When two or more cases of a definition are both satisfied, the uppermost one takes priority.

For a predicate $\varphi(x)$, I define $\hat{x}\varphi(x)$ to be the x that satisfies $\varphi(x)$; it is undefined if there's more then one such x. Then,

$$
\begin{array}{rcl}
A_0 & = & \{\varnothing\} \\
A_{n+1} & = & \{\{(\alpha_i, \beta_i)\}_{i\in k} \mid k \in \omega \setminus 1 \wedge \alpha_i \in \mathrm{Ord} \setminus 1 \wedge \beta_i \in A_n \wedge |\{\beta_i\}_{i\in k}| = k\} \\
A & = & \bigcup\limits_{n\in\omega} (A_n) \\
A' & = & A \setminus \{\varnothing\}
\end{array}
$$

$$
\begin{array}{rcl}
B_0 & = & \{\varnothing\} \\
B_{n+1} & = & \{\{(\alpha_i, \beta_i)\}_{i\in k} \mid k \in \omega \setminus 1 \wedge \alpha_i \in \mathrm{Ord} \wedge \beta_i \in B_n \wedge |\{\beta_i\}_{i\in k}| = k\} \\
B & = & \bigcup\limits_{n\in\omega} (A_n)
\end{array}
$$

$$
\begin{array}{rcccl}
a & : & A & \rightarrow & \bigcup\limits_{n\in\omega} (\{\{\alpha_i\}_{i\in n, \alpha_i \in \mathrm{Ord}}\}) \\
a(X) & : & X & \mapsto & \{X' \mid \exists X'' \in (\mathrm{Ord} \setminus 1)[(X'', X') \in X]\}
\end{array}
$$

$$
\begin{array}{rcccl}
b & : & A' & \rightarrow & A \\
b(X) & : & X & \mapsto & \hat{X}'(\nexists X'' \in a(X)[X' \prec X''])
\end{array}
$$

$$
\begin{array}{rcccl}
\overline{b} & : & A' & \rightarrow & A \\
\overline{b}(X) & : & X & \mapsto & \hat{X}'(\nexists X'' \in a(X)[X'' \prec X'])
\end{array}
$$

$$
\begin{array}{rcccllcl}
c_X(Y) & : & A^2 & \rightarrow & \mathrm{Ord} & & & \\
c_X(Y) & : & (X, Y) & \mapsto & \hat{X}'((X', Y) \in X) & \iff & Y \in a(X) \\
 & & (X, Y) & \mapsto & 0 & \iff & Y \notin a(X)
\end{array}
$$

$$
X \prec Y \iff \begin{cases}
\bot & [Y = \varnothing] \\
\top & [X = \varnothing \wedge Y \neq \varnothing] \\
b(X) \prec b(Y) & [X \neq \varnothing \wedge Y \neq \varnothing \wedge b(X) \neq b(Y)] \\
c_X(b(X)) \prec c_Y(b(Y)) & [X \neq \varnothing \wedge Y \neq \varnothing \wedge b(X) = b(Y) \wedge c_X(b(X)) \neq c_Y(b(Y))] \\
X \setminus \{(c_X(b(X)), b(X))\} \prec Y \setminus \{(c_Y(b(Y)), b(Y))\} & [X \neq \varnothing \wedge Y \neq \varnothing \wedge b(X) = b(Y) \wedge c_X(b(X)) = c_Y(b(Y))]
\end{cases}
$$

$$
\begin{array}{rcccl}
d & : & B & \rightarrow & A \\
d(X) & : & X & \mapsto & \{(X', X'') \mid (X', X'') \in X \wedge X' > 0\}
\end{array}
$$

$$
\begin{array}{rcccl}
e & : & A & \rightarrow & A \\
e(X) & : & X & \mapsto & \begin{cases}
X & [c_X(\varnothing) \in \mathrm{Lim} \cup 1] \\
d(X \setminus \{(c_X(\varnothing), \varnothing)\} \cup \{(\hat{\alpha}(\alpha + 1 = c_X(\varnothing)), \varnothing)\}) & [c_X(\varnothing) \in \mathrm{Succ}]
\end{cases}
\end{array}
$$

