# Blockchains & Distributed Ledgers

Lecture 09

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Slide credits: AK, Michele Ciampi, Dimitris Karakostas

# Security critical computations

- How to obtain the output of a security critical computation
- Deterministic with public inputs?
  - Repeat multiple times and consensus can be reached about its output
  - Example: blockchain systems with smart contracts
- What if it is probabilistic with public inputs?
  - Coin flipping protocol
- What if it uses private data?
  - Secure Multiparty Computation (MPC)

# Secure Multiparty Computation and Applications

- Sharing responsibility for signatures and cryptographic keys
  - Secret sharing
- Security critical computations
  - Coin flipping and verifiable secret-sharing
  - Secure multiparty computation (MPC)
- Fair swaps and fair MPC

# Secret sharing

# Overarching question

- How to protect security critical operations?
- Key idea: share responsibility and somehow tolerate faulty participants
  - Cryptographic keys?
  - Cryptocurrency addresses?
  - Computations?
  - What about computations on private data?

# Multi-sig transactions

- Multi-sig: a tx that can be redeemed if n parties sign it
- A payment to a script (P2SH) can facilitate a multi-signature transaction

```
scriptPubKey: OP_HASH160 <redeemscriptHash> OP_EQUAL
```

```
scriptSig: OP_0 <sig_Ai> … <sig_An> <redeemscript>
```

```
redeemscript = OP_m <A1 pubkey> <A2 pubkey>… <An pubkey>
<OP_n> <OP_CHECKMULTISIG>
```

# Secret-Sharing

Main question:

- How to share a secret *s* to *n* shareholders so that:
  - Any subset including *t* of them can recover the secret
  - Any subset including *less than t* of them knows nothing about the secret

- Relative questions:
  - Can we solve this for any *n* and *t* <= *n*?
  - What is the relation between the size of *s* and the size of each share?

# Finite fields

- Finite *sets* equipped with *two operations*, behaving similarly to addition and multiplication over the real numbers (which is an infinite field)
- Finite fields exist with number of elements equal to $p^k$, for:
  - any prime number p
  - any positive integer k

*Example*. A binary finite field {0, 1} with:

| + | 0 |
|---|---|
| 0 | 0 |
| 1 | 1 |

(a+b) mod 2

| * | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

(a*b) mod 2

# Secret-Sharing over a finite field

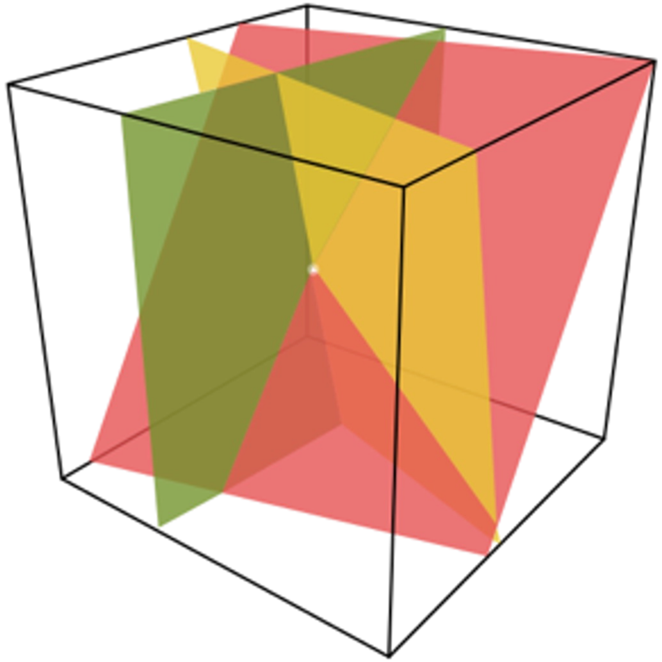
- Consider a secret x and *N* random values, subject to the constraint:

$$\sum_{i=1}^{N} x_i = x \quad \text{(over a finite field)}$$

- This is called (additive) secret-sharing
- Knowledge of *any* N-1 values cannot be used to infer any information about *x*

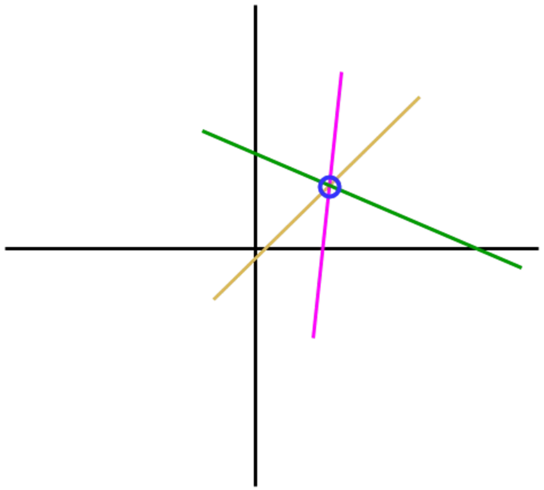
# Analysis

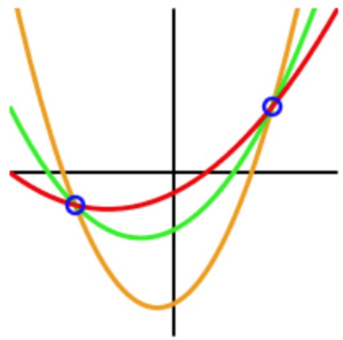
- Example: binary field
- If you hold only *N-1* values $[x_2, \ldots, x_N]$:
  - Two unknowns: $x_1$, $s$
  - One equation: $x_1 + x_2 + \ldots + x_N = s$
- $s$ cannot be undetermined
  - $s = 0 + x_2 + \ldots + x_N$ (if $x_1 = 0$)
  - $s = 1 + x_2 + \ldots + x_N$ (if $x_1 = 1$)



