Practice

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Chapter 4 Practice

Practice 4.1. Prove the Central Limit Theorem by Simulation Simulate multiple samples (say, 100) of the same size (say, 30) from the same distribution (try uniform, discrete uniform, Poisson and lognormal). Plot the distribution of the sample means. Does it look normal? What are its mean and standard deviation? How do they compare to the mean and the standard deviation of the underlying distribution?

Practice 4.2. Implement the Bootstrap Technique (section 3.10.3) to Estimate a Confidence Interval for the Percentile of a Distribution of Portfolio returns Columns B–D of the file **Ch4-BootstrapExercise.xlsx** contain a sample of 100 monthly returns on three stocks. Suppose that you invest one-third of your portfolio in each of the three stocks for one month. The portfolio return in each month can be computed as a sum of the returns of the individual stocks multiplied by their portfolio weights. It is computed in column E. The 5th percentile of the distribution of the 100 portfolio returns can be computed to be −12.43%. Compute a 95% CI for the percentile of the distribution using bootstrapping. Namely, draw many random samples (say, 1,000) of size 100 from the original sample, and record the 5th percentile realized with each of these 1,000 samples. Plot a histogram of the 1,000 values for portfolio percentile, and make a note of the 2.5th and 97.5th percentiles in the distribution of portfolio percentiles.

Hints: In @RISK, you only need to fill out all cells with yellow borders in the file (columns I–L). In column H, you can record a random number between 1 and 100, which will indicate the observation number from the original sample. Then, use the Excel function `VLOOKUP` to look up what values for the three stock returns correspond to that observation number from the data in columns B–D. In cell I5, write the Excel formula for computing the 5th percentile of the bootstrapped portfolio returns in column L, and make cell I5 the output cell for @RISK. Your final spreadsheet would look something like the spreadsheet in Exhibit P4.1.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Portfolio	Stock 1	Stock 2	Stock 3								
2	weights	0.33	0.33	0.33								
3												
4	Portfolio return 5th percentile						Portfolio return 5th percentile					
5		-12.43%							-10.78%			
6	(original sample)							(bootstrapped)				
7												
8	Month No.	Stock 1	Stock 2	Stock 3	Portfolio return	Boostrapping:		Observation drawn	Stock 1	Stock 2	Stock 3	Portfolio return
9	1	-0.48%	-0.44%	-1.32%	-0.74%		Trial 1	2	2.08%	-6.17%	4.09%	0.00%
10	2	2.08%	-6.17%	4.09%	0.00%		Trial 2	56	5.29%	12.90%	4.09%	7.43%
11	3	2.29%	-24.88%	8.15%	-4.81%		Trial 3	95	-2.20%	4.88%	-0.56%	0.71%
12	4	-9.88%	-21.25%	-6.15%	-12.43%		Trial 4	87	3.91%	-15.72%	8.67%	-1.05%
13	5	7.71%	23.02%	2.20%	10.97%		Trial 5	56	5.29%	12.90%	-6.62%	3.86%
14	6	-7.85%	-18.71%	-11.23%	-12.59%		Trial 6	39	-4.74%	-6.02%	-0.56%	-3.77%
15	7	6.09%	-12.70%	-0.94%	-2.52%		Trial 7	62	9.91%	10.20%	-10.42%	3.23%
16	8	8.85%	13.64%	-2.98%	6.50%		Trial 8	25	0.00%	30.68%	7.38%	12.69%
17	9	4.37%	-12.00%	0.25%	-2.46%		Trial 9	72	-2.96%	-3.40%	3.23%	-1.04%
18	10	16.57%	27.27%	-1.71%	14.04%		Trial 10	58	11.55%	48.72%	-4.62%	18.55%

EXHIBIT P4.1 Implementation of Practice.4.2 with @RISK.

In MATLAB, read in the data from the Excel file using the `xlsread` function in MATLAB. MATLAB's Statistics Toolbox has a bootstrap function (`bootstrp`) that can generate multiple samples from one original sample, and you can then use its `prctile` function to compute the 5th percentile for each sample. Store these percentiles in a vector array, and analyze the summary statistics of the values. See the MATLAB help for more details.

Practice 4.3. The Flaw of Averages Suppose you are tasked with finding the optimal level of inventory to have on hand every week. Your company sells perishable goods, and if your inventory exceeds actual demand in any particular week, you incur a $30 perishable cost per item, while if your inventory falls short of demand, you incur $20 per item in express mail cost. You collect data on weekly demand over the past year, and it turns out that the demand follows a lognormal probability distribution with an average demand of 100 units per week and a standard deviation of 15 units per week. Is it reasonable to assume that 100 units per week is the optimal number to stock? Why or why not? Use simulation to estimate the expected cost and its variability if you order (a) 80 units, (b) 90 units, (c) 100 units, (d) 110 units, and (e) 120 units.[1]

Hints: Use the RiskSimtable function in @RISK for implementing the different decisions available to the manager. In MATLAB, use loops to iterate through different decisions.

[1]See also the example in Appendix B on the companion web site. This is another version of the famous Newsvendor Problem. The beauty of the problem is in its unintuitive outcome.

Practice 4.4. Practice Finance Concepts Use ideas from section 4.2.1 of Chapter 4 combined with VBA or MATLAB to create a retirement planning model. Your inputs should include:

- Your current age
- The age at which you plan to retire
- Your expected life span
- Your current salary
- Your salary's annual growth rate
- The percentage of salary that is contributed to the retirement account during your working years
- The percentage of your current income that you would like to spend per year after retirement
- Your desired terminal wealth, that is, the amount of money you would like to keep in reserve if your life exceeds your life expectancy, or the amount of money you would like to leave to your heirs
- A distribution for annual investment returns. For example, assume that every year, returns are normally distributed with mean 8.79% and standard deviation 14.65%. For now, assume that returns in different years are independent (i.e., they are not autocorrelated).

Analyze the distribution of your terminal wealth for different percentage contributions to your retirement account. For example, what happens when you contribute 5%, 10%, or 15% of your salary? Are you able to achieve your terminal wealth goals? How large is the risk of missing your terminal wealth goals with each of the percentage contributions?

Chapter 5 Practice

Practice 5.1. IP formulations Implement the following modification of the portfolio optimization problem in section 5.3.1. The optimal solution was to invest in Funds 1, 3, and 4. Suppose now that the manager would like to invest in all four funds. All other constraints still apply. How can this condition be formulated?

Hints: Refer to the capital budgeting example formulation in section 5.3.3. Introduce four binary variables (y_1, y_2, y_3, y_4) that correspond to the decision to invest in Fund i ($y_i = 1$)or not ($y_i = 0$). The other decision variable definitions stay as before. You need several constraints to state that the number of investments should be four, and that the variables y_i cannot be 1 unless the amount invested in Fund i is strictly greater than 0.

Practice 5.2. Duality Derive the dual problem of the portfolio allocation problem in section 5.3.1 using the information in the table in Exhibit 5.9. Solve it with Excel Solver or MATLAB's Optimization Toolbox, and verify that the optimal value of the objective function is the same as the optimal value of the objective function of the primal problem.

Hint: The original (primal) problem is a linear maximization problem. The dual problem should be a linear minimization problem.

Practice 5.3. Optimal Liability Management under Transaction Costs and Taxes Suppose you are the manager of Hartford Insurance's (HI) investments in Atlantic Oil, Benson Oil, Carnegie Steel, and Draper Steel. Exhibit P5.1 shows the current number of shares HI owns in each company, the prices at which the shares were purchased (the *basis*), the current price of each share, and the price that you expect the shares to have at the end of the year.

HI just received an insurance claim for $1,000,000. The company decides to raise the money by selling some of the stocks in its investment portfolio of oil and steel stocks. Your task as an investment manager is to

EXHIBIT P5.1 Data for problem Practice 5.3.

