For a predicate $\varphi(x)$, I define $\hat{x}\varphi(x)$ to be the x that satisfies $\varphi(x)$; it is undefined if there's more then one such x. Then,

$$\begin{array}{rcl}
A_0 &=& \{\varnothing\} \\
A_{n+1} &=& \{\{(\alpha_i,\beta_i)\}_{i\in k} \mid k\in\omega\setminus 1 \wedge \alpha_i \in \mathrm{Ord}\setminus 1 \wedge \beta_i \in A_n \wedge |\{\beta_i\}_{i\in k}| = k\} \\
A &=& \bigcup\limits_{n\in\omega}(A_n) \\
A' &=& A\setminus\{\varnothing\} \\
B_0 &=& \{\varnothing\} \\
B_{n+1} &=& \{\{(\alpha_i,\beta_i)\}_{i\in k} \mid k\in\omega\setminus 1 \wedge \alpha_i \in \mathrm{Ord} \wedge \beta_i \in B_n \wedge |\{\beta_i\}_{i\in k}| = k\} \\
B &=& \bigcup\limits_{n\in\omega}(A_n)
\end{array}$$

$$\begin{array}{rcccl}
a &:& A &\to& \bigcup\limits_{n\in\omega}(\{\{\alpha_i\}_{i\in n,\alpha_i\in\mathrm{Ord}}\}) \\
a(X) &:& X &\mapsto& \{X' \mid \exists X'' \in (\mathrm{Ord}\setminus 1)[(X'',X')\in X]\}
\end{array}$$

$$\begin{array}{rcccl}
b &:& A' &\to& A \\
b(X) &:& X &\mapsto& \hat{X}'(\nexists X'' \in a(X)[X' \prec X''])
\end{array}$$

$$\begin{array}{rcccl}
\overline{b} &:& A' &\to& A \\
\overline{b}(X) &:& X &\mapsto& \hat{X}'(\nexists X'' \in a(X)[X'' \prec X'])
\end{array}$$

$$\begin{array}{rcccccl}
c_X(Y) &:& A^2 &\to& \mathrm{Ord} && \\
c_X(Y) &:& (X,Y) &\mapsto& \hat{X}'((X',Y)\in X) &\iff& Y\in a(X) \\
&& (X,Y) &\mapsto& 0 &\iff& Y\notin a(X)
\end{array}$$

$$X \prec Y \iff \begin{cases}
\bot & [Y=\varnothing] \\
\top & [X=\varnothing \wedge Y\neq\varnothing] \\
b(X)\prec b(Y) & [X\neq\varnothing \wedge Y\neq\varnothing \wedge b(X)\neq b(Y)] \\
c_X(b(X))\prec c_Y(b(Y)) & [X\neq\varnothing \wedge Y\neq\varnothing \wedge b(X)=b(Y) \wedge c_X(b(X))\neq c_Y(b(Y))] \\
X\setminus\{(c_X(b(X)),b(X))\} \prec Y\setminus\{(c_Y(b(Y)),b(Y))\} & [X\neq\varnothing \wedge Y\neq\varnothing \wedge b(X)=b(Y) \wedge c_X(b(X))= c_Y(b(Y))]
\end{cases}$$

$$\begin{array}{rcccl}
d &:& B &\to& A \\
d(X) &:& X &\mapsto& \{(X',X'') \mid (X',X'')\in X \wedge X'>0\}
\end{array}$$

$$\begin{array}{rcccl}
e &:& A &\to& A \\
e(X) &:& X &\mapsto& \begin{cases}
X & [c_X(\varnothing)\in \mathrm{Lim}\cup 1] \\
d(X\setminus\{(c_X(\varnothing),\varnothing)\}\cup\{(\hat{\alpha}(\alpha+1=c_X(\varnothing)),\varnothing)\}) & [c_X(\varnothing)\in\mathrm{Succ}]
\end{cases}
\end{array}$$

$$\begin{array}{rcccl}
f &:& A &\to& \mathrm{Ord}\cup\{\mathrm{Ord}\} \\
f(X) &:& X &\mapsto& \begin{cases}
0 & [X=\varnothing] \\
1 & [e(X)\neq X] \\
c_X(\overline{b}(X)) & [c_X(\overline{b}(X))\in\mathrm{Lim}] \\
\mathrm{Ord} & [e(\overline{b}(X))\neq\overline{b}(X)] \\
f(\overline{b}(X)) & [e(\overline{b}(X))=\overline{b}(X)]
\end{cases}
\end{array}$$

$$\begin{array}{rcccl}
g &:& A\times\mathrm{Ord} &\to& A \\
g(X,\alpha) &:& (X,\alpha) &\mapsto& \begin{cases}
\varnothing & [X=\varnothing] \\
e(X) & [e(X)\neq X] \\
d(X\setminus\{(c_X(\overline{b}(X)),\overline{b}(X))\}\cup\{(\hat{\beta}(\beta+1=c_X(\overline{b}(X))),\overline{b}(X)),(\alpha,e(\overline{b}(X)))\}) & [e(\overline{b}(X))\neq\overline{b}(X)\wedge c_X(\overline{b}(X))\in\mathrm{Succ}] \\
d(X\setminus\{(c_X(\overline{b}(X)),\overline{b}(X))\}\cup\{(\hat{\beta}(\beta+1=c_X(\overline{b}(X))),\overline{b}(X)),(1,g(\overline{b}(X),\alpha))\}) & [e(\overline{b}(X))=\overline{b}(X)\wedge c_X(\overline{b}(X))\in\mathrm{Succ}] \\
d(X\setminus\{(c_X(\overline{b}(X)),\overline{b}(X))\}\cup\{(\alpha,\overline{b}(X))\}) & [c_X(\overline{b}(X))\in\mathrm{Lim}]
\end{cases}
\end{array}$$

$$\begin{array}{rcccl}
h &:& A &\to& A \\
h(X) &:& X &\mapsto& \begin{cases}
\varnothing & [X=\varnothing] \\
X\setminus\{(c_X(\varnothing),\varnothing)\} & [\varnothing\in a(X)] \\
X & [\varnothing\notin a(X)]
\end{cases}
\end{array}$$

$$\begin{array}{rcccl}
\varphi &:& A &\to& \mathrm{Ord} \\
\varphi X &:& (X,\alpha) &\mapsto& \begin{cases}
1 & [X=\varnothing] \\
\omega^{c_X(\varnothing)} & [a(X)=\{\varnothing\}] \\
\mathrm{enum}(\{\alpha \mid \forall\beta<f(h(X))[\alpha=\varphi(g(h(X),\beta)\cup\{(\alpha,\varnothing)\})]\})(c_X(\varnothing)) & [f(h(X))\in\mathrm{Ord}] \\
\mathrm{enum}(\{\alpha \mid \alpha=g(h(X),\alpha)\})(c_X(\varnothing)) & [f(h(X))=\mathrm{Ord}]
\end{cases}
\end{array}$$

$$\begin{array}{rcl}
\{(\alpha_i,i)\}_{i\in n,n\in\omega} &=& (\alpha_i)_{i\in n,n\in\omega} \\
\{(\alpha_i,X_i)\}_{i\in n,n\in\omega} &=& \begin{pmatrix}\alpha_i\\ X_i\end{pmatrix}_{i\in n,n\in\omega}
\end{array}$$