
```

8.2 Original Problem Data Code

```
set row := 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15;
set col := 1 2;
param per:
    1       2 :=
1   0.22    0.22
2   0.22    0.22
3   0.22    0.22
4   0.22    0.22
5   0.18    0.18
6   0.18    0.18
7   0.18    0.18
8   0.18    0.18
9   0.2     0.2
10  0.2     0.2
11  0.2     0.2
12  0.2     0.2
13  0.25    0.25
14  0.25    0.25
15  0.25    0.25;
param p:
    1       2 :=
1   960000  700000
2   934000  680000
3   895000  650000
4   817000  590000
5   460000  420000
6   0       380000
7   0       320000
8   0       280000
9   330000  300000
10  0       270000
11  0       240000
12  0       200000
13  0       220000
14  0       160000
15  0       140000;

param lot_size :
    1       2 :=
1   21780   21780
2   21780   21780
3   21780   21780
4   21780   21780
5   10890   4950
6   0       4356
7   0       4356
8   0       4356
9   7260    4356
10  0       4356
11  0       4356
12  0       4356
```

```
13  0       1500
14  0       1500
15  0       1500;
param parking :
    1       2 :=
1   400     400
2   200     200
3   200     200
4   0       0
5   200     200
6   0       400
7   0       200
8   0       200
9   400     400
10  0       200
11  0       200
12  0       0
13  0       600
14  0       400
15  0       400;
```

8.3 Modified Problem Model Code

```
reset;
option solver cplex;
option cplex_options 'sensitivity';
option presolve 0;

# initialize row, col index
set row;
set col;

# parameters
param per{row, col}; #percentage of selling price to profit
param p{row, col}; #selling price, calculation detail showing in the report
param lot_size{row, col}; #lot size constant, detail showing in report
param parking{row, col}; #parking constant, detail showing in report
param addition_profit{row, col}; #addition profit constant, detail showing in report
param M; # the big number

# decision variable
var x{row, col} >= 0; #the number of units of homes in each plan
var products{i in 1..4} >= 0; #the total unit of homes in each series

# addtional decision variable for modified problem
var TaxC >= 0; #the Luxury Tax on each unit sold of the Country Condominiums
var Net_Profit >= 0; #the total net profit the corporation gains
var z {i in 1..3} binary; #whether the number of Country Condominiums is larger than product i, i = 1,2,3
var rank integer, >=0; #the rank of country condominiums in four products
var position {i in 1..4} binary; # whether the rank is equal to the position i
var sports binary; #whether the corporation build the sports/recreational complex
var w {i in 1..4} binary; #whether the number of units of the plan built on the lake is greater than 8 units

# objective function
maximize Profit: Net_Profit - 0.08*(sum{i in 1..12} sum{j in col} x[i,j]*p[i,j]) - TaxC - 8000000*sports;

# Constraints
# Total Net Profit
subject to Total_Net_Profit1:
    Net_Profit - sum{i in row} sum{j in col} per[i, j] * p[i,j] * x[i,j] <= M * sports;
subject to Total_Net_Profit2:
    Net_Profit - sum{i in row} sum{j in col} per[i, j] * p[i,j] * x[i,j] * (1 + addition_profit[i,j]) <= M * (1 - sports);
```

```
# calculation of number of products
subject to products_definition1:
    products[1] = sum{i in 1..4} sum{j in col} x[i,j];
subject to products_definition2:
    products[2] = sum{i in 5..8} sum{j in col} x[i,j];
subject to products_definition3:
    products[3] = sum{i in 9..12} sum{j in col} x[i,j];
subject to products_definition4:
    products[4] = sum{i in 13..15} sum{j in col} x[i,j];

# Tax of Country Condominiums
# use z[i] to define the rank of Country Condominiums in four products
subject to z1_1:
    products[4] - products[1] <= M * z[1];
subject to z1_2:
    products[4] - products[1] >= M * (z[1]-1);

subject to z2_1:
    products[4] - products[2] <= M * z[2];
subject to z2_2:
    products[4] - products[2] >= M * (z[2]-1);

subject to z3_1:
    products[4] - products[3] <= M * z[3];
subject to z3_2:
    products[4] - products[3] >= M * (z[3]-1);

subject to Rank:
    rank = 4-z[1]-z[2]-z[3];