$$
\begin{array}{rcccl}
f & : & A & \rightarrow & \mathrm{Ord} \\
f(X) & : & X & \mapsto & \begin{cases}
0 & [X = \varnothing] \\
1 & [e(X) \neq X] \\
2 & [e(\overline{b}(X)) \neq \overline{b}(X) \wedge c_X(\overline{b}(X)) \in \mathrm{Succ}] \\
f(\overline{b}(X)) & [e(\overline{b}(X)) = \overline{b}(X) \wedge c_X(\overline{b}(X)) \in \mathrm{Succ}] \\
c_X(\overline{b}(X)) & [c_X(\overline{b}(X)) \in \mathrm{Lim}]
\end{cases}
\end{array}
$$

$$
\begin{array}{rcccl}
g & : & A \times \mathrm{Ord} & \rightarrow & A \\
g(X, \alpha) & : & (X, \alpha) & \mapsto & \begin{cases}
\varnothing & [X = \varnothing] \\
e(X) & [e(X) \neq X] \\
d(X \setminus \{(c_X(\overline{b}(X)), \overline{b}(X))\} \cup \{(\hat{\beta}(\beta + 1 = c_X(\overline{b}(X))), \overline{b}(X)), (\alpha, e(\overline{b}(X)))\}) & [e(\overline{b}(X)) \neq \overline{b}(X) \wedge c_X(\overline{b}(X)) \in \mathrm{Succ}] \\
d(X \setminus \{(c_X(\overline{b}(X)), \overline{b}(X))\} \cup \{(\hat{\beta}(\beta + 1 = c_X(\overline{b}(X))), \overline{b}(X)), (1, g(\overline{b}(X), \alpha))\}) & [e(\overline{b}(X)) = \overline{b}(X) \wedge c_X(\overline{b}(X)) \in \mathrm{Succ}] \\
d(X \setminus \{(c_X(\overline{b}(X)), \overline{b}(X))\} \cup \{(\alpha, \overline{b}(X))\}) & [c_X(\overline{b}(X)) \in \mathrm{Lim}]
\end{cases}
\end{array}
$$

$$
\begin{array}{rcccl}
h & : & A & \rightarrow & A \\
h(X) & : & X & \mapsto & \begin{cases}
\varnothing & [X = \varnothing] \\
X \setminus \{(c_X(\varnothing), \varnothing)\} & [\varnothing \in a(X)] \\
X & [\varnothing \notin a(X)]
\end{cases}
\end{array}
$$

$$
\begin{array}{rcccl}
\varphi & : & A & \rightarrow & \mathrm{Ord} \\
\varphi X & : & (X, \alpha) & \mapsto & \begin{cases}
1 & [X = \varnothing] \\
\omega^{c_X(\varnothing)} & [a(X) = \{\varnothing\}] \\
\mathrm{enum}(\{\alpha \mid \forall \beta < f(h(X))[\alpha = \varphi(g(h(X), \beta) \cup \{(\alpha, \varnothing)\})]\})(c_X(\varnothing)) & [f(h(X)) > 2] \\
\mathrm{enum}(\{\alpha \mid \alpha = \varphi g(h(X), \alpha)\})(c_X(\varnothing)) & [f(h(X)) = 2]
\end{cases}
\end{array}
$$

$$
\begin{array}{rcl}
\underline{\alpha} & = & \{(\alpha, 0)\} \\
\{(\alpha_i, \underline{i})\}_{i\in n, n\in\omega} & = & (\alpha_i)_{i\in n, n\in\omega} \\
\{(\alpha_i, X_i)\}_{i\in n, n\in\omega} & = & \begin{pmatrix} \alpha_i \\ X_i \end{pmatrix}_{i\in n, n\in\omega}
\end{array}
$$