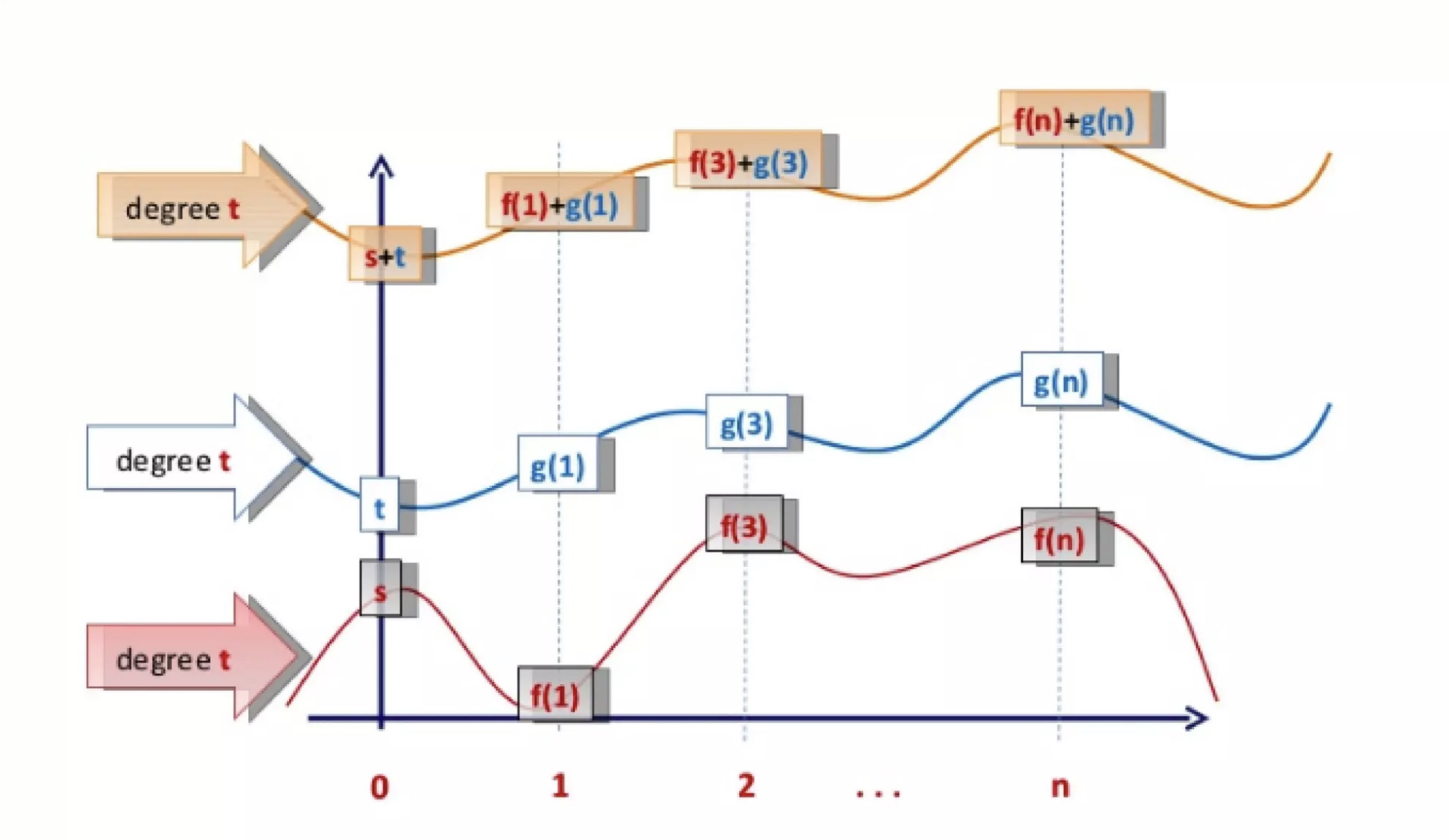
# Generalisation t-out-of-n

- Consider a polynomial of degree *d*: $\boldsymbol{p(x) = a_0 + a_1x + \ldots + a_d x^d}$
- Any *d+1* points of the polynomial completely determine it
- With *d* or less points, at least one degree of freedom remains
  - *p* cannot be fully determined
- We can use that idea to solve secret-sharing for any t, n





# Generalisation t-out-of-n



# Example

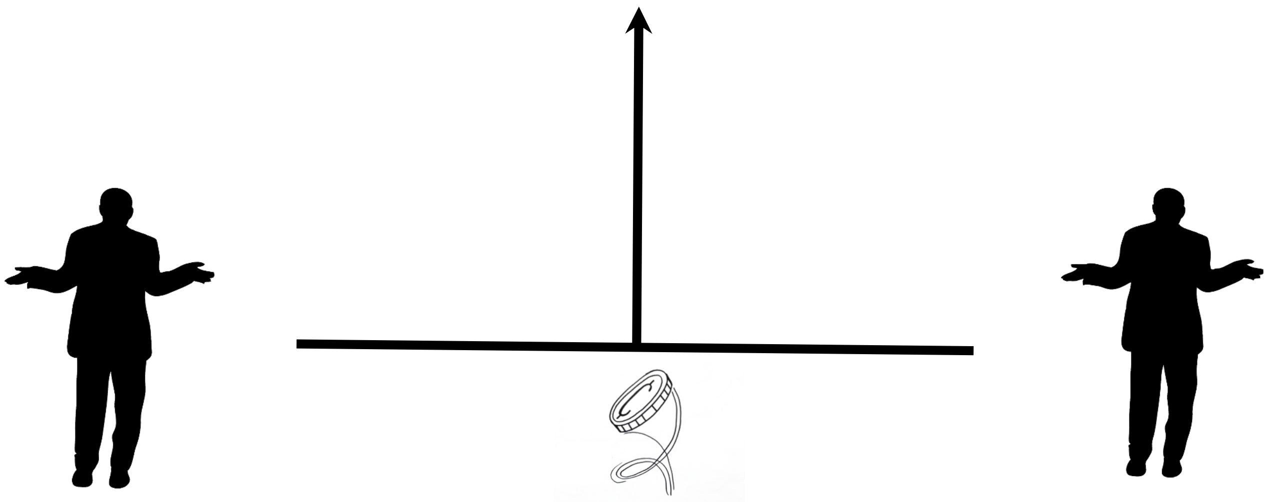
- 5 parties
- Polynomials of degree 2
- Any three parties (who hold 3 points) can interpolate such polynomials
- Any two parties have no information about the shared secret

# Secret-sharing cryptographic keys

- Using polynomial secret-sharing, a cryptographic key can be split to multiple shareholders
  - Each shareholder gets a point on the plane
  - The secret/key is the solution to the polynomial problem
- Additional points to consider:
  - How should the value of *t* be determined:
    - in comparison to *d*?
    - in comparison to n?
  - To engage in the cryptographic operation, is it necessary to reconstruct the original key?
  - How to accomodate an evolving set of shareholders?

# Distributed Randomness Generation

$$b \xleftarrow{\$} \{0, 1\}$$



# Application: coin-flipping

- Alice and Bob want to flip a coin remotely
  - output a bit uniformly at random
- Alice doesn't trust Bob and vice versa
  - neither Alice nor Bob should be able to bias the bit choice

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- Alice and Bob want to flip a coin remotely
  - output a bit uniformly at random
- Alice doesn’t trust Bob and vice versa
  - neither Alice nor Bob should be able to bias the bit choice
- Solution:
  - Alice commits to a random coin
  - Bob commits to a random coin
  - Alice and Bob open the commitments
  - Output = XOR of (committed) values
- Consider:
  - Can the situation be improved in an *N* party coin flip?
  - What about when >N/2 parties are honest?
  - How do you deal with (selective) aborts?

# A first step towards multi-party coin flipping

- Each player commits to their coin (publicly)
- Each player publishes a secret-sharing of the opening to their commitment
  - Any subset of at least (N/2 + 1) players can reconstruct the opening
  - Shares should be encrypted with the respective public-keys of the parties
- If some parties abort the protocol: assuming that a subset of $>N/2$ parties continue, they can recover the share and terminate
- Any number of parties up to N/2 cannot gain any advantage over the honest parties

# What if some parties announce incorrect shares?

- A secret cannot be retrieved from incorrect shares
- Selective aborts possible, as remaining parties cannot reconstruct the secret
- Possible solution: require that all commitments open at the end irrespectively of aborts
  - deviating players will be caught, but still have the option to selectively abort if they wish
  - other parties will only know of the abort when it is too late
- One possible approach: issue monetary penalties to those that abort

# Publicly Verifiable Secret-sharing (PVSS)

- The dealer creates shares that are distributed in encrypted form
- The shares can be **publicly verified** as correct
- Verifiability should not leak information about the secret

- PVSS enables parties to detect improper share distribution at the onset
- Protocol can still be aborted, but any abort would be independent of the (random) coin!