# Use value of rank to define the value of position[i]
subject to position1_1:
    rank <= position[1] + M * (1 - position[1]);
subject to position1_2:
    rank >= position[1] - M * (1 - position[1]);

subject to position2_1:
    rank <= 2*position[2] + M * (1 - position[2]);
subject to position2_2:
    rank >= 2*position[2] - M * (1 - position[2]);

subject to position3_1:
    rank <= 3*position[3] + M * (1 - position[3]);
subject to position3_2:
    rank >= 3*position[3] - M * (1 - position[3]);

subject to position4_1:
    rank <= 4*position[4] + M * (1 - position[4]);
subject to position4_2:
    rank >= 4*position[4] - M * (1 - position[4]);

subject to sum_positions:
    sum{i in 1..4}position[i] = 1;
```

```
# Set the value of Tax for Country Condominiums
subject to TaxC_1:
    0.02*sum{i in 13..15} sum{j in col} p[i,j] * x[i,j] - TaxC <= M * (1-position[1]);
subject to TaxC_2:
    0.04*sum{i in 13..15} sum{j in col} p[i,j] * x[i,j] - TaxC <= M * (1-position[2]);
subject to TaxC_3:
    0.06*sum{i in 13..15} sum{j in col} p[i,j] * x[i,j] - TaxC <= M * (1-position[3]);
subject to TaxC_4:
    0.08*sum{i in 13..15} sum{j in col} p[i,j] * x[i,j] - TaxC <= M * (1-position[4]);

# At least three of the Grand Estate series plan must have at least eight units on the lake
subject to Grand_Estate_series_x11:
    x[1,1] >= 8 * w[1];
subject to Grand_Estate_series_x21:
    x[2,1] >= 8 * w[2];
subject to Grand_Estate_series_x31:
    x[3,1] >= 8 * w[3];
subject to Grand_Estate_series_x41:
    x[4,1] >= 8 * w[4];
subject to At_least_three:
    w[1] + w[2] + w[3] + w[4] >= 3;

# Total land space constraint modified
subject to total_land_space_c:
    sum{i in row} sum{j in col} x[i,j] * (lot_size[i, j] + parking[i, j] + 1000) <= 300 * 43560 - 10 * 43560 * sports;

# permium cypress and Bayview constraint
subject to amount_c2:
    x[5,1] <= 0.25 * (x[5,1] + x[5,2]);
subject to amount_c3:
    x[9,1] <= 0.25 * (x[9,1] + x[9,2]);

# lot size constraint
subject to lot_size_c:
    (x[1,1] + x[2,1] + x[3,1] + x[4,1]) * 0.5 * 43560 <= 50 * 0.5 * 43560;

# parking constraint
subject to parking_c:
    sum{i in row} sum{j in col} x[i,j] * parking[i,j] <= 15 * 43560;
```

```
# product variety constraints
subject to product1_max_c:
    products[1] <= 0.35 * sum{i in row} sum{j in col} x[i, j];
subject to product1_min_c:
    products[1] >= 0.15 * sum{i in row} sum{j in col} x[i, j];
subject to product2_max_c:
    products[2] <= 0.35 * sum{i in row} sum{j in col} x[i, j];
subject to product2_min_c:
    products[2] >= 0.15 * sum{i in row} sum{j in col} x[i, j];
subject to product3_max_c:
    products[3] <= 0.35 * sum{i in row} sum{j in col} x[i, j];
subject to product3_min_c:
    products[3] >= 0.15 * sum{i in row} sum{j in col} x[i, j];
subject to product4_max_c:
    products[4] <= 0.35 * sum{i in row} sum{j in col} x[i, j];
subject to product4_min_c:
    products[4] >= 0.15 * sum{i in row} sum{j in col} x[i, j];

# constraints for each series
########### Grand Estate
subject to variety_c1:
    x[1,1] + x[1,2] <= 0.35 * products[1];
subject to variety_c2:
    x[1,1] + x[1,2] >= 0.2 * products[1];
subject to variety_c3:
    x[2,1] + x[2,2] <= 0.35 * products[1];
subject to variety_c4:
    x[2,1] + x[2,2] >= 0.2 * products[1];
subject to variety_c5:
    x[3,1] + x[3,2] <= 0.35 * products[1];
subject to variety_c6:
    x[3,1] + x[3,2] >= 0.2 * products[1];
subject to variety_c7:
    x[4,1] + x[4,2] <= 0.35 * products[1];
subject to variety_c8:
    x[4,1] + x[4,2] >= 0.2 * products[1];
```