	Atlantic Oil	Benson Oil	Carnegie Steel	Draper Steel
Current number of shares	200,000	20,000	300,000	5,000
Basis/share	\$20	\$30	\$25	\$40
Current price/share	\$25	\$35	\$30	\$44
Expected end-of-year price	\$23	\$25	\$28	\$39

figure out how to sell the shares so that you can generate enough cash to cover this claim, while at the same time maximizing the expected value of this stock portfolio at the end of the year.

In making your decision, you should take into consideration the costs associated with rebalancing an investment portfolio. If HI sells any shares, it will incur a transaction cost equal to 1% of the dollar amount sold. In addition, HI needs to pay capital gains tax at the rate of 30% of any capital gains at the time of sale. Capital gains are based on the difference between the money received from the sale of stock and the original cost of the shares. For example, suppose that HI sells 1,000 shares of a stock today at \$50 a share, which HI originally purchased for \$30 a share. HI would receive \$50,000. However, HI would need to pay capital gains taxes of $0.30 \cdot (50{\cdot}1{,}000\ 30{\cdot}1{,}000) = \$6{,}000$, and HI would also need to pay $0.01 \cdot (50 \cdot 1{,}000) = \$500$ in transaction costs. Therefore, by selling 1,000 shares of stock, HI would have a net cash flow of $\$50{,}000 - \$6{,}000 - \$500 = \$43{,}500$.

(a) Provide a linear programming formulation of the problem of rebalancing the portfolio so that the future expected return is maximized, while the necessary cash is generated. State clearly your decision variables, your objective function, and your constraints.
(b) Implement the model with Excel or MATLAB. What is the optimal solution? Give a concise interpretation of the answer report.
(c) Would the optimal solution remain the same if there were no taxes and transaction costs? Create a new model to find the optimal solution in this case.
(d) Go back to your model in (3). Suppose the current price of Benson Oil is \$31 instead of \$30. Repeat part (3), and determine if this new price changes your answer to part (3).
(e) Consider your model in (3) again. Make sure the current price of Benson oil is back to \$30, and suppose that the investment team needs to satisfy the following additional investment guideline: The value of the steel

stocks sold must be at least 50% of the value of the oil stocks sold. How would you write this constraint mathematically? Add it to your optimization model, and resolve the problem. What is the optimal solution with this added constraint?

Practice 5.5. Deterministic Dynamic Programming Suppose you have purchased a 3-year lease on a gold mine. The price of gold for the three years of your lease is expected to be \$600/oz, \$870/oz, and \$450/oz, respectively.

(**a**) In each of the three years, you have three options: (1) You can do nothing; (2) You can mine 10,000 oz of gold at a cost of $\$500 \cdot u_t^2/x_t$, where $u_t = 10{,}000$ is the amount you mine and x_t is the level of current reserves, and (3) You can mine 20,000 oz of gold using an enhanced method at a cost of $\$900 \cdot u_t^2/x_t$, where $u_t = 20{,}000$ is the amount you mine and x_t is the level of current reserves. What is the optimal amount of gold to mine in each of the three years of the lease?

(**b**) Suppose that each year, you can mine as much gold as you want at a cost of $\$900 \cdot u_t^2/x_t$, where u_t is the amount you mine and x_t is the level of current reserves. What is the optimal amount of gold to mine in each of the three years of the lease?

Chapter 6 Practice

Practice 6.1. Dynamic Programming Solve the infinite-state version of the oil well problem from section 5.6.1 of Chapter 5. Namely, suppose that you have purchased a 3-year lease for the oil well. The estimated reserves of the well are 600,000 barrels. You can use an enhanced method to pump oil that allows you to pump as much oil as you want during the year at a cost of $20 \cdot u_t^2/x_t$, where u_t is the amount of oil that was pumped, and x_t is the amount of available reserves at time t. The evolution of the price of a barrel of oil over the first, second, and third year of your lease is shown in Exhibit 6.1, and the discount factor per year is 0.9. How much oil should you pump in each year to maximize your profit over the three years?

Practice 6.2. Stochastic Programming Suppose that you are a portfolio manager, and would like to invest the assets under your supervision so that your loss over the next year, with 5% probability, is not more than \$$L$. You will be investing in N assets, with unknown returns $\tilde{\mathbf{r}} = \tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_N$.

(a) Your research team has presented you with S possible scenarios for the returns of the assets in the portfolio. Write an optimization formulation that can be passed to an optimization solver, whose optimal solution will give you the optimal portfolio allocation under these conditions.

(b) Now suppose that you make the assumption that returns on all assets in the portfolio will be normally distributed with means $\boldsymbol{\mu} = \mu_1, \mu_2, \ldots, \mu_N$ and a covariance matrix of $\boldsymbol{\Sigma}$ over the next year. Write the formulation of your portfolio allocation problem in such a way that it can be passed to an optimization solver.

Practice 6.3. Robust Optimization Suppose that you are considering investing in N assets with random returns $\tilde{\mathbf{r}} = \tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_N$ for the next year. You would like to invest percentages $w_1, w_2, \ldots, w_N$ of your capital so as to maximize return. The problem can be formulated as

$$\begin{aligned} \max_{\mathbf{w}} \quad & \sum_{i=1}^{N} \tilde{r}_i w_i \\ \text{s.t.} \quad & w_1 + \ldots + w_N = 1 \\ & \mathbf{w} \geq 0. \end{aligned}$$

The problem as stated currently is not well defined. We can view the returns $\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_N$ as uncertain parameters in an optimization problem. In robust optimization, one would assume that they vary in a particular set defined by one's knowledge about their probability distributions, and take a worst-case (min-max) approach: Find portfolio weights such that the portfolio return is maximized even when the vector of realizations for all asset returns takes its "worst" value over the uncertainty set. Write the robust counterpart of the problem if the vector of returns varies in the ellipsoidal uncertainty set

$$\sqrt{(\mathbf{r} - \boldsymbol{\mu}_{\mathbf{r}})' \Sigma_{\mathbf{r}}^{-1} (\mathbf{r} - \boldsymbol{\mu}_{\mathbf{r}})} \leq \kappa.$$

Hint: In Chapter 6, we only discussed how to write the robust counterpart of a constraint, not of the objective function in an optimization problem. Note, however, that the objective function can be stated as a constraint. Let us introduce a new decision variable v. We can rewrite the problem as

$$\begin{aligned} \max_{\mathbf{w}} \quad & v \\ \text{s.t.} \quad & \sum_{i=1}^{N} \tilde{r}_i w_i \geq v \\ & w_1 + \ldots + w_N = 1 \\ & \mathbf{w} \geq 0. \end{aligned}$$

Then, we derive the robust counterpart of the constraint

$$-\sum_{i=1}^{N} \tilde{r}_i w_i \leq -v$$

in a similar way as we did in section 6.3.

Practice 6.4. Multistage Robust Optimization Consider the multistage robust optimization formulation of the oil well example in section 6.3.2. Suppose that instead of intervals, the uncertainty sets for the oil prices in years 1 and 2, $\tilde{p}_1$ and $\tilde{p}_2$, are ellipsoids. For simplicity, assume that the oil prices in different years are not correlated, that is, each of the parameters belongs to a separate uncertainty set:

$$U(p_1) = \left\{ p_1 \,\middle|\, \sqrt{\frac{(p_1 - \hat{p}_1)}{\sigma_1}} \leq \kappa_1 \right\}$$

$$U(p_2) = \left\{ p_2 \,\middle|\, \sqrt{\frac{(p_2 - \hat{p}_2)}{\sigma_2}} \leq \kappa_2 \right\}$$

where $\hat{p}_1$ and $\hat{p}_2$ are estimates of the expected values of the oil prices in year 1 and year 2, respectively, and σ_1 and σ_2 are their standard deviations. Formulate the robust counterpart of the multistage oil well problem when $\tilde{p}_1$ and $\tilde{p}_2$ vary in the uncertainty sets above.