# PVSS Design Challenges

- Assuming:

$$\sum_{i=1}^{N} x_i = x \qquad \begin{array}{r} \psi_i = \mathcal{E}_i(x_i) \\ \psi = \mathbf{Com}(x) \end{array}$$

- Verify that the value encrypted in $\psi_i$ satisfies the equation w.r.t. the values encrypted in $\psi$
- This problem can be solved using a **zero-knowledge proof**

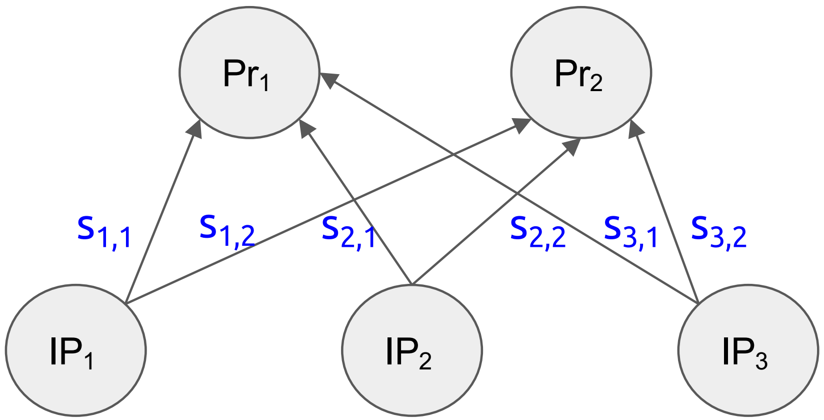
# Secure MPC

# Secure Multiparty Computation

- (Secure) Multiparty Computation (MPC)
- Parameterized by function $f$
- A set of $n$ parties $P_i$ contribute inputs $x_1, x_2, \ldots, x_n$
- At the end of the protocol they compute $f(x_1, x_2, \ldots, x_n)$
  - Everyone receives output $f(x_1, x_2, \ldots, x_n)$
  - No party except $P_i$ obtains information about $x_i$

# MPC Construction Idea

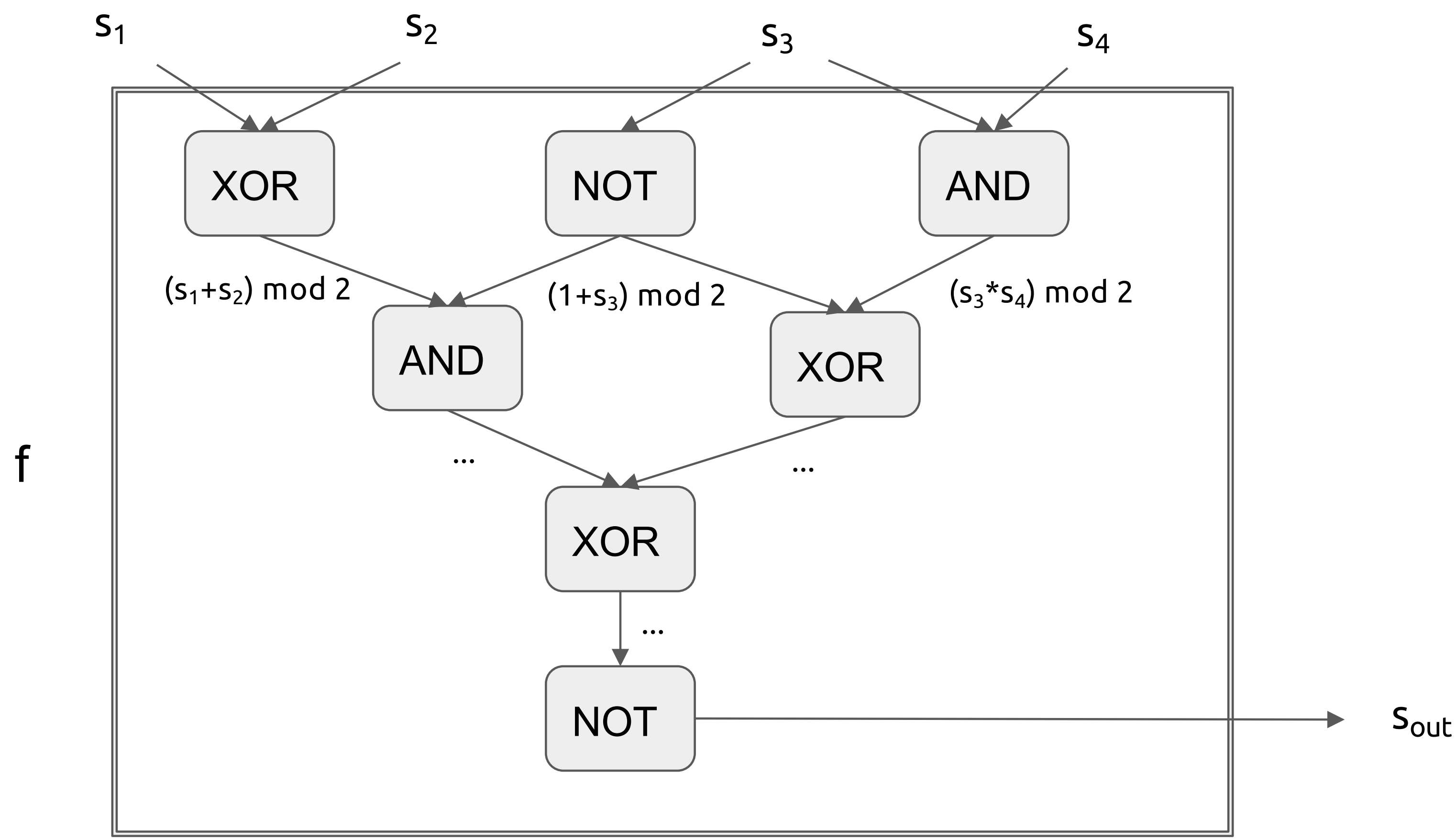
- Consider three roles
  - Input providers
  - Processors
  - Output-receivers
- Input providers secret-share their input to the processors
  - Additive secret-sharing



# MPC Construction Idea

- Any function *f* can be expressed as a Boolean circuit
  - Fixed-size input
  - Upper-bound on number of steps (circuit depth)
  - Example: any boolean function can be implemented as a combination of NAND gates
- XOR, AND, NOT gates
- Arithmetic representation of gates
  - AND: Input: a, b; Output: (a*b) mod 2
  - XOR: Input: a,b; Output: (a+b) mod 2
  - NOT: Input: a; Output: (1+a) mod 2
- Each processor executes the circuit with their shares as input
  - How to implement the gates s.t. operations on shares, when combined, produce the correct aggregate output?

