$$A_0 = \{\varnothing\}$$
$$A_{n+1} = \{\{(\alpha_i, \beta_i)\}_{i \in k} \mid k \in \omega \setminus 1 \land \alpha_i \in \mathrm{Ord} \setminus 1 \land \beta_i \in A_n \land |\{\beta_i\}_{i \in k}| = k\}$$
$$A = \bigcup_{n \in \omega} (A_n)$$
$$A' = A \setminus \{\varnothing\}$$

$$B_0 = \{\varnothing\}$$
$$B_{n+1} = \{\{(\alpha_i, \beta_i)\}_{i \in k} \mid k \in \omega \setminus 1 \land \alpha_i \in \mathrm{Ord} \land \beta_i \in B_n \land |\{\beta_i\}_{i \in k}| = k\}$$
$$B = \bigcup_{n \in \omega} (A_n)$$

$$a : A \to \bigcup_{n \in \omega} (\{\{\alpha_i\}_{i \in n, \alpha_i \in \mathrm{Ord}}\})$$
$$a(X) : X \mapsto \{X' \mid \exists X'' \in (\mathrm{Ord} \setminus 1)[(X'', X') \in X]\}$$

$$b : A' \to A$$
$$b(X) : X \mapsto \hat{X}'(\nexists X'' \in a(X)[X' \prec X''])$$

$$\overline{b} : A' \to A$$
$$\overline{b}(X) : X \mapsto \hat{X}'(\nexists X'' \in a(X)[X'' \prec X'])$$

$$c_X(Y) : A^2 \to \mathrm{Ord}$$
$$c_X(Y) : (X, Y) \mapsto \hat{X}'((X', Y) \in X) \iff Y \in a(X)$$
$$(X, Y) \mapsto 0 \iff Y \notin a(X)$$

$$X \prec Y \iff \begin{cases} \bot & [Y = \varnothing] \\ \top & [X = \varnothing \land Y \neq \varnothing] \\ b(X) \prec b(Y) & [X \neq \varnothing \land Y \neq \varnothing \land b(X) \neq b(Y)] \\ c_X(b(X)) \prec c_Y(b(Y)) & [X \neq \varnothing \land Y \neq \varnothing \land b(X) = b(Y) \land c_X(b(X)) \neq c_Y(b(Y))] \\ X \setminus \{(c_X(b(X)), b(X))\} \prec Y \setminus \{(c_Y(b(Y)), b(Y))\} & [X \neq \varnothing \land Y \neq \varnothing \land b(X) = b(Y) \land c_X(b(X)) = c_Y(b(Y))] \end{cases}$$

$$d : B \to A$$
$$d(X) : X \mapsto \{(X', X'') \mid (X', X'') \in X \land X' > 0\}$$

$$e : A \to A$$
$$e(X) : X \mapsto \begin{cases} X & [c_X(\varnothing) \in \mathrm{Lim} \cup 1] \\ d(X \setminus \{(c_X(\varnothing), \varnothing)\} \cup \{(\hat{\alpha}(\alpha + 1 = c_X(\varnothing)), \varnothing)\}) & [c_X(\varnothing) \in \mathrm{Succ}] \end{cases}$$

$$f : A \to \mathrm{Ord}$$
$$f(X) : X \mapsto \begin{cases} 0 & [X = \varnothing] \\ 1 & [e(X) \neq X] \\ 2 & [e(\overline{b}(X)) \neq \overline{b}(X) \land c_X(\overline{b}(X)) \in \mathrm{Succ}] \\ f(\overline{b}(X)) & [e(\overline{b}(X)) = \overline{b}(X) \land c_X(\overline{b}(X)) \in \mathrm{Succ}] \\ c_X(\overline{b}(X)) & [c_X(\overline{b}(X)) \in \mathrm{Lim}] \end{cases}$$

$$g : A \times \mathrm{Ord} \to A$$
$$g(X, \alpha) : (X, \alpha) \mapsto \begin{cases} \varnothing & [X = \varnothing] \\ e(X) & [e(X) \neq X] \\ d(X \setminus \{(c_X(\overline{b}(X)), \overline{b}(X))\} \cup \{(\hat{\beta}(\beta + 1 = c_X(\overline{b}(X))), \overline{b}(X)), (\alpha, e(\overline{b}(X)))\}) & [e(\overline{b}(X)) \neq \overline{b}(X) \land c_X(\overline{b}(X)) \in \mathrm{Succ}] \\ d(X \setminus \{(c_X(\overline{b}(X)), \overline{b}(X))\} \cup \{(\hat{\beta}(\beta + 1 = c_X(\overline{b}(X))), \overline{b}(X)), (1, g(\overline{b}(X), \alpha))\}) & [e(\overline{b}(X)) = \overline{b}(X) \land c_X(\overline{b}(X)) \in \mathrm{Succ}] \\ d(X \setminus \{(c_X(\overline{b}(X)), \overline{b}(X))\} \cup \{(\alpha, \overline{b}(X))\}) & [c_X(\overline{b}(X)) \in \mathrm{Lim}] \end{cases}$$

$$h : A \to A$$
$$h(X) : X \mapsto \begin{cases} \varnothing & [X = \varnothing] \\ X \setminus \{(c_X(\varnothing), \varnothing)\} & [\varnothing \in a(X)] \\ X & [\varnothing \notin a(X)] \end{cases}$$

$$\varphi : A \to \mathrm{Ord}$$
$$\varphi X : (X, \alpha) \mapsto \begin{cases} 1 & [X = \varnothing] \\ \omega^{c_X(\varnothing)} & [a(X) = \{\varnothing\}] \\ \mathrm{enum}(\{\alpha \mid \forall \beta < f(h(X))[\alpha = \varphi d(g(h(X), \beta) \cup \{(\alpha, \varnothing)\})]\})(c_X(\varnothing)) & [f(h(X)) > 2] \\ \mathrm{enum}(\{\alpha \mid \alpha = \varphi g(h(X), \alpha)\})(c_X(\varnothing)) & [f(h(X)) = 2] \end{cases}$$

$$\underline{\alpha} = \{(\alpha, 0)\}$$
$$\{(\alpha_i, \underline{i})\}_{i \in n, n \in \omega} = (\alpha_i)_{i \in n, n \in \omega}$$
$$\{(\alpha_i, X_i)\}_{i \in n, n \in \omega} = \binom{\alpha_i}{X_i}_{i \in n, n \in \omega}$$