Chapter 7 Practice

Practice 7.1. Evaluate Optimized Portfolio Risk with Simulation The worksheet **Data and Simulation** in the file **Ch7-MVBootstrap-Template.xlsx** contains data on 60 projected scenarios for the monthly returns of three stocks.

(a) Compute the expected return of each stock and the covariance (or correlations) matrix from the data.

Hint: To compute empirical covariance or correlation matrices with Excel, you can use the `COVARIANCE` or `CORRELATION` functions in Excel's **Data Analysis** tools. If you do not see **Data Analysis** group under the **Data** tab in the Excel ribbon, click on the Microsoft Excel button, click **Excel Options,** click **Add-Ins** in the Excel Options window, and then select the **Analysis Toolpak** in the Add-Ins pane. Click **Go** to install it. To compute covariance or correlation matrices with MATLAB, use the functions `cov` and `corrcoef`.

(b) Use these estimates as inputs to an optimization problem whose goal is to minimize the portfolio standard deviation. (Worksheet **MV Optimization** is set up for Excel Solver.) Set the target expected return to something small, e.g., 0%. The solver will find the portfolio with the minimum possible standard deviation.

(c) Generate 1000 scenarios for the return of two portfolios: the minimum standard deviation portfolio and an equally weighted portfolio (cells E2:E3 in worksheet **Data and Simulation**). Compare the empirical standard deviations of the two portfolios.

Chapter 8 Practice

Practice 8.1. Stochastic programming formulation of the problem of minimizing portfolio semideviation At the beginning of Chapter 8, we introduced semideviation as a downside portfolio risk measure. This problem asks you to solve the portfolio allocation problem when the goal is to minimize the portfolio semideviation subject to a constraint on the portfolio expected return. Semideviation looks at the shortfall below the mean, and can be expressed mathematically as

$$E\left[\max\left(E\left[\tilde{\mathbf{r}}'\mathbf{w}\right] - \tilde{\mathbf{r}}'\mathbf{w}, 0\right)\right].$$

(a) Using the expression above, show that the portfolio semideviation minimization problem can be formulated as

$$\begin{aligned}
\max_{\mathbf{w}} \quad & (1-\kappa)\sum_{i=1}^{N}\mu_i w_i + \kappa E\left[\min\left(\sum_{i=1}^{N}\mu_i w_i, \sum_{i=1}^{N} r_i w_i\right)\right] \\
\text{s.t.} \quad & \boldsymbol{\mu}'\mathbf{w} \geq r_{\text{target}} \\
& \mathbf{w}'\iota = 1
\end{aligned}$$

(b) Suppose you are given a set of S possible scenarios for returns. Let r_i^s be the realization of the return of security i in scenario s, and let π_s be the probability of scenarios s. Show that the portfolio semideviation minimization problem can be formulated as the following stochastic optimization problem:

$$\max_{\mathbf{w},\mathbf{R}} \quad (1-\kappa)\sum_{i=1}^{N}\mu_i w_i + \kappa\sum_{s=1}^{S} p_s R_s$$

$$\text{s.t.} \quad R_s \leq \sum_{i=1}^{N} \mu_i w_i, s = 1, \ldots, S$$

$$R_s \leq \sum_{i=1}^{N} r_i^s w_i, s = 1, \ldots, S$$

$$\boldsymbol{\mu}'\mathbf{w} \geq r_{\text{target}}$$

$$\mathbf{w}'\boldsymbol{\iota} = 1$$

Hint: The auxiliary variables R_s, $s = 1, \ldots, S$, keep track of the smaller of the mean return and the realized return in each scenario.

Practice 8.2. Computing VaR Consider two zero-coupon bonds, A and B, each of which has a 3% probability of default over the next year. The prices of both bonds today are \$100, and the bonds will pay \$110 at the end of the year. (If any of the bonds defaults, the payment will be 0, that is, you will lose your original investment.) Assuming that the bonds' defaults are independent events, what is the VaR of a portfolio consisting of a long position in each of the two bonds?

Practice 8.3. VaR and CVaR subadditivity Test whether VaR and CVaR are subadditive risk measures in the following situation. Suppose that we can invest in two assets, A1 and A2, the losses on which follow Pareto distributions with location parameters 1.1 and 1.05, and shape parameters 1 and 1, respectively. Calculate the 95% VaR and the 95% CVaR of a portfolio consisting of \$1 invested in each asset. Next, calculate the sum of the 95% VaRs for each individual asset, and the sum of the 95% CVaRs for each individual asset. Discuss whether the VaR/CVaR of the portfolio is greater than or less than the sum of the VaRs/CVaRs of the individual assets. What are the implications of your findings?

Hints: See the Excel template **Ch8-VaRCVaRSubadditivity.xlsx**. Use the formula `=RiskPareto(1.1,1)` (respectively, `=RiskPareto(1.05,1)`) in @RISK to simulate the losses for each asset.

Practice 8.4. CVaR under normality In section 8.3.1, we stated that the CVaR can be computed as

$$\text{CVaR}_{(1-\varepsilon)} = -\mu_{P/L} + \frac{\varphi(z_{(1-\varepsilon)})}{\varepsilon} \cdot \sigma_{P/L}$$

if we assume that the P/L distribution is normal. Derive this expression from the basic definition of CVaR given at the beginning of section 8.3.

Hints: CVaR is defined as

$$\mathrm{CVaR}_{(1-\varepsilon)}(\tilde{r}) = E\left[-\tilde{r} \mid -\tilde{r} \geq \gamma\right]$$

where γ is the $100(1-\varepsilon)\%$ VaR, that is, the $(1-\varepsilon)$th percentile of the distribution of losses. Based on this definition and the definition of expectation in section 3.6.1, we can write

$$\mathrm{CVaR}_{(1-\varepsilon)}(\tilde{r}) = \int_{-\infty}^{-\gamma} (-r) \cdot \varphi(r) dr$$

where $\varphi(r)$ is the density function of the normal distribution. Recall from section 3.4 that the probability density function for the normal distribution is given by

$$\varphi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where μ is the mean, and σ is the standard deviation of the normal random variable $\tilde{x}$. Plug in the expression for $\varphi(r)$ in the expression for the CVaR, add and subtract μ from the expression $(-r)$ inside the integral, and split the integral into two parts that can be computed separately.

Practice 8.5. CVaR sample optimization In section 8.3.2, we gave the following definition of the $100(1\text{-}\varepsilon)\%$ sample CVaR:

$$\mathrm{CVaR}_{(1-\varepsilon)} = \frac{1}{\lfloor \varepsilon \cdot S \rfloor} \cdot \sum_{s=S-\lfloor \varepsilon \cdot S \rfloor+1}^{S} l_s$$

Here we assumed that the scenarios are organized in increasing order, i.e., scenario S has the largest portfolio loss. Let us think of this CVaR formulation in terms of the distribution of portfolio returns, rather than the distribution of losses. The expression for the $100(1-\varepsilon)\%$ CVaR in that case is

$$\mathrm{CVaR}_{(1-\varepsilon)} = \frac{1}{\lfloor \varepsilon \cdot S \rfloor} \cdot \sum_{s=S-\lfloor \varepsilon \cdot S \rfloor+1}^{S} \left(-r_p^{(s)}\right)$$

where $r_p^{(s)}$ is the portfolio return in scenario s. Now consider the sample CVaR portfolio optimization problem, in which the goal is to find a set of weights $\mathbf{w}$ so that the portfolio CVaR over the S scenarios is minimal. It can be formulated as

$$\begin{aligned} \min_{\mathbf{w}} \quad & \frac{1}{\lfloor \varepsilon \cdot S \rfloor} \cdot \sum_{s=S-\lfloor \varepsilon \cdot S \rfloor+1}^{S} (-\mathbf{r}^{(s)})'\mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}'\iota = 1 \end{aligned}$$