# MPC Construction Idea, Example



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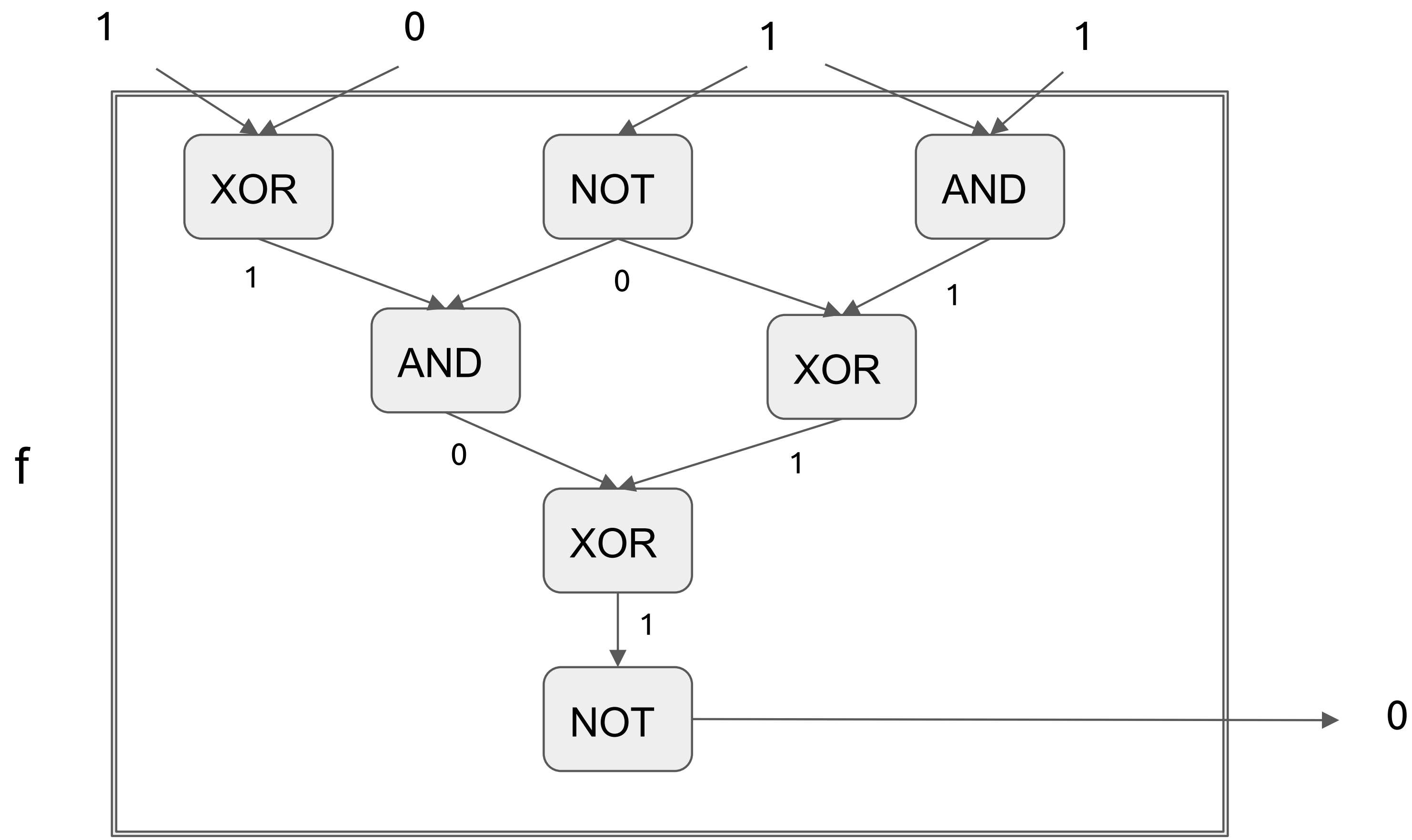

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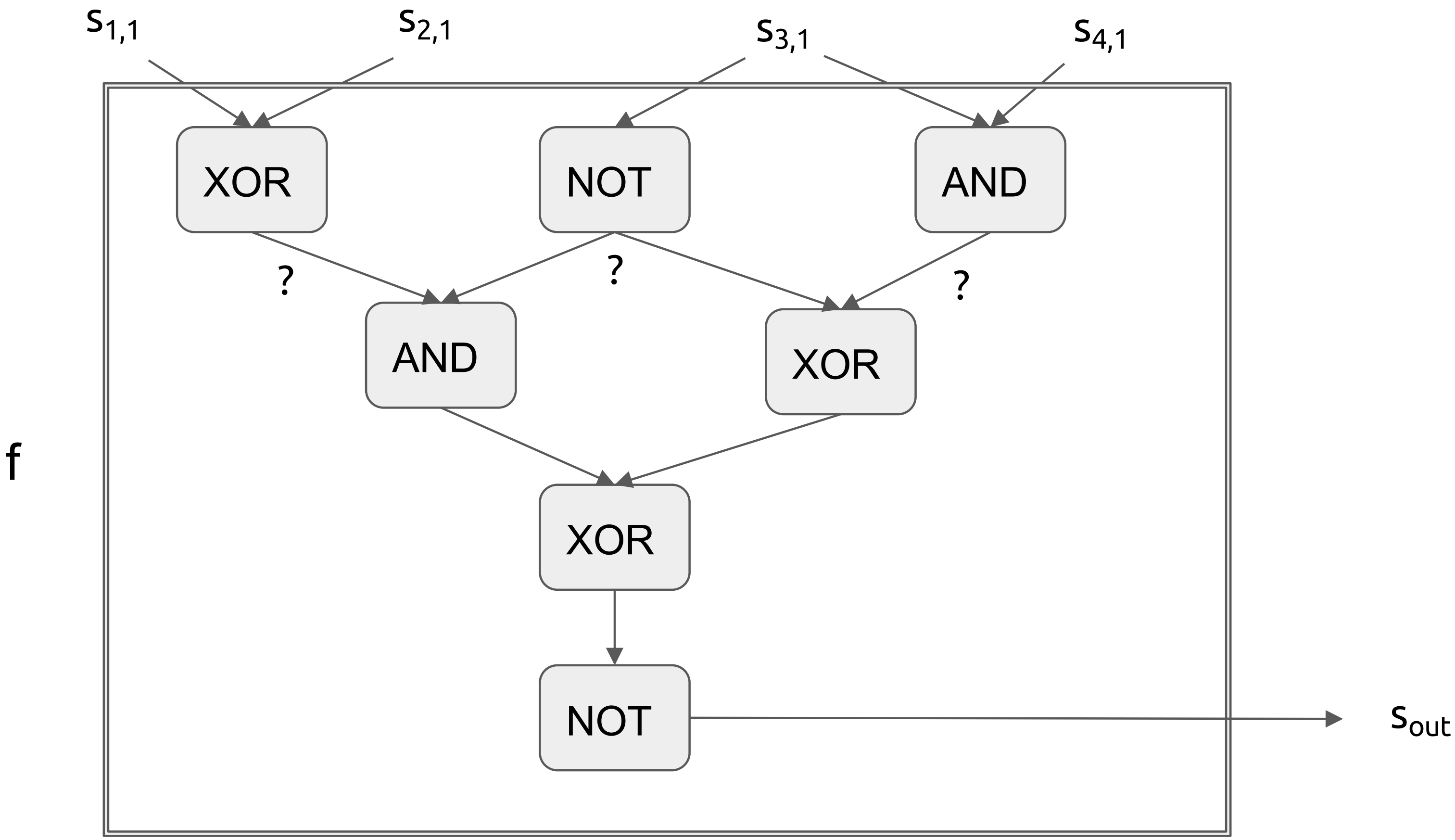


