For a predicate $\varphi(x)$, I define $\hat{x}\varphi(x)$ to be the x that satisfies $\varphi(x)$; it is undefined if there's more then one such x. Then,

$$\begin{aligned}
A_0 &= \{\varnothing\} \\
A_{n+1} &= \{\{(\alpha_i,\beta_i)\}_{i\in k} \mid k\in\omega\setminus 1 \wedge \alpha_i\in \mathrm{Ord}\setminus 1 \wedge \beta_i \in A_n \wedge |\{\beta_i\}_{i\in k}| = k\} \\
A &= \bigcup_{n\in\omega}(A_n) \\
A' &= A\setminus\{\varnothing\} \\
B_0 &= \{\varnothing\} \\
B_{n+1} &= \{\{(\alpha_i,\beta_i)\}_{i\in k} \mid k\in\omega\setminus 1 \wedge \alpha_i\in \mathrm{Ord} \wedge \beta_i \in B_n \wedge |\{\beta_i\}_{i\in k}| = k\} \\
B &= \bigcup_{n\in\omega}(A_n)
\end{aligned}$$

$$\begin{aligned}
a &: A \to \bigcup_{n\in\omega}(\{\{\alpha_i\}_{i\in n,\alpha_i\in\mathrm{Ord}}\}) \\
a(X) &: X \mapsto \{X' \mid \exists X'' \in (\mathrm{Ord}\setminus 1)[(X'',X')\in X]\}
\end{aligned}$$

$$\begin{aligned}
b &: A' \to A \\
b(X) &: X \mapsto \hat{X}'(\nexists X'' \in a(X)[X' \prec X''])
\end{aligned}$$

$$\begin{aligned}
\overline{b} &: A' \to A \\
\overline{b}(X) &: X \mapsto \hat{X}'(\nexists X'' \in a(X)[X'' \prec X'])
\end{aligned}$$

$$\begin{aligned}
c_X(Y) &: A^2 \to \mathrm{Ord} \\
c_X(Y) &: (X,Y) \mapsto \hat{X}'((X',Y)\in X) \iff Y\in a(X) \\
& (X,Y) \mapsto 0 \iff Y\notin a(X)
\end{aligned}$$

$$X \prec Y \iff \begin{cases} \bot & [Y=\varnothing] \\ \top & [X=\varnothing \wedge Y\neq\varnothing] \\ b(X)\prec b(Y) & [X\neq\varnothing\wedge Y\neq\varnothing\wedge b(X)\neq b(Y)] \\ c_X(b(X)) \prec c_Y(b(Y)) & [X\neq\varnothing\wedge Y\neq\varnothing\wedge b(X)= b(Y)\wedge c_X(b(X))\neq c_Y(b(Y))] \\ X\setminus\{(c_X(b(X)),b(X))\} \prec Y\setminus\{(c_Y(b(Y)),b(Y))\} & [X\neq\varnothing\wedge Y\neq\varnothing\wedge b(X)= b(Y)\wedge c_X(b(X))= c_Y(b(Y))] \end{cases}$$

$$\begin{aligned}
d &: B \to A \\
d(X) &: X \mapsto \{(X',X'') \mid (X',X'')\in X \wedge X' > 0\}
\end{aligned}$$

$$\begin{aligned}
e &: A \to A \\
e(X) &: X \mapsto \begin{cases} X & [c_X(\varnothing)\in \mathrm{Lim}\cup 1] \\ d(X\setminus\{(c_X(\varnothing),\varnothing)\}\cup\{(\hat{\alpha}(\alpha+1=c_X(\varnothing)),\varnothing)\}) & [c_X(\varnothing)\in\mathrm{Succ}] \end{cases}
\end{aligned}$$

$$\begin{aligned}
f &: A \to \mathrm{Ord}\cup\{\mathrm{Ord}\} \\
f(X) &: X \mapsto \begin{cases} 0 & [X=\varnothing] \\ 1 & [e(X)\neq X] \\ c_X(b(X)) & [c_X(b(X))\in\mathrm{Lim}] \\ \mathrm{Ord} & [e(b(X))\neq b(X)] \\ f(b(X)) & [e(b(X)) = b(X)] \end{cases}
\end{aligned}$$

$$\begin{aligned}
g &: A\times\mathrm{Ord} \to A \\
g(X,\alpha) &: (X,\alpha) \mapsto \begin{cases} \varnothing & [X=\varnothing] \\ e(X) & [e(X)\neq X] \\ d(X\setminus\{(c_X(b(X)),b(X))\}\cup\{(\hat{\beta}(\beta+1=c_X(b(X))),b(X)),(\alpha,e(b(X)))\}) & [e(b(X))\neq b(X)\wedge c_X(b(X))\in\mathrm{Succ}] \\ d(X\setminus\{(c_X(b(X)),b(X))\}\cup\{(\hat{\beta}(\beta+1=c_X(b(X))),b(X)),(1,g(b(X),\alpha))\}) & [e(b(X))= b(X)\wedge c_X(b(X))\in\mathrm{Succ}] \\ d(X\setminus\{(c_X(b(X)),b(X))\}\cup\{(\alpha,b(X))\}) & [c_X(b(X))\in\mathrm{Lim}] \end{cases}
\end{aligned}$$

$$\begin{aligned}
h &: A \to A \\
h(X) &: X \mapsto \begin{cases} \varnothing & [X=\varnothing] \\ X\setminus\{(c_X(\varnothing),\varnothing)\} & [\varnothing\in a(X)] \\ X & [\varnothing\notin a(X)] \end{cases}
\end{aligned}$$

$$\begin{aligned}
\varphi &: A \to \mathrm{Ord} \\
\varphi X &: (X,\alpha) \mapsto \begin{cases} 1 & [X=\varnothing] \\ \omega^{c_X(\varnothing)} & [a(X)=\{\varnothing\}] \\ \mathrm{enum}(\{\alpha \mid \forall\beta< f(h(X))[\alpha=\varphi(g(h(X),\beta)\cup\{(\alpha,\varnothing)\})]\})(c_X(\varnothing)) & [f(h(X))\in\mathrm{Ord}] \\ \mathrm{enum}(\{\alpha \mid \alpha = g(h(X),\alpha)\})(c_X(\varnothing)) & [f(h(X))=\mathrm{Ord}] \end{cases}
\end{aligned}$$

$$\begin{aligned}
\{(\alpha_i,i)\}_{i\in n,n\in\omega} &= (\alpha_i)_{i\in n,n\in\omega} \\
\{(\alpha_i,X_i)\}_{i\in n,n\in\omega} &= \begin{pmatrix}\alpha_i \\ X_i\end{pmatrix}_{i\in n,n\in\omega}
\end{aligned}$$