The problem is that we do not know the order of the scenarios for portfolio return in advance. Every combination of weights will result in a different ordering of scenarios in terms of realized losses. We could consider every possible ordering of scenarios, find weights that result in the minimum CVaR in each case, and then select the set of weights that resulted in the minimum CVaR among all estimated CVaRs. This is, however, an optimization problem with an enormous number of constraints. Instead, you are asked to use duality theory to formulate the optimization problem as one with a manageable number of constraints. The final result should be the CVaR optimization formulation from section 8.3.3:

$$\begin{aligned} \min_{\mathbf{w},\xi,\mathbf{y}} \quad & \xi + \frac{1}{\lfloor \varepsilon \cdot S \rfloor} \cdot \sum_{s=1}^{S} y_s \\ \text{s.t.} \quad & y_s \geq -(\mathbf{r}^{(s)})'\mathbf{w} - \xi, s = 1, \ldots, S \\ & y_s \geq 0, s = 1, \ldots, S \\ & \mathbf{w}'\iota = 1 \end{aligned}$$

Hints: Follow the following steps.

1. The problem of finding the K largest components of a vector $\mathbf{v}$ of dimension S has the following linear optimization formulation:

$$\begin{aligned} \max_{\mathbf{z}} \quad & \mathbf{v}'\mathbf{z} \\ \text{s.t.} \quad & \mathbf{z}'\iota = K \\ & \mathbf{z} \leq 1 \\ & \mathbf{z} \geq 0 \end{aligned}$$

2. Explain why. (Note that in the context of CVaR minimization, $K = \lfloor \varepsilon \cdot S \rfloor$ and the S-dimensional vector $\mathbf{v}$ is the vector of portfolio returns $(-\mathbf{r}^{(s)})'\mathbf{w}$.)
3. Find the dual of the problem in (1) (see section 5.5 of Chapter 5). Let ξ be the dual variable corresponding to the constraint

$$\mathbf{z}'\iota = K$$

and let $\mathbf{y}$ be the S-dimensional vector of dual variables corresponding to the constraints

$$\mathbf{z} \leq 1$$

4. Note that since the primal is a maximization problem, the dual should be a minimization problem.

According to optimization duality theory, the maximum of the objective function of the optimization problem in (1) should be equal to the minimum of the objective function in (2). Replace the expression in the objective function of the primal problem with the expression in the objective function of the dual problem. Remember also to add the constraints of the dual problem into the primal problem. You should obtain the formulation from section 8.3.3.

Chapter 9 Practice

Practice 9.1. Index Tracking Using a CVaR Constraint Implement the index tracking problem with CVaR constraint on the tracking error (section 9.3.2) with Excel Solver or MATLAB using the data in the file **Ch9-IndexTracking.xlsx.**

Practice 9.2. Resampling Write VBA (or MATLAB) code for finding the optimal mean-variance weights with resampling (section 9.8) for the portfolio of three stocks using the data in worksheet **Data** the file **Ch9-Cardinality.xlsx.**

Hints: One can include @RISK functions directly in VBA code by selecting **Tools | References** from the main menu in the VBA editor, and checking "AtRisk" in the list of references. However, one can also generate the scenarios with @RISK in advance, save the output to a worksheet, and run through the different estimates of the mean and the covariance matrix as part of the VBA code.

Practice 9.3. Robust Counterpart Derivation Using the optimization duality technique described in section 6.3.1 of Chapter 6 and the discussion in section 9.8, derive the robust counterpart of the mean-variance optimization problem

$$\begin{aligned} \max_{\mathbf{w}} \quad & \mathbf{w}'\boldsymbol{\mu} - \lambda \cdot \mathbf{w}'\Sigma\mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}'\iota = 1 \end{aligned}$$

if the expected returns are assumed to lie in the ellipsoidal uncertainty set

$$U_\delta(\hat{\boldsymbol{\mu}}) = \left\{ \boldsymbol{\mu} \,|\, (\boldsymbol{\mu} - \hat{\boldsymbol{\mu}})' \, \Sigma_{\boldsymbol{\mu}}^{-1} \, (\boldsymbol{\mu} - \hat{\boldsymbol{\mu}}) \le \delta^2 \right\}.$$

Practice 9.4. Robust Counterpart Derivation for the Zero-Net Alpha Adjustment Uncertainty Set Ceria and Stubbs (2006) introduced a structured uncertainty set for the expected returns that they claimed worked particularly well in practice. Their uncertainty set is given by

$$U_\delta(\hat{\boldsymbol{\mu}}) = \left\{ \boldsymbol{\mu} \,\middle|\, \begin{array}{c} (\boldsymbol{\mu} - \hat{\boldsymbol{\mu}})' \, \Sigma_{\boldsymbol{\mu}}^{-1} \, (\boldsymbol{\mu} - \hat{\boldsymbol{\mu}}) \leq \delta^2 \\ \iota' \mathbf{D} \, (\boldsymbol{\mu} - \hat{\boldsymbol{\mu}}) = 0 \end{array} \right\}$$

Note that this uncertainty set is a combination of an ellipsoidal uncertainty set and an additional condition on the expected returns. It was named *zero-net alpha adjustment uncertainty set*, because, instead of assuming that all estimates of the expected returns will take their worst-case values, it assumes that some of them will be below their expected values, and some of them will be above. The matrix $\mathbf{D}$ could be the identity matrix $\mathbf{I}$, in which case the total deviations of the expected returns from their estimates is assumed to be zero, or can be selected to incorporate also information about the variability of each estimate. Using the optimization duality technique described in section 6.3.1 of Chapter 6, show that the robust counterpart of the mean-variance optimization problem if the expected returns ellipsoidal uncertainty fall in the zero-net alpha adjustment uncertainty set is given by

$$\begin{aligned} \max_{\mathbf{w}} \quad & \mathbf{w}'\boldsymbol{\mu} - \lambda \cdot \mathbf{w}'\Sigma\mathbf{w} - \delta \left\| \left(\boldsymbol{\Sigma}_{\boldsymbol{\mu}} - \frac{1}{\iota'\mathbf{D}\boldsymbol{\Sigma}_{\boldsymbol{\mu}}\mathbf{D}'\iota} \Sigma_{\boldsymbol{\mu}}\mathbf{D}'\iota\iota'\mathbf{D}\Sigma_{\boldsymbol{\mu}} \right)^{1/2} \mathbf{w} \right\| \\ \text{s.t.} \quad & \mathbf{w}'\iota = 1 \end{aligned}$$

Chapter 10 Practice

Practice 10.1. Modeling insurance risks[1] Risk measurement and management problems for insurance companies bear similarities to risk management problems of credit risky bond portfolios. Let us consider one specific problem: estimating the VaR of an insurance portfolio pool of identical contracts (e.g., car warranties). Suppose there are N contracts, and the company expects to make a profit of θ on each contract. Suppose also that the insurance losses are independent, and each occurs with probability p. When a loss occurs, its magnitude L is drawn from a lognormal distribution with mean μ and standard deviation σ. Each contract has a deductible D. When a loss occurs, the insurance company will bear a loss of $L - D$ if $L > D$ and a loss of 0 if $L \leq D$.

Write a program in VBA or MATLAB that simulates S scenarios for losses and computes the $100(1 - \varepsilon)\%$ VaR of portfolio losses. Test your program for different values of the input parameters N, θ, p, μ, σ, L, D, and S.