# MPC Construction Idea

## NOT GATE

- Suppose *m* parties hold shares of two inputs to a NOT gate.

$$[a] = \langle a_1, \ldots, a_m \rangle$$

- How do they calculate shares of the output of the NOT gate?

$$[\overline{a}] = \langle 1 + a_1 \bmod 2, a_2, \ldots, a_m \rangle$$

# MPC Construction Idea

### XOR GATE

- Suppose *m* parties hold shares of two inputs to an XOR gate.

$$[a], [b] = \langle a_1, \ldots, a_m \rangle, \langle b_1, \ldots, b_m \rangle$$

- How do they calculate shares of the output of the XOR gate?

$$[a] + [b] \bmod 2$$

# MPC Construction Idea

- Suppose *m* parties hold shares of two inputs to an AND gate.

AND GATE

$$[a],[b] = \langle a_1, \ldots, a_m \rangle, \langle b_1, \ldots, b_m \rangle$$

- How do they calculate shares of the output of the AND gate?

$$[a] \cdot [b] = \langle a_1 b_1 \bmod 2, \ldots, a_m b_m \bmod 2 \rangle$$

but we want: $$s_1 + \ldots + s_m = (\sum_{i=1}^{m} a_i)(\sum_{i=1}^{m} b_i)$$

# MPC Construction Idea

- A Beaver triple is an initial secret-sharing of random values

$$x \cdot y = z$$

$$[x] = \langle x_1, \ldots, x_m \rangle, [y] = \langle y_1, \ldots, y_m \rangle, [z] = \langle z_1, \ldots, z_m \rangle$$

AND GATE :

publish $d_i = a_i - x_i$ reconstruct $d, e$

$e_i = b_i - y_i$

define $s_i = de \quad + dy_i + ex_i + z_i$ share calculation

$$\sum s_i = de + d \sum y_i + e \sum x_i + xy$$ ( assuming m is odd )

$$= de + dy + ex + xy = (a - x)(b - y) + (a - x)y + (b - y)x + xy$$

$$= ab$$

# Constructing Beaver Triples

- The above construction idea requires the setup of all servers with a sufficient number of Beaver triples (how many?)
- Constructing Beaver triples can be done via special-purpose cryptographic protocols

# MPC strengths and weaknesses

- Possible to compute any function *f* privately on parties' inputs
- Unless *honest majority* is present, there is no way to provide:
    - fairness: either all parties learn the output or none
    - guaranteed output delivery

# Fairness

# Workarounds for fairness

- Optimistic fairness (by involving a third party):
  - The protocol is basically not fair
  - A third party is guaranteed to be able to engage and amend the execution in case of deviation
- Gradual/timed release:
  - Protocols engage in many rounds
  - Parties gradually come closer to computing the output
  - “gradual closeness” can be measured in terms of:
    - probability of guessing the output
    - number of computational steps remaining to compute the output
  - Example:
    - At each round $l = 1,...,n$ the two parties can compute the output in $2^{n-l}$ steps
    - If a party aborts the interaction, the other party will be 2 times more steps “behind” in the calculation of the output

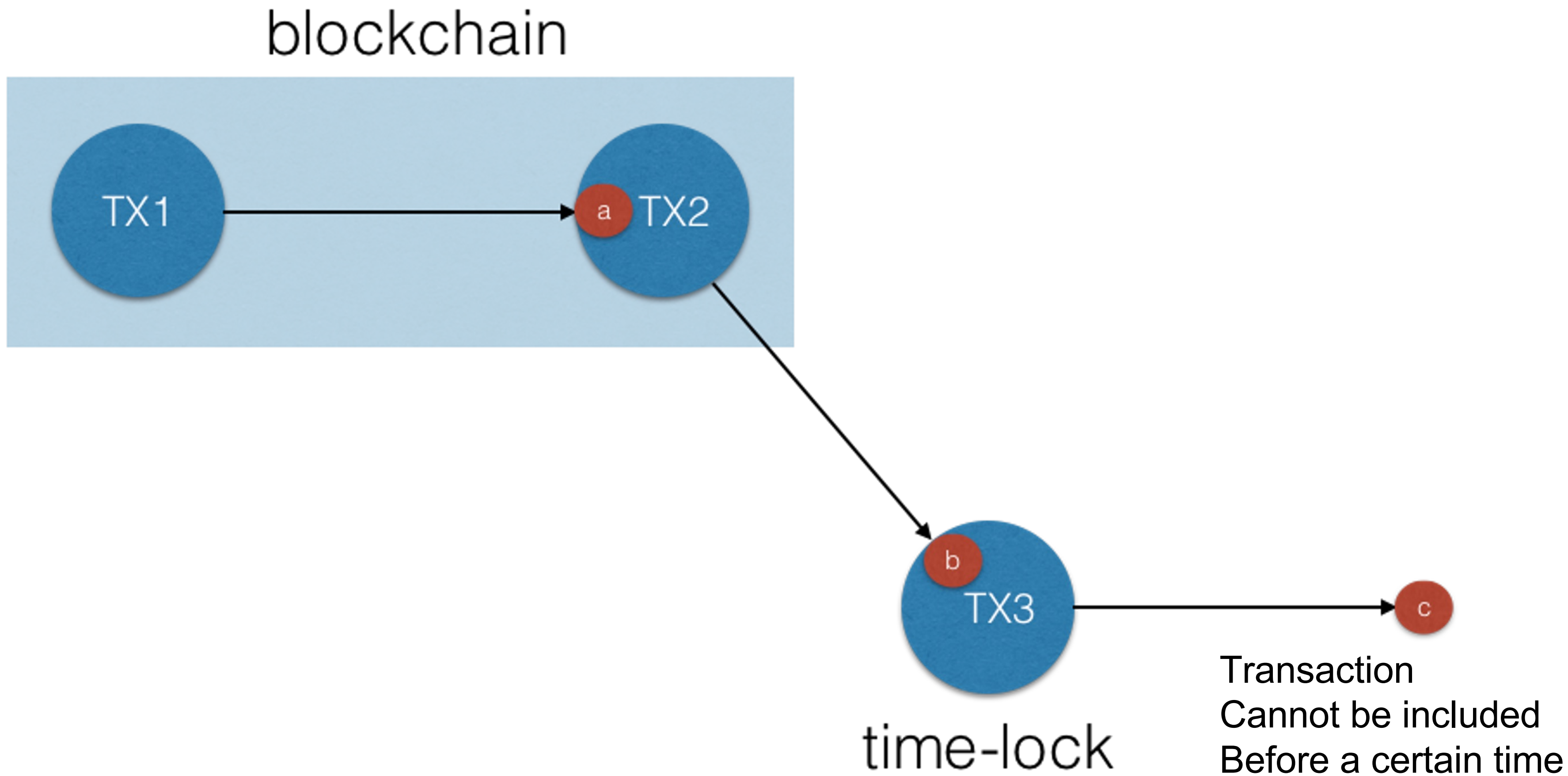
# Using a blockchain

- Along the lines of optimistic fairness, but substituting the trusted third party with the blockchain
- How is that possible?
  - Blockchain cannot keep secrets
  - Rationale: penalize parties that deviate from the protocol