```
########### Glen Wood Collection
subject to variety_c9:
    x[5,1] + x[5,2] <= 0.35 * products[2];
subject to variety_c10:
    x[5,1] + x[5,2] >= 0.2 * products[2];
subject to variety_c11:
    x[6,1] + x[6,2] <= 0.35 * products[2];
subject to variety_c12:
    x[6,1] + x[6,2] >= 0.2 * products[2];
subject to variety_c13:
    x[7,1] + x[7,2] <= 0.35 * products[2];
subject to variety_c14:
    x[7,1] + x[7,2] >= 0.2 * products[2];
subject to variety_c15:
    x[8,1] + x[8,2] <= 0.35 * products[2];
subject to variety_c16:
    x[8,1] + x[8,2] >= 0.2 * products[2];

##########Lakeview Patio Homes
subject to variety_c17:
    x[9,1] + x[9,2] <= 0.35 * products[3];
subject to variety_c18:
    x[9,1] + x[9,2] >= 0.2 * products[3];
subject to variety_c19:
    x[10,1] + x[10,2] <= 0.35 * products[3];
subject to variety_c20:
    x[10,1] + x[10,2] >= 0.2 * products[3];
subject to variety_c21:
    x[11,1] + x[11,2] <= 0.35 * products[3];
subject to variety_c22:
    x[11,1] + x[11,2] >= 0.2 * products[3];
subject to variety_c23:
    x[12,1] + x[12,2] <= 0.35 * products[3];
subject to variety_c24:
    x[12,1] + x[12,2] >= 0.2 * products[3];

########### Country Condominiums
subject to variety_c25:
    x[13,1] + x[13,2] <= 0.35 * products[4];
subject to variety_c26:
    x[13,1] + x[13,2] >= 0.2 * products[4];
subject to variety_c27:
    x[14,1] + x[14,2] <= 0.35 * products[4];
subject to variety_c28:
    x[14,1] + x[14,2] >= 0.2 * products[4];
subject to variety_c29:
    x[15,1] + x[15,2] <= 0.35 * products[4];
subject to variety_c30:
    x[15,1] + x[15,2] >= 0.2 * products[4];
```

```
subject to appearence_c:
    0.7 * sum{i in 1..3} products[i] >= sum{j in col} (x[1,j] + x[2,j]
    + x[5,j] + x[6,j] + x[7,j] + x[9,j] + x[10,j]);

subject to none1:
x[6,1] = 0;
subject to none2:
x[7,1] = 0;
subject to none3:
x[8,1] = 0;
subject to none4:
x[10,1] = 0;
subject to none5:
x[11,1] = 0;
subject to none6:
x[12,1] = 0;
subject to none7:
x[13,1] = 0;
subject to none8:
x[14,1] = 0;
subject to none9:
x[15,1] = 0;
```

8.4 Modified Problem Data Code

```
set row := 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15;
set col := 1 2;

param M := 999999999999;

param addition_profit:
    1       2 :=
1   0.05    0.05
2   0.05    0.05
3   0.05    0.05
4   0.05    0.05
5   0.03    0.03
6   0.03    0.03
7   0.03    0.03
8   0.03    0.03
9   0.02    0.02
10  0.02    0.02
11  0.02    0.02
12  0.02    0.02
13  0.03    0.03
14  0.03    0.03
15  0.03    0.03;
```

```
param per:
    1       2 :=
1   0.22    0.22
2   0.22    0.22
3   0.22    0.22
4   0.22    0.22
5   0.18    0.18
6   0.18    0.18
7   0.18    0.18
8   0.18    0.18
9   0.2     0.2
10  0.2     0.2
11  0.2     0.2
12  0.2     0.2
13  0.25    0.25
14  0.25    0.25
15  0.25    0.25;

param p:
    1       2 :=
1   960000  700000
2   934000  680000
3   895000  650000
4   817000  590000
5   460000  420000
6   0       380000
7   0       320000
8   0       280000
9   330000  300000
10  0       270000
11  0       240000
12  0       200000
13  0       220000
14  0       160000
15  0       140000;

param lot_size :
    1       2 :=
1   21780   21780
2   21780   21780
3   21780   21780
4   21780   21780
5   10890   4950
6   0       4356
7   0       4356
8   0       4356
9   7260    4356
10  0       4356
11  0       4356
12  0       4356
13  0       1500
14  0       1500
15  0       1500;
```

```
param parking :
    1       2 :=
1   400     400
2   200     200
3   200     200
4   0       0
5   200     200
6   0       400
7   0       200
8   0       200
9   400     400
10  0       200
11  0       200
12  0       0
13  0       600
14  0       400
15  0       400;
```