[1] This practice problem is modeled after the discussion in section 9.6 of Dowd (2002).

Chapter 11 Practice

Practice 11.1. Predicting Corporate Bond Spreads[1] In Chapter 11, we discussed mainly applications of factor models in equity portfolio management. However, factor models are widely used in fixed-income portfolio management as well. This exercise asks you to use regression to create a model for predicting corporate bond spreads. The necessary data is in the file **Ch11-BondSpreads.xlsx**.

Part 1. Consider a regression model of the kind where

$$\text{Spread}_i = \beta_0 + \beta_1 \cdot \text{Coupon}_i + \beta_2 \cdot \text{CoverageRatio}_i + \beta_3 \cdot \text{LoggedEBIT}_i + \varepsilon_i$$

where Spread_i is option-adjusted spread (in basis points) for the bond issue of company *i*.
Coupon_i is coupon rate for the bond of company *i*, expressed without considering percentage sign (i.e., 7.5% = 7.5).
CoverageRatio_i is earnings before interest, taxes, depreciation and amortization (EBITDA) divided by interest expense for company *i*.
LoggedEBIT_i is logarithm of earnings (earnings before interest and taxes, EBIT, in millions of dollars) for company *i*.

The dependent variable, Spread, is not measured by the nominal spread but by the option-adjusted spread. This spread measure adjusts for any embedded options in a bond.

Theory would suggest the following properties for the estimated coefficients:

- The higher the coupon rate, the greater the issuer's default risk and hence the larger the spread. Therefore, a positive coefficient for the coupon rate is expected.

[1]This practice is based on data from Rachev, Mittnik, Fabozzi, Focardi, and Jai (2006).

- A coverage ratio is a measure of a company's ability to satisfy fixed obligations, such as interest, principal repayment, or lease payments. There are various coverage ratios. The one used in this illustration is the ratio of the earnings before interest, taxes, depreciation, and amortization (EBITDA) divided by interest expense. Since the higher the coverage ratio the lower the default risk, an inverse relationship is expected between the spread and the coverage ratio; that is, the estimated coefficient for the coverage ratio is expected to be negative.
- There are various measures of earnings reported in financial statements. Earnings in this illustration is defined as the trailing 12-months earnings before interest and taxes (EBIT). Holding other factors constant, it is expected that the larger the EBIT, the lower the default risk and therefore an inverse relationship (negative coefficient) is expected.

Part 2. Now let us include a dummy variable in the regression model. A dummy variable is used to represent categories of data. In this data set, we are given information about whether each bond is investment grade or not. The column "CCC+ and below" in the spreadsheet contains 0 if the bond is investment grade, and 1 if it is not. Including this variable in the regression is simple—it is just added as a regular explanatory variable. The interpretation of its p-value in the final regression output is also the same as for standard numerical explanatory variables. The only distinction is in the interpretation of its coefficient (beta). The coefficient in front of the variable in the final regression equation will tell us if there is a fixed average difference in the spreads of investment-grade and noninvestment grade bonds. Run the regression from part 1 with the added dummy variable, and discuss the output.

Chapter 12 Practice

Practice 12.1. What are the main assumptions of arithmetic random walks? What are their main disadvantages when modeling asset prices?

Practice 12.2. What are the main assumptions of geometric random walks? What is the probability distribution of asset prices that follow geometric random walks?

Practice 12.3. What are the main assumptions of mean reversion? When would you model asset prices using mean-reverting walks instead of arithmetic or geometric random walks?

Practice 12.4. Portfolio VaR and CVaR Estimation The file **Ch12-Portfolio.xlsx** contains data on 100 monthly returns of four stocks: GE, DUK, AIR, and AMD. Use the data to answer the following questions:

(a) Suppose that you are holding an equally weighted portfolio of the four stocks, and that their prices follow correlated random walks. In the right hand side of the spreadsheet, set up a simulation model for the total portfolio return over the next six months (cell N12) by estimating the parameters (drifts, volatilities, correlations) of the 100 historical returns. The starting prices for each stock at the beginning of the 6 months (cells I4:L4) are provided in the spreadsheet.

(b) Using the simulation output from (1), estimate the 95% Value at Risk (VaR) and the 95% CVaR of the portfolio. (Assume that the initial portfolio value is $100,000.)

Practice 12.5. Evaluating a Simple Trading Strategy Let us evaluate a simple trading strategy whose goal is to limit downside risk. (Such strategies are called stop-loss strategies.) Suppose you are holding one share of stock. You sell the stock as soon your loss (relative to the original price of stock) exceeds x% of the original stock value; otherwise you keep the stock. Once you have sold the stock, you do not buy it back for three months. If you have not sold the stock during the three months, you sell it at the end. A template for the model is provided in the file **Ch12-StopLoss.xlsx**.

1. Complete the model for 63 days ahead (a quarter of a year in terms of trading days), and compute the simulated total profit from the trading strategy (cell E4) over 1,000 scenarios for the stock price. (Ignore discounting.) All cells with colored borders should be filled out. (Some of them already have been filled out for you.) Skip cell E3 for now. Assume the following: During those 63 trading days, the stock price will follow a geometric random walk. The annual drift μ, however (in cell C3), is unknown to you, and may take any of three values with equal probabilities: -0.2, 0 and 0.1. The annual volatility σ of the process is 0.5.
2. You are considering three possible values for the threshold percentage x for the trade (x = 5%, 1%, or 0.5%). Simulate the outcomes of the trading strategy for the three different values of x, and discuss which "optimal" x you would select. (Do not discuss whether or not you would implement the trading strategy itself; assume that you need to implement this strategy, and specify which x you would pick.) Recall that there are multiple ways to define "optimal" when we are dealing with uncertainty; please present as broad discussion as possible.
3. In cell E3, store the profit from a *holding strategy*, that is, a strategy of holding the stock until the end of the 63-day period, and selling the stock in day 63. Discuss the advantages and disadvantages of the holding strategy versus your "optimal" trading strategy from (1). Use as many simulation and statistical tools that are relevant for your analysis as you can remember.

Chapter 13 Practice

Practice 13.1. Bond Arbitrage Let us make the example in section 13.4.1 more realistic. Consider a collection of five bonds with bid and ask prices as shown in Exhibit P13.1. (See also file **Ch13-Arbitrage.xlsx**, worksheet **Arbitrage (2)**.) Is there an arbitrage opportunity? State the exact strategy you would pursue.

Practice 13.2. Using Optimization to Identify Arbitrage Opportunities in the Currency Market Suppose you are given the exchange rates for four currencies: U.S. dollar, Euro, British pound, and Japanese yen. An arbitrage opportunity in this context would be, for example, if you could convert one dollar to euro, then to yen, then to pound, then back to dollar, and end up with more than one dollar. Write an optimization problem formulation that will allow you to detect such arbitrage opportunities.

Hint: Introduce variables D (number of dollars that can be generated by arbitrage), DE (quantity of dollars to convert to euro), DP (quantity of dollars to convert to pounds), etc. Your objective function should be to maximize D, and the constraints should involve the observed exchange rates.

Practice 13.3. Constructing a Hedging Strategy A landlord typically purchases $10,000 gallons of heating oil for the coming winter in September. New rent rates are negotiated at the end of June, and take effect on July 1every year. (Heating is included in the rent.) Obviously, if the price of heating oil increases between June and September, the landlord will lose money. How can the landlord protect himself against adverse movements in the price of heating oil? Describe several possible hedging strategies, and their advantages and disadvantages.