# Basic tool: time-lock transactions

- Time-lock transactions
  - part of transaction data
  - specifies the earliest time that a transaction can be included in a block
- Key observation: if a conflicting transaction has already being included in the ledger, the time-lock transaction will be rejected

# Time-lock example



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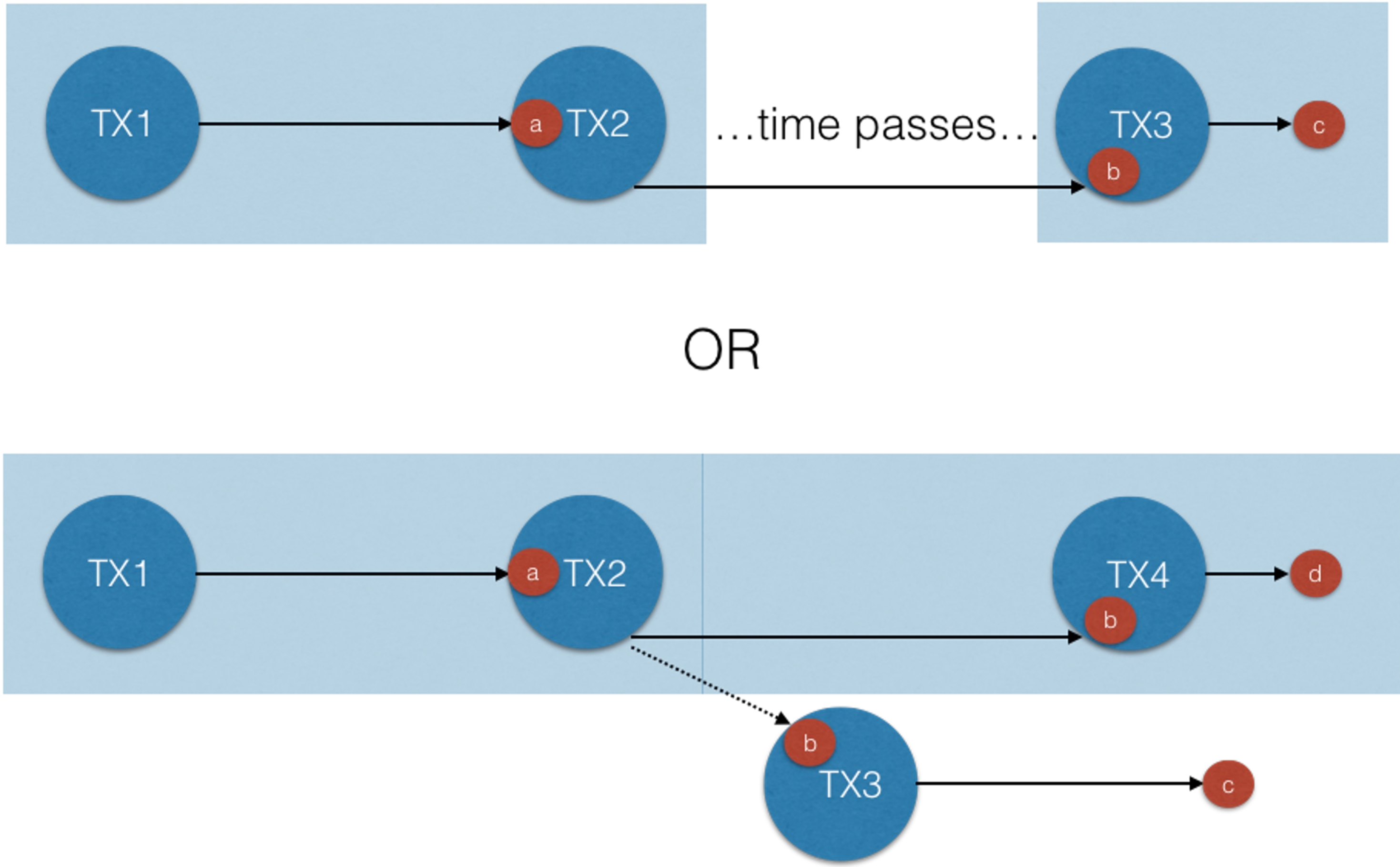


# Fair swap of values using time-locks, Setup

- $P_1$ holds $\mathbf{w_1}$, $\mathbf{h_2=H(w_2)}$
- $P_2$ holds $\mathbf{w_2}$, $\mathbf{h_1=H(w_1)}$
- *They want to exchange $w_1$, $w_2$*

# Fair swap of values using time-locks, Setup

**$TX_A$** *P2SH*

```
Value: 5 $
Pay Bob if
1)  two values w1, w2, s.t.
    H(w1) = h1 and H(w2) = h2
2)  Or P1 and P2 sign, as 2-
    out-of-2 multisignature
```

**$TX_B$** *P2SH*

```
Value: 5 $
Pay Alice if
1)  The value w1 s.t. H(w1) =
    h1 is provided
2)  Or P1 and P2 sign, as 2-
    out-of-2 multisignature
```