	A	B	C	D	E	F	G	H	I
1	Arbitrage								
2									
3	**Decision variables**	Bond 1	Bond 2	Bond 3	Bond 4	Bond 5			
4	long								
5	short								
6									
7	**Objective function**								
8	ask prices	$102.81	$111.28	$97.39	$115.10	$97.08	cash flow today		
9	bid prices	$102.16	$110.33	$96.64	$114.55	$96.43	$ -		
10									
11	**Constraints**						Total		Required
12	*t*=1	$ 2.50	$ 5.00	$ 3.00	$ 4.00	$ 3.50	$ -	>=	$ -
13	*t*=2	$ 2.50	$ 5.00	$ 3.00	$ 4.00	$ 3.50	$ -	>=	$ -
14	*t*=3	$ 2.50	$ 5.00	$ 3.00	$ 4.00	$ 3.50	$ -	>=	$ -
15	*t*=4	$ 2.50	$ 5.00	$ 3.00	$ 4.00	$ 3.50	$ -	>=	$ -
16	*t*=5	$ 102.50	$ 5.00	$ 3.00	$ 4.00	$ 3.50	$ -	>=	$ -
17	*t*=6	$ -	$ 105.00	$ 3.00	$ 4.00	$ 3.50	$ -	>=	$ -
18	*t*=7	$ -	$ -	$ 103.00	$ 4.00	$ 3.50	$ -	>=	$ -
19	*t*=8	$ -	$ -	$ -	$ 104.00	$ 103.50	$ -	>=	$ -
20	bound						$ -	>=	$ (100,000.00)

EXHIBIT P13.1 Data for problem Practice 13.1.

Cases

Case 13.1 Evaluating the Risk of a Futures Hedging Strategy[1]

Topics: Simulation, Hedging, Futures
Data file: **Ch13-HedgingOil.xlsx**

Suppose that a landlord needs to purchase 10,000 gallons of heating oil for the year on November 1. There are futures contracts on heating oil with delivery dates on September 1 and December 1 that year. In order to hedge his exposure, the landlord decides to purchase futures contracts with the December delivery date. (This is because if he purchases futures contracts with a delivery date that is before the date on which he needs to purchase heating oil, he is not protected against adverse movements in the price of oil between the time the futures contract expires and the time he needs to purchase the oil.) Of course, the hedge with December futures will not be perfect. However, we can see that holding a futures contract with a December delivery date makes the changes in heating oil price between July 1 and November 1 largely irrelevant by playing out the different scenarios. Suppose the price of heating oil increases, and the landlord closes his futures contract on November 1. The increase in the cost of heating oil is bad for the landlord, but the value of the futures contract on heating oil he holds increases as well, because he has the right to receive a more valuable commodity. So, he can

[1]This case is based on Winston (2000).

	A	B	C	D	E	F	G
1	**Hedging purchase of heating oil**						
2							
3	Oil price per gallon, July 1	$ 2.00					
4	Number of gallons to buy on 1 Nov	10,000					
5	Risk-free rate r	5.00%					
6	Volatility	0.0848					
7	Oil price drift	1.4450					
8	Oil price speed of adjustment	0.0340					
9	December futures price on 1 July	$ 2.04					
10	Duration of futures (in years)	0.4167					
11	# of long futures positions	25,000					
12	Spot oil price on 1 Nov	$ 1.85					
13	Theoretical price of December future on 1 Nov	$ 1.86					
14	Std dev of percentage variation of futures from theoretical price	0.05					
15	Actual futures price on 1 Nov	$ 1.77					
16							
17	PV of cost of buying oil on 1 Nov	$ 18,478.84					
18	PV of revenue from selling futures on 1 Nov	$ 44,281.01					
19	Cost of buying futures on June 8	$ 51,052.59					
20	PV of total cost	$ 25,250.42	<-----	This is a cost, so the goal is to minimize it.			

EXHIBIT P13.2 Oil hedging model in file Ch13-HedgingOil.xlsx.

offset the increased cost of heating oil by selling the futures on November 1. Conversely, if the price of heating oil decreases between July and November, the drop in the cost of heating oil decreases the value of the futures the landlord holds, but it also decreases his cost of purchasing oil.

How many futures contracts should the landlord buy to create a hedging strategy with a minimum risk and as low cost as possible? Assume that the price of heating oil on July 1 is $2 per gallon. Based on historical data, the price of heating oil follows a mean reverting process with drift μ =1.4450, volatility $\sigma = 0.0848$, and speed of adjustment $\kappa = 0.0340$. The annual risk-free interest rate is $r = 5\%$.

The file **Ch13-HedgingOil.xlsx** has been set up to create a model for computing the optimal hedging strategy (see Exhibit P13.2.) We consider several possible quantities for the number of futures to buy (cell B11): 25000, 20000, 15000, 10000, 5000, and 0. In @RISK, this can be specified with the formula

```
=RiskSimtable({25000,20000,15000,10000,5000,0}).
```

Based on the quantity of futures to buy, the price of the futures today (the cost of the hedging strategy), the value of the futures on November 1 (when they will be sold), and the price of oil on November 1, we can compute the total cost to the landlord (cell B20). Note that the latter two quantities are unknown, and need to be simulated.

The price of oil on November 1 (cell B12) can be found as the future value of a mean reverting process. The theoretical price of the December futures on November 1 (cell B13) is the spot price of oil (cell B12) times the factor $e^{r\cdot(1/12)}$. (The factor of 1/12 is there, because there is one month left between November 1 and December 1 when the futures expire. The price of the futures on 1 November should be determined by how long is left on the contract.) Since there will be other factors influencing the futures price several months away from 1 July, we include some variability in the theoretical price estimate—we assume that on average, the actual market price of the futures will be equal to the theoretical futures price, but may deviate from it by 5% on average.

Assignment

(a) Study the spreadsheet with the heating oil hedging example, and make sure you understand how all quantities are determined.
(b) Run a simulation to evaluate the number of futures to buy so that the landlord can achieve the hedge with the minimum risk and as low expected cost as possible. (There are different definitions of risk; try to use a couple of different ones.)
(c) Repeat the exercise if the price of oil is assumed to follow a geometric random walk with drift parameter $\mu = 0.12$ and volatility $\sigma = 0.15$ (rather than a mean reversion process).

Case 13.2 Analysis of Option Hedging Strategies[2]

Topics: Simulation, Hedging, Options, Option Delta

Suppose that you are in the position of a financial institution that has sold a European call option on 100,000 shares of a nondividend-paying stock. (For simplicity, assume that the financial institution has sold 100,000 European call options, each of which entitles the option holder to buy a single share of stock.) Assume that the stock price is $49, the strike price is $50, the risk-free rate is 5% per annum, the stock price volatility is 20% per annum, the time to maturity is 20 weeks (0.3846 years), and the expected return from the stock is 13% per annum.[3] The Black-Scholes price for each option is $2.40 (verify!), and the contracts sold for a total of $240,052.73.

[2]This case is based on Hull (2008) and work by Nick Kyprianou, Jason Aronson, and Rohan Duggal.
[3]Financial institutions typically do not write call options on individual stocks; however, this example is deliberately simple in order to illustrate some important concepts and terminology in option hedging strategies.

The financial institution is faced with the problem of hedging the risk from its short position in the call options. In this case, you will examine several strategies:

- Nonhedged, "naked," position
- Covered call strategy
- Stop-loss hedging strategy
- Static delta strategy
- Dynamic delta strategy without transaction costs
- Dynamic delta strategy with transaction costs

Let us explain each strategy in more detail:

- *Naked position*. A naked position consists of selling the call options and doing nothing; this position is not a hedge. It is a good idea to simulate the payoff of this strategy, so that the others can be benchmarked against it.