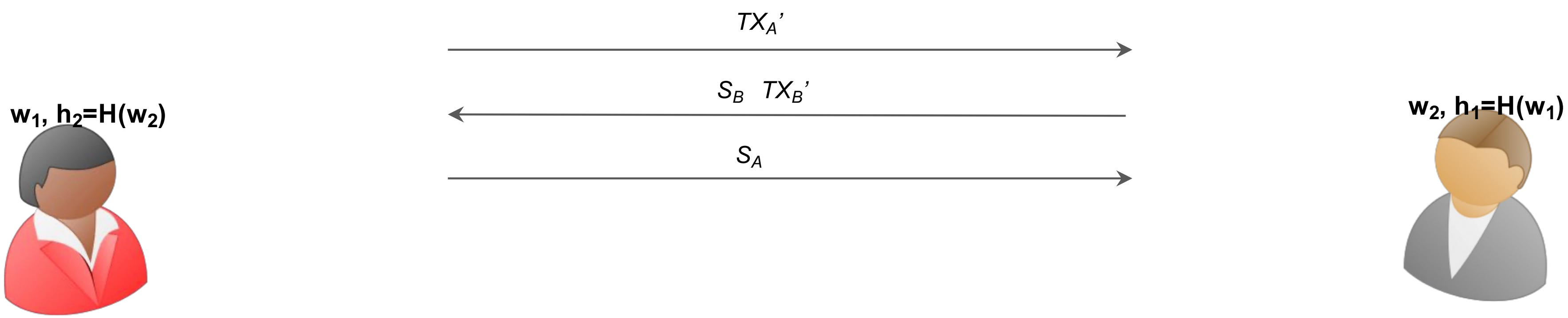
**$TX_A'$** *P2PKH*

Give the money of $TX_A$ to Alice after time $t_A$

**$TX_B'$** *P2PKH*

Give the money of $TX_B$ to Bob After time $t_B$

Refund transactions using time-locks



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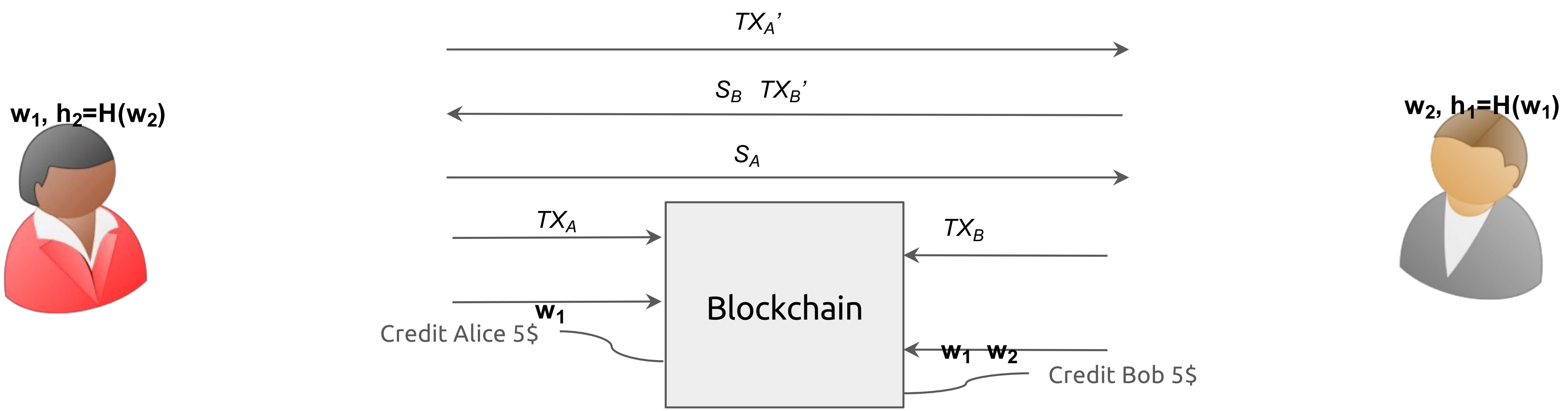

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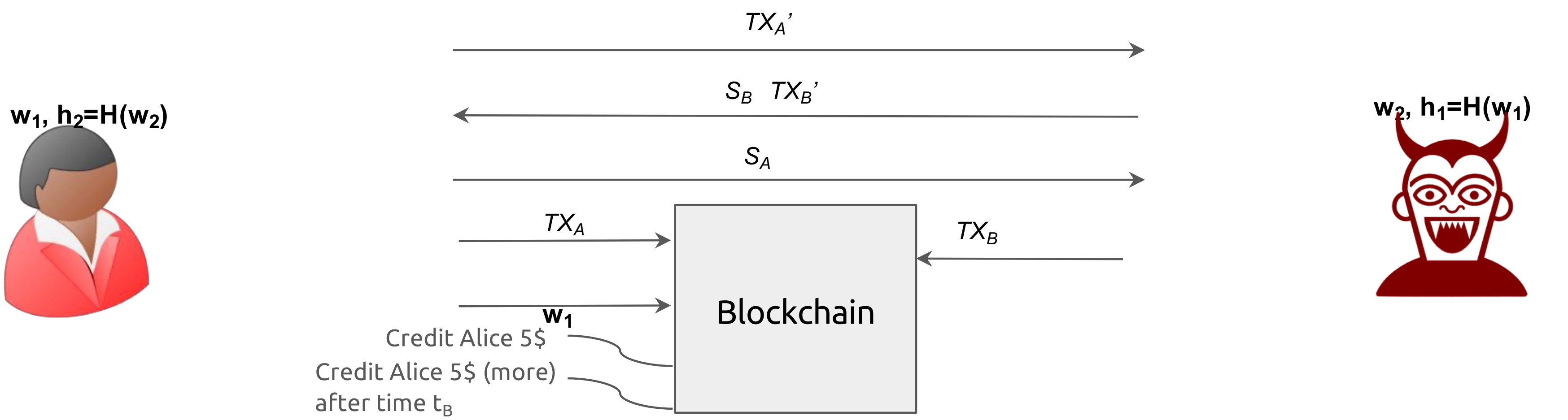

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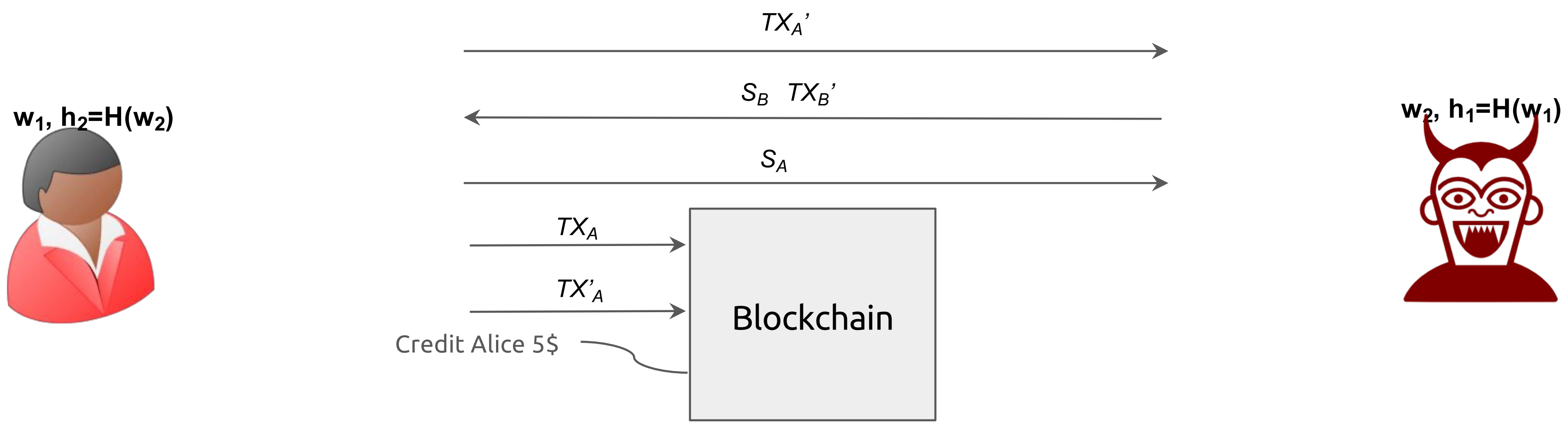

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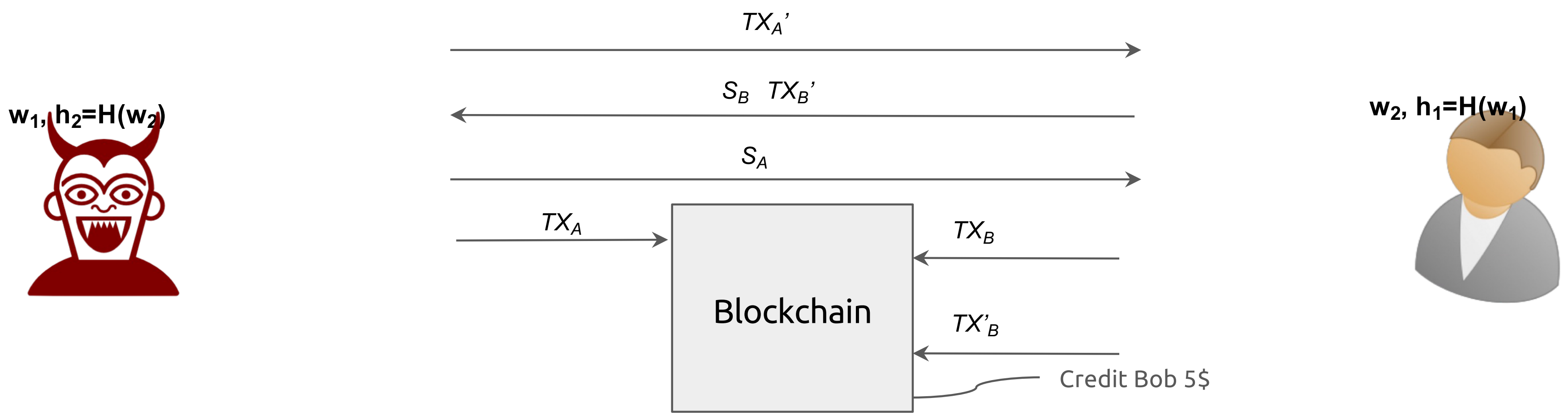

# Fair swap of values using time-locks, Execution

Pay to script hash (P2SH)
Pay-to-Public-Key-Hash (P2PKH)

- $P_1$:
  - Creates a P2SH transaction TX for $X provided that:
    i. ($P_1$ and $P_2$ sign, as 2-out-of-2 multisignature) *or*
    ii. ($P_2$ signs and reveals $\mathbf{w_1}$, $\mathbf{w_2}$, s.t. $H(\mathbf{w_1}) = \mathbf{h_1}$ and $H(\mathbf{w_2}) = \mathbf{h_2}$)
  - Creates a P2PKH transaction TX' that spends the output of TX with a time-lock in the near future
  - Sends TX' to $P_2$ to sign it ($P_2$ does not see TX, only the tx id is needed to refer to it)
- $P_2$ acts in the same way:
  - Create a TX that can be redeemed via (2-out-of-2 multisig) or ($P_1$ signs and reveals $\mathbf{w_1}$, s.t. $H(\mathbf{w_1})=\mathbf{h_1}$)
  - Create a corresponding time-locked TX' and send to $P_1$ to sign
- Completion:
  - $P_1$ publishes its TX, so $P_2$ can redeem $X by revealing $\mathbf{w_1}$**,** $\mathbf{w_2}$
  - $P_2$ publishes its TX, so $P_1$ can redeem $X by revealing $\mathbf{w_1}$
  - $P_1$ reveals $\mathbf{w_1}$ and redeems $X (from $P_2$'s TX)
  - $P_2$ reveals $\mathbf{w_1}$, $\mathbf{w_2}$ and redeems $X (from $P_1$'s TX)
- If either party aborts, the other can claim $X (from their TX) after time-lock fires, by publishing their TX'

# Fair swap of values using time-locks, Notes

- If $P_1$'s TX could be redeemed by "$H(\mathbf{w_2}) = \mathbf{h_2}$ and $P_2$ signs it":
  - $P_2$ could reveal $\mathbf{w_2}$ and obtain payment of \$X, without publishing its own TX transaction
  - $P_1$ would obtain the output $\mathbf{w_2}$ but lose \$X
  - (note that we cannot ensure that the TX transactions will appear concurrently in the blockchain)
- If a multisig was not used for the refunds, a player could:
  - Submit its value
  - Rush to obtain its refund, invalidating the TX payment of the other player
- The time-lock for $P_1$ should be less than that for $P_2$; if equal, $P_1$ could:
  - Wait for the very last minute to reveal $\mathbf{w_1}$
  - Hope that time-lock fires before $P_2$ can publish $\mathbf{w_2}$ on the chain
  - Claim \$X even if $P_2$ tries to act honestly (and reveals $\mathbf{w_2}$ out of time)

# Fair Computation

- The two parties use MPC to compute a secret sharing of the output of the computation
  - **$w_1$ + $w_2$ = MPC_output**
- Subsequently parties do a fair swap of values, to obtain the MPC_output:
  - If a party aborts, the other will be compensated

# N-party ladder construction, I

- Uses N-out-of-N multisig for refunds
- $P_N$ can redeem \$X from each player if it reveals $\mathbf{w_1}$, $\mathbf{w_2}$, …, $\mathbf{w_N}$ (i.e., the N-1 parties prepare these “roof” **TX** transactions)
- For i = 1, …, N-1, player $P_{N-i}$ can redeem from player $P_{N-i+1}$ an amount equal to \$X(N-i) if it reveals $\mathbf{w_1}$**,** $\mathbf{w_2}$**, …,** $\mathbf{w_{N-i}}$ (the N-1 parties also prepare these “ladder” **TX** transactions)
- Redeeming follows the sequence $P_1$, $P_2$, …, $P_N$

# N-party ladder construction, II

- $P_1$ will redeem \$X from $P_2$ for publishing $\mathbf{w_1}$
- $P_2$ will redeem \$2X from $P_3$ for publishing $\mathbf{w_1, w_2}$
- …
- $P_{N-1}$ will redeem \$(N-1)X from $P_N$ for publishing $\mathbf{w_1, w_2, \ldots, w_{N-1}}$
- $P_N$ will redeem \$X from each of $P_1, \ldots, P_{N-1}$ for publishing $\mathbf{w_1, w_2, \ldots, w_N}$

Note before $P_N$ acts, $P_1$ is at \$X, $P_2$ is at -\$X+\$2X =\$X, $P_3$ is at -\$2X+\$3X =\$X, etc.

# References

- For secret sharing and multi-party computation in general, look at Chapter 3, until Section 3.3.2 of the following book (you can access to the book with your university account.
  - Cramer, R., Damgård, I., & Nielsen, J. (2015). *Secure Multiparty Computation and Secret Sharing*. Cambridge: Cambridge University Press. doi:10.1017/CBO9781107337756.
- For fair swap, and in particular for how to achieve fairness with compensation in multi-party computation, please look at this paper and follows the references when something is not clear.
  - Marcin Andrychowicz, Stefan Dziembowski, Daniel Malinowski, and Lukasz Mazurek. Fair two-party computations via the bitcoin deposits. In 1st Workshop on Bitcoin Research 2014 (in Assocation with Financial Crypto), 2014. http://eprint.iacr.org/2013/837.