  In week 0 (think of it as end of week 0), we sell 100,000 call options for a total revenue of $240,052.73. At the end of week 20, one of two things can happen. If the stock price is less than the strike price, the call options will not be exercised and we will retain the $240,052.73. However, if the stock price is greater than the strike price, the call options will be exercised. We (the seller) will be obligated to purchase 100,000 shares at the current market price in week 20 and immediately sell the shares to the option holder at the lower strike price of $50 per share. We will lose a dollar amount equal to the difference between the current share price and the stock price, multiplied by the number of shares that need to be purchased.
- *Covered position*. A covered position involves hedging call options bought or sold with the purchase or sale of shares equal to the number of call options sold. In our case, in week 0, 100,000 shares of stock are purchased at the current market price of $49 per share, for a total of $4,900,000. In order to purchase the stock, $4,900,000 must be borrowed, and interest must be paid each period on the cumulative debt held. We can assume, however, that at least some of the amount received from selling the options will be applied towards buying the shares. For simplicity, let us assume that all of it ($240,052.73) will be used for that purpose.

  If the stock price is below the strike price of $50 in week 20, we keep the option proceeds and sell the 100,000 shares at market price in week 20. However, if the stock price in week 20 is above the strike price, the call options will be exercised. We will keep the option proceeds, and sell the 100,000 shares at the strike price.

- *Stop-loss strategy*. A stop-loss strategy in this context involves buying one unit of stock as soon as the stock's price rises above the strike price, and selling a unit of stock as soon as its price falls below the strike price. The trading scheme is designed in such a way that the institution owns the stock if the option closes in the money, and does not own it if the option closes out of the money. Of course, the delay in the purchasing and sales of stock incurs additional costs. Ideally, the purchases and sales will happen within some very small interval after the stock price changes from above the strike price to below the strike price, or vice versa. In this case, assume that the time interval is one week, to be consistent with the implementation of the other strategies.
- *Static delta hedging*. Static delta hedging involves setting up an initial hedge and not adjusting for future changes in delta resulting from changes in the stock price. If we use the Black-Scholes pricing model and the definition of delta as the change in option price divided by the change in stock price, it can be shown that the value of delta is given by $\Phi(d_1)$.

  The initial position in the static delta hedge is set by multiplying the delta by the number of call options sold, which determines the number of shares to purchase in week 0. Interest is paid for the money borrowed, to cover the purchase of the stock each week. In week 20, the position is closed out and the shares are sold at the current market value or at the strike price if the options expire in-the-money. The gains or losses from the sale of the stock are added to the interest paid to determine the effectiveness of the hedge.
- *Dynamic delta hedging*. On an intuitive level, static hedging will not provide a good hedge, as market prices are always changing. Therefore, a position must be constantly adjusted to retain an optimal hedge. This is known as rebalancing. It is easy to compute from the Black-Scholes formula that an increase in the stock price will lead to an increase in the delta. The investor will then need to add more shares to his portfolio.

  In our example, the dynamic delta hedge's initial position is calculated in the same manner as a static delta hedge. However, the dynamic approach requires delta to be calculated each week due to the changing stock prices. Each stock position, following the initial, is calculated by multiplying the change in delta between the current and previous period by the number of call options sold. Buy orders for shares are placed if delta increases, while sell orders are placed if delta decreases. Shares are purchased to offset the losses from the call options through price appreciation; conversely, the exposure of holding a long position is decreased by selling shares in case the stock price drops below the strike price.

As with static delta hedging, stock is purchased with loans that require weekly interest payments. The stock position is closed out in week 20, and the gains or losses of the stock are added to the gains or losses of the call options.

- *Dynamic delta hedging with transaction costs.* The final assignment is to implement the dynamic delta hedge, but assuming that each stock trade requires a $7 flat fee. In addition, a $0.01 transaction cost per share is incurred when a share is traded. The transaction costs have a strong impact on the effectiveness of the trading strategy.

Assignment Your goal is to find the strategy that will minimize the risk of the financial institution you represent (risk could be variability, probability of loss, amount of loss, etc.), and will achieve that at a relatively low cost. For the purposes of this exercise, assume that the stock price follows a geometric random walk with the parameters given at the beginning of the case. Simulate the stock price in each of the 20 weeks, and keep track of the (random) payoff of the different strategies. Create a table with the simulation output for the different strategies, and analyze their advantages and disadvantages.

Chapter 14 Practice

Practice 14.1. European Put option Using the crude Monte Carlo code for computing the price of a European call option under the assumption that the underlying asset follows a geometric random walk, create a VBA/MATLAB function that computes the price of a European put option under the same assumption.

Practice 14.2. Lookback Option A lookback option gives the option holder the right to buy the underlying stock at any price that occurred during the life of the option. A call option holder will therefore choose to exercise at the highest price that occurred during the life of the option, $S_{\max}$, and the payoff to the option holder will be

$$\max\{S_{\max} - K, 0\}.$$

A put option holder will choose to exercise at the lowest price that occurred during the life of the option, $S_{\min}$, and the payoff to the option holder will be

$$\max\{K - S_{\min}, 0\}.$$

A lookback option contract can have additional features, such as an early exercise provision. It is generally an expensive option, since the option holder has a lot of flexibility, whereas the option writer takes on a lot of risk.

Create a spreadsheet model/VBA/MATLAB program that computes the value of a lookback call option if the underlying asset is assumed to follow a geometric random walk with prespecified parameters, and compare the value of the lookback call option with the value of a European call option with the same strike price and time to maturity.

Practice 14.3. European Option Pricing under Stochastic Interest Rates Recall that when we price a European call option, we simulate the price of the underlying stock so that it grows at the risk-free rate. This risk free rate is assumed to be constant over the life of the option. Now let us consider an extension of this model in which the interest rate r follows a mean reverting process of the kind

$$r_{t+1} = r_t + \kappa_r \cdot (\mu_r - r_t) + \sigma_r \cdot \tilde{\varepsilon}_t^{(r)},$$

whereas the stock price is assumed to follow a geometric random walk as usual:

$$S_{t+1} = S_t + (r_t - 0.5 \cdot \sigma^2) \cdot S_t + \sigma \cdot S_t \cdot \tilde{\varepsilon}_t.$$

Under these conditions, we can find the value of a European call option with strike price K and time to maturity T as follows:

1. Simulate a path for r_t and a path for S_t given the realizations of r_t at each point in time.
2. Calculate the payoff of the option as $\max\{S_T - K, 0\}$.
3. Discount the payoff from (2) by using all the realizations of r_t along this path, that is, use the discount factor

$$e^{-(r_0+\ldots+r_{T-1})}.$$

4. Repeat (1)–(3) many times (e.g., 10,000 trials), and compute the average of the discounted payoffs in all the trials.

Assignment Consider a European call option with strike price 60 and time to maturity 1 year. Suppose the underlying stock has volatility 0.4 and initial price 55.

(a) Compute the price of a European call option by crude Monte Carlo simulation if the underlying annual interest rate is assumed constant at 0.05 per year.
(b) Compute the price of a European call option by crude Monte Carlo simulation if the underlying annual interest rate is assumed to follow a mean reversion process with long term mean 0.05 per annum, speed of adjustment 0.0045, and volatility 0.0065 per annum.

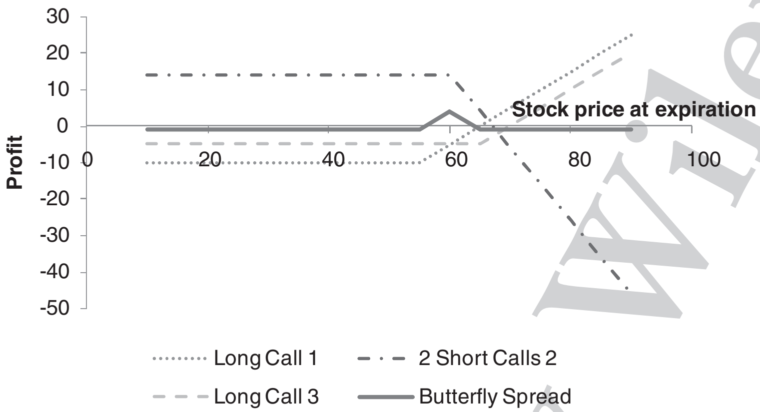

EXHIBIT P14.1 Butterfly spread using call options

Practice 14.4. Pricing a Butterfly Spread with Antithetic Variables[1] A butterfly spread is a trading strategy which consists of four European call options on the same asset and with the same maturity, but with different strike prices. It is created by:

- Buying a call option with a low strike price K_1.
- Buying a call option with a high strike price K_3 ($K_3 > K_1$).
- Selling two call options with a strike price K_2 that is midway between K_1 and K_3.

The payoff of a butterfly strategy is illustrated in Exhibit P14.1. Note that the strategy results in a profit if the final stock price is close to K_2, but leads to a small loss if the stock price is significantly away from K_2 in either direction. A butterfly strategy is appropriate for an investor who thinks that large stock price movements are unlikely.

Consider a butterfly spread for which $K_1 = 55$, $K_2 = 60$, and $K_3 = 65$. The time to maturity for all options is 6 months. The current price for the underlying stock is \$58, the annual volatility for the stock price is $\sigma = 0.5$, and the annual risk-free interest rate is 10%.

(a) Given that the butterfly spread is a combination of European call options, its payoff can be computed easily. Price the butterfly spread using the Black-Scholes formula.

[1]This practice problem is adapted for simulation from Hull (2008).

(b) Price the butterfly spread using simulation. Assume that the underlying stock price follows a geometric random walk. Repeat 100 times, and record the expected prices obtained in each run. Compute the variance of the expected price in the 100 runs.

(c) Price the butterfly spread using simulation with antithetic variables. In other words, generate a set of values for the standard normal variable $\tilde{\varepsilon}$ in the geometric random walk formula for the future stock price, and then generate a set consisting of the negative of these values. Compute the payoffs in both cases, and take the pairwise average of all the payoffs to compute the final payoff of the option. Repeat the procedure 100 times, and record the expected prices obtained in each run. Compute the variance of the expected price in the 100 runs. Is it larger or smaller than the variance you found in (2)?

Practice 14.5. Pricing an Arithmetic Average Asian Option with Control Variates In section 14.2.4, we showed how to price an arithmetic average Asian option using the price of a geometric average Asian option as a control variate. Price an arithmetic average Asian option using the sum of asset prices over the life of the option,

$$\sum_{m=0}^{M} S(t_m),$$

as a control variate, where M is the number of steps between the starting date and maturity.[2] This is a logical control variate, since the sum of the prices over the life of the option is clearly correlated with the option payoff. Assume that the asset price follows a geometric random walk. Is using the sum of the asset prices over the life of the option estimate better or worse for variance reduction than using the price of the geometric average Asian option? (See section 14.2.4 and Chapter 14's Software Hints.)*Hints*: In order to compute an estimate for the price of the arithmetic average Asian option, you need a closed-form solution for the sum of the prices over the life of the option. Note that

$$E\left[\sum_{m=0}^{M} S(t_m)\right] = \sum_{m=0}^{M} S_0 \cdot e^{r\cdot(m\cdot\Delta t)} = S_0 \cdot \sum_{m=0}^{M} \left(e^{r\cdot\Delta t}\right)^m = S_0 \cdot \frac{1 - e^{r\cdot\Delta t\cdot(M+1)}}{1 - e^{r\cdot\Delta t}}.$$

[2]Since we are trying to price an option, you should assume that the asset price paths are generated under the risk neutral measure, i.e., that the drift is the risk-free rate r.

Here we used the formula for a sum of geometric series,

$$\sum_{i=0}^{M} q^i = 1 + q^1 + \ldots + q^M = \frac{1 - q^{M+1}}{1 - q},$$

with

$$q = e^{r \cdot \Delta t}.$$

Practice 14.6. Pricing an Asian Call Option with Quasi–Monte Carlo Methods Implement the example in section 14.3.5. Create a function that computes the price of an arithmetic Asian call option with time to maturity of 1 year. Assume that the underlying asset follows a geometric random walk with a given drift μ and volatility σ, and we would like to sample the price monthly. Use the Halton sequence or the Sobol sequence.

Practice 14.7. Pricing an American Put Option with a Regression-Based Method When the Underlying Asset Follows a Mean-Reversion Process with Jumps[3] In section 12.5.2 of Chapter 12, we discussed how to simulate a geometric random walk with jumps. Assume that the underlying asset for an American put option follows a geometric random walk with jump intensity λ per year. Write VBA or MATLAB code that computes the fair price of the American option with a regression-based method using the same basis functions as in section 14.4.2.

Practice 14.8. Deriving a Pathwise Estimator for the Delta of an Arithmetic Asian Call Option In section 14.4.3, we gave a closed-form expression for the delta of an arithmetic Asian call option when the underlying asset price follows a geometric random walk. Derive this expression (the derivation is similar to the derivation of the delta for a European call option) and write VBA or MATLAB code to estimate it in a simulation.

[3]This example is modeled after an example in Longstaff and Schwartz (2001).

Chapter 15 Practice

Practice 15.1. Pricing Pass-Through Using Simulation Assume that the short rate follows the CIR model with particular parameters. Generate paths for the interest rates, assume the Arctangent prepayment model (see section 15.4.1), and find the price of the pass-through RMBS described in section 15.3.1. Analyze the distribution of the present value of the cash flows from the RMBS, the distribution of its average life, as well as the sensitivity of the RMBS price to the assumptions made on the process followed by the interest rate.

Practice 15.2. Structuring an MBS Using Dynamic Programming Suppose that we would like to find the optimal size of an RMBS with four tranches and the characteristics of Structure 1 from section 15.3.3, but assuming this is a nonagency deal and that the expected default rate is 0.075% per month. Implement the dynamic programming algorithm described in Chapter 15.6, and find the optimal tranche sizes. You will need to also research data on Treasury yields and spreads. Assume that the loss multiple for rating AAA is 6, for rating AA is 5, for rating A is 4, for rating BBB is 3, for rating BB is 2, for rating B is 1.5, and for rating CCC is 0.

Chapter 16 Practice

Practice 16.1. Performance of equity protective put strategy Assuming that the price of a stock follows a geometric random walk, analyze the risk profile of a protective put strategy. Run a simulation to estimate its risk and return compared to simply holding a long position in the stock. Use several different measures of risk.

Chapter 18 Practice

Practice 18.1. Gold Mine[1] Suppose your company is considering leasing a gold mine for eight years. Each year, up to 20,000 ounces of gold can be extracted at a cost of $450 per ounce. The current price of gold is $668.25 per ounce. The risk-free rate is 4.46%, and the historical volatility of gold prices is 15% per annum (both compounded annually).

(a) Create a binomial tree for the process followed by the gold price for eight years.

(b) Develop additional tree models, and use them to answer the question of how much this lease is worth to your company. Note that your company will have the flexibility not to mine in any year in which the cost per ounce of extracting gold is higher than the market price of the gold, so make sure this is incorporated in the decision model.

[1]This practice is based on Luenberger (1998) and Winston (2000).

Appendix D Practice

Practice D.1. To practice simple function and subroutine use in VBA, extend the factorial subroutines and user-defined function examples in Appendix D to computing binomial probabilities and cumulative probabilities.

Given a set of inputs (value for k, number of trials, and probability of success), create a subroutine and a user-defined function that compute the probability that the binomial random variable takes the value k and the cumulative probability up to the value k. The PMF for the binomial distribution was given in section 3.3 of Chapter 3.

Note that there is an Excel function (`BINOMDIST`) that computes the same quantities. Use the built-in Excel function to double-check the answers you obtain with your user-defined function or subroutine.